IEEE Open Journal of the
Computer Society

Received 5 April 2024; revised 30 April 2024; accepted 2 May 2024. Date of publication 7 May 2024;
date of current version 30 May 2024. The review of this paper was arranged by Associate Editor Song Guo.

Digital Object Identifier 10.1109/OJCS.2024.3397345

Performance Analysis of Gossip Algorithms for Large Scale Wireless Sensor Networks

SATEESHKRISHNA DHULI [1], FOUZUL ATIK [2], ANAMIKA CHHABRA [3], PREM SINGH [4], AND LINGA REDDY CENKERAMADDI [5] (Senior Member, IEEE)

[1]Department of Electronics and Communications Engineering, SRM University-AP, Amaravati, Andhra Pradesh 522502, India
[2]Department of Mathematics, SRM University-AP, Amaravati, Andhra Pradesh 522502, India
[3]IIT Madras, Chennai, Tamil Nadu 600036, India
[4]IIIT Bangalore, Bengaluru, Karnataka 560100, India
[5]ACPS Research Group, the Department of Information and Communication Technology, University of Agder, 4630 Kristiansand, Norway

Corresponding author: Linga Reddy Cenkeramaddi (e-mail: linga.cenkeramaddi@uia.no)

This work was supported in part by the Indo-Norwegian Collaboration in Autonomous Cyber-Physical Systems (INCAPS) through the International Partnerships for Excellent Education, Research and Innovation (INTPART) Program under Grant 287918, and in part by the Low-Altitude UAV Communication and Tracking (LUCAT) through the IKTPLUSS Program from the Research Council of Norway under Grant 280835.

ABSTRACT Gossip algorithms are often considered suitable for wireless sensor networks (WSNs) because of their simplicity, fault tolerance, and adaptability to network changes. They are based on the idea of distributed information dissemination, where each node in the network periodically sends its information to randomly selected neighbors, leading to a rapid spread of information throughout the network. This approach helps reduce the communication overhead and ensures robustness against node failures. They have been commonly employed in WSNs owing to their low communication overheads and scalability. The time required for every node in the network to converge to the average of its initial value is called the average time. The average time is defined in terms of the second-largest eigenvalue of a stochastic matrix. Thus, estimating and analyzing the average time required for large-scale WSNs is computationally complex. This study derives explicit expressions of average time for WSNs and studies the effect of various network parameters such as communication link failures, topology changes, long-range links, network dimension, node transmission range, and network size. Our theoretical expressions substantially reduced the computational complexity of computing the average time to $O\left(n^{-3}\right)$. Furthermore, numerical results reveal that the long-range links and node transmission range of WSNs can significantly reduce average time, energy consumption, and absolute error for gossip algorithms.

INDEX TERMS Gossip algorithms, large-scale WSNs, computational complexity, average time.

I. INTRODUCTION

Wireless sensor networks (WSNs) consist of numerous tiny nodes with limited computation and communication capabilities due to severe resource constraints [1], [2], [3]. Node failures and communication link failures are common occurrences owing to resource limitations and environmental issues, leading to frequent changes in network topology as nodes join or leave during network operation. Despite these challenges, large-scale WSNs can collaboratively produce reliable and robust information for measuring physical phenomena. However, they face severe operational issues such as the absence of a centralized node for computation, communication, and time-synchronization, network topology changes, node dynamics, and limited computational energy resources [4], [5]. These challenges drive the need for simple distributed algorithms, where every node communicates with only a few local immediate neighbors at a time. Research on large-scale WSNs has gained considerable interest in distributing computation, decentralized estimation, and distributed learning algorithms [6], [7]. Consensus problems have been extensively examined across various domains, including distributed inference [8], load balancing [9], multi-agent collaboration [10], and data fusion [11]. To achieve consensus, the nodes involved in the process compute the average of their initial scalar values, which is commonly referred to as the average consensus [12]. Gossip algorithms [13], [14] have been predominantly investigated to address the distributed consensus problems. These algorithms ensure neighboring nodes exchange data and disseminate information throughout the entire network, making them resilient to network topology changes. These approaches use iterative-based algorithms, allowing nodes to reach a global consensus with their local one-hop neighbors and converge to the optimal solution. Gossip algorithms fall into two main categories: randomized and deterministic algorithms. In randomized gossiping [14], a randomly selected pair of neighbors exchange their current values in the network until every node converges to the average of the initial measurements. A random pair of nodes is active at every iteration, this scheme is termed as randomized and asynchronous. In synchronous and deterministic algorithms [15] nodes update their current measurement values with the average of their values and the values received from all their neighbors. Asynchronous gossip [16] is suitable for WSN applications, because synchronization is a highly challenging task. In asynchronous and randomized gossip algorithms, the time taken for each node's value to converge to the average value is characterized by the average time [14].

Achieving a global average quickly is a challenging task in WSNs with a large number of nodes. A fast average time ensures that the network can rapidly converge to a consensus that enables timely decision-making, reduces the resources required to reach consensus, and increases the robustness of the network against node and link failures. Overall, the average time of the gossip algorithm plays a pivotal role in the effectiveness, efficiency, and usability of large-scale WSNs, making it a crucial consideration in designing and deploying gossip algorithms for large-scale WSNs. The average time of the gossip algorithms is defined in terms of the second largest eigenvalue of a doubly stochastic matrix [14]. Hence, estimating the average time required for large-scale networks requires significant computational resources. Moreover, it only provides limited insights into the impact of various network parameters on average time. In this study, these challenges are addressed by deriving explicit formulas for the average time in terms of the WSN parameters. To derive these explicit expressions, we utilized the characteristics of the ring topology while considering the key parameters and conditions commonly encountered in WSNs.

Our proposed formulas reduce the computational complexity to $O(n^{-3})$ and provide key insights into the effects of various WSN parameters on the average time of gossip algorithms. For instance, communication link failures can result in a network communication delay which leads to a prolonged average time. The addition and deletion of nodes lead to network topology changes that affect the average time. The incorporation of long-range links can significantly affect the average time because they can provide alternative communication paths that affect the overall network performance. Similarly, the transmission range of the node is one of the crucial factors that can significantly influence the average time as it enables communication with a larger number of nodes. A larger transmission range can reduce the average time because it provides more opportunities for communication with other nodes. Overall, the analytical expressions derived in this study can provide a comprehensive understanding of the design and optimization of the performance of gossip algorithms for large-scale WSNs based on specific constraints.

II. RELATED WORK

This section presents relevant literature related to the study of gossip algorithms for WSNs. The authors in [14] proposed a framework for the design of gossip algorithms for WSNs and Internet graphs. They also established a relationship between the average time of the gossip algorithm and the mixing time of a random walk. The work therein defined the average time of the gossip algorithm in terms of the second-largest eigenvalue of the stochastic matrix. We used this expression of the average time to derive the explicit expressions of the average time for gossip algorithms. In [19], the authors studied the average time of gossip algorithms for WSNs. They showed the agreement between analytic and simulation results for the average time and proposed an optimization technique to improve the performance of gossip algorithms. However, this study did not analyze the effect of WSN parameters on the average time. The authors of [24] proposed an algorithm for computing distributed averages for ring, grid, and geographic networks by exploiting geographic information. They showed that energy consumption can be improved using geographic gossip algorithms over standard nearest-neighbor gossip algorithms. However, this work could provide much information for analyzing the average time required for real-world systems.

In [20], the authors introduced the greedy gossip algorithm and demonstrated that adopting greedy updates results in accelerated convergence. They also showed that the convergence of these algorithms can be easily analyzed for connected network topologies. An algorithm rooted in greedy gossiping principles was introduced, and it has been demonstrated that the convergence of this algorithm is ensured for connected topologies. Greedy gossip with eavesdropping outperforms randomized gossip algorithms. This could not provide any insight into the direction of studying the average time of gossip algorithms for large-scale WSNs. A Support Vector Machine-based optimal gossip algorithm was proposed in [23] for WSNs with minimal local communications. A new gossip algorithm was proposed in [30] for directed graph-modelled sensor networks. They showed that the proposed algorithm achieves a better convergence speed and energy savings than standard gossip algorithms. In [25], the authors presented an energy consumption model for heterogeneous WSNs and proposed a distributed gossip-based algorithm to increase network lifetime. They experimentally proved the advantages of a distributed approach over a centralized approach. Regular graphs play a prominent role in studying consensus and gossip algorithms [31]. In [32], the authors considered the communication link failures in interconnected networks and studied the distributed fault diagnosis problem using consensus algorithms. To improve the efficiency of neighbor discovery in vehicular ad-hoc networks, authors proposed a distributed algorithm in [33] that utilizes the sensing ability of the radar. In [34], the authors proposed various neighbor discovery algorithms with successive interference cancellation technology to unpack multiple collision packets and improve the speed of wireless ad-hoc networks. By integrating the average gossip algorithm with a distributed community detection algorithm, a novel gossip algorithm was proposed in [35].

Furthermore, they verified the efficiency of the proposed scheme on synthetic and real-world peer-to-peer networks. The frequent propagation of redundant information in gossip algorithms renders them inefficient in terms of computational resources. Sirocchi et al. [36] investigated, the influence of topological network attributes on the efficiency of gossip protocols, and showed that adjustments in the network structure can lead to increased convergence rates. Sharing information with neighboring nodes can raise privacy issues that lead to security threats in gossip networks. The authors in [37] derived privacy guarantees for gossip networks in both synchronous and asynchronous cases. This study derived differential privacy and prediction uncertainty guarantees in terms of closed forms for asynchronous settings. Furthermore, we quantify the tradeoff between differential privacy guarantees and information-spreading efficiency in unreliable communications. A robust and distributed gossip monitoring service for a 6 G network architecture was proposed in [38]. They showed that their method provides significant implementation flexibility in 6 G networks by interconnecting the data centers of different cloud providers. A fully connected wireless gossip network was considered in [39] and a distributed opportunistic gossiping scheme was proposed. In this scheme, the nodes in the network that have a higher age remain silent, and only the low-age nodes participate in gossip to increase the total gossip rate. By using gossiping, the authors proposed communication efficient protocols in [40] for single-sender broadcast and parallel broadcast under dishonest majority settings. Gossip protocols can ensure data integrity and consistency in large-scale Blockchain networks. To make gossip protocols more efficient, the authors in [41] proposed a new method by integrating fail-proof, opportunistic, and checking algorithms. In [42], a gossip-based routing algorithm was implemented to improve communications in a LoRaWAN network where all nodes have NLOS conditions. The authors proposed a novel asynchronous Laplacian in [43] that was adapted to a network of heterogeneous lattices, and showed that the resulting gossip algorithm converges asymptotically. In [44], the authors considered an undirected network to analyze the gossip-based average consensus algorithm with some initial values. Next, they characterized the precise conditions of information exchange that guarantee the privacy of all nodes.

In [27], the authors modeled the WSN as a random geometric graph and analyzed consensus algorithms for synchronous and asynchronous cases. Reference [18] proved that the gossiping technique can be used to improve the performance of the angular and spanning tree protocols in WSNs. An energy-efficient gossiping protocol was presented in [21], where data routing was performed by choosing the optimal neighbors using the Chebyshev distance, sink distance, and residual energy. They demonstrated that the total energy consumption of the network decreased the computational overhead. However, this study did not discuss the impact of different network topologies on the performance of gossip algorithms and did not provide any analytical results. The authors provided closed-form expressions for various network properties for consensus algorithms [45]. This study focused on the consensus parameters for m-dimensional prism networks. In our work, we studied the average time required by randomized gossip algorithms for m-dimensional prism networks. Power consumption is one of the important resources in WSNs.

Much of the research could not study the impact of power or energy consumption on convergence. A recent study [26] investigated the convergence of the gossip algorithm for energy-efficient WSNs. They considered the power consumption and second-largest eigenvalue of the average update matrix to optimize the convergence speed and lifetime of WSNs. However, this study did not provide an analysis of network topologies on the convergence speed. The authors in [22] derived closed-form expressions for the convergence rate of periodic gossip algorithms for one-dimensional lattice networks. In this study, the effect of the gossip weight on the convergence rate of the gossip algorithms was investigated. In Table 1, we present research works that studied gossip algorithms for WSN scenarios. To the best of our knowledge, this is the first work to analytically study the effects of communication link failures, topology changes, network size, network dimension, node degree, and long-range links on the average time of gossip algorithms. In this work, we model the WSN as a ring network [46], [47] and incorporate the small-world, r-regular ring, random, and m-dimensional prism networks to develop theoretical tools for gossip algorithms. Ring networks are extremely useful for characterizing the geographical properties of WSNs and deriving theoretical bounds for network parameters. Furthermore, we analytically studied the effect of various significant network parameters on the average time of gossip algorithms for WSNs. The main contributions of this study are as follows.

A. CONTRIBUTIONS

- First, we model the WSNs using r-regular, small-world, random, and m-dimensional prism networks. Subsequently, we derive explicit expressions for the average time to optimize the performance of gossip algorithms in large-scale WSNs.

TABLE 1. Summary of Literature Focusing on Gossip Algorithms for WSNs

References	Energy Efficiency	Graph Models	WSN Parameters	Average Time
[17], [18], [19]	✗	✗	✓	Simulation
[20]	✗	RGG	✓	Analytical
[21]	✓	✗	✓	✗
[22]	✗	Path Graph	✓	Analytical
[23]	✓	✗	✓	✗
[24]	✗	RGG	✗	Analytical
[25], [26]	✓	✗	✓	Simulation
[27]	✗	RGG	✓	Simulation
[28], [29]	✗	✗	✓	Simulation
[30]	✗	Random Graph	✗	Simulation
Proposed Work	✓	Regular Networks, Small-world network, Prism Network, RGG	✓	Analytical, Simulation

- Next, we analyze the effect of node transmission ranges, long-range links, network size, dimension, and communication link failures on the average time. This analysis aims to study the performance of gossip algorithms for large-scale WSNs.
- Furthermore, we investigate the energy consumption and absolute error aspects associated with gossip algorithms in WSN scenarios.
- Finally, we conduct extensive simulation experiments to analyze average time in real-world networks. These results reveal the performance of gossip algorithms in the practical applications of WSN.

III. REVIEW OF GOSSIP ALGORITHMS

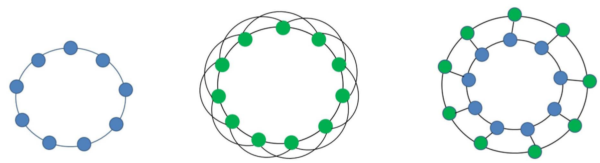
We begin by considering graph G with vertex $V = \{1, 2, 3 \ldots n\}$. We consider a nearest neighbor gossip algorithm, in which only direct neighbors exchange observed data and replace the previous node's data values with the average of the current pair of node values. Let each node obtain a real-valued scalar through observations at time $k = 0$. The objective of the gossip algorithm is to compute the average $\bar{x} = \frac{1}{n}\sum_{j=1}^{n} x_j(0)$ at every node of the graph, where $x_1(0)$, $x_2(0)$, $..x_n(0)$ are the initial values observed by n nodes. The vector of observed values at the k^{th} slot can be expressed as $\mathbf{x}(k) = \mathbf{W}\mathbf{x}(k-1)$. It follows from [14] that the time required for every node to converge to $\bar{x}$ is calculated as the average time $T = \frac{1}{\log \frac{1}{\lambda_2(\mathbf{W})}}$, where λ_2 is the second largest eigenvalue of $\mathbf{W}$. The weight matrix $\mathbf{W}$ is defined as follows:

$$\mathbf{W} \triangleq \mathbf{I} - \frac{1}{2n}\mathbf{D} + \frac{\mathbf{P} + \mathbf{P}^T}{2n}, \tag{1}$$

where $\mathbf{I}$ is an identity matrix and $\mathbf{D}$ is a diagonal matrix with its i^{th} elements

$$D_i = \sum_{j=1}^{n} \left[P_{ji} + P_{ij}\right]. \tag{2}$$

Here, scalar P_{ij} gives the probability that node i chooses to average with node j. It takes a value of 0 if $|i - j| = 1$, otherwise it is $1/2$ [14]. The gossip matrix $\mathbf{P}$ is a symmetric circulant matrix generated by the n-vector $\left[0, \frac{1}{2}, 0, 0, \ldots \frac{1}{2}\right]$ [14]. The average time T is defined as the time taken by each node to converge to the average value. It depends on the second largest eigenvalue of the weight matrix $\lambda_2(\mathbf{W})$ which characterizes the speed of the gossip algorithm. Hence, to study the average time of the gossip algorithms, we need to compute the second largest eigenvalue of the weight matrix $\mathbf{W}$ [14], [24].



(a) 2-Regular Ring (b) 4-Regular Ring (c) Prism Network

FIGURE 1. Regular networks.

IV. CLOSED FORM EXPRESSIONS OF λ_2(W) FOR WSNs

In this section, we model the WSN as an r-regular ring, small-world, random, and prism network and derive the closed-form expressions of the second largest eigenvalue of the stochastic matrix W.

A. R-REGULAR RING NETWORK

In this subsection, we study the gossip algorithms for r-regular ring network. A 2-regular ring network and 4-regular ring network are shown in the Fig. 1(a) and (b) respectively, for which the gossip matrix $\mathbf{P}$ can be written as

$$\mathbf{P} = \frac{1}{2}\mathbf{A}\left(R_n^r\right), \tag{3}$$

where $\mathbf{A}$, r, n, and R_n^r denote the adjacency matrix, degree, number of nodes, and the r-regular ring network, respectively.

The degree matrix $\mathbf{D}$ for r-regular ring network can be written as

$$\mathbf{D} = 2r\mathbf{I}. \tag{4}$$

In this case, the weight matrix $\mathbf{W}$ is a circulant matrix generated by the vector $\left[1 - \frac{r}{n}, \frac{1}{2n}, \ldots, \frac{1}{2n}, 0, 0, \ldots, 0, \frac{1}{2n}, \ldots, \frac{1}{2n}\right]$.

Theorem 4.1: Let $\mathbf{C}$ be a circulant matrix generated by the vector $[c_1, c_2, c_3, \ldots, c_n]$. The eigenvalues of $\mathbf{C}$ are given as [48]

$$\lambda_k = c_1 + c_2\, w^{k-1} + c_3\, w^{2(k-1)} + \cdots + c_n w^{(n-1)(k-1)}, \tag{5}$$

where $w = \exp\left(\frac{2\pi}{n} i\right)$ and $k = 1, 2, \ldots, n$.

From the above theorem, the eigenvalues of $\mathbf{W}$ are given by

$$\lambda_k(\mathbf{W}) = 1 - \frac{r}{n} - \frac{1}{2n} + \frac{1}{2n}\frac{\sin\left(\frac{\pi(2r+1)k}{n}\right)}{\sin\left(\frac{\pi k}{n}\right)}, \tag{6}$$

where $k = 0, 1, 2, 3, \ldots, n-1$. From the above expression, we observe that the second largest eigenvalues of $\mathbf{W}$ are obtained for $k = 1$. Hence the second largest eigenvalue of $\mathbf{W}$ is given by

$$\lambda_2(\mathbf{W}) = 1 - \frac{r}{n} - \frac{1}{2n} + \frac{1}{2n}\frac{\sin\left(\frac{\pi(2r+1)}{n}\right)}{\sin\left(\frac{\pi}{n}\right)}. \tag{7}$$

B. m-DIMENSIONAL PRISM NETWORK WITH BASE GRAPH R_n:

Next, we consider an m-dimensional prism network with base graph R_n, denoted by $(R_n)^m$, where m represents the dimension. The networks for $(R_n)^1$ and $(R_n)^2$ are illustrated in Fig. 1(c). We see that $(R_n)^m$ is a regular graph of regularity $2 + m - 1 = m + 1$. Additionally, the number of vertices in $(R_n)^m$ is $N = n2^{m-1}$. For this purpose, the gossip matrix $\mathbf{P}$ is expressed as

$$\mathbf{P} = \frac{1}{2}\mathbf{A}((R_n)^m). \tag{8}$$

The degree matrix $\mathbf{D}$ is expressed as

$$\mathbf{D} = (m+1)\mathbf{I}. \tag{9}$$

Since the number of vertices is N, we get

$$\begin{aligned}\mathbf{W} &= \left(1 - \frac{m+1}{2N}\right)\mathbf{I} + \frac{1}{N}\mathbf{P}\\ &= \left(1 - \frac{m+1}{2N}\right)\mathbf{I} + \frac{1}{2N}\mathbf{A}((C_n)^m)\\ &= \left(1 - \frac{m+1}{n2^m}\right)\mathbf{I} + \frac{1}{n2^m}\mathbf{A}((C_n)^m).\end{aligned} \tag{10}$$

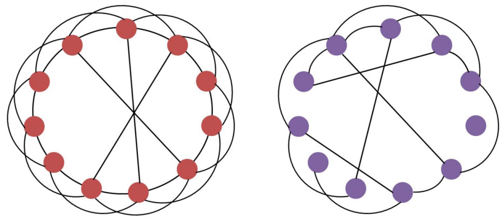
From [45], we get that the second largest eigenvalue of $\mathbf{A}((R_n)^m)$ is as $m - 1 + 2\cos\frac{2\pi}{n}$. Hence the second largest eigenvalue of $\mathbf{W}$ is given by

$$\lambda_2(\mathbf{W}) = 1 - \frac{m+1}{n2^m} + \frac{1}{n2^m}\left(m - 1 + 2\cos\frac{2\pi}{n}\right). \tag{11}$$

C. m-DIMENSIONAL PRISM NETWORK WITH BASE GRAPH R_n^r, $(r \geq 2)$:

We next consider an m-dimensional prism network with the base graph R_n^r, and is denoted by $\left(R_n^r\right)^m$. Then $\left(R_n^r\right)^m$ is a regular graph of regularity $2r + m - 1$. Also number of vertices in $\left(R_n^r\right)^m$ is $N = n2^{m-1}$. For this, the gossip matrix $\mathbf{P}$ is expressed as

$$\mathbf{P} = \frac{1}{2}\mathbf{A}\left(\left(R_n^r\right)^m\right), \tag{12}$$



(a) Small-World Graph (b) Random Graph

FIGURE 2. Network topologies.

where $\mathbf{A}$ denotes the adjacency matrix. The degree matrix $\mathbf{D}$ is expressed as

$$\mathbf{D} = (2r + m - 1)\mathbf{I}. \tag{13}$$

As the number of vertices is N, we get

$$\begin{aligned}\mathbf{W} &= \left(1 - \frac{2r+m-1}{2N}\right)\mathbf{I} + \frac{1}{2N}\mathbf{A}\left(\left(R_n^r\right)^m\right)\\ &= \left(1 - \frac{2r+m-1}{n2^m}\right)\mathbf{I} + \frac{1}{n2^m}\mathbf{A}\left(\left(R_n^r\right)^m\right).\end{aligned} \tag{14}$$

Because the second largest eigenvalue of $\mathbf{A}\left(\left(R_n^r\right)^m\right)$ is given by $2r + m - 3$ for $r \geq 2$, the second largest eigenvalue of $\mathbf{W}$ is given by

$$\lambda_2(\mathbf{W}) = 1 - \frac{2r+m-1}{n2^m} + \frac{1}{n2^m}(2r + m - 3). \tag{15}$$

D. SMALL-WORLD NETWORKS

Networks with smaller average path length and large clustering coefficients are called small-world networks [49]. These networks are designed to optimize the graphs between regular and random cases. In these networks, nodes establish a few long-range links each time with a small-world probability ε [50], as shown in Fig. 2(a). Let us assume that the nodes form long-range links with a probability $\varepsilon \propto n^{-\beta}$, where n is the network size and β is a natural number. The $(i, j)^{th}$ element $\mathbf{P}_{ij}$ in the gossip matrix is given by

$$\mathbf{P}_{ij} = \begin{cases} 0 & i = j,\\ \frac{1}{2} & |i - j| = 1,\\ \frac{1}{2} & (i, j) = (1, n) \text{ or } (i, j) = (n, 1),\\ \varepsilon & \text{otherwise.}\end{cases} \tag{16}$$

For this case, the degree matrix $\mathbf{D}$ is expressed as

$$\mathbf{D} = (2 + 2(n-3)\varepsilon)\mathbf{I}. \tag{17}$$

The weight matrix $\mathbf{W}$ is a circulant matrix generated by the vector $\left[1 - \frac{1}{n} - \frac{(n-3)}{n}, \frac{1}{2n}, \frac{\varepsilon}{n}, \frac{\varepsilon}{n}, \ldots, \frac{\varepsilon}{n}, \frac{1}{2n}\right]$. Then, by the Theorem 4.1, the eigenvalues of $\mathbf{W}$ for $k = 1, 2, 3, \ldots, n$ are given by

$$\lambda_k=\begin{cases}\left(1-\frac{1}{n}-\frac{(n-3)\varepsilon}{n}\right)+\frac{1}{n}\cos\left(\frac{2\pi(k-1)}{n}\right)\\ \quad +\frac{2\varepsilon}{n}\sum_{j=2}^{\frac{n-1}{2}}\cos\left(\frac{2\pi(k-1)j}{n}\right), & \text{when n is odd}\\ \left(1-\frac{1}{n}-\frac{(n-3)\varepsilon}{n}\right)+\frac{1}{n}\cos\left(\frac{2\pi(k-1)}{n}\right)\\ \quad +\frac{\varepsilon}{n}\cos(\pi(k-1))\\ \quad +\frac{2\varepsilon}{n}\sum_{j=2}^{\frac{n-2}{2}}\cos\left(\frac{2\pi(k-1)j}{n}\right), & \text{when n is even.}\end{cases} \tag{18}$$

We observe that the second largest eigenvalue of $\mathbf{W}$ is obtained for $k=\frac{n+1}{2}$ when n is odd, and for $k=\frac{n+2}{2}$ when n is even. Using these k values, the second largest eigenvalue of $\mathbf{W}$ is

$$\lambda_2(\mathbf{W})=\begin{cases}\left(1-\frac{1}{n}-\frac{(n-3)\varepsilon}{n}\right)+\frac{1}{n}\cos\left(\frac{\pi(n-1)}{n}\right)\\ +\frac{2\varepsilon}{n}\sum_{j=2}^{\frac{n-1}{2}}\cos\left(\frac{\pi(n-1)j}{n}\right), & \text{when n is odd}\\ \left(1-\frac{1}{n}-\frac{(n-3)\varepsilon}{n}\right)-\frac{1}{n}+\frac{\varepsilon}{n}\cos\left(\frac{n\pi}{2}\right)\\ +\frac{2\epsilon}{n}\sum_{j=2}^{\frac{n-2}{2}}\cos(\pi j), & \text{when n is even.}\end{cases} \tag{19}$$

By using Dirichlet kernel $1+2\sum_{j=1}^{r}\cos(jx)=\frac{\sin\left(r+\frac{1}{2}\right)x}{\sin\frac{x}{2}}$, the above expression can be further simplified as

$$\lambda_2(\mathbf{W})=\begin{cases}\left(1-\frac{1}{n}-\frac{(n-3)\varepsilon}{n}\right)+\frac{1}{n}\cos\left(\frac{\pi(n-1)}{n}\right)\\ -\frac{2\varepsilon}{n}\left(\cos\frac{(n-1)\pi}{n}\right)+1, & \text{when n is odd}\\ \left(1-\frac{1}{n}-\frac{(n-3)\varepsilon}{n}\right)-\frac{1}{n}+\frac{\varepsilon}{n}\cos\left(\frac{n\pi}{2}\right)\\ +\frac{\varepsilon}{n}\left(\sin\frac{(n-1)\pi}{n}+1\right), & \text{when n is even.}\end{cases} \tag{20}$$

E. RANDOM GRAPHS

In this subsection, we derive the second largest eigenvalue of weight matrix $\mathbf{W}$ for random graphs. A random graph is shown in Fig. 2(b). This figure was generated by randomly adding links to the ring network. We studied the effects of adding random communication links, communication link failures, and topology changes in WSNs.

1) RANDOM COMMUNICATION LINKS

Let a communication link exist between the two nodes with probability q. The gossip matrix $\mathbf{P}$ is expressed as follows:

$$\mathbf{P}=q\begin{bmatrix}0 & \frac{1}{2} & 1 & \cdots & \frac{1}{2}\\ \frac{1}{2} & 0 & \frac{1}{2} & \cdots & 1\\ 1 & \frac{1}{2} & 0 & \cdots & 1\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ \frac{1}{2} & 1 & 1 & \cdots & 0\end{bmatrix}. \tag{21}$$

Next, the degree matrix $\mathbf{D}$ is expressed as

$$\mathbf{D}=2q(n-2)\mathbf{I}. \tag{22}$$

The corresponding weight matrix $\mathbf{W}$ is expressed as

$$\mathbf{W}=\left(1-q+\frac{2q}{n}\right)\mathbf{I}+\frac{1}{n}\mathbf{P}. \tag{23}$$

The weight matrix $\mathbf{W}$ is a circulant matrix generated by the vector $\left[1-q+\frac{2q}{n},\frac{q}{2n},\frac{q}{n},\frac{q}{n},\ldots,\frac{q}{n},\frac{q}{2n}\right]$. Using the Theorem 4.1, the eigenvalues of $\mathbf{W}$, for $k=1,2,3,\ldots,n$, are given by

$$\lambda_k=\begin{cases}1-q+\frac{2q}{n}+\frac{q}{n}\cos\left(\frac{2\pi(k-1)}{n}\right)\\ \quad +\frac{2q}{n}\sum_{j=2}^{\frac{n-1}{2}}\cos\left(\frac{2\pi(k-1)j}{n}\right), & \text{for n is odd}\\ 1-q+\frac{2q}{n}+\frac{q}{n}\cos\left(\frac{2\pi(k-1)}{n}\right)\\ \quad +\frac{q}{n}\cos(\pi(k-1))\\ \quad +\frac{2q}{n}\sum_{j=2}^{\frac{n-2}{2}}\cos\left(\frac{2\pi(k-1)j}{n}\right), & \text{for n is even,}\end{cases} \tag{24}$$

We observe that the second largest eigenvalue of $\mathbf{W}$ is obtained for $k=\frac{n+1}{2}$ when n is odd, and for $k=\frac{n+2}{2}$, for n is even. Consequently, the second largest eigenvalue of $\mathbf{W}$ is

$$\lambda_2(\mathbf{W})=\begin{cases}1-q+\frac{2q}{n}+\frac{q}{n}\cos\left(\frac{\pi(n-1)}{n}\right)\\ \quad +\frac{2q}{n}\sum_{j=2}^{\frac{n-1}{2}}\cos\left(\frac{\pi(n-1)j}{n}\right), & \text{for n is odd}\\ 1-q+\frac{2q}{n}-\frac{q}{n}+\frac{q}{n}\cos\left(\frac{n\pi}{2}\right)\\ \quad +\frac{2q}{n}\sum_{j=2}^{\frac{n-2}{2}}\cos(\pi j), & \text{for n is even.}\end{cases} \tag{25}$$

Using the Dirichlet kernel, we can further simplify the above expression as

$$\lambda_2(\mathbf{W})=\begin{cases}1-q+\frac{2q}{n}+\frac{q}{n}\cos\frac{\pi(n-1)}{n}\\ +\frac{2q}{n}\left(2\cos\frac{(n-1)\pi}{n}+1\right), & \text{for n is odd}\\ 1-q+\frac{2q}{n}-\frac{q}{n}+\frac{q}{n}\cos\frac{\pi n}{2}\\ +\frac{2q}{n}\left(\sin\frac{(n-1)\pi}{n}+1\right), & \text{for n is even.}\end{cases} \tag{26}$$

2) COMMUNICATION LINK FAILURES

In this section, we derive closed-form expressions of $\lambda_2(\mathbf{W})$ for a ring network by considering the effects of communication link failures and topology changes. The expression is subsequently used to study the effect of link failures and topology changes on average time. We begin by studying the effect of communication link failures on the average time of the gossip algorithm. Let a communication link in a ring network fail with a probability p [50]. Thus, the elements of gossip matrix $\mathbf{P}$ are obtained as

$$P_{ij}=\begin{cases}\frac{1-p}{2} & |i-j|=1\\ \frac{1-p}{2} & (i,j)=(1,n)\text{ or }(i,j)=(n,1)\\ 0 & \text{otherwise.}\end{cases} \tag{27}$$

The corresponding degree matrix $\mathbf{D}$ is expressed as

$$\mathbf{D}=2(1-p)\mathbf{I}, \tag{28}$$

and the weight matrix $\mathbf{W}$ is expressed as

$$\mathbf{W} = \left(1 - \frac{1-p}{n}\right)\mathbf{I} + \frac{1}{n}\mathbf{P}. \tag{29}$$

The circulant matrix $\mathbf{W}$ is generated by the vectors $\left[1 - \frac{1-p}{n}, \frac{1-p}{2n}, 0, 0, \ldots, 0, \frac{1-p}{2n}\right]$. By using Theorem 4.1. The eigenvalues of $\mathbf{W}$ are given by:

$$\lambda_k(\mathbf{W}) = 1 - \frac{1-p}{n} + \frac{1-p}{n}\cos\frac{2\pi(k-1)}{n}, \tag{30}$$

where $k = 1, 2, 3, \ldots, n$. We observe that the second largest eigenvalue of $\mathbf{W}$ is obtained for $k = 2$, and is given as

$$\lambda_2(\mathbf{W}) = 1 - \frac{1-p}{n} + \frac{1-p}{n}\cos\frac{2\pi}{n}. \tag{31}$$

3) TOPOLOGY CHANGES

We now study the effect of topology changes on the average time of gossip algorithms. Here, nodes randomly choose neighbors and exchange information with only two neighbors [50]. In this case, the gossip matrix $\mathbf{P}$ can be written as

$$\mathbf{P} = \begin{bmatrix} 0 & \frac{1}{n-1} & \frac{2}{n-1} & \cdots & \frac{1}{n-1} \\ \frac{1}{n-1} & 0 & \frac{1}{n-1} & \cdots & \frac{2}{n-1} \\ \frac{2}{n-1} & \frac{1}{n-1} & 0 & \cdots & \frac{2}{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{1}{n-1} & \frac{2}{n-1} & \frac{1}{n-1} & \cdots & 0 \end{bmatrix}, \tag{32}$$

and the degree matrix $\mathbf{D}$ is expressed as

$$\mathbf{D} = \frac{4(n-2)}{n-1}\mathbf{I}. \tag{33}$$

The weight matrix $\mathbf{W}$ is expressed as

$$\mathbf{W} = \left(\frac{(n^2-3n+4)}{n(n-1)}\right)\mathbf{I} + \frac{1}{n}\mathbf{P}. \tag{34}$$

The circulant matrix $\mathbf{W}$ is generated by the vector $\left[\frac{n^2-3n+4}{n(n-1)}, \frac{1}{n(n-1)}, \frac{2}{n(n-1)}, \frac{2}{n(n-1)}, \ldots, \frac{2}{n(n-1)}, \frac{1}{n(n-1)}\right]$. From Theorem 1, the eigenvalues of $\mathbf{W}$ are given by

$$\lambda_k = \begin{cases} \frac{n^2-3n+4}{n(n-1)} + \frac{2}{n(n-1)}\cos\left(\frac{2\pi(k-1)}{n}\right) \\ \quad + \frac{4}{n(n-1)}\sum_{j=2}^{\frac{n-1}{2}}\cos\left(\frac{2\pi(k-1)j}{n}\right), & \text{when n is odd} \\ \frac{n^2-3n+4}{n(n-1)} + \frac{2}{n(n-1)}\cos\left(\frac{2\pi(k-1)}{n}\right) \\ \quad + \frac{2}{n(n-1)}\cos(\pi(k-1)) \\ \quad + \frac{4}{n(n-1)}\sum_{j=2}^{\frac{n-2}{2}}\cos\left(\frac{2\pi(k-1)j}{n}\right), & \text{when n is even,} \end{cases} \tag{35}$$

where $k = 1, 2, 3, \ldots, n$. The second largest eigenvalue of $\mathbf{W}$ is obtained for $k = \frac{n+1}{2}$ when n is odd, and for $k = \frac{n+2}{2}$ when n is even. Hence, we get

$$\lambda_2(\mathbf{W}) = \begin{cases} \frac{n^2-3n+4}{n(n-1)} + \frac{2}{n(n-1)}\cos\left(\frac{\pi(n-1)}{n}\right) \\ \quad + \frac{4}{n(n-1)}\sum_{j=2}^{\frac{n-1}{2}}\cos\left(\frac{\pi(n-1)j}{n}\right), \\ \qquad \text{when n is odd} \\ \frac{n^2-3n+4}{n(n-1)} - \frac{2}{n(n-1)} + \frac{2}{n(n-1)}\cos\left(\frac{n\pi}{2}\right) \\ \quad + \frac{4}{n(n-1)}\sum_{j=2}^{\frac{n-2}{2}}\cos(\pi j), \\ \qquad \text{when n is even.} \end{cases} \tag{36}$$

Using the Dirichlet kernel, we get

$$\lambda_2(\mathbf{W}) = \begin{cases} \frac{n^2-3n+4}{n(n-1)} + \frac{2}{n(n-1)}\cos\frac{\pi(n-1)}{n} \\ - \frac{2}{n(n-1)}\left(2\cos\frac{(n-1)\pi}{n} + 1\right), \\ \qquad \text{when n is odd} \\ \frac{n^2-3n+4}{n(n-1)} - \frac{2}{n(n-1)} + \frac{2}{n(n-1)}\cos\frac{n\pi}{2} \\ + \frac{2}{n(n-1)}\left(\sin\frac{(n-1)\pi}{n} + 1\right), \\ \qquad \text{when n is even.} \end{cases} \tag{37}$$

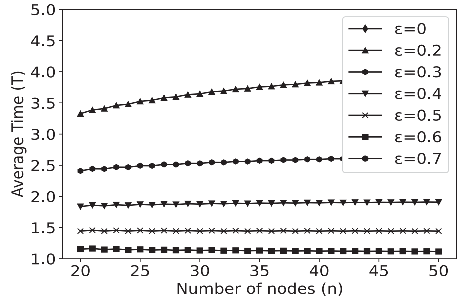


FIGURE 3. Average time versus *n* for small-world networks.

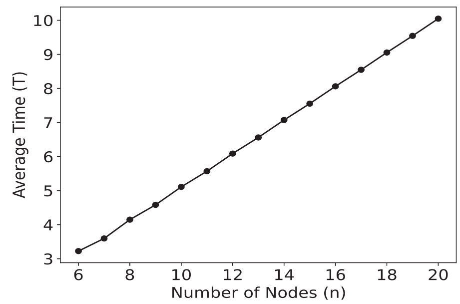


FIGURE 4. Effect of topology changes on average time.

V. RESULTS AND DISCUSSION

This section presents the numerical results to examine the effect of communication link failures, topology changes, long-range links, network dimension, node transmission range, and network size on the average time of gossip algorithms. We substituted the derived explicit values of $\lambda_2(\mathbf{W})$ in $T = \frac{1}{\log\frac{1}{\lambda_2(\mathbf{W})}}$ to plot Figs. 3 to 20.

Fig. 3 shows the average time of small-world networks as a function of the number of nodes. Here, we see that the average

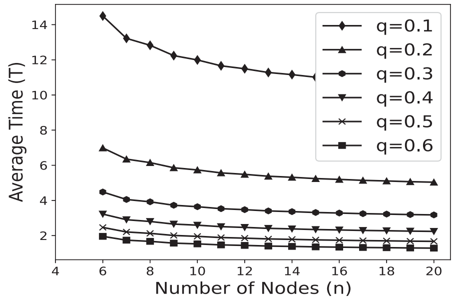


FIGURE 5. **Average time versus *n* for random networks.**

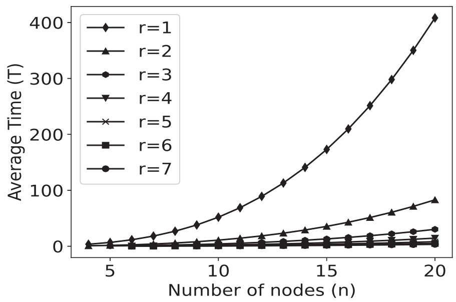


FIGURE 6. **Average time versus *n* for *r*-regular ring networks.**

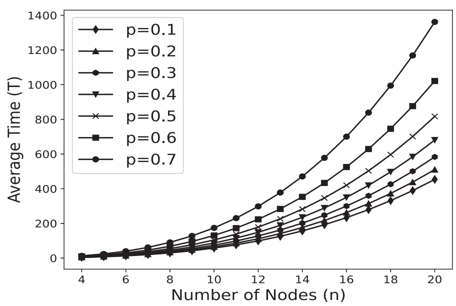


FIGURE 7. **Effect of communication link failures on average time.**

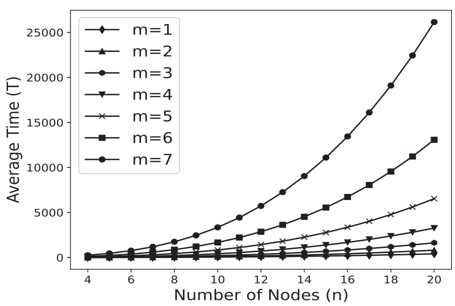


FIGURE 8. **Average time versus *n* for *m*-dimensional prism networks.**

time decreases with ε. For $\varepsilon>0.5$, the average time was independent of network size. This is because of the establishment of the long-range links between non-adjacency nodes which results in fast information exchange between nodes. This is an interesting phenomenon that provides a trade-off between energy consumption and average time. Fig. 4 shows the effect of topology changes on the average time as a function of number of nodes. We observed that the average time linearly

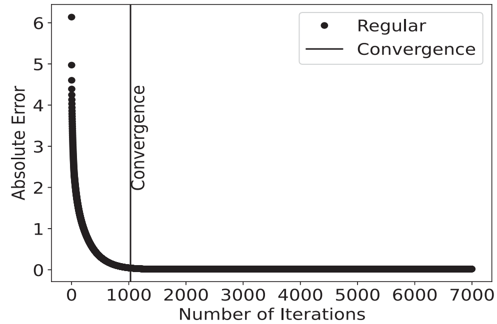


FIGURE 9. **Average consensus difference versus iterations for regular networks.**

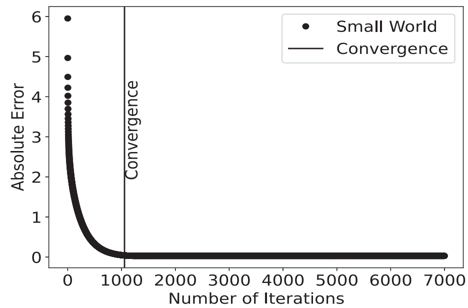


FIGURE 10. **Average consensus difference versus iterations for small world networks.**

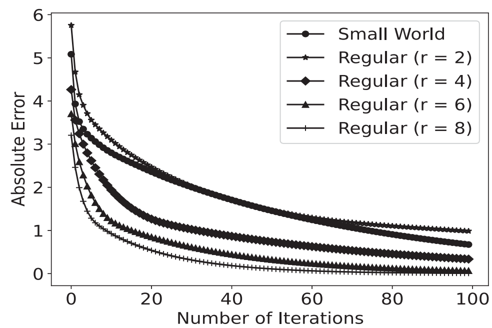


FIGURE 11. **Comparison of regular and small world networks for average consensus difference.**

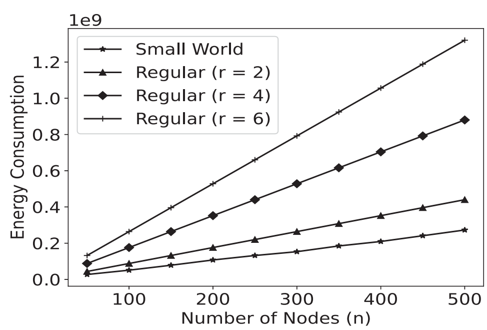


FIGURE 12. **Energy consumption versus nodes for regular and small world networks.**

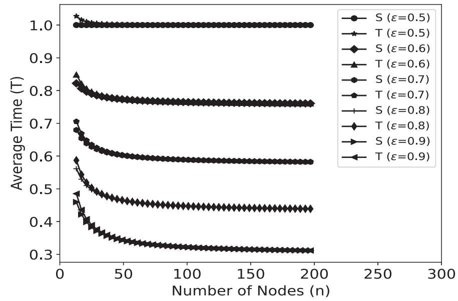


FIGURE 13. Comparison of theoretical (T) and simulation results (S).

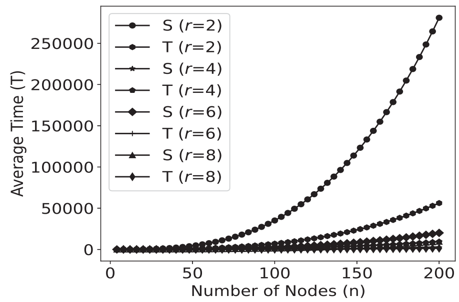


FIGURE 14. Comparison of theoretical (T) and simulation results (S) for regular networks.

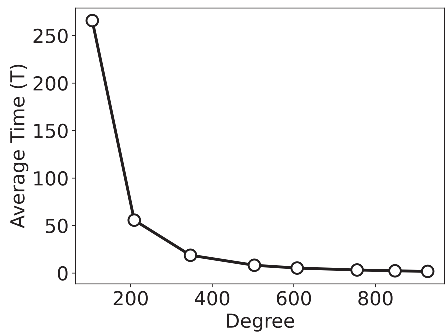


FIGURE 15. Average time versus node degree for random geometric graphs.

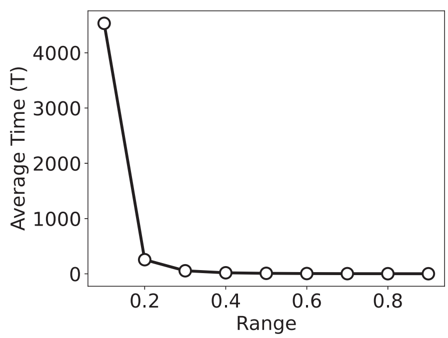


FIGURE 16. Average time versus node range for random geometric graphs.

TABLE 2. Energy Dissipation Values for Radio Model

Components	Energy Dissipation Values
E_{elec}	50 nJ/bit
ε_{amp}	100 pJ/bit/m^2

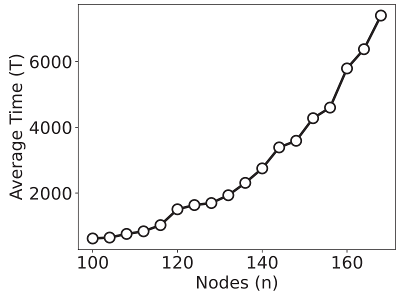


FIGURE 17. Average time versus number of nodes for random geometric graphs.

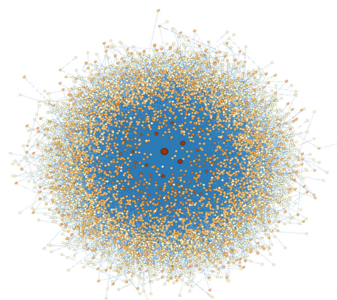


FIGURE 18. Graph visualization for real-world autonomous systems.

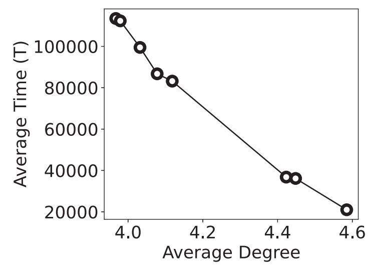


FIGURE 19. Average time versus node degree for real-world autonomous systems.

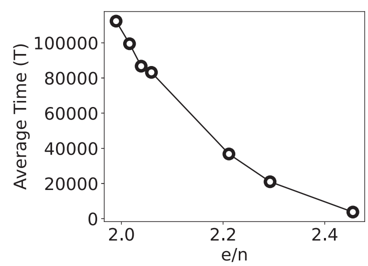


FIGURE 20. Average time versus e/n for real world autonomous systems.

increased with network size. This occurs because an increase in n leads to slow convergence in the gossip process. This result reveals a delay in information gathering in large-scale networks. Fig. 5 shows the average time of the random networks as a function of the number of nodes. As shown, the average time reduces with network size with an increase in q value and it becomes independent of the network size n. Here, q represents the probability that a link exists. Here, the

TABLE 3. Effect of WSN Parameters on Average Time for Graph Structures and Applications

Graph Structure	Parameters	Average Time	Applications
Ring	n	Increases with n	Wireless Mesh Networks [51]
r-Regular	r, n	Increases with n, decreases with r	Intermittently Connected Networks [52]
Prism	m, n	Increases with m and n	Scale-Free Networks [45]
Random	q, p, n	Increases with p and n, decreases with q	Wireless Sensor Networks[53]
Small World	ε, n	Decreases with ε and constant for large n	Wireless Adhoc Networks[54]
RGG	r, d, n	Decreases with r and d, Increases with n	Wireless Sensor Networks [55]

TABLE 4. Brief Comparison With the Recent Works

References	Computational Complexity	Network Parameters
[19]	$O(kn^3)$	n, N
[24]	$O(n^{-3})$	n, r
[59]	$O(n^3)$	n
[60]	$O(n^5)$	n, N
[20]	$O(n^{-3})$	n, N
[61]	$O(n^3)$	n, d
[26]	$O(n)$	n, N
[36]	$O(n^3)$	d, r, n
Proposed Work	$O(n^{-3})$	d, n, m, ε, E, q, p

probability of the existence of communication links improves the network connectivity, which naturally reduces the average time due to more gossiping among the nodes. Fig. 6 shows the average time of r-regular ring networks as a function of the number of nodes. To verify the normal case (ring network), we have considered $r = 1$. The average time increases exponentially with the network size. However, an increase in r value was reduced. This occurs because the increase in r-values improves the network connectivity, which naturally decreases the convergence time. This r can also model the node's transmission range or node overhead in WSNs. Fig. 7 shows the effect of link failures on the average time of the ring network. We observe that the average time increases exponentially with network size. The average time increased with the increase in p. Here p represents the probability of a link failure. Communication link failures are the most common phenomenon in WSNs. If p increases, the communication among nodes is affected, which delays the convergence process. For $p = 0$, a ring network can be observed. Fig. 8 shows the average time of an m-dimensional Prism Network as a function of the number of nodes. Here, the average time increased with n and m. As m increased, the number of nodes also increased, which eventually increased the average time.

To understand the accuracy of the gossip algorithm, we defined the absolute error as the difference between the current values of the nodes and the average value of the initial values. Figs. 9 and 10 show the variation in the absolute error concerning the number of iterations (N) for regular graphs and small-world networks respectively. It is observed that the absolute error exponentially decreases with the number of iterations, and convergence is reached at approximately 5800 iterations and 2000 iterations for regular graphs and small-world graphs respectively. The number of iterations required to achieve consensus is drastically reduced in small-world graphs due to the long-range connections. In Fig. 11, we plot the absolute error versus the number of iterations for the small-world graph, ring network (2-regular ring graph), and r-regular graphs. Here, we can observe that the absolute error exponentially decreases in the small-world and r-regular graphs in comparison with the ring network (2-regular graph).

Next, we study the energy consumed by the nodes in the gossip algorithm. For this study, we considered the radio model in [56] and the energy consumption values, as shown in Table 1. According to this model, the node's transmit energy consumption is $E_{Tx}(d) = E_{elec} * k + \varepsilon_{amp} * k * d^2$ and the received energy consumption is $E_{Rx}(k) = E_{elec} * k$. Here, E_{elec} and ε_{amp} represent the energy consumption of the electronics and amplifiers respectively as shown in Table 2. The parameters k and d denote the number of bits and the distance between sensor nodes respectively. The energy consumed by the gossip algorithm for $k = 8$ and $d = 10$ is shown in Fig. 12. We observe that the total energy consumption E required to reach consensus linearly increases with the number of nodes for both small world and regular graphs. Here, the energy consumption also increases with an increase in r values. However, we observed that small-world graphs consume less energy than the r-regular graphs. Figs. 13 and 14 validate the accuracy of the theoretical results derived in this study. We see that in both figures, the theoretical values of the average time match perfectly with the simulated counterparts.

A. RANDOM GEOMETRIC GRAPH

In this subsection, we study the average time of the gossip algorithms for random geometric graphs (RGG). As shown in Figs. 15 and 16, we plot the average time with respect to the degree for $n = 100$ and node range for $n = 1000$ respectively. As shown, we observed that the average time exponentially decreased with both the degree and range. Network connectivity increases with node degree and communication range. These parameters help the nodes to participate in the gossip process and achieve a fast consensus. Fig. 17 depicts the effect of the number of nodes on the average time for the range=0.2. We observed that the average time increased exponentially with the number of nodes.

B. REAL-WORLD AUTONOMOUS SYSTEMS

To examine a real-world scenario, we used a dataset that consists of routers organized into sub-networks called consensus systems (AS) [57]. Each AS exchanges traffic with some of its peers, as shown in Fig. 18, which was constructed from Border Gateway Protocol (BGP) logs by extracting who-talks-to-whom data. Data were collected from the University of Oregon. It contains 733 daily instances spanning an interval of 785 days from November 8, 1997, to January 2, 2000. During this time, new nodes were added, and some previous nodes were deleted. In addition to a few exceptions, the number of nodes varied from 3015 to 6474. To investigate the average time in the real world, we used real-world consensus systems. In Figs. 19 and 20, the average time of the real-world systems decreases with the node degree and e/n respectively. We have observed a similar trend in the results of the r-nearest ring and small-world networks' when we increase r or ε. In Table 3, we have shown the effect of WSN parameters on the average time for various graph models studied in this work. Here, n, r, m, q, p, ε, d, e/n denote the number of nodes, degree, network dimension, probability of link exists, probability of link failures, probability of long-range links, node's range, edges to number of nodes ratio respectively.

C. COMPARISON OF COMPUTATIONAL COMPLEXITY

Computing the average time $T = \frac{1}{\log \frac{1}{\lambda_2(\mathbf{W})}}$ is a computationally challenging task with a time complexity of $O\left(n^3\right)$. As shown in Table 4, there are very few studies in the literature that focus on studying computationally efficient gossip algorithms for large-scale WSNs. In particular, the works in [24] and [20] exhibit lower computational complexity, but offer limited insights in terms of network parameters. The remaining studies mentioned in the table exhibit high computational complexity. As shown in Table 4, their work could not provide more insights into gossip algorithms in terms of WSN parameters. To fill this gap in the literature, the theoretical results developed in this study address this issue. The complexity required to calculate the $\cos\left(\frac{2\pi}{n}\right)$ is $O\left(n^{-2}\right)$. Hence, the complexity of (7) is $O\left(n^{-2}\right)$. Similarly, we can compute the complexities of (11), (20), (25), (26), and (31) as $O\left(n^{-3}\right)$. The complexities of (15) and (37) are as $O(1)$ and $O\left(n^{-1}\right)$ respectively. For large-scale networks, this complexity asymptotically converges to $O(1)$.

D. APPLICATIONS

We present a summary of the applications of the network models and gossip algorithms in Table 4, in which wireless mesh networks can be theoretically analyzed using the framework of ring graphs [51]. Further exploration can be performed by leveraging regular graphs [52] to increase the applicability of ring networks to investigate the network overhead and node transmission radius in intermittently connected networks. Prism graph modeling offers great insight into characterizing high-dimensional and robust networks, thereby facilitating scale-free properties [45]. For WSNs, random graph models are often employed because of uncertainty inherent in wireless communication links [53]. The utilization of small-world graph modeling is anticipated to be extensive for enhancing communication speeds in wireless ad-hoc networks [54]. Among the existing graph models, Random Geometric Graph (RGG) is the most suitable graph model for capturing geographical proximity in WSNs [55].

E. ADVANTAGES

Gossip algorithms have shown widespread advantages in large-scale WSNs for aggregating and disseminating information [58]. These algorithms operate in a decentralized manner, eliminating the need for a centralized fusion center. They are specifically well-suited for networks with nodes that have limited computational resources and are prone to failures. Gossip algorithms demonstrate resilience to dynamic topology changes that are typical of mobile WSNs. They operate in asynchronous communication scenarios and are effective in distributed computations. However, while these algorithms excel in rapidly routing information, achieving global convergence can be time-consuming, specifically in large-scale and sparse WSNs. The proposed theoretical tools play a major role in controlling and optimizing the convergence times in large-scale WSNs.

VI. CONCLUSION

In this study, we exploited the properties of regular, small-world, and scale-free networks to derive explicit expressions of average time in WSNs. Next, we analytically study the effect of communication link failures, topology changes, long-range links, network dimension, node transmission range, and network size on average time. Our numerical results reveal that the average time of the gossip algorithms can be significantly reduced by the node's transmission range and long-range communication links. We also demonstrated that the increase in the average time for scale-free WSNs is attributed to their structure, which can be effectively controlled by the node's transmission range values.

VII. FUTURE WORK

In this study, we exploited the properties of ring networks to derive explicit expressions of convergence time for gossip algorithms. However, the exploration of random geometric graphs as a tool for investigating the spatial characteristics of WSNs can provide further valuable insights. A promising future research direction involves modeling WSNs as directed ring graphs and studying the effect of asymmetric modeling on the convergence time. Additionally, employing expander graphs, such as Ramanujan graphs can significantly reduce the convergence time of gossip algorithms. Furthermore, scale-free modeling techniques can improve our understanding of the performance of gossip algorithms in large-scale WSNs and their robustness to future challenges.

REFERENCES

[1] I. F. Akyildiz, W. Su, Y. Sankarasubramaniam, and E. Cayirci, "Wireless sensor networks: A survey," *Comput. Netw.*, vol. 38, no. 4, pp. 393–422, 2002.

[2] J. Yick, B. Mukherjee, and D. Ghosal, "Wireless sensor network survey," *Comput. Netw.*, vol. 52, no. 12, pp. 2292–2330, 2008.

[3] X. Wang and I. Akilydiz, "A survey on sensor networks," *IEEE Commun. Mag.*, vol. 40, no. 8, pp. 102–114, Aug. 2002.

[4] M. Younis, I. F. Senturk, K. Akkaya, S. Lee, and F. Senel, "Topology management techniques for tolerating node failures in wireless sensor networks: A. survey," *Comput. Netw.*, vol. 58, pp. 254–283, 2014.

[5] S. Chouikhi, I. El Korbi, Y. Ghamri-Doudane, and L. A. Saidane, "A survey on fault tolerance in small and large scale wireless sensor networks," *Comput. Commun.*, vol. 69, pp. 22–37, 2015.

[6] J. B. Predd, S. B. Kulkarni, and H. V. Poor, "Distributed learning in wireless sensor networks," *IEEE Signal Process. Mag.*, vol. 23, no. 4, pp. 56–69, Jul. 2006.

[7] W. Yu, G. Chen, Z. Wang, and W. Yang, "Distributed consensus filtering in sensor networks," *IEEE Trans. Syst., Man, Cybern., Part B.*, vol. 39, no. 6, pp. 1568–1577, Dec. 2009.

[8] S. Kar, S. Aldosari, and J. M. Moura, "Topology for distributed inference on graphs," *IEEE Trans. Signal Process.*, vol. 56, no. 6, pp. 2609–2613, Jun. 2008.

[9] G. Cybenko, "Dynamic load balancing for distributed memory multiprocessors," *J. Parallel Distrib. Comput.*, vol. 7, no. 2, pp. 279–301, 1989.

[10] A. Jadbabaie, J. Lin, and A. S. Morse, "Coordination of groups of mobile autonomous agents using nearest neighbor rules," *IEEE Trans. Autom. Control*, vol. 48, no. 6, pp. 988–1001, Jun. 2003.

[11] V. Saligrama and D. A. Castanon, "Reliable distributed estimation with intermittent communications," in *Proc. IEEE 45th Conf. Decis. Control*, 2006, pp. 6763–6768.

[12] R. Olfati-Saber, J. A. Fax, and R. M. Murray, "Consensus and cooperation in networked multi-agent systems," *Proc. IEEE*, vol. 95, no. 1, pp. 215–233, Jan. 2007.

[13] A. G. Dimakis, S. Kar, J. M. F. Moura, M. G. Rabbat, and A. Scaglione, "Gossip algorithms for distributed signal processing," *Proc. IEEE*, vol. 98, no. 11, pp. 1847–1864, Nov. 2010.

[14] S. Boyd, A. Ghosh, B. Prabhakar, and D. Shah, "Gossip algorithms: Design, analysis and applications," in *Proc. IEEE 24th Annu. Joint Conf. IEEE Comput. Commun. Societies.*, 2005, pp. 1653–1664.

[15] L. Xiao, S. Boyd, and S.-J. Kim, "Distributed average consensus with least-mean-square deviation," *J. Parallel Distrib. Comput.*, vol. 67, no. 1, pp. 33–46, 2007.

[16] J. Tsitsiklis, D. Bertsekas, and M. Athans, "Distributed asynchronous deterministic and stochastic gradient optimization algorithms," *IEEE Trans. Autom. Control*, vol. 31, no. 9, pp. 803–812, Sep. 1986.

[17] L. Altoaimy, H. Kurdi, A. Alromih, A. Alomari, E. Alrogi, and S. H. Ahmed, "Enhanced distance-based gossip protocols for wireless sensor networks," in *Proc. 16th IEEE Annu. Consum. Commun. Netw. Conf.*, 2019, pp. 1–4.

[18] C. Patra and N. Botezatu, "Effect of gossiping on some basic wireless sensor network protocols," in *Proc. 21st Int. Conf. Syst. Theory, Control Comput.*, 2017, pp. 309–314.

[19] E. Zanaj, M. Baldi, and F. Chiaraluce, "Efficiency of the gossip algorithm for wireless sensor networks," in *Proc. 15th Int. Conf. Softw., Telecommun. Comput. Netw.*, 2007, pp. 1–5.

[20] D. Ustebay, B. N. Oreshkin, M. J. Coates, and M. G. Rabbat, "Greedy gossip with eavesdropping," *IEEE Trans. Signal Process.*, vol. 58, no. 7, pp. 3765–3776, Jul. 2010.

[21] A. Alromih and H. Kurdi, "An energy-efficient gossiping protocol for wireless sensor networks using Chebyshev distance," *Procedia Comput. Sci.*, vol. 151, pp. 1066–1071, 2019.

[22] S. Kouachi, S. Dhuli, and Y. N. Singh, "Convergence rate analysis of periodic gossip algorithms for one-dimensional lattice WSNs," *IEEE Sensors J.*, vol. 20, no. 21, pp. 13150–13160, Nov. 2020.

[23] K. Flouri, B. Beferull-Lozano, and P. Tsakalides, "Optimal gossip algorithm for distributed consensus SVM training in wireless sensor networks," in *Proc. 16th Int. Conf. Digit. Signal Process.*, 2009, pp. 1–6.

[24] A. D. G. Dimakis, A. D. Sarwate, and M. J. Wainwright, "Geographic gossip: Efficient averaging for sensor networks," *IEEE Trans. Signal Process.*, vol. 56, no. 3, pp. 1205–1216, Mar. 2008.

[25] V. Pilloni, M. Franceschelli, L. Atzori, and A. Giua, "Deployment of applications in wireless sensor networks: A gossip-based lifetime maximization approach," *IEEE Trans. Control Syst. Technol.*, vol. 24, no. 5, pp. 1828–1836, Sep. 2016.

[26] J. Zhang, "Power optimized and power constrained randomized gossip approaches for wireless sensor networks," *IEEE Wireless Commun. Lett.*, vol. 10, no. 2, pp. 241–245, Feb. 2021.

[27] F. Peper, K. Leibnitz, T. Shimokawa, and M.-A. Remiche, "Average consensus in asymmetric broadcasting wireless sensor networks through gossiping," in *Proc. 13th Int. Conf. Mobile Ubiquitous Syst.: Comput. Netw. Serv.*, 2016, pp. 171–176.

[28] J. Mertens, L. Galluccio, and G. Morabito, "Centrality-aware gossiping for distributed learning in wireless sensor networks," in *Proc. IFIP Netw. Conf.*, 2022, pp. 1–6.

[29] J. Mertens, L. Galluccio, and G. Morabito, "MGM-4-FL: Combining federated learning and model gossiping in WSNs," *Comput. Netw.*, vol. 214, 2022, Art. no. 109144.

[30] M. Franceschelli, A. Giua, and C. Seatzu, "Distributed averaging in sensor networks based on broadcast gossip algorithms," *IEEE Sensors J.*, vol. 11, no. 3, pp. 808–817, Mar. 2011.

[31] S. Dhuli, K. Gaurav, and Y. N. Singh, "Convergence analysis for regular wireless consensus networks," *IEEE Sensors J.*, vol. 15, no. 8, pp. 4522–4531, Aug. 2015.

[32] H. Wang, H. Luo, Y. Jiang, and M. Huo, "Distributed fault diagnosis approach for large-scale interconnected systems with communication link failures," in *Proc. IEEE 6th Int. Conf. Ind. Cyber- Phys. Syst.*, 2023, pp. 1–6.

[33] Y. Liu, S. Sun, and R. Zhang, "Sensing-assisted neighbor discovery for vehicular ad hoc networks," in *Proc. IEEE Wireless Commun. Netw. Conf.*, 2023, pp. 1–6.

[34] Z. Wei, Y. Liang, Z. Meng, Z. Feng, K. Han, and H. Wu, "Fast neighbor discovery for wireless ad hoc network with successive interference cancellation," *IEEE Trans. Veh. Technol.*, vol. 73, no. 1, pp. 1322–1336, Jan. 2024.

[35] C. Sirocchi and A. Bogliolo, "Community-based gossip algorithm for distributed averaging," in *Proc. IFIP Int. Conf. Distrib. Appl. Interoperable Syst.*, 2023, pp. 37–53.

[36] C. Sirocchi and A. Bogliolo, "Topological network features determine convergence rate of distributed average algorithms," *Sci. Rep.*, vol. 12, no. 1, 2022, Art. no. 21831.

[37] R. Jin, Y. Huang, Z. Zhang, and H. Dai, "On the privacy guarantees of gossip protocols in general networks," *IEEE Trans. Netw. Sci. Eng.*, vol. 10, no. 6, pp. 3114–3130, Nov./Dec. 2023.

[38] M. Femminella and G. Reali, "Gossip-based monitoring protocol for 6G networks," *IEEE Trans. Netw. Service Manag.*, vol. 20, no. 4, pp. 4126–4140, Dec. 2023.

[39] P. Mitra and S. Ulukus, "Age-aware gossiping in network topologies," *IEEE Trans. Commun.*, 2024.

[40] G. Tsimos, J. Loss, and C. Papamanthou, "Gossiping for communication-efficient broadcast," in *Proc. Annu. Int. Cryptol. Conf.*, 2022, pp. 439–469.

[41] G. Saldamli, C. Upadhyay, D. Jadhav, R. Shrishrimal, B. Patil, and L. Tawalbeh, "Improved gossip protocol for blockchain applications," *Cluster Comput.*, vol. 25, no. 3, pp. 1915–1926, 2022.

[42] A. Osorio, M. Calle, J. Soto, and J. E. Candelo-Becerra, "Routing in LoRa for smart cities: A gossip study," *Future Gener. Comput. Syst.*, vol. 136, pp. 84–92, 2022.

[43] H. Riess and R. Ghrist, "Diffusion of information on networked lattices by gossip," in *Proc. IEEE 61st Conf. Decis. Control*, 2022, pp. 5946–5952.

[44] N. E. Manitara, A. I. Rikos, and C. N. Hadjicostis, "Privacy-preserving distributed average consensus in finite time using random gossip," in *Proc. Eur. Control Conf.*, 2022, pp. 1282–1287.

[45] S. Dhuli, F. Atik, F. Parveen, and O. J. Pandey, "Performance analysis of consensus algorithms over prism networks using Laplacian spectra," *IEEE Netw. Lett.*, vol. 4, no. 3, pp. 142–146, Sep. 2022.

[46] G. J. Pottie and W. J. Kaiser, "Wireless integrated network sensors," *Commun. ACM*, vol. 43, no. 5, pp. 51–58, 2000.

[47] B. Bamieh, M. R. Jovanovic, P. Mitra, and S. Patterson, "Coherence in large-scale networks: Dimension-dependent limitations of local feedback," *IEEE Trans. Autom. Control*, vol. 57, no. 9, pp. 2235–2249, Sep. 2012.

[48] I. Kra and S. R. Simanca, "On circulant matrices," *Notices AMS*, vol. 59, no. 3, pp. 368–377, 2012.

[49] D. J. Watts and S. H. Strogatz, "Collective dynamics of 'small-world'networks," *Nature*, vol. 393, no. 6684, pp. 440–442, 1998.

[50] P. Hovareshti, J. S. Baras, and V. Gupta, "Average consensus over small world networks: A probabilistic framework," in *Proc. IEEE 47th Conf. Decis. Control*, 2008, pp. 375–380.

[51] A. Alasaad, H. Nicanfar, S. Gopalakrishnan, and V. C. Leung, "A ring-based multicast routing topology with QoS support in wireless mesh networks," *Wireless Netw.*, vol. 19, pp. 1627–1651, 2013.

[52] C.-K. Chau and P. Basu, "Analysis of latency of stateless opportunistic forwarding in intermittently connected networks," *IEEE/ACM Trans. Netw.*, vol. 19, no. 4, pp. 1111–1124, Aug. 2011.

[53] S. D. Servetto and G. Barrenechea, “Constrained random walks on random graphs: Routing algorithms for large scale wireless sensor networks,” in *Proc. 1st ACM Int. Workshop Wireless Sensor Netw. Appl.*, 2002, pp. 12–21.
[54] P. K. Biswas, S. J. Mackey, D. H. Cansever, M. P. Patel, and F. B. Panettieri, “Context-aware smallworld routing for wireless ad-hoc networks,” *IEEE Trans. Commun.*, vol. 66, no. 9, pp. 3943–3958, Sep. 2018.
[55] M. Haenggi, J. G. Andrews, F. Baccelli, O. Dousse, and M. Franceschetti, “Stochastic geometry and random graphs for the analysis and design of wireless networks,” *IEEE J. Sel. Areas Commun.*, vol. 27, no. 7, pp. 1029–1046, Sep. 2009.
[56] W. B. Heinzelman, A. P. Chandrakasan, and H. Balakrishnan, “An application-specific protocol architecture for wireless microsensor networks,” *IEEE Trans. Wireless Commun.*, vol. 1, no. 4, pp. 660–670, Oct. 2002.
[57] J. Leskovec, J. Kleinberg, and C. Faloutsos, “Graphs over time: Densification laws, shrinking diameters and possible explanations,” in *Proc. 11th ACM SIGKDD Int. Conf. Knowl. Discov. Data Mining*, 2005, pp. 177–187.
[58] M. Jelasity, A. Montresor, and O. Babaoglu, “Gossip-based aggregation in large dynamic networks,” *ACM Trans. Comput. Syst.*, vol. 23, no. 3, pp. 219–252, 2005.
[59] F. He, S. Mou, J. Liu, and A. S. Morse, “Convergence rate on periodic gossiping,” *Inf. Sci.*, vol. 364, pp. 111–125, 2016.
[60] M. Zheng, M. Goldenbaum, S. Stańczak, and H. Yu, “Fast average consensus in clustered wireless sensor networks by superposition gossiping,” in *Proc. IEEE Wireless Commun. Netw. Conf.*, 2012, pp. 1982–1987.
[61] S. Dhuli, Y. N. Singh, and P. Ranjan, “Effect of nearest neighbors on convergence rate of periodic gossip algorithms in WSNs,” in *Proc. IEEE 9th Int. Conf. Commun. Syst. Netw. Technol.*, 2020, pp. 96–101.