Received 18 September 2023; revised 6 October 2023; accepted 7 October 2023. Date of publication 12 October 2023; date of current version 2 November 2023.
Digital Object Identifier 10.1109/OJCOMS.2023.3324064

Asymptotic Performance Analysis of Large-Scale Active IRS-Aided Wireless Network

YAN WANG[1], FENG SHU[1,2,5] (Member, IEEE), ZHIHONG ZHUANG[2], RONGEN DONG[1], QI ZHANG[1], DI WU[1], XUEHUI WANG[1], LIANG YANG[3], AND JIANGZHOU WANG[4] (Fellow, IEEE)

[1]School of Information and Communication Engineering, Hainan University, Haikou 570228, China
[2]School of Electronic and Optical Engineering, Nanjing University of Science and Technology, Nanjing 210094, China
[3]College of Computer Science and Electronic Engineering, Hunan University, Changsha 410082, China
[4]School of Engineering, University of Kent, CT2 7NT Canterbury, U.K.
[5]Communication Engineering and Collaborative Innovation Center of Information Technology, Hainan University, Haikou 570228, China

CORRESPONDING AUTHOR: F. SHU (e-mail: shufeng0101@163.com)

This work was supported in part by the National Natural Science Foundation of China under Grant U22A2002 and Grant 62071234; in part by the Hainan Province Science and Technology Special Fund under Grant ZDKJ2021022; in part by the Scientific Research Fund Project of Hainan University under Grant KYQD(ZR)-21008; and in part by the Collaborative Innovation Center of Information Technology, Hainan University under Grant XTCX2022XXC07.

ABSTRACT In this paper, the dominant factor affecting the performance of active intelligent reflecting surface (IRS) aided wireless communication networks in Rayleigh fading channel, namely the average signal-to-noise ratio (SNR) γ_0 at IRS, is studied. Making use of the weak law of large numbers, the simple asymptotic expressions for the received SNR at user and the average SNR at IRS are derived as the number N of IRS elements goes to medium-scale and large-scale. When N tends to large-scale, the asymptotic received SNR at user is proved to be a linear increasing function of a product of γ_0 and N. Subsequently, when the base station (BS) transmit power is fixed, there exists an optimal limited reflective power at IRS. At this point, more IRS reflect power will degrade the SNR performance. Additionally, under the total power sum constraint of the BS transmit power and the power reflected by the IRS, an optimal power allocation (PA) strategy is derived and shown to achieve 0.83 bit rate gain over equal PA. Finally, an IRS with finite phase shifters being taken into account, generates phase quantization errors, and further leads to a degradation of receive performance. The corresponding closed-form performance loss expressions for user's asymptotic SNR, achievable rate (AR), and bit error rate (BER) are derived for active IRS. Numerical simulation results show that a 3-bit discrete phase shifter is required to achieve a trivial performance loss for a large-scale active IRS.

INDEX TERMS Active IRS, finite phase shifter, quantization error, performance loss, the law of large numbers.

I. INTRODUCTION

FROM the 1st generation (1G) wireless networks to the 5th generation (5G) wireless networks, the rapid evolution of wireless communication technology has changed many aspects of people's lives in a convenient and intelligent way. Now, researchers are focusing their attention on the investigation of candidate technologies related to the 6th generation (6G) wireless network. For example, the application scenarios, supporting technologies, and future opportunities and challenges of 6G were discussed in [1]. In the traditional communication diagram, the wireless transmission channel cannot be reconfigured smartly and automatically, which limits the further improvement of wireless communication system performance. Due to the fact that intelligent reflecting surface (IRS) can overcome the uncontrollability of traditional wireless transmission environments, and its significant advantages in terms of cost, energy efficiency, reliability, and energy conservation. Therefore, researchers have made an extensive analysis and research on IRS assisted wireless communication networks [2], [3], [4], [5].

Compared with other existing transmission technologies, the channel model and characteristics of IRS-aided wireless communication systems have a significant distinction [6], [7]. A main difference is that numerous IRS reflective elements introduce a large number of communication links. In addition, when receive radio frequency (RF) chains are not installed at the IRS, it is infeasible for the IRS to estimate the channels. Therefore, the channel estimation of IRS assisted wireless networks has attracted widespread attention. There have been some research works on channel estimation for IRS-aided wireless communication. For instance, the authors of [8] proposed a novel least-squares estimator to estimate the direct and cascading channels. Moreover, the transmission from the transmitter to the receiver includes direct channels and non-direct channels reflected by IRS, and the received signal is susceptible to both the channel. In order to obtain strict and comprehensive theoretical analysis of system performance, and further reveal the key factors and internal mechanisms affecting the performance, it is necessary to conduct in-depth research on the performance analysis of IRS-aided communication systems.

The performance evaluation and analysis of IRS-assisted communication systems is becoming a focus issue for researchers [9]. On the one hand, existing research has proposed multiple joint beamforming schemes for various IRS-aided communication systems using optimization theories and methods [10], [11], [12], [13], [14], [15]. These research works were done by deploying the IRS in directional modulation (DM), decode-and-forward (DF) relay networks, covert communication, and spatial modulation networks. For example, in order to address the limitation of traditional DM systems that can only send a single confidential bit stream, the authors of [11] introduced IRS into the DM system to create multi-stream transmission to improve the secrecy rate (SR). To further improve the SR of the system, two alternative optimization schemes for joint receiver beamforming and IRS phase shift matrix were proposed in [12]. Subsequently, a new enhanced receive beamforming scheme for DM networks with full duplex malicious attackers was proposed in [13]. For instance, in order to maximize the received power at the relay, the authors of [14] jointly optimized the beamforming vector at the relay and the phase shift at the IRS. Simulation results showed that IRS assisted DF relay networks can achieve better rate performance and coverage. The authors of [15] analyzed the performance gain obtained by deploying IRS in covert communication. It demonstrated that joint design of transmission power and IRS reflection coefficient can achieve significant performance improvement. Moreover, analytical expressions for transmission power and IRS reflection coefficient were provided. In [16], for the IRS-aided secure spatial modulation system, the average SR is maximized by jointly optimizing passive beamforming at IRS and the base station (BS) transmit power.

To obtain more rigorous and universal properties regarding system performance, existing research has made a detailed analysis of the signal-to-noise ratio (SNR) [17], achievable rate (AR), bit error rate (BER), energy efficiency [18], delay outage rate [19], IRS location placement [20], and other performance aspects of typical IRS-aided communication systems. The initial design of IRS phase shift was based on the assumption of an ideal continuous phase shift, in which case there is no phase quantization error (QE) in the IRS [21], [22], [23], [24]. Under the assumption of continuous phase shift, using the central limit theorem in [19], the amplitude of the Rayleigh composite channel is approximated as a complex Gaussian distribution, and then the received SNR is approximated as a non-central Chi-square distribution. In addition, the authors of [25] analyzed the tight upper bound of the AR for hybrid relay and IRS-aided communication systems. Moreover, the authors of [26] derived a new expression of the average SNR based on the probability density function (PDF) and the cumulative distribution function (CDF). Subsequently, the IRS assisted wireless system was analyzed and verified to be superior to the corresponding amplify-and-forward relay system in terms of average SNR, outage probability, average symbol error rate and ergodic capacity. Furthermore, the authors of [27] derived the critical propagation characteristics of the double IRS-aided unmanned aerial vehicle (UAV)-to-ground communication channel model. Simulation results showed that the double-IRS aided UAV-to-ground communication has advantages over traditional one with single-IRS or the line-of-sight (LoS) links.

However, considering the high circuit cost of IRS-aided systems based on continuous phase shifters, especially when the number of IRS elements tends to large-scale, it is more reasonable and suitable to adopt some low-cost finite phase shifters in practice. Therefore, the performance analysis of the IRS-aided communication system with discrete phase shift was further studied in [28], [29], [30], [31], [32]. In [31], the author analyzed the impact of QE introduced by phase shifters with finite quantization bits, and derived a closed-form expression for the performance loss of received signals to interference plus noise ratio using the law of large numbers. The authors of [32] found the performance loss caused by IRS in LoS channels and Rayleigh channels. Simulation results showed that when the quantization bits number of passive IRS is larger than or equal to 3, the performance losses of SNR in LoS channels are less than 0.23 dB, and the corresponding degradation on BER is negligible.

Due to the "multiplicative fading" effect, the existing passive IRSs only achieve limited capacity gains in many scenarios with strong direct links. In [33], the concept of active IRSs was proposed to overcome this fundamental limitation. Unlike passive IRSs that reflect signals without amplification, active IRSs can amplify the reflected signals via amplifiers integrated into their elements. Due to the use of active components, the noise introduced by active IRS elements cannot be neglected as is done for passive IRSs. How does the amplifying ability and the noise of active IRS affect the system performance, particularly, under the power constraint or in the presence of phase quantization error.

In this paper, we will present a performance analysis of a large-scale active IRS-aided wireless communication network, and our main contributions are summarized as follows:

1) To find the dominant affecting factor of active IRS-aided communication network, a new factor, called average SNR γ_0 at IRS, is defined, and its asymptotic simple expression is derived by using the weak law of large numbers as the number N of elements of IRS goes to medium-scale and large-scale. Using this definition, when N tends to large-scale, the receive SNR at user is proven to be a linear increasing function of a product of N and γ_0. Considering parameter γ_0 is proportional to the ratio of P_s to σ_i^2, where P_s is the transmit power at BS and σ_i^2 is the noise power at active IRS. In other words, γ_0 will have a significant impact on system rate performance given a fixed number of active IRS elements.
2) To evaluate the influence of adjusting the transmit power P_s at BS or the reflected power P_i at IRS on rate performance, two situations are considered as follows: adjust P_i with fixed P_s and adjust any one of P_i and P_s under their sum constraint. In the first case, when P_s is fixed, there exists an optimal P_i and we give a closed-form expression of the optimal P_i in this case. In the second case, an efficient power allocation (PA) strategy is given in the presence of a constraint on the sum of P_i and P_s, and it shows a rate gain of 0.83 bit over equal PA (EPA).
3) To see the performance loss caused by finite phase shifters at active IRS, according to the law of large numbers, the closed-form expressions for the performance loss (PL) of user's asymptotic SNR, AR, and BER are derived firstly. Subsequently, expressions for the approximate performance loss (APL) of SNR, AR and BER are given based on the Taylor series expansions. Numerical simulations show that when the number of quantization bits is greater than or equal to 3, the loss of the asymptotic SNR and AR of the active IRS-aided wireless network are less than 0.22 dB and 0.08 bits/s/Hz, respectively, and the corresponding BER performance loss may be ignored.

The remainder of this paper is organized as follows. Section II constructs a system model of an active IRS-aided communication network. Section III derives the performance analysis of the active IRS with infinite phase shifter and reveals the key factors affecting the user received SNR. Subsequently, the SNR, AR, and BER performance loss analysis of large-scale active IRS-aided wireless networks with finite phase shifters is presented in Section IV. Simulation and numerical results are shown in Section V. Finally, we draw our conclusions in Section VI.

Notations: Throughout the paper, vectors and scalars are denoted by letters of bold lower case and lower case, respectively. The sign $|\cdot|$ represents modulus. The notation $\mathbb{E}\{\cdot\}$ represents expectation operation.

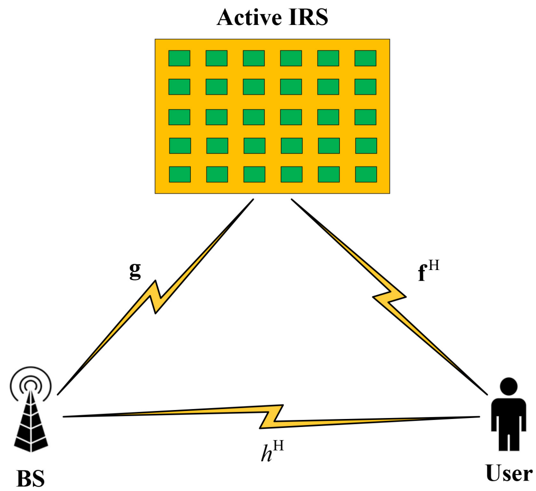


FIGURE 1. System model of an active IRS-aided communication network.

II. SYSTEM MODEL

A downlink communication system with the aid of an N-element active IRS is described in Fig. 1, where the BS and the user are equipped with single antenna. It is worth noting that a multi-antenna BS scenario can be viewed as a virtual single-antenna BS through transmit beamforming. Then, the subsequent derivation process of BS with multi antenna is similar to the single-antenna BS scenario. The BS→IRS, IRS→User, and BS→User channels are the Rayleigh channels.

Assume that x is the transmit signal at BS with $\mathbb{E}[|x|^2 = P_s$. The signal received at the n-th active IRS element can be modeled as

$$s_i(n) = \sqrt{L_g} g(n)x + w_i(n), \tag{1}$$

where $g(n) = |g(n)|e^{j\theta_g(n)}$ is the channel between the BS and the n-th active IRS element, wherein L_g is the channel path loss coefficient. $w_i(n)$ represents the additive white Gaussian noise (AWGN) at the n-th active IRS element with distribution $w_i(n) \sim \mathcal{CN}(0, \sigma_i^2)$.

The signal reflected by the n-th active IRS element can be expressed as

$$y_i(n) = \sqrt{L_g} p(n)g(n)x + p(n)w_i(n), \tag{2}$$

where the amplification factor at the n-th IRS element can be expressed as $p(n) = |p(n)|e^{j\theta_i(n)}$.

From (2), the average total power reflected by all N active IRS elements is

$$P_i = P_s L_g \sum_{n=1}^{N} |p(n)g(n)|^2 + \sigma_i^2 \sum_{n=1}^{N} |p(n)|^2, \tag{3}$$

where P_i represents the maximum reflecting power sum at active IRS.

The signal received at user can be written as

$$\begin{aligned} y_u &= \Big(\sqrt{L_h}h^* + \sqrt{L_f L_g}\sum_{n=1}^{N} f^*(n)p(n)g(n)\Big)x \\ &\quad + \sqrt{L_f}\sum_{n=1}^{N} f^*(n)p(n)w_i(n) + w_u \\ &= \Big(\sqrt{L_h}|h|e^{-j\theta_h} + \sqrt{L_f L_g}\sum_{n=1}^{N}|f(n)||p(n)||g(n)| \\ &\quad \cdot e^{j\left(-\theta_f(n)+\theta_i(n)+\theta_g(n)\right)}\Big)x \\ &\quad + \sqrt{L_f}\sum_{n=1}^{N} f^*(n)p(n)w_i(n) + w_u \\ &= e^{-j\theta_h}\Big(\sqrt{L_h}|h| + \sqrt{L_f L_g}\sum_{n=1}^{N}|f(n)||p(n)||g(n)| \\ &\quad \cdot e^{j\left(-\theta_f(n)+\theta_i(n)+\theta_g(n)+\theta_h\right)}\Big)x \\ &\quad + \sqrt{L_f}\sum_{n=1}^{N} f^*(n)p(n)w_i(n) + w_u, \end{aligned} \tag{4}$$

where $h^* = |h|e^{-j\theta_h} \sim \mathcal{CN}(0, \alpha_h^2)$ and $f^*(n) = |f(n)|e^{-j\theta_f(n)} \sim \mathcal{CN}(0, \alpha_f^2)$ represent channels from BS to the user and the n-th active IRS element to the user, respectively, and L_h and L_f are the corresponding channel path loss coefficient. w_u represents the AWGN at the user with distribution $w_u \sim \mathcal{CN}(0, \sigma_u^2)$.

If the phase shifter of the active IRS is continuous, which means that there is no phase QE, the transmit signal of the BS is perfectly reflected by the IRS to the user, so the phase shifter of the n-th active IRS element can be designed as

$$\theta_{ic}(n) = \theta_f(n) - \theta_g(n) - \theta_h, \tag{5}$$

where we assume $\theta_h = 0$ for the convenience of the subsequent derivation.

Assuming that the actual IRS implementation involves many finite phase shifters, where each discrete phase shifter uses a k-bit phase quantizer, the phase feasible set of each reflect element of the IRS is as follows

$$\Omega = \left\{\frac{\pi}{2^k}, \frac{3\pi}{2^k}, \ldots, \frac{\left(2^{k+1}-1\right)\pi}{2^k}\right\}. \tag{6}$$

The desired continuous phase of the n-th element of the IRS is shown in (5), and the final discrete phase chosen from the phase feasible set Ω is

$$\theta_i(n) = \mathop{\arg\min}_{\theta_i(n)\in\Omega} \|\theta_i(n) - \theta_{ic}(n)\|_2. \tag{7}$$

In general, the actual discrete phase is not equal to the desired continuous phase, and this phase mismatch leads to degraded receive performance. For the next analysis, we define the n-th phase QE at the IRS as

$$\Delta\theta(n) = \theta_i(n) - \theta_{ic}(n). \tag{8}$$

Assuming that the phase quantization error obeys a uniform distribution, it follows the PDF as follows

$$f(x) = \begin{cases} \frac{1}{2\Delta x}, & x \in [-\Delta x, \Delta x], \\ 0, & \text{otherwise}, \end{cases} \tag{9}$$

where

$$\Delta x = \frac{\pi}{2^k}, \tag{10}$$

where k is a positive integer.

In the presence of phase QE, the receive signal (4) can be converted into

$$\begin{aligned} \widehat{y}_u &= \Big(\sqrt{L_h}|h| + \sqrt{L_f L_g}\underbrace{\sum_{n=1}^{N}|f(n)||p(n)||g(n)|e^{j\Delta\theta(n)}}_{S}\Big) \\ &\quad \cdot x + \underbrace{\sqrt{L_f}\sum_{n=1}^{N} f^*(n)p(n)w_i(n)}_{N1} + \underbrace{w_u}_{N2} \\ &= \left(\sqrt{L_h}|h| + \sqrt{L_f L_g}S\right)x + N1 + N2. \end{aligned} \tag{11}$$

The SNR at the user is

$$\gamma_u = \frac{P_s\left(\sqrt{L_h}|h| + \sqrt{L_f L_g}S\right)^2}{\mathbb{E}\left(N_1^H N_1\right) + \mathbb{E}\left(N_2^H N_2\right)}, \tag{12}$$

where $w_i(n)$, w_u is independent and identically distributed, so $\mathbb{E}(N_1^H N_2) = \mathbb{E}(N_2^H N_1) = 0$.

In accordance with the weak law of large numbers, we have

$$\begin{aligned} S &= N \cdot \frac{1}{N}\sum_{n=1}^{N}|f(n)||p(n)||g(n)|e^{j\Delta\theta(n)} \\ &\approx N \cdot \mathbb{E}\Big(|f(n)||p(n)||g(n)|e^{j\Delta\theta(n)}\Big). \end{aligned} \tag{13}$$

The power of noise amplified by active IRS is

$$\begin{aligned} &\mathbb{E}\left(N_1^H N_1\right) \\ &= L_f \mathbb{E}\left\{\sum_{m=1}^{M}\sum_{n=1}^{N} f^*(m)f(n)p^*(m)p(n)w_i^*(m)w_i(n)\right\} \\ &= L_f \sum_{m=1}^{M}\sum_{n=1}^{N} f^*(m)f(n)p^*(m)p(n) \cdot \mathbb{E}\left\{w_i^*(m)w_i(n)\right\}, \end{aligned} \tag{14}$$

where

$$\mathbb{E}\left\{w_i^*(m)w_i(n)\right\} = \sigma_i^2\delta(m-n), \tag{15}$$

then

$$\begin{aligned} \mathbb{E}\left(N_1^H N_1\right) &= L_f \sum_{m=1}^{M}\sum_{n=1}^{N} f^*(m)f(n)p^*(m)p(n)\sigma_i^2\delta(m-n) \\ &= L_f\sigma_i^2 \sum_{n=1}^{N}|f(n)|^2|p(n)|^2. \end{aligned} \tag{16}$$

Similarly, according to the weak law of large numbers, (16) can be further converted to

$$\mathbb{E}\left(N_1^H N_1\right) \approx N L_f \sigma_i^2 \mathbb{E}\left(|f(n)|^2 |p(n)|^2\right). \tag{17}$$

The noise power at the user is

$$\mathbb{E}\left(N_2^H N_2\right) = \sigma_u^2. \tag{18}$$

III. PERFORMANCE ANALYSIS WITH INFINITE PHASE SHIFTERS

In order to show the main factors affecting the receive performance of the active IRS-aided wireless network, based on the system model constructed in Section II, we make an asymptotic performance analysis and derivation of the IRS-assisted wireless network with infinite phase shifter in this section. First, we define the average SNR at the active IRS in Section III-A, and subsequently uncover the relationship between receive SNR at user and average SNR at active IRS in Section III-B.

A. DEFINITION OF AVERAGE SNR AT ACTIVE IRS

From (1), the receive power at the n-th active IRS element is given by

$$\mathbb{E}\left\{s_i^H(n) s_i(n)\right\} = P_s L_g |g(n)|^2 + \sigma_i^2. \tag{19}$$

The average SNR at active IRS is defined as

$$\gamma_0 = \frac{P_s L_g \sum_{n=1}^{N} |g(n)|^2}{N \sigma_i^2}, \tag{20}$$

which will be shown to affect the final receive SNR at the user given a fixed transmit power constraint at BS. This is due to the fact that active IRS introduces a reflected noise unlike passive IRS. In accordance with the law of large numbers, (20) can be re-expressed as

$$\gamma_0 = \frac{P_s L_g}{\sigma_i^2} \cdot \frac{1}{N} \sum_{n=1}^{N} |g(n)|^2 \approx \frac{P_s L_g}{\sigma_i^2} \cdot \mathbb{E}\left\{|g(n)|^2\right\}, \tag{21}$$

where $g(n) \sim \mathcal{CN}(0, \alpha_g^2)$, then $|g(n)|$ obeys Rayleigh distribution and the corresponding PDF is as follows

$$f_\alpha(x) = \begin{cases} \frac{x}{\alpha^2} e^{-\frac{x^2}{2\alpha^2}}, & x \in [0, +\infty), \\ 0, & \text{otherwise}, \end{cases} \tag{22}$$

where $\alpha > 0$ stands for the Rayleigh distribution parameter.

According to [35], the k-order origin moment of corresponding (22) is

$$\mathbb{E}\left(x^k\right) = \begin{cases} \left(\frac{\pi}{2}\right)^{\frac{1}{2}} \cdot \frac{(2n)!\alpha^{2n-1}}{2^n \cdot n!}, & k = 2n-1, \\ 2^n \cdot n! \cdot \alpha^{2n}, & k = 2n. \end{cases} \tag{23}$$

From (23), we can get

$$\mathbb{E}\left\{|g(n)|^2\right\} = 2\alpha_g^2. \tag{24}$$

Substituting (24) into (21) yields

$$\gamma_0 = \frac{2 P_s L_g \alpha_g^2}{\sigma_i^2}. \tag{25}$$

B. RELATIONSHIP BETWEEN RECEIVED SNR AT USER AND AVERAGE SNR γ_0 AT IRS

If infinite phase shifters are deployed at IRS, implying that there is no phase QE at the IRS, we can obtain

$$\Delta\theta(n) = 0. \tag{26}$$

Then, (13) can be converted to

$$S_{\text{noQE}} = N \cdot \mathbb{E}(|f(n)||p(n)||g(n)|). \tag{27}$$

According to [33] (40c), the active IRS allocates the same amplification factor to all channels. From (3), the total power reflected by active IRS is P_i. let us define

$$|p(n)|_{\text{active}} = \lambda_a = \sqrt{\frac{P_i}{P_s L_g \sum_{n=1}^{N} |g(n)|^2 + N\sigma_i^2}}. \tag{28}$$

When the number of IRS elements tends to be large-scale, according to the law of large numbers, (28) can be re-expressed as

$$\begin{aligned} \lambda_a &= \sqrt{\frac{P_i}{P_s L_g N \cdot \frac{1}{N} \sum_{n=1}^{N} |g(n)|^2 + N\sigma_i^2}} \\ &\approx \sqrt{\frac{P_i}{P_s L_g N \cdot \mathbb{E}\left(|g(n)|^2\right) + N\sigma_i^2}}. \end{aligned} \tag{29}$$

Substituting (24) into (29) yields

$$\lambda_a = \sqrt{\frac{P_i}{N\left(2 P_s L_g \alpha_g^2 + \sigma_i^2\right)}}. \tag{30}$$

The signal amplified by the IRS with infinite phase shifters by substituting (28) into (27) is

$$S_{\text{noQE}} = \lambda_a N \cdot \mathbb{E}(|f(n)||g(n)|). \tag{31}$$

Since $|f(n)|$ and$|g(n)|$ are independent of each other, (31) can be further converted to

$$S_{\text{noQE}} = \lambda_a N \cdot \mathbb{E}(|f(n)|) \cdot \mathbb{E}(|g(n)|). \tag{32}$$

Since $|f(n)|$ and $|g(n)|$ follow the Rayleigh distribution with parameters α_f^2 and α_g^2, respectively, from (23), it can be obtained that

$$\begin{aligned} \mathbb{E}\left(|f(n)|\right) &= \left(\frac{\pi}{2}\right)^{\frac{1}{2}} \alpha_f, \\ \mathbb{E}\left(|g(n)|\right) &= \left(\frac{\pi}{2}\right)^{\frac{1}{2}} \alpha_g. \end{aligned} \tag{33}$$

Substituting (33) into (32) yields

$$S_{\text{noQE}} = \frac{\pi}{2} \lambda_a N \alpha_f \alpha_g. \tag{34}$$

Similarly, noise amplified by the IRS with infinite phase shifters by substituting (28) into (17) is

$$\mathbb{E}\left(N_1^H N_1\right)_{\text{noQE}} = \lambda_a^2 N L_f \sigma_i^2 \mathbb{E}\left(|f(n)|^2\right). \tag{35}$$

From (23), the second-order origin moment of $|f(n)|$ is

$$\mathbb{E}\left(|f(n)|^2\right) = 2\alpha_f^2, \tag{36}$$

then, we have

$$\mathbb{E}\left(N_1^H N_1\right)_{\text{noQE}} = 2\lambda_{\text{a}}^2 N L_f \alpha_f^2 \sigma_i^2. \tag{37}$$

From (23), it can be obtained that

$$\mathbb{E}(|h|) = \left(\frac{\pi}{2}\right)^{\frac{1}{2}} \alpha_h. \tag{38}$$

Substituting (34), (35) and (38) into (12) yields the expression of the SNR without performance loss

$$\gamma_u^{\text{noQE}} = \frac{P_s\left(\left(\frac{\pi}{2}\right)^{\frac{1}{2}}\sqrt{L_h}\alpha_h + \frac{\pi}{2}\lambda_{\text{a}} N\sqrt{L_f L_g}\alpha_f\alpha_g\right)^2}{2\lambda_{\text{a}}^2 N L_f \alpha_f^2\sigma_i^2 + \sigma_u^2}. \tag{39}$$

In order to conveniently analyze the key indicators affecting the received SNR, substituting (30) into (39) yields (40), shown at the bottom of the page.

To facilitate the subsequent derivation, (40) can be rewritten as

$$\gamma_u^{\text{noQE}} = \frac{A_1 P_s^2 + A_2 P_s \sigma_i^2 + A_3 N P_s P_i + A_4 P_s\sqrt{N P_i\left(A_5 P_s + A_6\sigma_i^2\right)}}{B_1 P_i \sigma_i^2 + B_2 P_s \sigma_u^2 + \sigma_u^2\sigma_i^2}, \tag{41}$$

where

$$\begin{aligned}
A_1 &= \pi L_h L_g \alpha_h^2 \alpha_g^2,\\
A_2 &= \frac{\pi}{2} L_h \alpha_h^2,\\
A_3 &= \frac{\pi^2}{4} L_f L_g \alpha_f^2 \alpha_g^2,\\
A_4 &= \pi\left(\frac{\pi}{2}\right)^{\frac{1}{2}} \alpha_h \alpha_f \alpha_g,\\
A_5 &= 2 L_h L_g^2 L_f \alpha_g^2,\\
A_6 &= L_f L_g L_h,\\
B_1 &= 2 L_f \alpha_f^2,\\
B_2 &= 2 L_g \alpha_g^2.
\end{aligned} \tag{42}$$

Assuming that the power $\bar{P}_i$ reflected by each IRS element is limited, when the number of IRS elements N tends to large-scale, namely when $P_i = N\bar{P}_i \rightarrow +\infty$, then

$$\gamma_u^{\text{noQE}} \rightarrow \frac{A_3 N P_s}{B_1\sigma_i^2} = \frac{\pi^2 N P_s L_g \alpha_g^2}{8\sigma_i^2}. \tag{43}$$

To further reveal the relationship between received SNR γ_m^{noQE} at user and average SNR γ_0 at active IRS, we substitute (25) into (43) to obtain

$$\gamma_u^{\text{noQE}} \rightarrow \frac{\pi^2}{16}\cdot N \cdot \gamma_0. \tag{44}$$

From the above expression, the receive SNR at user is shown to be a linear increasing function of a product of N and γ_0 given fixed P_s and $\bar{P}_i$. In other words, the relationship reveals: (a) Unlike passive IRS with array gain N^2, the array gain of active IRS is N and increasing the number of IRS elements may linearly improve the SNR performance at user for a fixed γ_0; (b) Fixing N, the receive SNR at the user is shown to be a linear increasing function of γ_0, that is, reducing the noise level (noise variance σ_i^2) or increasing the transmit power P_s at BS will linearly increase the receive SNR at user. This is mainly due to the noise at active IRS.

In general, the asymptotic SNR at the user decreases as the noise power at the active IRS increases. It is also important to consider that when $\sigma_i^2 \rightarrow +\infty$, we have

$$\gamma_u^{\text{noQE}} \rightarrow \frac{A_2 P_s}{B_1 P_i + \sigma_u^2}. \tag{45}$$

In this case, the asymptotic received SNR at user decreases as P_i increases when the BS transmit power P_s is fixed.

On the contrary, when $\sigma_i^2 \rightarrow 0$,

$$\gamma_u^{\text{noQE}} \rightarrow \frac{A_1 P_s^2 + A_3 N P_s P_i + A_4 P_s\sqrt{A_5 N P_s P_i}}{B_2 P_s \sigma_u^2}. \tag{46}$$

At this case, the asymptotic received SNR at user increases as P_i increases when the BS transmit power P_s is fixed.

In order to obtain the optimal IRS reflect power P_i^{opt} when the BS transmit power is fixed, (41) can be expressed as

$$\gamma_u^{\text{noQE}} = \frac{C_1 + C_2 P_i + C_3\sqrt{P_i}}{C_4 P_i + C_5}, \tag{47}$$

where

$$\begin{aligned}
C_1 &= A_1 P_s^2 + A_2 P_s \sigma_i^2,\\
C_2 &= A_3 N P_s,\\
C_3 &= A_4 P_s\sqrt{N\left(A_5 P_s + A_6\sigma_i^2\right)},\\
C_4 &= B_1\sigma_i^2,\\
C_5 &= B_2 P_s\sigma_u^2 + \sigma_u^2\sigma_i^2.
\end{aligned} \tag{48}$$

The derivative of (47) with respect to P_i yields

$$\frac{\mathrm{d}\gamma_u^{\text{noQE}}}{\mathrm{d}P_i} = \frac{a P_i^{\frac{1}{2}} + b P_i^{-\frac{1}{2}} + c}{2\left(C_4 P_i + C_5\right)^2}, \tag{49}$$

$$\gamma_u^{\text{noQE}} = \frac{\frac{\pi}{2}\alpha_h^2 P_s L_h\left(2P_s L_g\alpha_g^2 + \sigma_i^2\right) + \frac{\pi^2}{4} N P_s P_i L_f L_g \alpha_f^2\alpha_g^2 + \pi\left(\frac{\pi}{2}\right)^{\frac{1}{2}} P_s \alpha_h\alpha_f\alpha_g\sqrt{N P_i L_f L_g L_h\left(2P_s L_g \alpha_g^2 + \sigma_i^2\right)}}{2P_i L_f \alpha_f^2\sigma_i^2 + \sigma_u^2\left(2P_s L_g\alpha_g^2 + \sigma_i^2\right)}. \tag{40}$$

where

$$a = C_2C_4 - 2C_2C_5,\quad b = C_2C_5,\quad c = 2\big(C_3C_5 - C_1C_4\big). \tag{50}$$

Letting (49) equal 0, we get

$$P_i^{\mathrm{opt}} = \arg\max_{P_i \in S} (47), \tag{51}$$

where $S = \{P_i^{\mathrm{opt1}}, P_i^{\mathrm{opt2}}\}$ with

$$P_i^{\mathrm{opt1}} = \frac{-2ab + c^2 + \sqrt{-4abc^2 + c^4}}{2a^2}, \tag{52}$$

and

$$P_i^{\mathrm{opt2}} = \frac{-2ab + c^2 - \sqrt{-4abc^2 + c^4}}{2a^2}. \tag{53}$$

IV. PERFORMANCE LOSS DERIVATION AND ANALYSIS WITH FINITE PHASE SHIFTERS

The high circuit cost of the infinite phase shifter makes it difficult to implement in practice. It is more relevant to study IRS-aided wireless networks with finite phase shifters. We will conduct performance impact analysis on SNR, AR, and BER of a large-scale active IRS-aided wireless network in this section.

If the active IRS is equipped with finite phase shifters, this will inevitably result in phase QE, i.e.,

$$\Delta\theta(n) \neq 0, \tag{54}$$

in this case, the signal amplified by the active IRS by substituting (28) into (13) is

$$S_{\mathrm{active}} = \lambda_{\mathrm{a}} N \cdot \mathbb{E}\Big(|f(n)||g(n)|e^{j\Delta\theta(n)}\Big). \tag{55}$$

It is worth noting that $|f(n)|$, $|g(n)|$, and $\Delta\theta(n)$ are independent of each other. (55) can be further converted to

$$S_{\mathrm{active}} = \lambda_{\mathrm{a}} N \cdot \mathbb{E}(|f(n)|) \cdot \mathbb{E}(|g(n)|) \cdot \mathbb{E}\Big(e^{j\Delta\theta(n)}\Big). \tag{56}$$

The phase QE $\Delta\theta(n)$ follows uniform distribution, from (9), we can have

$$\begin{aligned}\mathbb{E}\Big(e^{j\Delta\theta(n)}\Big) &= \mathbb{E}\Big(\cos\Delta\theta(n)\Big) + j\mathbb{E}\Big(\sin\Delta\theta(n)\Big)\\ &= \int_{-\Delta x}^{+\Delta x} \cos(\Delta\theta(n)) f(\Delta\theta(n)) d(\Delta\theta(n)) + 0\\ &= \frac{1}{2\Delta x}\int_{-\Delta x}^{+\Delta x} \cos(\Delta\theta(n)) d(\Delta\theta(n))\\ &= \frac{\sin(\Delta x)}{\Delta x}\\ &= \mathrm{sinc}\Big(\frac{\pi}{2^k}\Big). \end{aligned} \tag{57}$$

Substituting (33) and (57) into (56) yields

$$S_{\mathrm{active}} = \frac{\pi}{2}\lambda_{\mathrm{a}} N \alpha_f \alpha_g\, \mathrm{sinc}\Big(\frac{\pi}{2^k}\Big). \tag{58}$$

Whether finite phase shifters or infinite phase shifters are deployed at the IRS, the noise amplified by the active IRS is equal. In this case, one obtains

$$\mathbb{E}\Big(N_1^H N_1\Big)_{\mathrm{active}} = \mathbb{E}\Big(N_1^H N_1\Big)_{\mathrm{noQE}} = 2\lambda_{\mathrm{a}}^2 N L_f \alpha_f^2 \sigma_i^2. \tag{59}$$

Substituting (58), (59) and (38) into (12) yields that the expression of the SNR with performance loss is

$$\widehat{\gamma}_u^{\mathrm{active}} = \frac{P_s\Big(\big(\frac{\pi}{2}\big)^{\frac{1}{2}}\sqrt{L_h}\alpha_h + \frac{\pi}{2}\lambda_{\mathrm{a}} N\sqrt{L_f L_g}\alpha_f\alpha_g\,\mathrm{sinc}\big(\frac{\pi}{2^k}\big)\Big)^2}{2\lambda_{\mathrm{a}}^2 N L_f \alpha_f^2\sigma_i^2 + \sigma_u^2}. \tag{60}$$

The asymptotic performance analysis of large-scale active IRS-aided wireless networks in Rayleigh fading channels can be extended to LoS or Rician fading channels. When the BS→IRS and IRS→User channels are the Rician channels, the SNR at user with performance loss is shown in (61), shown at the bottom of the page, where

$$\lambda_{\mathrm{a}}^{\mathrm{Rician}} = \sqrt{\frac{P_i(K+1)}{NKP_sL_g + 2NP_sL_g\alpha_g^2 + N(K+1)\sigma_i^2}}, \tag{62}$$

wherein K is the Rician factor. From (61), it can be seen that when $K = 0$, (61) can be simplified to be consistent with the SNR (60) at user with performance loss in Rayleigh fading channel. Correspondingly, when $K = 0$, $\lambda_{\mathrm{a}}^{\mathrm{Rician}}$ in (62) degenerates to be consistent with λ_{a} in (30). Similarly, when $K = \infty$, the received SNR at user in LoS fading channel can be derived.

Substituting (30) into (60) yields

$$\widehat{\gamma}_u^{\mathrm{active}} = \frac{P_s\Big(D_a + \frac{\pi}{2}N\sqrt{P_iL_fL_g}\alpha_f\alpha_g\,\mathrm{sinc}\big(\frac{\pi}{2^k}\big)\Big)^2}{2NP_iL_f\alpha_f^2\sigma_i^2 + N\sigma_u^2\Big(2P_sL_g\alpha_g^2 + \sigma_i^2\Big)}, \tag{63}$$

where $D_a = (\frac{\pi}{2})^{\frac{1}{2}}\alpha_h\sqrt{NL_h(2P_sL_g\alpha_g^2 + \sigma_i^2)}$.

To simplify (63), using the Taylor series expansion [36] to approximate $\cos(\Delta\theta(n))$ can be obtained

$$\cos\big(\Delta\theta(n)\big) \approx 1 - \frac{\Delta\theta^2(n)}{2}, \tag{64}$$

$$\gamma_u^{\mathrm{Rician}} = \frac{(K+1)P_s\Big[\sqrt{L_h}|h| + \frac{\lambda_{\mathrm{a}}^{\mathrm{Rician}} N\sqrt{L_fL_g}\,\mathrm{sinc}\big(\frac{\pi}{2^k}\big)}{K+1}\Big(K + \frac{\pi}{2}\alpha_f\alpha_g + \sqrt{K}\big(\frac{\pi}{2}\big)^{\frac{1}{2}}\alpha_f + \sqrt{K}\big(\frac{\pi}{2}\big)^{\frac{1}{2}}\alpha_g\Big)\Big]^2}{NL_f\big[\lambda_{\mathrm{a}}^{\mathrm{Rician}}\big]^2\sigma_i^2\Big(K + \sqrt{2\pi K}\alpha_f + 2\alpha_f^2\Big) + (K+1)\sigma_u^2}, \tag{61}$$

then (57) can be rewritten as

$$\mathbb{E}\Big\{\cos\left(\Delta\theta(n)\right)\Big\} = \frac{1}{2\Delta x}\int_{-\Delta x}^{\Delta x}\cos(\Delta\theta(n))d(\Delta\theta(n)) \approx \frac{1}{2\Delta x}\int_{-\Delta x}^{\Delta x}\left(1-\frac{\Delta\theta^2(n)}{2}\right)d(\Delta\theta(n)) = 1-\frac{1}{6}(\Delta x)^2 = 1-\frac{1}{6}\left(\frac{\pi}{2^k}\right)^2. \quad (65)$$

The receive SNR with approximate performance loss is

$$\widetilde{\gamma}_u^{\text{active}} = \frac{P_s\left(D_a+\frac{\pi}{2}N\sqrt{P_iL_fL_g}\alpha_f\alpha_g\left(1-\frac{1}{6}\left(\frac{\pi}{2^k}\right)^2\right)\right)^2}{2NP_iL_f\alpha_f^2\sigma_i^2+N\sigma_u^2\left(2P_sL_g\alpha_g^2+\sigma_i^2\right)}. \quad (66)$$

When $k\to+\infty$, the receive SNR at user with no PL is

$$\gamma_u^{\text{active}} = \frac{P_s\left(D_a+\frac{\pi}{2}N\sqrt{P_iL_fL_g}\alpha_f\alpha_g\right)^2}{2NP_iL_f\alpha_f^2\sigma_i^2+N\sigma_u^2\left(2P_sL_g\alpha_g^2+\sigma_i^2\right)}. \quad (67)$$

The performance loss of the receive SNR at user can be formulated as follows

$$\widehat{L}_u^{\text{active}} = \frac{\gamma_u^{\text{active}}}{\widehat{\gamma}_u^{\text{active}}} = \frac{\left(D_a+\frac{\pi}{2}N\sqrt{P_iL_fL_g}\alpha_f\alpha_g\right)^2}{\left(D_a+\frac{\pi}{2}N\sqrt{P_iL_fL_g}\alpha_f\alpha_g\,\text{sinc}\left(\frac{\pi}{2^k}\right)\right)^2} = \left(1+\frac{\frac{\pi}{2}\sqrt{P_iL_fL_g}\alpha_f\alpha_g\left(1-\text{sinc}\left(\frac{\pi}{2^k}\right)\right)}{\frac{1}{N}D_a+\frac{\pi}{2}\sqrt{P_iL_fL_g}\alpha_f\alpha_g\,\text{sinc}\left(\frac{\pi}{2^k}\right)}\right)^2. \quad (68)$$

The approximate performance loss of SNR at user is

$$\widetilde{L}_u^{\text{active}} = \frac{\gamma_u^{\text{active}}}{\widetilde{\gamma}_u^{\text{active}}} = \frac{\left(D_a+\frac{\pi}{2}N\sqrt{P_iL_fL_g}\alpha_f\alpha_g\right)^2}{\left(D_a+\frac{\pi}{2}N\sqrt{P_iL_fL_g}\alpha_f\alpha_g\left(1-\frac{1}{6}\left(\frac{\pi}{2^k}\right)^2\right)\right)^2} = \left(1+\frac{\frac{\pi}{12}\sqrt{P_iL_fL_g}\alpha_f\alpha_g\left(\frac{\pi}{2^k}\right)^2}{\frac{1}{N}D_a+\frac{\pi}{2}\sqrt{P_iL_fL_g}\alpha_f\alpha_g\left(1-\frac{1}{6}\left(\frac{\pi}{2^k}\right)^2\right)}\right)^2. \quad (69)$$

Observing (68) and (69), we find that $\widehat{L}_u^{\text{active}}$ and $\widetilde{L}_u^{\text{active}}$ gradually decrease as k increases, while they gradually increase with increases N.

The AR at user with PL, APL, and no PL are given by

$$\widehat{R}_u^{\text{active}} = \log_2\left(1+\widehat{\gamma}_u^{\text{active}}\right) = \log_2\left(1+\frac{P_s\left(D_a+\frac{\pi}{2}N\sqrt{P_iL_fL_g}\alpha_f\alpha_g\,\text{sinc}\left(\frac{\pi}{2^k}\right)\right)^2}{2NP_iL_f\alpha_f^2\sigma_i^2+N\sigma_u^2\left(2P_sL_g\alpha_g^2+\sigma_i^2\right)}\right), \quad (70)$$

$$\widetilde{R}_u^{\text{active}} = \log_2\left(1+\widetilde{\gamma}_u^{\text{active}}\right) = \log_2\left(1+\frac{P_s\left(D_a+\frac{\pi}{2}N\sqrt{P_iL_fL_g}\alpha_f\alpha_g\left(1-\frac{1}{6}\left(\frac{\pi}{2^k}\right)^2\right)\right)^2}{2NP_iL_f\alpha_f^2\sigma_i^2+N\sigma_u^2\left(2P_sL_g\alpha_g^2+\sigma_i^2\right)}\right), \quad (71)$$

and

$$R_u^{\text{active}} = \log_2\left(1+\gamma_u^{\text{active}}\right) = \log_2\left(1+\frac{P_s\left(D_a+\frac{\pi}{2}N\sqrt{P_iL_fL_g}\alpha_f\alpha_g\right)^2}{2NP_iL_f\alpha_f^2\sigma_i^2+N\sigma_u^2\left(2P_sL_g\alpha_g^2+\sigma_i^2\right)}\right), \quad (72)$$

respectively.

In accordance with [34], the expression of BER is

$$\text{BER}(z) \approx \lambda Q\left(\sqrt{\mu z}\right), \quad (73)$$

where λ represents the number of nearest neighbors of the constellation at the minimum distance, which depends on the modulation type. z denotes the SNR of each symbol, and μ is a constant, which related to the average symbol energy at the minimum distance. $Q(z)$ stands for the probability that a Gaussian random variable x with mean zero and variance one exceeds the value z, it can be expressed as follows

$$Q(z) = \int_z^{+\infty}\frac{1}{\sqrt{2\pi}}e^{\frac{-x^2}{2}}dx. \quad (74)$$

Assuming that the modulation scheme adopts quadrature phase shift keying (QPSK), in accordance with (67), (63) and (66), the BERs without PL, PL and APL are given by

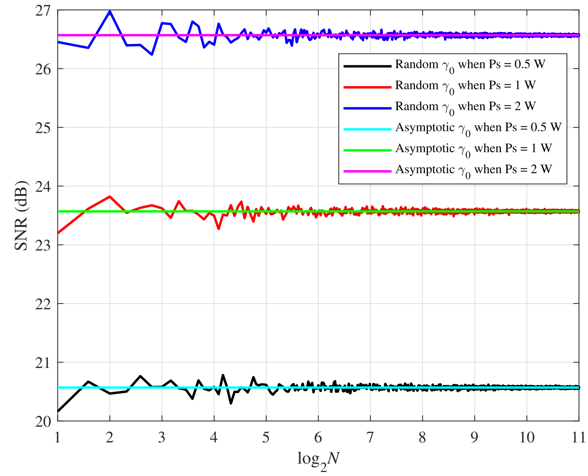
$$\text{BER}_u^{\text{active}} \approx Q\left(\sqrt{\frac{P_s\left(D_a+\frac{\pi}{2}N\sqrt{P_iL_fL_g}\alpha_f\alpha_g\right)^2}{2NP_iL_f\alpha_f^2\sigma_i^2+N\sigma_u^2\left(2P_sL_g\alpha_g^2+\sigma_i^2\right)}}\right), \quad (75)$$

$$\widehat{\text{BER}}_u^{\text{active}} \approx Q\left(\sqrt{\frac{P_s\left(D_a+\frac{\pi}{2}N\sqrt{P_iL_fL_g}\alpha_f\alpha_g\,\text{sinc}\left(\frac{\pi}{2^k}\right)\right)^2}{2NP_iL_f\alpha_f^2\sigma_i^2+N\sigma_u^2\left(2P_sL_g\alpha_g^2+\sigma_i^2\right)}}\right), \quad (76)$$

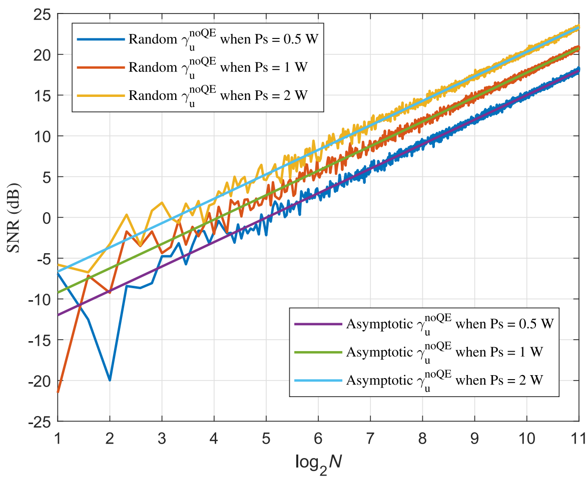
and

$$\widetilde{\text{BER}}_u^{\text{active}} \approx Q\left(\sqrt{\frac{P_s\left(D_a+\frac{\pi}{2}N\sqrt{P_iL_fL_g}\alpha_f\alpha_g\left(1-\frac{1}{6}\left(\frac{\pi}{2^k}\right)^2\right)\right)^2}{2NP_iL_f\alpha_f^2\sigma_i^2+N\sigma_u^2\left(2P_sL_g\alpha_g^2+\sigma_i^2\right)}}\right), \quad (77)$$

respectively.



(a) SNR at active IRS



(b) SNR at user

FIGURE 2. SNR versus the number N of IRS elements.

V. SIMULATION RESULTS AND DISCUSSION

In this section, due to the introduction of IRS with finite phase shifters, phase QE will be present. Below, the impact of discrete phase shifter IRS on SNR, AR, and BER will be simulated and analyzed. The path loss at distance d is modeled as $L(d) = \text{PL}_0 - 10a\log_{10}\frac{d}{d_0}$, where $\text{PL}_0 = -30$ dB represents the path loss reference distance $d_0 = 1$m, and a is the path loss exponent. The path loss exponents of BS→IRS, IRS→User, and BS→User channels are respectively chosen as 2.7, 2.7, and 3. Simulation parameters are set as follows: BS, user, and active IRS are located at (0 m, 0 m), (200 m, 0 m), and (50 m, 30 m), respectively. The Rayleigh distribution parameter is set to $\alpha_h^2 = \alpha_f^2 = \alpha_g^2 = \frac{1}{2}$.

To assess the value of N that the asymptotic SNR can approximate the actual SNR well in Rayleigh fading channel, the actual SNR and its asymptotic simple expression at active IRS are plotted in Figs. 2 (a), while the actual SNR and its asymptotic expression at user are plotted in Figs. 2 (b). From Figs. 2, it can be seen that the asymptotic SNR can be approximately equal to the actual SNR in Rayleigh fading channel when N is greater than or equal to 64. In addition, Fig. 2 (b) confirms this conclusion that the user receive SNR increases linearly with increasing N as derived from (44).

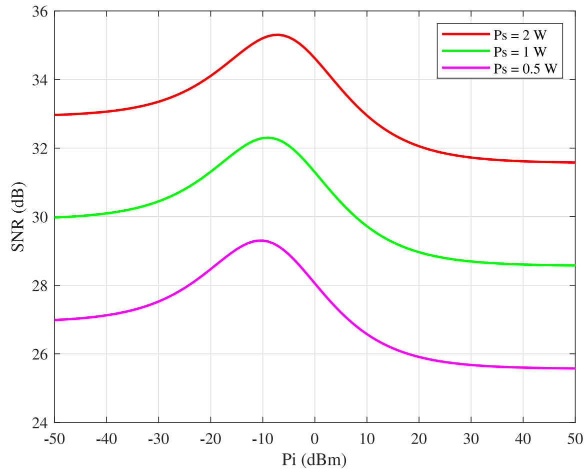


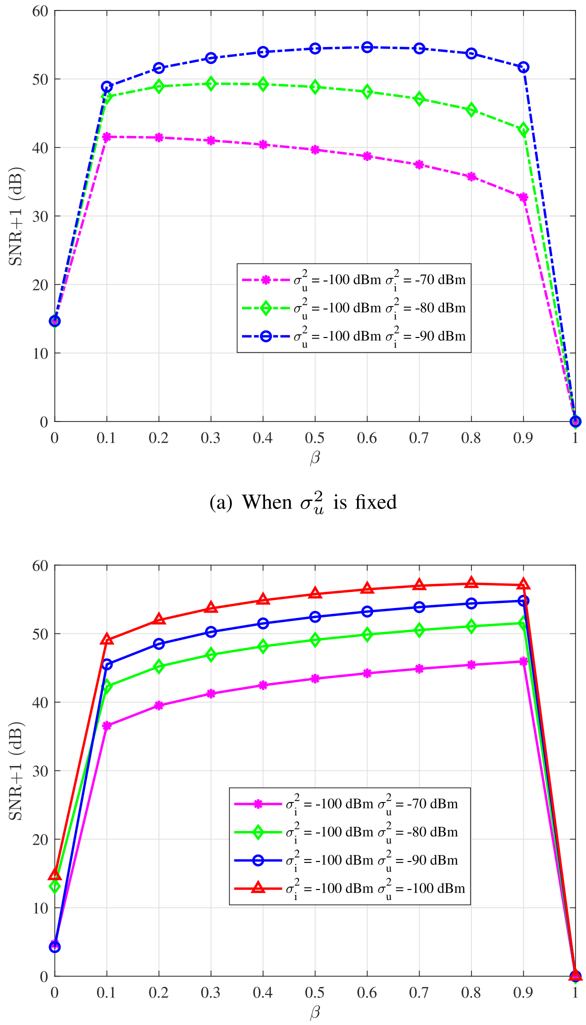
FIGURE 3. SNR versus reflect power at active IRS.

TABLE 1. Rate gains of optimal β_{opt} over EPA with $\sigma_i^2 = -100$ dBm.

σ_i^2(dBm)	σ_u^2(dBm)	β_{opt}	Rate gains (bit)
-100	-70	0.9	0.83
	-80	0.9	0.82
	-90	0.9	0.79
	-100	0.8	0.51

Fig. 3 plots the curves of the asymptotic received SNR versus P_i. From Fig. 3, it is seen: as $P_i = N\bar{P}_i$ varies from -60 dBm to 50 dBm, SNR at user firstly increases gradually, them reach the peak value, then decreases monotonically, and finally converges to a SNR floor. Observing three typical scenarios in this figure, we can conclude that the asymptotic received SNR may be viewed as a quasi-concave function of P_i. This figure also tells us that there is an optimal reflect power at IRS to achieve a maximum receive SNR at user given a fixed transmit power at BS. At this point, further increasing the value of P_i will harvest no SNR gain. This tendency is mainly due to the fact that the active IRS amplifies signal and at the same time noise.

Inspired by [37], under the total power sum constraint, i.e., $P_i + P_s = P_T$ with P_T being fixed. In order to evaluate the impact of PA on SNR performance, a PA factor β is defined as follows: $\beta = P_i/P_T$, and $P_s = (1-\beta)P_T$, where $0 \leq \beta \leq 1$. Fig. 4 (a) illustrates the curves of SNR at user versus β for three typical values of σ_i^2 and σ_u^2. In order to fully plot the received SNR at user versus β from 0 to 1, the vertical axis is set to "SNR+1". From this figure, it is seen that the SNR at user is a concave function of PA factor β. As β ranges from 0 to 1, there exists an optimal PA strategy. Table 1 lists the rate gains of optimal β_{opt} over classical EPA. When $\sigma_i^2 = -100$ dBm, $\sigma_u^2 = -70$ dBm, and $\beta_{\text{opt}} = 0.9$, the rate gain is 0.83 bit. Similarly, Fig. 4 (b) and Table 2 depict that when $\sigma_i^2 = -70$ dBm, $\sigma_u^2 = -100$ dBm, and $\beta_{\text{opt}} = 0.1$, the rate gain is 0.62 bit.



(a) When σ_u^2 is fixed

(b) When σ_i^2 is fixed

FIGURE 4. **SNR versus power allocation factor.**

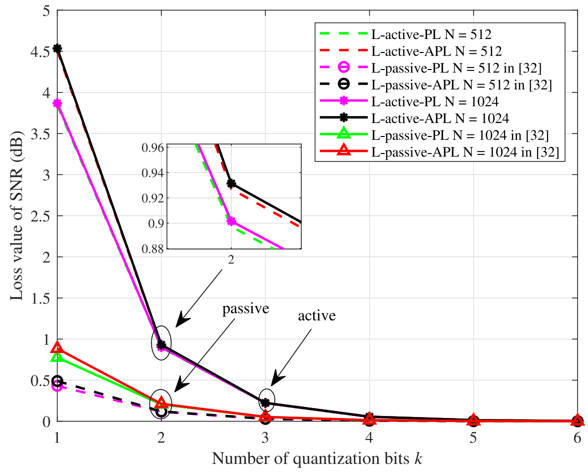


FIGURE 5. **Loss of SNR versus quantization bit numbers k.**

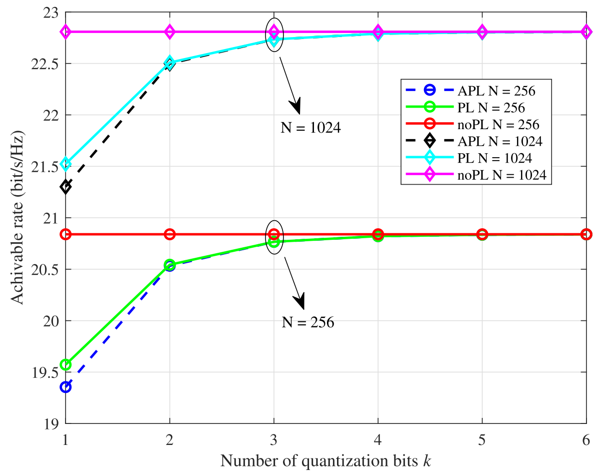


FIGURE 6. **AR versus quantization bit numbers k.**

TABLE 2. **Rate gains of optimal β_{opt} over EPA with $\sigma_u^2 = -100$ dBm.**

σ_u^2(dBm)	σ_i^2(dBm)	β_{opt}	Rate gains (bit)
-100	-70	0.1	0.62
	-80	0.3	0.16
	-90	0.6	0.06

Fig. 5 illustrates the loss of SNR versus quantization bit numbers k with k from 1 to 6. It can be seen that both SNR PL and APL decrease with the increase of k, while it increases with N increases. When k is greater than or equal to 3, the SNR loss of active IRS-aided wireless network is less than 0.22 dB when $N = 1024$. This indicates that for active IRS, about 3 bits is sufficient to achieve trivial performance loss.

Fig. 6 describes the AR versus k with k ranging from 1 to 6. It can be observed that the AR performance loss decreases with k increases, and increases with the increase of N. In addition, when $N = 1024$, the AR performance loss achieved by 3 quantization bits is less than 0.08 bits/Hz in the case of PL and APL compared to without PL.

Fig. 7 depicts the BER versus the number k of quantization bits varying from 1 to 6, where SNR is equal to 3 dB ($N = 256$) and 9 dB ($N = 1024$), respectively. From this figure we can find that, when k reaches 3 in the case of an active IRS, the BER performance of PL and APL is almost the same as that without PL, which means that using a discrete phase shifter with $k = 3$ in practice to achieve a negligible performance loss is feasible.

VI. CONCLUSION

In this paper, the asymptotic performance of large-scale active IRS-aided wireless communication networks has been investigated. The key factor γ_0 that affects the user receive SNR was defined. The simple asymptotic expression for γ_0 was derived when the number N of IRS elements tended to medium-scale and large-scale. As N reached large-scale, the asymptotic SNR at user was verified to be a linearly increasing function of the product of γ_0 and N. Subsequently, an optimal IRS reflect power exists for a fixed BS transmit

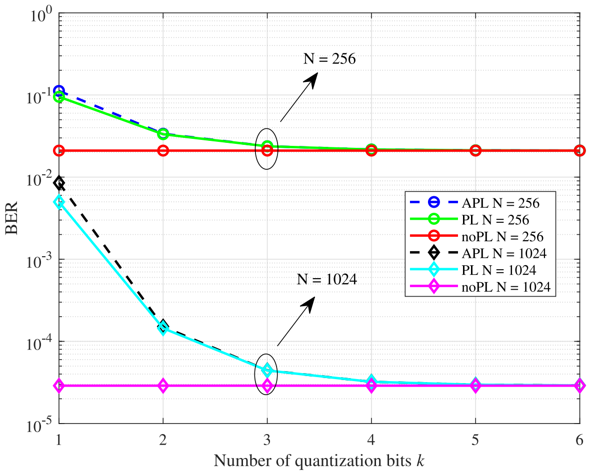


FIGURE 7. **BER versus quantization bit numbers *k*.**

power. At this point, more IRS reflect power will reduce SNR performance. Furthermore, an optimal PA strategy was obtained with the sum constraint of BS transmit power and IRS reflected power, and the rate gain of the optimal PA factor over EPA is up to 0.83 bit. To analyze the performance loss due to the finite phase shifter, we derived closed-form expressions for PL and APL for the user's asymptotic SNR, AR, and BER. Moreover, the expression for the approximate performance losses for SNR, AR, and BER were given based on the Taylor series expansions. Numerical simulations showed that when k is greater than or equal to 3, the losses of active asymptotic SNR and AR are less than 0.22 dB and 0.08 bits/s/Hz, respectively. This means that for active IRS, a 3-bit phase shifter is sufficient to achieve a trivial rate performance loss.

REFERENCES

[1] R. Liu, R. Yu-Ngok Li, M. Di Renzo, and L. Hanzo, "A vision and an evolutionary framework for 6G: Scenarios, capabilities and enablers," May 2023, *arXiv:abs/2305.13887*.

[2] M. Di Renzo et al., "Smart radio environments empowered by reconfigurable intelligent surfaces: How it works, state of research, and the road ahead," *IEEE J. Sel. Areas Commun.*, vol. 38, no. 11, pp. 2450–2525, Nov. 2020.

[3] E. Basar, M. Di Renzo, J. De Rosny, M. Debbah, M. Alouini, and R. Zhang, "Wireless communications through reconfigurable intelligent surfaces," *IEEE Access*, vol. 7, pp. 116753–116773, 2019.

[4] Q. Wu and R. Zhang, "Intelligent reflecting surface enhanced wireless network via joint active and passive beamforming," *IEEE Trans. Wireless Commun.*, vol. 18, no. 11, pp. 5394–5409, Nov. 2019.

[5] C. Huang, A. Zappone, G. C. Alexandropoulos, M. Debbah, and C. Yuen, "Reconfigurable intelligent surfaces for energy efficiency in wireless communication," *IEEE Trans. Wireless Commun.*, vol. 18, no. 8, pp. 4157–4170, Aug. 2019.

[6] Q. Wu and R. Zhang, "Towards smart and reconfigurable environment: Intelligent reflecting surface aided wireless network," *IEEE Commun Mag*, vol. 58, no. 1, pp. 106–112, Jan. 2020.

[7] R. Liu, J. Dou, P. Li, J. Wu, and Y. Cui, "Simulation and field trial results of reconfigurable intelligent surfaces in 5G networks," *IEEE Access*, vol. 10, pp. 122786–122795, 2022.

[8] Z. Sun, X. Wang, S. Feng, X. Guan, F. Shu, and J. Wang, "Pilot optimization and channel estimation for two-way relaying network aided by IRS with finite discrete phase shifters," *IEEE Trans. Veh. Technol.*, vol. 72, no. 4, pp. 5502–5507, Apr. 2023.

[9] R. Liu, Q. Wu, M. Di Renzo, and Y. Yuan, "A path to smart radio environments: An industrial viewpoint on reconfigurable intelligent surfaces," *IEEE Wireless Commun.*, vol. 29, no. 1, pp. 202–208, Feb. 2022.

[10] W. Shi, Q. Wu, F. Xiao, F. Shu, and J. Wang, "Secrecy throughput maximization for IRS-aided MIMO wireless powered communication networks," *IEEE Trans. Commun.*, vol. 70, no. 11, pp. 7520–7535, Nov. 2022.

[11] F. Shu et al., "Enhanced secrecy rate maximization for directional modulation networks via IRS," *IEEE Trans. Commun.*, vol. 69, no. 12, pp. 8388–8401, Dec. 2021.

[12] R. Dong, S. Jiang, X. Hua, Y. Teng, F. Shu, and J. Wang, "Low-complexity joint phase adjustment and receive beamforming for directional modulation networks via IRS," *IEEE Open J. Commun. Soc.*, vol. 3, pp. 1234–1243, Aug. 2022.

[13] Y. Teng et al., "Low-complexity and high-performance receive beamforming for secure directional modulation networks against an eavesdropping-enabled full-duplex attacker," *Sci. China Inf. Sci.*, vol. 65, Dec. 2021, Art. no. 119302.

[14] X. Wang et al., "Beamforming design for IRS-aided decode-and-forward relay wireless network," *IEEE Trans. Green Commun. Netw.*, vol. 6, no. 1, pp. 198–207, Mar. 2022.

[15] X. Zhou, S. Yan, Q. Wu, F. Shu, and D. W. Kwan Ng, "Intelligent reflecting surface (IRS)-aided covert wireless communications with delay constraint," *IEEE Trans. Wireless Commun.*, vol. 21, no. 1, pp. 532–547, Jan. 2022.

[16] F. Shu et al., "Beamforming and transmit power design for intelligent reconfigurable surface-aided secure spatial modulation," *IEEE J. Sel. Topics Signal Process.*, vol. 16, no. 5, pp. 933–949, Aug. 2022.

[17] R. Long, Y. C. Liang, Y. Pei, and E. G. Larsson, "Active reconfigurable intelligent surface-aided wireless communications," *IEEE Trans. Wireless Commun.*, vol. 20, no. 8, pp. 4962–4975, Aug. 2021.

[18] M. Ahsan, S. Jamil, M. T. Ejaz, and M. S. Abbas, "Energy efficiency maximization in RIS-assisted wireless networks," in *Proc. Int. Conf. Comput. Electron. Elect. Eng. (ICE Cube)*, Quetta, Pakistan, 2021, pp. 1–6.

[19] L. Yang, Y. Yang, M. Hasna, and M. Alouini. "Coverage, probability of SNR gain, and DOR analysis of RIS-aided communication systems," *IEEE Wireless Commun. Lett.*, vol. 9, no. 8, pp. 1268–1272, Aug. 2020.

[20] C. You and R. Zhang, "Wireless communication aided by intelligent reflecting surface: Active or passive?" *IEEE Wireless Commun. Lett.*, vol. 10, no. 12, pp. 2659–2663, Dec. 2021.

[21] M. Di Renzo, F. Danufane, and S. Tretyakov, "Communication models for reconfigurable intelligent surfaces: From surface electromagnetics to wireless networks optimization," *Proc. IEEE*, vol. 110, no. 9, pp. 1164–1209, Sep. 2022.

[22] S. Khaledian, F. Farzami, H. Soury, B. Smida, and D. Erricolo, "Active two-way backscatter modulation: An analytical study," *IEEE Trans. Wireless Commun.*, vol. 18, no. 3, pp. 1874–1886, Mar. 2019.

[23] W. Mei and R. Zhang, "Performance analysis and user association optimization for wireless network aided by multiple intelligent reflecting surface," *IEEE Trans. Commun.*, vol. 69, no. 9, pp. 6296–6312, Sep. 2021.

[24] H. Shen, T. Ding, W. Xu, and C. Zhao, "Beamforming design with fast convergence for IRS-aided full-duplex communication," *IEEE Commun. Lett.*, vol. 24, no. 12, pp. 2849–2853, Dec. 2020.

[25] Z. Abdullah, G. Chen, S. Lambotharan, and J. Chambers. "A hybrid relay and intelligent reflecting surface network and its ergodic performance analysis," *IEEE Wireless Commun. Lett*, vol. 9, no. 10, pp. 1653–1657, Oct. 2020.

[26] S. Atapattu, R. Fan, P. Dharmawansa, G. Wang, J. Evans, and T. A. Tsiftsis, "Reconfigurable intelligent surface assisted two–way communications: Performance analysis and optimization," *IEEE Trans. Commun*, vol. 68, no. 10, pp. 6552–6567, Oct. 2020.

[27] H. Jiang, B. Xiong, H. Zhang, and E. Basar, "Physics-based 3D end-to-end modeling for double-RIs assisted non-stationary UAV-to-ground communication channels," *IEEE Trans. Commun.*, vol. 71, no. 7, pp. 4247–4261, Jul. 2023.

[28] B. Di, H. Zhang, L. Song, Y. Li, Z. Han, and H. Vincent Poor, "Hybrid beamforming for reconfigurable intelligent surface based multi-user communications: Achievable rates with limited discrete phase shifts," *IEEE J. Sel. Areas Commun.*, vol. 38, no. 8, pp. 1809–1822, Aug. 2020.

[29] Q. Wu and R. Zhang, "Beamforming optimization for intelligent reflecting surface with discrete phase shifts," in *Proc. IEEE ICASSP*, May 2019, pp. 7830–7833.

[30] C. You, B. Zheng, and R. Zhang, "Channel estimation and passive beamforming for intelligent reflecting surface: Discrete phase shift and progressive refinement," *IEEE J. Sel. Areas Commun.*, vol. 38, no. 11, pp. 2604–2620, Nov. 2020.

[31] J. Li et al., "Performance analysis of directional modulation with finite-quantized RF phase shifters in analog beamforming structure," *IEEE Access*, vol. 7, pp. 97457–97465, 2019.

[32] R. Dong et al., "Performance analysis of wireless network aided by discrete-phase-shifter IRS," *J. Commun. Netw.*, vol. 24, no. 5, pp. 603–612, Aug. 2022.

[33] Z. Zhang et al., "Active RIS vs. Passive RIS: Which will prevail in 6G?," *IEEE Trans. Commun.*, vol. 71, no. 3, pp. 1707–1725, Mar. 2023.

[34] L. Yang, Y. Yang, D. B. da Costa, and I. Trigui, "Outage probability and capacity scaling law of multiple RIS-aided networks," *IEEE Commun. Lett.*, vol. 10, no. 2, pp. 256–260, Feb. 2021.

[35] L. Wasserman, *All of Statistics: A Concise Course in Statistical Inference*. New York, NY, USA: Springer, 2004.

[36] T. K. Moon and W. C. Stirling, *Mathematical Methods and Algorithms For Signal Processing*. Hoboken, NJ, USA: Prentice Hall, 1999.

[37] F. Shu, X. Liu, G. Xia, T. Xu, J. Li, and J. Wang, "High-performance power allocation strategies for secure spatial modulation," *IEEE Trans. Veh. Technol.*, vol. 68, no. 5, pp. 5164–5168, May 2019.

YAN WANG is currently pursuing the Ph.D. degree with the School of Information and Communication Engineer, Hainan University, China. Her research interests include the performance analysis of wireless communication systems and intelligent reflecting surface.

FENG SHU (Member, IEEE) was born in 1973. He received the B.S. degree from Fuyang Teaching College, Fuyang, China, in 1994, the M.S. degree from Xidian University, Xi'an, China, in 1997, and the Ph.D. degree from Southeast University, Nanjing, China, in 2002. From September 2009 to September 2010, he was a Visiting Postdoctoral Fellow with the University of Texas at Dallas, Richardson, TX, USA. From July 2007 to September 2007, he was a Visiting Scholar with the Royal Melbourne Institute of Technology (RMIT University), Australia. From October 2005 to November 2020, he was with the School of Electronic and Optical Engineering, Nanjing University of Science and Technology, Nanjing, where he was promoted from an Associate Professor to a Full Professor of supervising Ph.D. students in 2013. He is with the School of Information and Communication Engineering and the Collaborative Innovation Center of Information Technology, Hainan University, Haikou, China, and also with the School of Electronic and Optical Engineering, Nanjing University of Science and Technology. His research interests include wireless networks, wireless location, and array signal processing. He has authored or coauthored more than 300 in archival journals with more than 150 papers on IEEE journals and 220 SCI-indexed papers. His citations are more than 6000. He holds more than 40 Chinese patents and is also a PI or CoPI for seven national projects. He is awarded with the Leading-Talent Plan of Hainan Province in 2020, the Fujian Hundred-Talent Plan of Fujian Province in 2018, and the Mingjian Scholar Chair Professor in 2015. He was an IEEE TRANSACTIONS ON COMMUNICATIONS Exemplary Reviewer for 2020. He is currently an Editor of IEEE WIRELESS COMMUNICATIONS LETTERS. He was an Editor of the IEEE SYSTEMS JOURNAL from 2019 to 2021 and IEEE ACCESS from 2016 to 2018.

ZHIHONG ZHUANG received the M.S. and Ph.D. degrees in electrical engineering from the Nanjing University of Science and Technology, Nanjing, China, in 1994 and 1997, respectively, where he is currently a Professor with the School of Electronic and Optical Engineering. His research interests include digital signal processing, adaptive filtering, guidance, and flight control.

RONGEN DONG is currently pursuing the Ph.D. degree with the School of Information and Communication Engineer, Hainan University, China. Her research interests include physical layer security and directional modulation networks.

QI ZHANG received the M.S. degree in electronic and communication engineering from Hainan University, China, in 2021, where he is currently pursuing the Ph.D. degree with the School of Information and Communication Engineering. His research interests include wireless physical-layer security, massive MIMO, signal processing, and integrated sensing and communication.

DI WU was born in 1991. He received the M.S. degree in information and communication engineering from Hainan University, Haikou, China, in 2018. He is currently pursuing the Ph.D. degree in control science and engineering with the Department of Automation, Shanghai Jiao Tong University, Shanghai, China. He is currently a Lecturer with the School of Information and Communication Engineering, Hainan University. His research interests primarily revolve around nonlinear control of aerial vehicles, motion planning, and localization and mapping.

XUEHUI WANG received the M.S. degree from Hainan University, China, in 2020, where she is currently pursuing the Ph.D. degree with the School of Information and Communication Engineering. Her research interests include wireless communication, signal processing, and IRS-aided relay systems.

LIANG YANG was born in Hunan, China. He received the Ph.D. degree in electrical engineering from Sun Yat-sen University, Guangzhou, China, in 2006. From 2006 to 2013, he was a Teacher with Jinan University, Guangzhou. He joined the Guangdong University of Technology, Guangzhou, in 2013. He is currently a Professor with Hunan University, Changsha, China. His current research interest includes the performance analysis of wireless communication systems.

JIANGZHOU WANG (Fellow, IEEE) has been a Professor with the University of Kent, U.K., since 2005. He has published over 400 papers and four books in the areas of wireless communications. He was a recipient of the Best Paper Award from the IEEE GLOBECOM 2012. He was an IEEE Distinguished Lecturer from 2013 to 2014. He was the Technical Program Chair of the 2019 IEEE International Conference on Communications, Shanghai, the Executive Chair of the IEEE ICC 2015, London, and the Technical Program Chair of the IEEE WCNC 2013. He has served as an Editor for a number of international journals, including IEEE TRANSACTIONS ON COMMUNICATIONS from 1998 to 2013. He is a Fellow of the Royal Academy of Engineering, U.K., and IET.