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Resource Sharing Strategies for Point-to-Multipoint Distribution in Next-Generation DSL Networks

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ABSTRACT This paper investigates resource sharing strategies for point-to-multipoint (P2MP) distribution in next-generation digital subscriber line (DSL) networks. The latest DSL ITU standard, multi-gigabit fast access to subscriber terminals (MGfast) or G.9711, supports P2MP transmission as a new feature, which allows resource sharing among multiple customer premises equipments (CPEs). It offers an optimized user experience by efficiently utilizing available resources, thereby reducing the cost of service per user. The resource sharing can be done by allocating part of the available bandwidth to each CPE connected to the same MGfast transceiver unit (MTU-O) at the distribution point unit (DPU). An optimal solution to the grouping and per-group frequency-division multiple access (FDMA) allocation is necessary to optimally exploit the network resources. In this scenario, computing the optimal solutions involves significant computational complexity, especially when the network is dynamic, i.e., CPEs are frequently changing their activity status and traffic demands. Therefore, to overcome these issues, it is necessary to employ heuristic strategies that can provide comparable performance but with significantly reduced computational complexity. This paper proposes the optimal solutions to the grouping and per-group FDMA allocation for both upstream (US) and downstream (DS) P2MP transmission. Additionally, heuristic strategies with significantly lower computational complexity are proposed based on the optimal solutions. These heuristic strategies are shown to achieve comparable performance to the optimal solutions.

INDEX TERMS DSL, dynamic spectrum management, MGfast, point-to-multipoint, resource allocation, resource sharing.

I. INTRODUCTION

Over the last decades, with advancements in signal processing [1] and the deployment of optical-fiber cable (OFC) closer to the customer premises equipments (CPEs) [2], the digital subscriber line (DSL) network has evolved from the 144 kbps basic rate integrated service digital network (ISDN) [3] to the high-speed fast access to subscriber terminals (G.fast) network [4], [5] capable of providing aggregated data rates up to 2 Gbps. Continuing the evolution, a next generation DSL standard, multi-gigabit fast access to subscriber terminals (MGfast), has been defined under ITU-T project G.9711 [6].

MGfast will be capable of providing multi-gigabit aggregated data rates [7], [8], [9], with an aggregated bit rate of up to 8 Gbps. For a smooth transition from G.fast to MGfast, inherently MGfast conserves backward compatibility with G.fast [10]. Hence, it also employs discrete multi-tone (DMT) modulation [11] and has the same subcarrier tone spacing (51.75 kHz) and cyclic extension lengths as G.fast. Moreover, it inherits many other G.fast features like dynamic time assignment (DTA), impulse noise protection, reverse power feeding (RPF), frequency and time synchronization, etc. Also similar to G.fast, in the MGfast network multiple CPEs are connected to a distribution point unit (DPU) placed in close proximity to the customers. In turn, a DPU is connected to the central office (CO) via a passive optical network (PON) [12]. However, the difference between MGfast and earlier generations of DSL is that significantly higher peak data rates are offered. Additionally, resource sharing through point-to-multipoint (P2MP) transmission has been added as a new feature to MGfast [6], which is to be implemented without decreasing the user quality of experience (QoE). It provides an optimized user experience with an efficient utilization of available resources and reduces the cost of service per user. It is to be noted that although MGfast allows both full-duplex (FDX) and time division duplex (TDD) operation [6], this work focuses on TDD operation.

The MGfast transceiver units (MTU) are present in both the DPU (called MTU-O) and CPE (called MTU-R). In P2MP transmission, multiple CPEs are connected to a single MTU-O at the DPU. The group of CPEs connected to the same MTU-O is flexible, i.e., all subscriber lines may be connected to the analogous ports at the DPU and allocation of lines to a MTU-O may be maintained by a digital layer at the DPU. A grouping and vectoring control entity (GVCE) is responsible for the allocation of CPEs to MTU-Os at the DPU. CPEs connected to the same MTU-O are referred as CPEs belonging to the same group and the process of assigning CPEs to MTU-Os is referred as grouping. According to the MGfast standard G.9711, the maximum number of CPEs connected to the same MTU-O is restricted to 16 [6].

In P2MP transmission, grouping strategies have a significant impact on the overall performance of the network, especially on the sustained rates achieved by the lines connected to the DPU. The sustained rate for a network is defined as the data rate experienced by all the lines within the network, during peak usage periods.

In general, optimizing the assignment of CPEs into groups, to maximize the sustained rate, is a complex combinatorial problem. Due to the typically large number of combinations, a low complexity heuristic grouping strategy is required.

Within each group, the resource sharing in P2MP transmission can be done either using frequency-division multiple access (FDMA), i.e., allocating individual tones (or bands of tones) to each CPE in a group or using time-division multiple access (TDMA) by dedicating time slots to each CPE in a group. The MGfast standard specification specifies FDMA based resource sharing for P2MP transmission, such that only one CPE per group is active on each tone. This not only allows resource sharing among CPEs, but it also reduces the number of vectored lines (i.e., the size of the vectoring matrix) on each tone by the sharing factor (i.e., the number of CPEs per group).

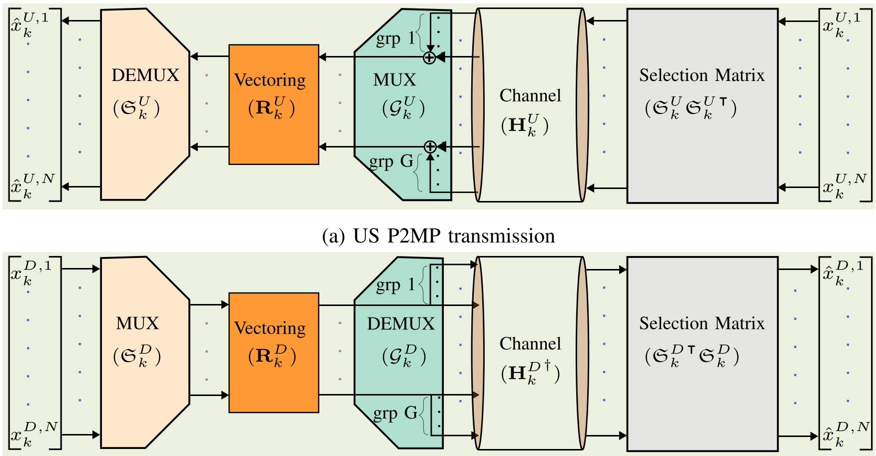
An optimal per-group FDMA allocation with optimal power allocation and vectoring ensures the rate maximization. However, solving the optimization problem for optimal resource allocation is generally an iterative process and involves significant computational complexity [13]. The computational complexity of optimal resource allocation may be too high to meet the requirements of the traffic control for a P2MP group to ensure quality of service (QoS) [7]. Therefore, solving the resource allocation optimization problem sufficiently frequently may not be achievable and a heuristic, near-optimal resource allocation strategy is required with significantly reduced computational complexity.

In the literature, there are existing resource allocation solutions for typical P2MP distribution [14], [15], [16], [17], [18]. However, a large part of the existing literature focuses on PONs which differ vastly from the copper telephone networks, used for DSL. For instance, [14] proposes and demonstrates a novel architectural concept for P2MP distribution in PONs but lacks any mention of resource allocation strategies. In [15], [16], [17], dynamic resource allocation is proposed for P2MP in PONs. However, given the characteristics of optical fibers, these strategies fail to take into account the existence of multiple groups which can cause far-end crosstalk (FEXT) and lack any vectoring technique, an inherent aspect of DSL. Moreover, they assume uniform capacity across all lines and frequencies and do not incorporate spectral power allocation as a decision variable. In [18], a joint resource allocation is proposed, which incorporates power allocation as a decision variable. However, the authors do not consider FEXT or any system model to optimize the data rate of the network. Consequently, to the best of our knowledge, no prior work has addressed the resource sharing problem for P2MP distribution in MGfast (or next-generation DSL) networks.

A. CONTRIBUTIONS

This paper proposes (1) optimal solutions to the grouping and per-group FDMA allocation for both US and DS P2MP transmission. However, computing the optimal solutions involves significant computational complexity and may not be a realistic choice for the practical implementation of MGfast with P2MP transmission, given its highly dynamic nature. (2) Therefore, additionally a heuristic strategy, with significantly reduced computational complexity, is proposed based on the obtained optimal solutions for both the grouping and per-group FDMA allocation. (3) The simulation results are provided to demonstrate that the proposed heuristic strategies achieve comparable performance to the optimal solutions.

The paper is organized as follows: In Section II the system and signal models for US and DS P2MP transmission are presented. In Section III, the grouping problem in P2MP transmission is discussed and a heuristic grouping strategy is proposed. In Section IV the proposed optimal per-group FDMA allocation algorithms separately for US and DS P2MP transmission are presented. Based on these optimal solutions, a heuristic strategy, with significantly lower



(a) US P2MP transmission

(b) DS P2MP transmission

FIGURE 1. System model for US P2MP transmission (Fig. 1a) and DS P2MP transmission (Fig. 1b), for tone k. The left side of the channel represents DPU while the right side of the channel represents CPE.

computational complexity is proposed for the per-group FDMA allocation. In Section V simulation results are provided to compare the performance of the optimal solutions with the proposed heuristic strategies. Finally, Section VI concludes the paper.

Notation: Lower-case and upper-case boldface letters are used to denote vectors and matrices, respectively. Further, $(.)^{\mathsf{T}}$ is used to represent the transpose operation, $(.)^{\dagger}$ for the Hermitian transpose operation and $\mathbb{E}[.]$ for the expected value operation.

II. SYSTEM MODEL

DMT based systems, like DSL, use a cyclic prefix (CP). With a properly chosen CP length and properly synchronized transmitters and receivers, the system can be assumed not to have any inter-carrier interference (ICI). Hence, the transmission on each tone (carrier) can be modelled independently [7]. The system model for US and DS transmission is provided in Sections II-A and II-B, respectively and is also shown in Fig. 1.

A. UPSTREAM TRANSMISSION

The US transmission in DSL with N users (N CPEs) and K tones, with no ICI, can be modelled independently on each tone as a multiple-access channel (MAC):

$$\mathbf{y}_k^U = \mathbf{H}_k^U \mathbf{x}_k^U + \mathbf{z}_k^U \tag{1}$$

where, $\mathbf{x}_k^U \triangleq \left[x_k^{U,1}, \ldots, x_k^{U,N}\right]^{\mathsf{T}}$ is the transmit signal vector on tone k with $x_k^{U,n}$ the transmit signal for user n, $\mathbf{y}_k^U \triangleq \left[y_k^{U,1}, \ldots, y_k^{U,N}\right]^{\mathsf{T}}$ is the receive signal vector on tone k with $y_k^{U,n}$ the receive signal for user n and $\mathbf{z}_k^U \triangleq \left[z_k^{U,1}, \ldots, z_k^{U,N}\right]^{\mathsf{T}}$ is the additive Gaussian noise signal vector with $z_k^{U,n}$ the noise signal for user n. $\mathbf{H}_k^U$ is the $N \times N$ channel matrix, which is assumed to be known, given the slowly time-varying characteristic of DSL channels, and is defined by

$$\mathbf{H}_k^U = \begin{bmatrix} h_k^{U,11} & \cdots & h_k^{U,1N} \\ \vdots & \ddots & \vdots \\ h_k^{U,N1} & \cdots & h_k^{U,NN} \end{bmatrix} \tag{2}$$

where, $h_k^{U,nm}$ represents transfer function from the transmitter of user m to the receiver of user n on tone k. Hence, the diagonal elements of $\mathbf{H}_k^U$ represent the direct channels, while the off-diagonal elements represent the crosstalk channels on tone k.

In P2MP transmission, the CPEs are organized in groups. For the US scenario, at the transmission side, only one CPE per group is active on tone k (corresponding to FDMA within each group). At the receiver side, the received signals corresponding to CPEs in a group are added up (corresponding to a row-wise folding of the channel matrix). Hence, the system equation in (1), for P2MP with G groups is replaced by

$$\underbrace{\mathcal{G}_k^U \mathbf{y}_k^U}_{\bar{\mathbf{y}}_k^U} = \mathcal{G}_k^U \mathbf{H}_k^U \mathfrak{S}_k^U \underbrace{\mathfrak{S}_k^{U\mathsf{T}} \mathbf{x}_k^U}_{\bar{\mathbf{x}}_k^U} + \mathcal{G}_k^U \mathbf{z}_k^U \tag{3}$$

where, $\bar{\mathbf{x}}_k^U$ is the transmit signal vector containing the transmit signal for users with active CPE on tone k. $\mathfrak{S}_k^U$ represents the $N \times G$ selection matrix (with a single "one" in each column and "zeros" elsewhere), which selects the corresponding columns of the channel matrix $\mathbf{H}_k$. At the receiver side, the addition of signals corresponding to CPEs belonging to the same group, is done by the grouping matrix $\mathcal{G}_k^U$ of size $G \times N$ (with in each row a number of "ones" for the addition and "zeros" elsewhere). Hence, the corresponding receive signal vector is defined as $\bar{\mathbf{y}}_k^U \triangleq \mathcal{G}_k^U \mathbf{y}_k^U$. In the US, signal coordination is possible at the DPU for FEXT cancellation. The estimated transmit signal vector for users on tone k, considering a **linear postcoder** is then given by

$$\begin{aligned}\hat{\mathbf{x}}_k^U &= \mathfrak{S}_k^U \mathbf{R}_k^{U\dagger} \mathcal{G}_k^U \mathbf{y}_k^U \\ &\triangleq \underbrace{\mathfrak{S}_k^U \mathbf{R}_k^{U\dagger}}_{\bar{\mathbf{R}}_k^{U\dagger}} \underbrace{\mathcal{G}_k^U \mathbf{H}_k^U \mathfrak{S}_k^U \mathfrak{S}_k^{U\intercal}}_{\bar{\mathbf{H}}_k^U} \mathbf{x}_k^U + \mathfrak{S}_k^U \mathbf{R}_k^{U\dagger} \underbrace{\mathcal{G}_k^U \mathbf{z}_k^U}_{\bar{\mathbf{z}}_k^U} \\ &\triangleq \bar{\mathbf{R}}_k^{U\dagger} \bar{\mathbf{H}}_k^U \mathbf{x}_k^U + \bar{\mathbf{R}}_k^{U\dagger} \bar{\mathbf{z}}_k^U \end{aligned} \tag{4}$$

where, $\bar{\mathbf{H}}_k^U$ is the $G \times N$ equivalent US P2MP channel matrix for tone k with $N-G$ zero columns and $\bar{\mathbf{R}}_k^{U\dagger}$ is the $N \times G$ equivalent US P2MP postcoder matrix for tone k with $N-G$ zero rows, assuming FDMA within each group. The system model for US P2MP transmission is shown in Fig. 1a.

The average symbol power for user n on tone k is given as $s_k^{U,n} = \Delta_f \mathbb{E}[|x_k^{U,n}|^2]$, where Δ_f is the tone spacing. Hence the total transmitted power by user n in US is

$$P^{U,n} = \sum_k s_k^{U,n}. \tag{5}$$

Based on the estimated transmit signal, with the linear postcoder, the signal-to-interference-plus-noise-ratio (SINR) for user $n = 1 \cdots N$ on tone k is given as

$$SINR_k^{U,n} = \frac{\left|\bar{\mathbf{r}}_k^{U,n\dagger} \bar{\mathbf{h}}_k^{U,n}\right|^2 s_k^{U,n}}{\sum_{m \neq n} \left|\bar{\mathbf{r}}_k^{U,n\dagger} \bar{\mathbf{h}}_k^{U,m}\right|^2 s_k^{U,m} + \bar{\mathbf{r}}_k^{U,n\dagger} \boldsymbol{\phi}_k^U \bar{\mathbf{r}}_k^{U,n}} \tag{6}$$

where, $\bar{\mathbf{r}}_k^{U,n}$ and $\bar{\mathbf{h}}_k^{U,n}$ represents the n^{th} column of the equivalent US P2MP postcoder matrix $\bar{\mathbf{R}}_k^U$ and the equivalent US P2MP channel matrix $\bar{\mathbf{H}}_k^U$, respectively. Moreover, $\boldsymbol{\phi}_k^U$ is the covariance matrix of the additive Gaussian noise on tone k, $\boldsymbol{\phi}_k^U = \Delta_f \mathbb{E}[\bar{\mathbf{z}}_k^U (\bar{\mathbf{z}}_k^U)^\dagger]$. Thus, the first term in the denominator of (6) ($|\bar{\mathbf{r}}_k^{U,n\dagger} \bar{\mathbf{h}}_k^{U,m}|^2 s_k^{U,m}$) denotes the interference power from user m to user n on tone k due to FEXT, after the postcoder. On the other hand, the second term ($\bar{\mathbf{r}}_k^{U,n\dagger} \boldsymbol{\phi}_k^U \bar{\mathbf{r}}_k^{U,n}$) represents the noise power for user n on tone k, after the postcoder.

It is noted that for a user n with inactive CPE on tone k, the effective channel gain $\bar{\mathbf{h}}_k^{U,n}$ is zero due to the selection matrix $\mathfrak{S}_k^U$. Hence, the $SINR_k^{U,n}$ for users with inactive CPE will be zero, corresponding to zero bit-loading.

For a **non-linear postcoder**, such as the generalized decision feedback equalizer (GDFE) [19], [20], the SINR expression in (6) can be modified by simply replacing the summation operator in the denominator over $m \neq n$ with a summation operator over $m > n$, assuming the decoding order is given by the user index (i.e., user 1 is decoded first and user N decoded last). The replacement of the summation operator over $m \neq n$ with a summation operator over $m > n$ reflects the non-linear postcoder's capability to successively decode transmitted symbols, while effectively removing any interference from already decoded symbols.

From (3), it can be noted that at the receiver side, since the addition of signals corresponding to CPEs belonging to the same group is done by the grouping matrix $\mathcal{G}_k^U$, this operation also adds the noise received in each group ($\bar{\mathbf{z}}_k^U = \mathcal{G}_k^U \mathbf{z}_k^U$). This increases the noise power received by the CPEs and may decrease the signal-to-noise-ratio (SNR), as shown in Section V.

B. DOWNSTREAM TRANSMISSION

Similar to the US, the DS transmission in DSL with N users and K tones, with no ICI, can be modelled independently on each tone k as a broadcast channel (BC):

$$\mathbf{y}_k^D = \mathbf{H}_k^{D\dagger} \mathbf{x}_k^D + \mathbf{z}_k^D \tag{7}$$

where, $\mathbf{y}_k^D$, $\mathbf{x}_k^D$ and $\mathbf{z}_k^D$ are the receive signal vector, transmit signal vector and noise signal vector, respectively, with similar size and structure as their upstream counterparts. The matrix $\mathbf{H}_k^D$ represents the Hermitian of the DS channel matrix with $[\mathbf{H}_k^D]_{mn} = h_k^{U,nm*}$ as the complex conjugate of the transfer function from the transmitter of user m to the receiver of user n on tone k.

In DS P2MP transmission again, only one CPE per group is active on tone k (corresponds to FDMA within each group). Since signal coordination is now possible at the transmitter side, vectoring is applied to the users with an active CPE on tone k and by using a precoder matrix. In addition, the signal intended for the user with the active CPE in a group, is transmitted by and to all the users in the group. Hence, the system equation in (7), for P2MP with G groups is replaced by

$$\mathbf{y}_k^D = \mathbf{H}_k^{D\dagger} \mathcal{G}_k^D \mathbf{R}_k^D \mathfrak{S}_k^D \mathbf{x}_k^D + \mathbf{z}_k^D \tag{8}$$

where, $\mathfrak{S}_k^D$ is the $G \times N$ DS selection matrix (with a single "one" in each row, and "zeros" elsewhere), $\mathbf{R}_k^D$ is the $G \times G$ precoder matrix for tone k and $\mathcal{G}_k^D$ is the $N \times G$ repetition matrix (with in each column a number of "ones" corresponding to the users in a group and "zeros" elsewhere).

Finally, since only one user has an active CPE per group on tone k, the estimated transmit signal vector for users on tone k in DS P2MP is given as

$$\begin{aligned}\underbrace{\mathfrak{S}_k^{D\intercal} \mathfrak{S}_k^D \mathbf{y}_k^D}_{\hat{\mathbf{x}}_k^D} &= \underbrace{\mathfrak{S}_k^{D\intercal} \mathfrak{S}_k^D \mathbf{H}_k^{D\dagger} \mathcal{G}_k^D}_{\bar{\mathbf{H}}_k^{D\dagger}} \underbrace{\mathbf{R}_k^D \mathfrak{S}_k^D}_{\bar{\mathbf{R}}_k^D} \mathbf{x}_k^D + \mathfrak{S}_k^{D\intercal} \underbrace{\mathfrak{S}_k^D \mathbf{z}_k^D}_{\bar{\mathbf{z}}_k^D} \\ &\triangleq \bar{\mathbf{H}}_k^{D\dagger} \bar{\mathbf{R}}_k^D \mathbf{x}_k^D + \mathfrak{S}_k^{D\intercal} \bar{\mathbf{z}}_k^D \end{aligned} \tag{9}$$

where, $\bar{\mathbf{H}}_k^{D\dagger}$ is the $N \times G$ equivalent DS P2MP channel matrix for tone k with $N-G$ zero rows and $\bar{\mathbf{R}}_k^D$ is the $G \times N$ equivalent DS P2MP precoder matrix for tone k with $N-G$ zero columns, assuming FDMA within each group. The system model for DS P2MP transmission is shown in Fig. 1b.

The average symbol power for user n on tone k before precoding is given as $s_k^{D,n} = \Delta_f \mathbb{E}[|x_k^{D,n}|^2]$. With precoding, the average signal power transmitted by user n on tone k is defined as

$$P_k^{D,n} = \sum_m \left| \left[\mathcal{G}_k^D \mathbf{R}_k^D \mathfrak{S}_k^D \right]_{nm} \right|^2 s_k^{D,n} \tag{10}$$

where, $[\mathcal{G}_k^D \mathbf{R}_k^D \mathfrak{S}_k^D]_{nm}$ represents the element in the n^{th} row and m^{th} column of the extended equivalent DS P2MP precoder matrix for tone k. Therefore, the total transmitted power by user n in DS is given as

$$P^{D,n} = \sum_k P_k^{D,n} = \sum_k \left(\sum_m \left| \left[\mathcal{G}_k^D \mathbf{R}_k^D \mathfrak{S}_k^D \right]_{nm} \right|^2 s_k^{D,n} \right). \quad (11)$$

For a **linear precoder**, the SINR for the user $n = 1 \cdots N$ on tone k, is given as

$$SINR_k^{D,n} = \frac{\left| \bar{\mathbf{h}}_k^{D,n^\dagger} \bar{\mathbf{r}}_k^{D,n} \right|^2 s_k^{D,n}}{\sum_{m \neq n} \left| \bar{\mathbf{h}}_k^{D,n^\dagger} \bar{\mathbf{r}}_k^{D,m} \right|^2 s_k^{D,m} + \sigma_k^{D,n}} \quad (12)$$

where, $\bar{\mathbf{r}}_k^{D,n}$ and $\bar{\mathbf{h}}_k^{D,n}$ represent the n^{th} column of the DS P2MP precoder matrix $\bar{\mathbf{R}}_k^D$ and Hermitian of the equivalent DS P2MP channel matrix $\bar{\mathbf{H}}_k^D$, respectively. $\sigma_k^{D,n}$ is the average received noise power for user n on tone k and is defined as $\sigma_k^{D,n} = \Delta_f \mathbb{E}[|z_k^{D,n}|^2]$. Similar to US P2MP transmission, it is noted that for a user l with inactive CPE on tone k, the effective channel gain $\bar{\mathbf{h}}_k^{D,l^\dagger}$ is zero due to the selection matrix $\mathfrak{S}_k^D$. Hence, the $SINR_k^{D,l}$ for users with inactive CPE will be zero, corresponding to zero bit-loading.

For a **non-linear precoder**, such as the multiple user Tomlinson-Harashima precoder (THP) [21], [22], [23] which is based on the theoretical concept of dirty paper coding (DPC) [24], distortions are added to the user data signals in order to pre-subtract the crosstalk from the previously decoded symbols at the receiver [25]. Hence, for non-linear precoding with user index based reverse decoding order, the SINR expression in (12) can be modified by replacing the summation over $m \neq n$ in the denominator with summation over $m < n$.

C. DATA RATES

A common performance metric used in DSL is the maximum data rate of the system. For a large number of users in a DSL network, the received interference plus noise is well approximated by a Gaussian distribution, due to the well-known central limit theorem [11]. Under this assumption, the theoretical maximum data rate achieved by a user n, with an arbitrarily small error probability (assuming ideal coding techniques) [26] is given as

$$B^{U/D,n} = \sum_k b_k^{U/D,n} = \sum_k \log_2\left(1 + SINR_k^{U/D,n}\right). \quad (13)$$

However, the implementation of ideal coding techniques (corresponding to the so-called Gaussian signaling) in order to achieve the theoretical data rates is not possible in a practical scenario, without infinite delay and decoding complexity. Therefore, an SNR-gap to capacity (denoted by Γ) is added to (13) to account for the difference between the ideal and the practically achievable data rates. The Γ is typically a function of the desired error probability, coding gain and noise margin [27], [28]. With the SNR-gap in consideration the practical achievable data rate of user n becomes

$$B^{U/D,n} = \sum_k b_k^{U/D,n} = \sum_k \log_2\left(1 + \frac{SINR_k^{U/D,n}}{\Gamma}\right). \quad (14)$$

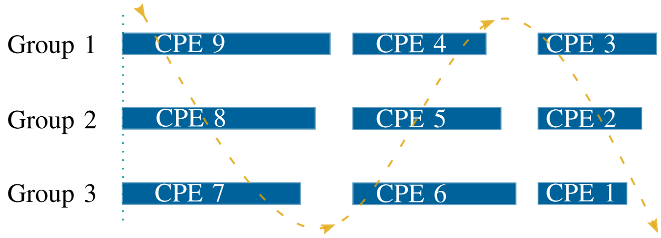


FIGURE 2. Heuristic grouping strategy for CPEs with the assumption that the corresponding lines are ordered according to their physical length (or direct channel data rates) such that length(CPE N) > $\cdots$ length(CPE 2) > length(CPE 1).

III. GROUPING STRATEGY

As discussed in Section I, optimizing the assignment of CPEs into groups, to maximize the sustained rate is a huge combinatorial problem. With a typical binder size of $N = 20$ and a desired number of groups $G = 5$, the number of possible grouping combinations exceeds 2×10^9. Hence, exhaustively searching through all possible grouping combinations is not feasible, and so a low complexity heuristic grouping strategy is required.

The proposed heuristic grouping strategy considers the physical length of the lines as decision variable. The physical length of the line is related to the electrical length, which is a parameter that is estimated by the CPE during initialization. The CPEs are ordered according to their physical line length and are subsequently assigned to groups, following the strategy shown in Fig. 2, where $N = 9$ and $G = 3$. The assignment of CPEs to groups (approximately) follows a pattern as depicted by the yellow curve in Fig. 2. Specifically, CPEs 9, 8, and 7 are assigned to groups 1, 2, and 3, respectively. CPEs 6, 5, and 4 are then assigned to groups 3, 2, and 1, respectively, following the order of decreasing physical length. This pattern continues for the remaining CPEs. Therefore, the number of CPEs per group is equal to the total number of active CPEs divided (and rounded up or down) by the number of groups, and is restricted to a maximum of 16 CPEs per group [6].

The proposed approach aims to achieve uniform distribution of different physical lines within each group, as longer lines are typically subject to higher attenuation levels than their shorter counterparts. The physical length of the lines is frequently estimated in existing access nodes and is therefore a suitable decision variable for heuristic strategies in the context of network optimization problems. By ensuring a uniform distribution of physical line lengths (and hence data rates) within each group, the method maximizes the sustained rate, contributing to the optimization of network performance. It is to be noted that the proposed heuristic

Algorithm 1 Heuristic Grouping Strategy

Input G, N, Relative physical length of lines *or* direct data rates precomputed for CPEs (data rate without crosstalk)

Initialisation $g = 0$, $g_{next} = 1$
Order CPEs based on their decreasing physical line length or increasing direct data rates
for $n = 1 : N$ **do**
    $g \leftarrow g + g_{next}$
    assign CPE n to group g
    if $g = G$ OR $g = 1$ **then**
        $g \leftarrow g + g_{next}$
        $g_{next} = -g_{next}$
    end if
end for

grouping strategy does not depend on the FEXT between the lines, since the effect of FEXT is minimized by the use of postcoders and precoders in US and DS transmission respectively.

For the scenarios where physical line lengths are unknown or cannot be determined, direct data rates of lines (without crosstalk) can be precomputed and used instead of the physical line lengths for grouping. This is based on the observation that the attenuation introduced by a line is directly proportional to its physical length. Hence, the data rate that can be achieved on a line is inversely related to its physical length [29]. The proposed heuristic grouping strategy is summarized in **Algorithm 1**.

The proposed heuristic grouping strategy is identical for both US and DS. Moreover, the grouping of US and DS is coupled, meaning that a CPE is connected to the same MTU-O for both US and DS. The performance of the proposed heuristic grouping strategy is discussed later in Section V.

IV. PER-GROUP FDMA ALLOCATION STRATEGY

In this section, dynamic spectrum management (DSM) strategies are discussed for the DSL network employing US and DS P2MP transmission. The problem considered here is focused on maximizing the sustained rate (i.e., considering rate-adaptive DSM [30]). Based on the optimal resource allocation, a heuristic per-group FDMA allocation strategy is proposed. The performance of the proposed heuristic per-group FDMA allocation strategy is compared with the optimal DSM strategy in Section V.

A. UPSTREAM TRANSMISSION

1) OPTIMAL POWER ALLOCATION AND VECTORING BASED PER-GROUP FDMA

The per-group FDMA allocation can be optimized by solving a sustained rate maximization problem considering vectoring and power allocation. The power allocation reflects the FDMA, i.e., a user allocated with zero power on a tone corresponds to that tone not being allocated to that user. However, allowing the sustained rate maximization problem to allocate power to all the users, without pre-assuming FDMA within groups, requires modifications in equation (3). Without a pre-assumed FDMA the selection matrix ($\mathfrak{S}_k^U$) is no longer required and hence, the modified equation becomes

$$\mathcal{G}_k^U \mathbf{y}_k^U = \underbrace{\mathcal{G}_k^U \mathbf{H}_k^U}_{\tilde{\mathbf{H}}_k^U} \mathbf{x}_k^U + \mathcal{G}_k^U \mathbf{z}_k^U. \tag{15}$$

In the US scenario, the optimal **non-linear postcoder** is the minimum mean squared error (MMSE) GDFE [31]. Substituting the modified system equation (15) in (4), the MMSE GDFE postcoder vector for user $n = 1 \cdots N$ on tone k is given as

$$\bar{\mathbf{r}}_k^{U,n} = \left(\underbrace{\sum_{m>n} s_k^{U,m} \tilde{\mathbf{h}}_k^{U,m} \left(\tilde{\mathbf{h}}_k^{U,m} \right)^{\dagger} + \boldsymbol{\phi}_k^U}_{\boldsymbol{\Psi}_k^{U,n}} \right)^{-1} \tilde{\mathbf{h}}_k^{U,n} \tag{16}$$

where, $\tilde{\mathbf{h}}^{U,m}$ represents the m^{th} column of the $G \times N$ channel matrix $\tilde{\mathbf{H}}_k^U$ and $\boldsymbol{\Psi}_k^{U,n}$ represents the interference-plus-noise covariance matrix for user n on tone k. Using expression (16) in the US SINR expression (6), the SINR for the modified system equations for user n on tone k (15) is given as

$$SINR_k^{U,n} = s_k^{U,n} \tilde{\mathbf{h}}_k^{U,n^{\dagger}} \left(\boldsymbol{\Psi}_k^{U,n} \right)^{-1} \tilde{\mathbf{h}}_k^{U,n}. \tag{17}$$

Hence, the achieved data rate ($b_k^{U,n}$) for user n on tone k, as defined in (14), is solely a function of all user powers on tone k, i.e., $\mathbf{s}_k^U \triangleq \left[s_k^{U,1}, \ldots, s_k^{U,N} \right]^{\mathsf{T}}$.

The sustained rate maximization problem is then given as

$$\underset{B^U, \mathbf{s}_k^U, \forall k}{\text{maximize}} \quad B^U \tag{18a}$$

$$\text{subject to} \quad B^U \leq \sum_k b_k^{U,n}\left(\mathbf{s}_k^U\right) \quad \forall n, \tag{18b}$$

$$0 \leq s_k^{U,n} \leq P_k^{mask} \quad \forall k, n, \tag{18c}$$

$$\sum_{n \in g} s_k^{U,n} \leq P_k^{mask} \quad \forall k, g \tag{18d}$$

where, B^U is a free variable representing the achievable sustained rate of the network, (18b) is the sustained rate constraint, which makes sure that all the users will attain a data rate higher than the sustained rate B^U, and (18c) constrains the per-tone power for each user, with P_k^{mask} the predefined spectral mask, to ensure spectral compatibility with other DSL services. Therefore, the constrained maximization problem through (18a) to (18c) maximizes the sustained rate under spectral mask constraint. The constraint (18d) then attempts to enforce a per-group FDMA allocation (this will be checked in the sequel). For the sake of simplicity, we have omitted the dependence of variable $b_k^{U,n}(\mathbf{s}_k^U)$, denoting it simply as $b_k^{U,n}$. In the remainder of the paper, we will only show the dependence of variables explicitly in the formulation of the optimization problems. This ensures a simplified presentation of mathematical equations and improves readability.

In [32], it has been shown that for multi-carrier systems like orthogonal frequency-division multiplexing (OFDM) and DMT, a strong duality exists between a primal problem maximizing the data rates with constraints on system resources and its dual problem, for continuous frequencies ($K = \infty$). It has been further proved in [33] that an asymptotic strong duality exists for discrete frequencies, such that the duality gap becomes zero as the number of frequencies K goes to ∞. In practical DSL scenarios, the number of discrete tones is high enough to have a sufficiently small duality gap. Therefore, to solve the primal problem in (18), its dual problem is formulated.

The dual secondary problem[1] for (18) is defined as

$$\begin{aligned} &\underset{\mathbf{w},\boldsymbol{\lambda}}{\text{minimize}} \quad q(\mathbf{w},\boldsymbol{\lambda}) \\ &\text{subject to} \quad \mathbf{w},\boldsymbol{\lambda} \geq 0 \end{aligned} \tag{19}$$

where, $q(\mathbf{w},\boldsymbol{\lambda})$ is the Lagrange dual function with $\mathbf{w}$ and $\boldsymbol{\lambda}$ as the vectors containing the Lagrange multipliers $\mathbf{w} \triangleq [w^1,\ldots,w^N]$ and $\boldsymbol{\lambda} \triangleq [\lambda_1^1,\ldots,\lambda_K^G]$ associated with the sustained rate constraint (18b) and the power constraint (18d), respectively. The Lagrange dual function $q(\mathbf{w},\boldsymbol{\lambda})$ is defined as

$$q(\mathbf{w},\boldsymbol{\lambda}) = \underset{B^U,\mathbf{s}_k^U,\forall k}{\text{maximize}} \ \mathcal{L}\left(\mathbf{w},\boldsymbol{\lambda},B^U,\mathbf{s}_k^U,\forall k\right) \tag{20a}$$

$$\text{subject to} \ \ 0 \leq s_k^{U,n} \leq P_k^{mask} \quad \forall k,n \tag{20b}$$

where $\mathcal{L}$ is the Lagrangian function, which is defined as

$$\begin{aligned} \mathcal{L}\left(\mathbf{w},\boldsymbol{\lambda},B^U,\mathbf{s}_k^U,\forall k\right) &= B^U\left(1-\sum_n w^n\right) + \sum_n w^n \sum_k b_k^{U,n} \\ &\quad + \sum_g \sum_k \lambda_k^g \left(P_k^{mask} - \sum_{n\in g} s_k^{U,n}\right) \end{aligned} \tag{21}$$

The Lagrangian function can be further simplified as

$$\begin{aligned} \mathcal{L}\left(\mathbf{w},\boldsymbol{\lambda},B^U,\mathbf{s}_k^U,\forall k\right) &= B^U\left(1-\sum_n w^n\right) \\ &\quad + \sum_k \sum_g \lambda_k^g P_k^{mask} + \sum_k \underbrace{\sum_n \left(w^n b_k^{U,n} - \lambda_k^{g\ni n} s_k^{U,n}\right)}_{\mathcal{L}_k} \end{aligned} \tag{22}$$

where, $\lambda_k^{g\ni n}$ denotes the Lagrange multiplier corresponding to the group g to which user n has been assigned, for tone k. The Lagrange multiplier $\mathbf{w}$ can be interpreted as the weights given to the different users. The term $\sum_k \sum_g \lambda_k^g P_k^{mask}$ for a fixed $\boldsymbol{\lambda}$ is constant. The maximization of the Lagrangian involves the term $B^{U,g}(1-\sum_{n\in g} w^n)$, where $B^{U,g}$ is a free variable. Hence, the maximization results in

$$B^{U,g} = \begin{cases} +\inf \text{ if } \left(1-\sum_{n\in g} w^n\right) > 0 \\ 0 \qquad \text{if } \left(1-\sum_{n\in g} w^n\right) < 0. \end{cases} \tag{23}$$

Therefore, a non-trivial solution is obtained only with $(1-\sum_{n\in g} w^n) = 0$. Therefore, the Lagrangian function can be maximized for a fixed set of Lagrange multipliers $(\mathbf{w},\boldsymbol{\lambda})$, by independently maximizing $\mathcal{L}_k$ for each tone k separately, also known as dual decomposition [34]. With this dual decomposition the dual secondary problem for tone k can be written as

$$q_k(\mathbf{w},\boldsymbol{\lambda}) = \underset{\mathbf{s}_k^U}{\text{maximize}} \ \sum_n \left(w^n b_k^{U,n} - \lambda_k^{g\ni n} s_k^{U,n}\right) \tag{24a}$$

$$\text{subject to} \ \ 0 \leq s_k^{U,n} \leq P_k^{mask} \quad \forall n. \tag{24b}$$

To solve the dual secondary problem, in [35], [36], [37] an exhaustive search over all possible discrete bit-loading (or discrete power-loading) combinations for all the users performed for each tone separately, has been proposed. However, pertaining to the exhaustive search, the computational complexity of these algorithms grows exponentially with the number of users and hence these are not suitable for P2MP transmission, where a large number of users are connected to the network. Therefore, to solve the dual secondary problem in (24), a distributed spectrum balancing (DSB) algorithm [38], [39], based on successive convex approximation (SCA) [40], [41], [42] and block coordinate descent (BCD) [43], is employed. The DSB algorithm solves maximization problems of the following form:

$$\begin{aligned} &\underset{\mathbf{X}}{\text{maximize}} \quad \mathfrak{f}(\mathbf{X}) = \sum_{i=1}^{n} \mathfrak{f}_i(\mathbf{X}) \\ &\text{subject to} \quad \mathbf{X}_i \in \mathcal{X}_i, \ i = 1,2,\ldots,n \end{aligned} \tag{25}$$

where the optimization variable $\mathbf{X}$ comprises of multiple coordinate blocks $\mathbf{X} \triangleq \left[\mathbf{X}_1,\ldots,\mathbf{X}_n\right]$. The cost function $\mathfrak{f}_i$ is concave over its (possibly empty) coordinate block $\mathbf{X}_i$ and convex over the other coordinate blocks. The above problem can be solved iteratively, such that at each iteration the function is maximized w.r.t its associated coordinate block, keeping the other coordinate blocks fixed. This approach is popularly known as BCD. With the BCD, the subproblem for the i^{th} user coordinate block ($\mathbf{X}_i$) can be solved independently and is given as

$$\begin{aligned} \mathbf{X}_i^{(l+1)} &= \arg \max_{\mathbf{X}_i\in\mathcal{X}_i} \mathfrak{f}\left(\mathbf{X}_i^{(l)}|\bar{\mathbf{X}}\right) = \sum_{i=1}^{n} \mathfrak{f}_i\left(\mathbf{X}_i^{(l)}|\bar{\mathbf{X}}\right) \\ &= \arg \max_{\mathbf{X}_i\in\mathcal{X}_i} \underbrace{\mathfrak{f}_i\left(\mathbf{X}_i^{(l)}|\bar{\mathbf{X}}\right)}_{\text{concave}} + \underbrace{\sum_{j\neq i}^{n} \mathfrak{f}_j\left(\mathbf{X}_i^{(l)}|\bar{\mathbf{X}}\right)}_{\text{convex}} \end{aligned} \tag{26}$$

where $\bar{\mathbf{X}}$ represents the current (fixed) value of $\mathbf{X}$ and $\mathbf{X}_i^{(l)}$ represents the value of the coordinate block $\mathbf{X}_i$ at the l^{th} iteration. It can be observed that the function $\mathfrak{f}(\mathbf{X}_i^{(l)}|\bar{\mathbf{X}})$ in (26) is not concave and a unique maximizer (if $\mathbb{R}^M \to \mathbb{R}$ is a function, then there exists a unique $x^* \in \mathbb{R}^M$ that maximizes

1. Please note that in this manuscript, the terms "dual primary" and "dual secondary" are used as substitutes for the conventional terms "dual master" and "dual slave" problem, respectively.

the output of the function) $\mathbf{X}_i^*$ does not exist. However, the convergence of BCD typically requires uniqueness of the maximizer, such that the sub-problem of each coordinate block variable achieves its unique global optimum. A popular approach to tackle this problem is to replace the convex part of the function with its (linear) concave approximation (CA), such that

$$\mathbf{X}_i^{(l+1)} = \arg\max_{\mathbf{X}_i \in \mathcal{X}_i} \tilde{\mathfrak{f}}\left(\mathbf{X}_i^{(l)} | \bar{\mathbf{X}}\right) \tag{27}$$

where, $\tilde{\mathfrak{f}}(\mathbf{X}_i|\bar{\mathbf{X}})$ is the approximate function (or the so-called surrogate function) defined as

$$\tilde{\mathfrak{f}}(\mathbf{X}_i|\bar{\mathbf{X}}) \triangleq \mathfrak{f}_i\left(\mathbf{X}_i^{(l)}|\bar{\mathbf{X}}\right) - \underbrace{tr\left(\mathbf{A}_i^{\dagger}\mathbf{X}_i\right)}_{\text{CA}} \tag{28}$$

where $\mathbf{A}_i = -\nabla_{\mathbf{X}_i^*}(\sum_{j\neq i} \mathfrak{f}_j(\mathbf{X}))|_{\mathbf{X}=\bar{\mathbf{X}}}$ is the negative conjugate gradient of $\sum_{j\neq i} \mathfrak{f}_j(\mathbf{X})$.

While solving the sub-problem for the i^{th} user coordinate block, the approximate function $\tilde{\mathfrak{f}}(\mathbf{X}_i|\bar{\mathbf{X}})$ depends only on its coordinate block $\mathbf{X}_i$ and is concave w.r.t $\mathbf{X}_i$. Hence, the sum maximization problem in (25) can be solved by iteratively solving a sequence of surrogate problems (27) for $i = 1, 2, \ldots, n$ [34], [39].

Applying the DSB algorithm to the dual secondary problem in (24). Comparing (24) and (25) the function $\mathfrak{f}_k^n$ for the problem is defined as

$$\mathfrak{f}_k\left(\mathbf{s}_k^U\right) = \sum_n \mathfrak{f}_k^n\left(\mathbf{s}_k^U\right) = \sum_n \left(w^n b_k^{U,n} - \lambda_k^{g\ni n} s_k^{U,n}\right). \tag{29}$$

The function $(\mathfrak{f}_k^n(\mathbf{s}_k^U))$ meets the requirement of the DSB algorithm, as it is concave in its associated coordinate block $(s_k^{U,n})$ and convex over other coordinate blocks $(s_k^{U,m\neq n})$ [44]. Similar to (27), the surrogate problems can be solved individually and independently for each user $n = 1, \ldots, N$:

$$s_k^{U,n} = \arg\max_{s_k^{U,n} \in \text{(24b)}} \tilde{\mathfrak{f}}_k\left(s_k^{U,n}|\bar{\mathbf{s}}_k^U\right) \tag{30}$$

where, $\bar{\mathbf{s}}_k^U$ represents the current fixed value of $\mathbf{s}_k^U$. Applying the Karush–Kuhn–Tucker (KKT) stationarity condition on (30):

$$\frac{\partial}{\partial s_k^{U,n}}\left(\tilde{\mathfrak{f}}_k\left(s_k^{U,n}|\bar{\mathbf{s}}_k^U\right)\right) = 0. \tag{31}$$

The optimal user power for user n on tone k is obtained by isolating $s_k^{U,n}$ on one side of the equation and taking bounds to satisfy the mask constraint (24b):

$$s_k^{U,n} = \left[\frac{w^n}{log(2)\left(\lambda_k^{g\ni n} + a_k^n\right)} - \frac{\Gamma}{\left(\bar{\mathbf{h}}_k^{U,n}\right)^{\dagger}\mathbf{\Psi}_k^{U,n}\bar{\mathbf{h}}_k^{U,n}}\right]_0^{P_k^{mask}} \tag{32}$$

where a_k^n is the negative conjugate gradient for user n on tone k as defined for the surrogate function (28) and is given in (33), shown at the bottom of the next page [38].

Algorithm 2 Optimal Power Allocation and Vectoring Based Per-Group FDMA

Input: User groups

Initialisation $\mathbf{s}_k^U \in \mathbb{R}_+^N\ \forall k$, $\mathbf{w} \in \mathbb{R}_+^N$, $\mu \in \mathbb{R}_+$
while COND_w $= 0$ **do**
  while $\sum_k \sum_n w^n b_k^{U,n}$ does not converge **do**
    for $k = 1 \cdots K$ **do**
      $\lambda_{g\ni n}^{k,min} = 0$, $\lambda_{g\ni n}^{k,max} \in \mathbb{R}_+$, COND_s $= 0\ \forall n$
      $a_k^n \leftarrow$ DPU_F1$(\bar{\mathbf{h}}_k^{U,n}, w^n, s_k^{U,n})\quad \forall n$
      while COND_s $= 0$ **do**
        $\lambda_k^{g\ni n} = (\lambda_{g\ni n}^{k,min} + \lambda_{g\ni n}^{k,max})/2$
        Update $s_k^{U,n}$ using (32) $\quad\forall n$
        Grp_sum $\leftarrow$ DPU_F1$(\bar{\mathbf{h}}_k^{U,n}, w^n, s_k^{U,n})$
        if Grp_sum $> P_k^{mask}$ **then**
          $\lambda_{g\ni n}^{k,min} = \lambda_k^{g\ni n}$
        else
          $\lambda_{g\ni n}^{k,max} = \lambda_k^{g\ni n}$
        end if
        COND_s $\leftarrow$ DPU_F1$(\bar{\mathbf{h}}_k^{U,n}, w^n, s_k^{U,n})$
      end while
    end for
  end while
  Calculate $b^{U,n} = \sum_k b_k^{U,n}$ using (17) and (14) $\quad\forall n$
  [COND_w, $\mathbf{w}$]$\leftarrow$ DPU_F2$(w^n, b^{U,n})$
end while
function DPU_F1$(\bar{\mathbf{h}}_k^{U,n}, w^n, s_k^{U,n})$
  Receive $\bar{\mathbf{h}}_k^{U,n}, w^n, s_k^{U,n}$ from all users n
  Find the group g to which user n belongs
  Calculate a_k^n using (33)
  Grp_sum $= \sum_{m\in g} s_k^{U,m}$
  if $|$ Grp_sum $-P_k^{mask}| \leq \delta_s$ OR $\lambda_g^{k,min} \leq \delta_\lambda$ **then**
    COND_s $= 1$
  end if
end function
function DPU_F2$(w^n, b^{U,n})$
  Receive $b^{U,n}$ and w^n from all users n
  $B^U = \min_n\{b^{U,n}\}$
  if $|B^U - b^{U,n}| \leq \delta_b$ OR $w^n \leq \delta_w \quad \forall n$ **then**
    COND_w $= 1$
  else
    $\acute{w}^n = \left[w^n + \mu\left(B^U - b^{U,n}\right)\right]_0 \quad \forall n$
    $w^n = \frac{\acute{w}^n}{\sum_n \acute{w}^n} \quad \forall n$
  end if
end function

Moreover, $[\,\cdot\,]_\alpha^\gamma \triangleq \min\{\max\{\cdot, \alpha\}, \gamma\}$. Finally, the sustained rate (B^U) is defined as $B^U = \min_n\{\sum_k b_k^{U,n}\}$. A bisection search algorithm and subgradient update approach is employed to converge to the globally optimum solution of the Lagrange multipliers $\boldsymbol{\lambda}$ and $\mathbf{w}$ respectively in the dual primary problem (19). The algorithm for the optimal power allocation and vectoring based per-group FDMA is summarized in **Algorithm 2**. It is noted that the functions DPU_F1 and DPU_F2 are to be computed at the DPU as they need data from other users in the network.

For a MMSE GDFE postcoder (assuming $\Gamma = 1$ and weight ($\mathbf{w}$) based decoding order such that the user with the highest weight is decoded last), problem (18) is convex [45] and the transmit powers $s_k^{U,n}$ converge to a unique optimal solution with Algorithm 2. However, simulations show that within each group, on each tone, more than one user (belonging to that group) may be allocated a non-zero transmit power, despite the constraint (18d). This indicates that the optimal allocated powers for an MMSE GDFE postcoder do not correspond to an FDMA allocation. This behaviour can be explained by the successive interference cancellation (SIC) nature of a non-linear GDFE postcoder.

A **linear postcoder** is hence a more ideal choice for the optimal power allocation and vectoring based per-group FDMA. Algorithm 2 can be adapted for a linear MMSE postcoder by simply replacing the summation over $m < n$ with $m \neq n$, in the calculation of a_k^n in line 26 and $b^{U,n}$ in line 20. However, with a linear MMSE postcoder, the sustained rate maximization problem in (18a) is no longer convex and contains multiple stationary points. Hence, the solution of the convex surrogate problem (30) for optimal power allocation and vectoring (Algorithm 2, line 7) may correspond to one of the stationary points of the cost function (18) rather than the global optimum. In that scenario the outer loop (Algorithm 2, line 3) responsible for updating the weights to maximize the sustained rate may not converge. However, all simulations that have been carried out with a linear MMSE postcoder and with different user weights, converge to a per-group FDMA power allocation, with the active user having a transmit power equal to P_k^{mask}. A similar observation and statement has been made in [46].

To ensure convergence to the global optimum of the non-convex optimization problem (18) for a linear MMSE postcoder, an exhaustive search method is needed, which has a high computational complexity. However, all the solutions for power allocation (inner loop of Algorithm 2) in the sustained rate maximization problem (18) for a linear MMSE postcoder, follow a structure with non-zero allocated powers equal to the power mask and per-group FDMA. Therefore, the resulting power allocation for user n on tone k can be represented as $s_k^{U,n} = f_k^n P_k^{mask}$, where f_k^n is the FDMA decision variable for user n on tone k, which is either 1 or 0, representing whether tone k is allocated or not allocated to user n, respectively. It should be noted that typical DSM algorithms are based on maximizing the data rates under a (total) power constraint coupled across all the tones [47], which results in a continuous power allocation between $[0, P_k^{mask}]$ for a linear MMSE postcoder. However, in the defined sustained rate maximization problem (18), the tones are coupled only under the sustained rate constraint (18b). For a linear MMSE postcoder, it is observed that this results in power allocation which is either zero or P_k^{mask}.

The observed structure in the solutions allows for a low complexity exhaustive search, by substituting $s_k^{U,n} = f_k^n P_k^{mask}$. The sustained rate maximization problem in (18a) can be modified to perform an exhaustive search at lower complexity as

$$\underset{B^U, f_k^n}{\text{maximize}} \quad B^U \tag{34a}$$
$$\text{subject to} \quad B^U \leq \sum_k \breve{b}_k^{U,n}(\mathbf{f}_k) \quad \forall n, \tag{34b}$$
$$f_k^n \in \{0, 1\} \qquad \forall k, n, \tag{34c}$$
$$\sum_{n \in g} f_k^n \leq 1 \qquad \forall k, g \tag{34d}$$

where, $\breve{b}_k^{U,n}$ is a function of $\mathbf{f}_k \triangleq [f_k^1, f_k^2, \ldots, f_k^N]$ given as

$$\breve{b}_k^{U,n} = \log_2\left(1 + \frac{S\breve{I}NR_k^{U,n}}{\Gamma}\right), \tag{35a}$$

$$S\breve{I}NR_k^{U,n} = f_k^n \tilde{\mathbf{h}}_k^{U,n^\dagger}\left(\sum_{m \neq n} f_k^m \tilde{\mathbf{h}}_k^{U,m}\left(\tilde{\mathbf{h}}_k^{U,m}\right)^\dagger + \frac{\boldsymbol{\phi}_k^U}{P_k^{mask}}\right)^{-1} \tilde{\mathbf{h}}_k^{U,n}. \tag{35b}$$

With the dual problem formulation, the dual primary problem of (34a) is given as

$$\begin{aligned} &\underset{\mathbf{w}}{\text{minimize}} \quad q(\mathbf{w}) \\ &\text{subject to} \quad \mathbf{w} \geq 0 \end{aligned} \tag{36}$$

where, $q(\mathbf{w})$ is the Lagrange dual function defined as

$$\begin{aligned} q(\mathbf{w}) = &\underset{B^U, f_k^n}{\text{maximize}} \quad B^U\left(1 - \sum_n w^n\right) + \sum_k \underbrace{\sum_n w^n \breve{b}_k^{U,n}}_{\mathcal{L}_k} \quad \forall k \\ &\text{subject to} \quad f_k^n \in \{0, 1\} \quad \forall k, n, \\ &\qquad \sum_{n \in g} f_k^n \leq 1 \quad \forall k, g\,. \end{aligned} \tag{37}$$

Similar to (23), B^U is a free variable and a non-trivial solution to (37) exists only if $(1 - \sum_n w^n) = 0$. With this constraint, the Lagrangian can again be decoupled over tones $\mathcal{L}_k$, defined in (37). Finally, the dual secondary problem for (34) for tone k can be written as

$$\begin{aligned} q_k(\mathbf{w}) = &\underset{f_k^n}{\text{maximize}} \quad \mathcal{L}_k\left(\mathbf{w}, f_k^n\right) \\ &\text{subject to} \quad f_k^n \in \{0, 1\} \quad \forall n, \\ &\qquad \sum_{n \in g} f_k^n \leq 1 \quad \forall g\,. \end{aligned} \tag{38}$$

$$a_k^n = log(2)^{-1}\left(\bar{\mathbf{h}}_k^{U,n}\right)^\dagger\left(\sum_{m<n} \frac{w^m s_k^{U,m}\left(\boldsymbol{\Psi}_k^{U,m}\right)^{-1}\bar{\mathbf{h}}_k^{U,m}\left(\bar{\mathbf{h}}_k^{U,m}\right)^\dagger\left(\boldsymbol{\Psi}_k^{U,m}\right)^{-1}}{\Gamma + s_k^{U,m}\left(\bar{\mathbf{h}}_k^{U,m}\right)^\dagger\left(\boldsymbol{\Psi}_k^{U,m}\right)^{-1}\bar{\mathbf{h}}_k^{U,m}}\right)\bar{\mathbf{h}}_k^{U,n} \tag{33}$$

Algorithm 3 Exhaustive Search Based Optimal FDMA for Linear MMSE Postcoder

Input: User groups

Initialisation $\mathbf{w} \in \mathbb{R}_+^N$, $\mu > 0$
while $w^n > \delta_w$ & $|B^U - \sum_k \breve{b}_k^{U,n}| > \delta_R$ for any n **do**
  for $k = 1 \cdots K$ **do**
    $\left\{\mathbf{f}_k, \breve{\mathbf{b}}_k^U\right\} \leftarrow$ ExhSrch_f_k($\mathbf{w}$, k)
  end for
  $B^U = \min_n \left\{\sum_k \breve{b}_k^{U,n}\right\}$
  $\mathbf{w} \leftarrow$ Optimize_w($\mathbf{w}$, B^U, $\breve{\mathbf{b}}^{U,n}$, μ)
end while
function ExhSrch_f_k($\mathbf{w}$, k)
  Initialize $\mathcal{L}_k^{opt} \leftarrow -\inf$
  for all $\mathbf{f}_k \in \mathbb{F}$ **do**
    Calculate $\breve{\mathbf{b}}_k^U$ using (35)
    Calculate $\mathcal{L}_k$ using (37)
    if $\mathcal{L}_k > \mathcal{L}_k^{opt}$ **then**
      $\mathcal{L}_k^{opt} \leftarrow \mathcal{L}_k$, $\mathbf{f}_k^{opt} \leftarrow \mathbf{f}_k$, $\breve{\mathbf{b}}_k^{U^{opt}} \leftarrow \breve{\mathbf{b}}_k^U$
    end if
  end for
end function
function Optimize_w($\mathbf{w}$, B^U, $\breve{\mathbf{b}}^{U,n}$, μ)
  for $n = 1 \cdots N$ **do**
    $\acute{w}^n = \left[w^n + \mu\left(B^U - \sum_k \breve{b}_k^{U,n}\right)\right]_0$
  end for
  $w^n = \frac{\acute{w}^n}{\sum_n \acute{w}^n} \quad \forall n$
end function

An exhaustive search is performed for the maximization of the Lagrangian over a search grid of possible $\mathbf{f}_k$ values, for a fixed set of Lagrange multipliers ($\mathbf{w}$), for each tone separately. The search grid ($\mathbb{F}$) contains all possible combinations of f_k^n such that the decision variable f_k^n is 1 for only one user in each group for each tone. The maximum value of B^U is defined as $B^U = \min_n\{\sum_k \breve{b}_k^{U,n}\}$. For the dual primary problem (36), a subgradient update approach is used to converge to the globally optimal value of the Lagrange multipliers ($\mathbf{w}$). **Algorithm 3** summarizes the proposed exhaustive search based optimal power allocation and vectoring based per-group FDMA for a linear MMSE postcoder.

Finally, a linear zero-forcing (ZF) postcoder can be used to further reduce the computational complexity. With linear ZF postcoder the SINR expression for user n on tone k can be written as

$$\breve{SINR}_k^{U,n} = \frac{f_k^n P_k^{mask}}{\left\|\left[\mathfrak{S}_k^U\left(\widetilde{\mathbf{H}}_k^U\right)^{-1}\mathfrak{S}_k^{U\top}\right]_{\text{row n}}\right\|_2^2 [\boldsymbol{\phi}_k^U]_{g\ni n, g\ni n}} \tag{39}$$

where, $\widetilde{\mathbf{H}}_k^U = \mathcal{G}_k^U \mathbf{H}_k^U \mathfrak{S}_k^U$ which represents the equivalent US P2MP channel matrix ($\mathbf{H}_k^U$) for tone k following the removal of $N - G$ zero columns and $[\boldsymbol{\phi}_k^U]_{g\ni n, g\ni n}$ represents the diagonal component of the noise covariance matrix $\boldsymbol{\phi}_k^U$ at position g corresponding to the group g to which user n has been assigned. Moreover, the FDMA decision variable f_k^n is related to the selection matrix $\mathfrak{S}_k^U$, such that for each set of FDMA decision variable $\mathbf{f}_k \in \mathbb{R}^N$ for tone k, that satisfies (34c) and (34d), a corresponding selection matrix $\mathfrak{S}_k^U$ exists. The changes can be made in Algorithm 3 accordingly, in the calculation of $\breve{b}_k^{U,n}$, to accommodate the linear ZF postcoder.

2) INDEPENDENT PER-GROUP FDMA ALLOCATION

The computational complexity of the optimal power allocation and vectoring based per-group FDMA can be further reduced by considering an independent per-group FDMA allocation, that maximizes the sustained rate for each group separately. In this way, the optimization problem can be solved independently for each group. This low-complexity optimization problem solely optimizes the tone allocation and assumes that the transmit power of all the users is set to the power spectral density (PSD) mask defined in the standardisation and that there is no crosstalk (FEXT, i.e., all crosstalk channels are assumed to be zero). Without loss of generality, the optimization problem for group g is then defined as

$$\underset{B^{U,g}, f_k^n}{\text{maximize}} \quad B^{U,g} \tag{40a}$$

$$\text{subject to} \quad B^{U,g} \leq \sum_k f_k^n \cdot \tilde{b}_k^{U,n} \quad \forall n \in g, \tag{40b}$$

$$f_k^n \in \{0, 1\} \quad \forall n \in g, \forall k, \tag{40c}$$

$$\sum_{n \in g} f_k^n \leq 1 \quad \forall k \tag{40d}$$

where $B^{U,g}$ represents the sustained rate for users in group g, $\tilde{b}_k^{U,n}$ represents the precomputed direct data rate for user n on tone k (under the above assumptions), and f_k^n represents the FDMA decision variable for user n on tone k. The constraint (40b) makes sure that all the users in group g will attain a higher data rate than the sustained rate $B^{U,g}$ for that group. The constraints on f_k^n in (40c) and (40d) ensure that only one user in group g is active on a tone.

To solve (40a) a dual problem is formulated. Since the problem is convex with linear inequality constraints (except (40c), which is dealt with later in the dual secondary problem (44)), the Slater's condition holds true, thus strong duality exists between the primal and the dual problem. The dual primary problem is defined as

$$\begin{aligned} &\underset{\mathbf{w}^g}{\text{minimize}} \quad q^g\left(\mathbf{w}^g\right) \\ &\text{subject to} \quad \mathbf{w}^g \geq 0 \end{aligned} \tag{41}$$

where, $q^g(\mathbf{w}^g)$ is the Lagrange dual function for group g and $\mathbf{w}^g \in \mathbb{R}^{N_g}$ is a sub-vector of $\mathbf{w} \triangleq [w^1, \ldots, w^N] \in \mathbb{R}^N$ containing Lagrange multipliers (w^n) only for $n \in g$ with N_g representing the number of users assigned to group g. The dual secondary problem maximizes the Lagrangian over the decision variables and is given as

$$q^g\left(\mathbf{w}^g\right) = \underset{B^{U,g}, f_k^n \ \forall n \in g, \forall k}{\text{maximize}} \mathcal{L}^g\left(\mathbf{w}^g, B^{U,g}, f_k^n\right). \tag{42}$$

The Lagrangian $\mathcal{L}^g$ is defined in (43). It is noted that only the constraint on the data rates (40b) is considered when defining the Lagrangian. The constraints on f_k^n (40c) and (40d) are applied separately to the dual secondary problem later in (44).

$$\mathcal{L}^g\left(\mathbf{w}^g, B^{U,g}, f_k^n\right) = B^{U,g}\left(1 - \sum_{n\in g} w^n\right) + \sum_k \underbrace{\left(\sum_{n\in g} w^n f_k^n \tilde{b}_k^{U,n}\right)}_{\mathcal{L}_k^g(\mathbf{w}^g, f_k^n)}. \quad (43)$$

The Lagrangian involves the term $B^{U,g}$ which is a free variable and similar to (23) a non-trivial solution exists only if $(1 - \sum_{n\in g} w^n) = 0$. With this constraint, a dual decomposition [34] can indeed be considered in order to decompose the dual secondary problem (42) into independent per-tone problems. The Lagrangian decoupled over tones ($\mathcal{L}_k^g$) is defined in (43). Finally, the dual secondary problem of (40a) for tone k can be written as

$$\begin{aligned} q_k^g\left(\mathbf{w}^g\right) = \underset{f_k^n,\, \forall n\in g}{\text{maximize}} \quad & \mathcal{L}_k^g\left(\mathbf{w}^g, f_k^n\right) \\ \text{subject to} \quad & f_k^n \in \{0, 1\} \quad \forall n \in g, \\ & \sum_{n\in g} f_k^n \leq 1\,. \end{aligned} \quad (44)$$

For the maximization of the Lagrangian in the dual secondary problem, since the constraints are discrete, an exhaustive search is done over f_k^n $\forall n \in g$, for a fixed set of Lagrange multipliers ($\mathbf{w}^g$), for each tone separately. The number of possible combinations to be searched is equal to the group size. Since the group size in P2MP transmission is generally small, the exhaustive search has a low complexity. The maximum value of $B^{U,g}$ is defined as $B^{U,g} = \min_{n\in g}\{\sum_k f_k^n \cdot \tilde{b}_k^{U,n}\}$. It is worth mentioning that the exhaustive search in the dual secondary problem can be avoided by changing the discrete constraint in (40c) to $f_k^n \geq 0$ $\forall n \in g, \forall k$ and including the constraints over f_k^n in the definition of the Lagrangian itself.

The dual primary problem (41) is convex over the Lagrange multipliers $\mathbf{w}^g$, thus a subgradient update approach can be used to converge to the global optimum. **Algorithm 4** summarizes the proposed independent per-group FDMA allocation, where $\tilde{\mathbf{b}}_k^{U,g} \in \mathbb{R}^{N_g}$ and $\mathbf{f}_k^g \in \mathbb{R}^{N_g}$ is a sub-vector of $\tilde{\mathbf{b}}_k^U \triangleq \left[\tilde{b}_k^{U,1}, \ldots, \tilde{b}_k^{U,N}\right]^{\mathsf{T}} \in \mathbb{R}^N$ and $\mathbf{f}_k \triangleq [f_k^1, \ldots, f_k^N]^{\mathsf{T}}$, respectively, containing precomputed direct data rates ($\tilde{b}_k^{U,n}$) and FDMA decision variables (f_k^n) only for $n \in g$.

The independent per-group FDMA allocation, uses static power allocation (PSD mask) and depends on the precomputed direct data rates ($\tilde{b}_k^{U,n}$) without considering the crosstalk (FEXT) and hence the vectoring. Therefore, while Algorithm 4 has a low complexity, it is also suboptimal.

Algorithm 4 Independent Per-Group FDMA Allocation

for each group g **do**
  Input: $\tilde{\mathbf{b}}_k^{U,g}$
  Initialisation $\mathbf{w}^g \in \mathbb{R}_+^{N_g}$, $\mu > 0$
  while $w^n > \delta_w$ & $|B^{U,g} - \sum_k f_k^n \cdot \tilde{b}_k^{U,n}| > \delta_R$ for any $n \in g$ **do**
    for $k = 1 \cdots K$ **do**
      $\left[\mathbf{f}_k^g\right] \leftarrow$ OPTIMIZE_f_k($\mathbf{w}^g$, k, $\tilde{\mathbf{b}}_k^{U,g}$)
    end for
    $B^{U,g} = \min_{n\in g}\left\{\sum_k f_k^n \cdot \tilde{b}_k^{U,n}\right\}$
    $\mathbf{w}^g \leftarrow$ OPTIMIZE_w($\mathbf{w}^g$, $B^{U,g}$, $\mathbf{f}_k^g$, $\tilde{\mathbf{b}}_k^{U,g}$, μ)
  end while
end for
function OPTIMIZE_f_k($\mathbf{w}^g$, k, $\tilde{\mathbf{b}}_k^{U,g}$)
  Initialize $\mathcal{L}_k^{g,opt} \leftarrow -\inf$
  for $n \in g$ **do**
    $\mathbf{f}_k^g \leftarrow 0$, $f_k^n \leftarrow 1$
    Calculate $\mathcal{L}_k^g$ using (43)
    if $\mathcal{L}_k^g > \mathcal{L}_k^{g,opt}$ **then**
      $\mathcal{L}_k^{g,opt} \leftarrow \mathcal{L}_k^g$, $\mathbf{f}_k^{g,opt} \leftarrow \mathbf{f}_k^g$
    end if
  end for
end function
function OPTIMIZE_w($\mathbf{w}^g$, $B^{U,g}$, $\mathbf{f}_k^g$, $\tilde{\mathbf{b}}_k^{U,g}$, μ)
  for $n \in g$ **do**
    $\acute{w}^n = \left[w^n + \mu\left(B^{U,g} - \sum_k f_k^n \cdot \tilde{b}_k^{U,n}\right)\right]_0$
  end for
  $w^n = \frac{\acute{w}^n}{\sum_{n\in g} \acute{w}^n}$ $\quad \forall n \in g$
end function

B. DOWNSTREAM TRANSMISSION

1) OPTIMAL POWER ALLOCATION AND VECTORING BASED PER-GROUP FDMA

For the optimal FDMA allocation for the DS scenario, a problem similar to (18) in US, can be defined. However, in US with the (linear or non-linear) MMSE postcoder, the SINR and hence the bitrate can be written as a function of the transmit powers ($s_k^{U,n}$) only. This is not a case in the DS scenario with a (linear or non-linear) MMSE precoder, where the SINR and hence the bitrate is a function of both the precoder matrix ($\mathbf{R}_k^D$) and the transmit powers ($s_k^{D,n}$). Hence, the equivalent of the sustained rate maximization problem in (18), for the DS scenario has both the precoder matrix and the transmit powers as the decision variables:

$$\begin{aligned} \underset{B^D, s_k^{D,n}, \bar{\mathbf{r}}_k^{D,n}}{\text{maximize}} \quad & B^D \\ \text{subject to} \quad & \text{Sustained rate constraints,} \\ & \text{Power constraints.} \end{aligned} \quad (45)$$

A conventional way to solve this problem is by using MAC-BC duality [48], [49], which states that it is possible to achieve the same set of data rates in the MAC and BC under the same total power constraint [50]. Hence, instead of directly solving (45) for the DS, its dual US problem is solved, which is only a function of the US transmit powers [51], [52], and the optimal transmit powers in the DS can then be derived from the US transmit powers. However, finding the optimal FDMA through power allocation in DS P2MP transmission using MAC - BC duality is not efficient, due to the complexity involved, in particular due to the optimal power allocation and sustained rate constraints. Specially for the linear receive structure, where an exhaustive search is required for each possible combination of the decision variables contained in the search grid $\mathbb{F}$, within each possible combination of the decision variables, a dual MAC problem is to be solved and the corresponding optimal BC precoding matrix is to be calculated for each tone.

Hence, instead of using a (linear or non-linear) MMSE precoder, a (linear or non-linear) ZF precoder is used in the DS scenario. Furthermore, in the US scenario for the measured channels representing a practical use-case scenario, simulations confirm that the difference in the data rates achieved by all the users by solving the sustained rate maximization problem in (18) for MMSE based postcoder and ZF based postcoder, is marginal. With the MAC-BC duality, a similar behaviour can be expected in the DS scenario and hence this justifies the use of a ZF precoder.

The **non-linear ZF precoder** can be seen as a multi-user extension of the Tomlinson-Harashima precoder (THP) [20], which is a low-complexity and efficient implementation of DPC. THP relies on the QR factorization of the channel matrix:

$$\widetilde{\mathbf{H}}_k^{D^\dagger} = \mathbf{Q}_k^{THP}\mathbf{E}_k^{THP} \tag{46}$$

where, $\widetilde{\mathbf{H}}_k^{D^\dagger} = \mathfrak{S}_k^D \mathbf{H}_k^{D^\dagger} \mathcal{G}_k^D$ which represents the equivalent DS P2MP channel matrix ($\bar{\mathbf{H}}_k^{D^\dagger}$) for tone k following the removal of $N-G$ zero rows, $\mathbf{Q}_k^{THP}$ is a unitary matrix and $\mathbf{E}_k^{THP}$ is an upper triangular matrix. At the transmitter side, a feedback loop is employed based on matrix $\mathbf{E}_k^{THP}$ followed by the feed- forward matrix $\mathbf{Q}_k^{THP}$, to generate the transmit signals. The spectral mask compliance is maintained by applying modulus operations within the feedback loop. At the receiver side, the received signals are scaled by the noise enhancement factors $[\mathbf{E}_k^{THP}]_{nn}$, followed by a modulus operation to generate the estimated signal. With the THP, the SINR for the user n on tone k is given as

$$SINR_k^{D,n,THP} = \frac{\left|\left[\mathfrak{S}_k^{D^\mathsf{T}} \mathbf{E}_k^{THP} \mathfrak{S}_k^D\right]_{nn}\right|^2 s_k^{D,n}}{\sigma_k^D} \tag{47}$$

which is only a function of user transmit power $s_k^{D,n}$. Hence, the achieved data rate ($b_k^{D,n}$) for user n on tone k, as defined in (14), is solely a function of its transmit power $s_k^{D,n}$. The sustained rate maximization problem for DS P2MP transmission can be written as

$$\begin{aligned}
&\underset{B^D, s_k^{D,n}, f_k^n}{\text{maximize}} \quad B^D && \text{(48a)}\\
&\text{subject to} \quad B^D \le \sum_k b_k^{D,n}\left(s_k^{D,n}\right) \quad \forall n, && \text{(48b)}\\
&\qquad f_k^n \in \{0,1\} \quad \forall k, n, && \text{(48c)}\\
&\qquad \sum_{n\in g} f_k^n \le 1 \quad \forall k, g, && \text{(48d)}\\
&\qquad \sum_{n\in g}\sum_k f_k^n s_k^{D,n} \le P_g \quad \forall g, && \text{(48e)}\\
&\qquad s_k^{D,n} \ge 0 \quad \forall k, n && \text{(48f)}
\end{aligned}$$

where B^D represents the sustained rate, and f_k^n represents the FDMA decision variable for user n on tone k. The FDMA decision variable f_k^n is related to the selection matrix $\mathfrak{S}_k^D$, such that for each set of FDMA decision variable $\mathbf{f}_k \in \mathbb{R}^N$ for tone k, that satisfies (48c) and (48d), a corresponding selection matrix $\mathfrak{S}_k^D$ exists. (48e) represents the total group power constraint for each group, which provides an upper bound to the achievable constrained sustained rate in the DS P2MP transmission. Since in DS P2MP transmission the users in a group transmit the same signal, each group can be represented by a virtual line. P_g refers to the total available power budget for all the users in a group for P2MP transmission. Intuitively, it is similar to the total power constraint for a line in a point-to-point (P2P) scenario ($P_g = \sum_k P_k^{mask}$). The constraint applied to that virtual line is hence applied to all the users within that group (due to the repetition matrix $\mathcal{G}_k^D$), which leads to the total group power constraint for each group as an equivalent version of sum power constraint for each line in a P2P scenario.

With the dual problem formulation, the dual primary problem of the cost function (48a), with the sustained rate constraint (48b) and the total group power constraint (48e) is given as

$$\begin{aligned}
&\underset{\mathbf{w},\boldsymbol{\lambda}}{\text{minimize}} \quad q(\mathbf{w},\boldsymbol{\lambda})\\
&\text{subject to} \quad \mathbf{w},\boldsymbol{\lambda} \ge 0
\end{aligned} \tag{49}$$

where, $q(\mathbf{w},\boldsymbol{\lambda})$ is the Lagrange dual function with $\mathbf{w}$ and $\boldsymbol{\lambda}$ as the vectors containing the Lagrange multipliers $\mathbf{w} \triangleq [w^1,\ldots,w^N]$ and $\boldsymbol{\lambda} \triangleq [\lambda_1,\ldots,\lambda_G]$ associated with the sustained rate constraint (48b) and the total group power constraint (48e), respectively. The Lagrange dual function ($q(\mathbf{w},\boldsymbol{\lambda})$) is defined as

$$\begin{aligned}
&q(\mathbf{w},\boldsymbol{\lambda}) = \underset{B^D, f_k^n, s_k^{D,n}}{\text{maximize}} \quad \mathcal{L}\left(\mathbf{w},\boldsymbol{\lambda},B^D,f_k^n,s_k^{D,n}\right) && \text{(50a)}\\
&\qquad \text{subject to} \quad f_k^n \in \{0,1\} \quad \forall k,n, && \text{(50b)}\\
&\qquad \sum_{n\in g} f_k^n \le 1 \quad \forall k, g && \text{(50c)}
\end{aligned}$$

where, $\mathcal{L}$ is the Lagrangian function, which is defined as

$$\mathcal{L} = B^D\left(1 - \sum_n w^n\right) + \sum_k \left(\underbrace{\sum_n w^n b_k^{D,n} - \sum_g \lambda_g \sum_{m\in g} f_k^m s_k^{D,m}}_{\mathcal{L}_k\left(\mathbf{w},\boldsymbol{\lambda},f_k^n,s_k^{D,n}\right)} \right). \quad (51)$$

Similar to (23), B^D is a free variable and a non-trivial solution exists only if $(1 - \sum_n w^n) = 0$. With this constraint, the Lagrangian function can be maximized for a fixed set of Lagrange multipliers $(\mathbf{w}, \boldsymbol{\lambda})$, by independently maximizing $\mathcal{L}_k$ for each tone k separately. The obtained Lagrangian function $\mathcal{L}_k$ is similar to the Lagrangian of maximising a weighted sum rate (WSR) cost function with total group power constraint (48e). Since the cost function is concave and the total group power constraint (48e) forms a convex set, the KKT conditions are sufficient for optimality. By the KKT stationarity condition we have that

$$\frac{\partial}{\partial s_k^{D,n}}\left(\sum_n w^n b_k^{D,n} - \sum_g \lambda_g \sum_{m\in g} f_k^m s_k^{D,m}\right) = 0. \quad (52)$$

The closed form expression for transmit power $s_k^{D,n}$ can be obtained by isolating $s_k^{D,n}$ on one side, leading to

$$s_k^{D,n} = \begin{cases} \left[\frac{w^n}{\lambda_{g\ni n}} - \left(\frac{\left|\left[\mathfrak{S}_k^{D\intercal}\mathbf{E}_k^{THP}\mathfrak{S}_k^D\right]_{nn}\right|^2}{\Gamma\cdot\sigma_k^D}\right)^{-1}\right]_0, & \text{if } f_k^n = 1 \\ 0, & \text{if } f_k^n = 0 \end{cases} \quad (53)$$

where, $\lambda_{g\ni n}$ represents λ_g corresponding to the group containing user n. Finally the dual secondary function for (48a), with $\mathcal{L}_k$ in (51) as Lagrangian, for tone k is defined as

$$\begin{aligned} q_k(\mathbf{w}, \boldsymbol{\lambda}) = \underset{s_k^{D,n},\, f_k^n}{\text{maximize}} \quad & \sum_n w^n b_k^{D,n} - \sum_g \lambda_g \sum_{m\in g} f_k^m s_k^{D,m} \\ \text{subject to} \quad & f_k^n \in \{0, 1\} \quad \forall n, \\ & \sum_{n\in g} f_k^n \le 1 \quad \forall g. \end{aligned} \quad (54)$$

The maximum value of B^D is defined as $B^D = \min_n\{\sum_k b_k^{D,n}\}$. For the dual primary problem (49), a sub-gradient update approach is used to converge to the global optimal value of the Lagrange multipliers $(\mathbf{w}, \boldsymbol{\lambda})$. The procedure is summarized in **Algorithm 5**.

Finally, for a **linear ZF precoder**, the SINR expression for user n on tone k in (47) can be written as

$$SINR_k^{D,n,ZF} = \frac{s_k^{D,n}}{\beta_k^2 \sigma_k^D} \quad (55)$$

where, β_k is the scaling factor for tone k, defined as

$$\beta_k \triangleq \max_n \left\| \left[\left(\widetilde{\mathbf{H}}_k^{D^\dagger}\right)^{-1}\right]_{\text{row n}} \right\|_2. \quad (56)$$

Algorithm 5 Optimal Power Allocation With Non-Linear ZF Precoder Based Per-Group FDMA

Input: User groups

Initialisation $\mathbf{s}_k^D \in \mathbb{R}_+^N\ \forall k$, $\mathbf{w} \in \mathbb{R}_+^N$, $\boldsymbol{\lambda} \in \mathbb{R}_+^G$, $\mu > 0$, $\eta > 0$
while $w^n > \delta_w$ & $|B^D - \sum_k b_k^{D,n}| > \delta_R$ for any n **do**
    $\{\mathbf{s}_k^D, \mathbf{b}_k^D\ \forall k\} \leftarrow$ OPTIMIZE_s($\mathbf{s}_k^D\ \forall k$, $\boldsymbol{\lambda}$, $\mathbf{w}$, η)
    $B^D = \min_n\left\{\sum_k b_k^{D,n}\right\}$
    $\mathbf{w} \leftarrow$ OPTIMIZE_w($\mathbf{w}$, B^D, $\mathbf{b}^{D,n}$)
end while
function OPTIMIZE_s($\mathbf{s}_k^D\ \forall k$, $\boldsymbol{\lambda}$, $\mathbf{w}$, η)
    while $\lambda_g > \delta_\lambda$ & $|\sum_{n\in g}\sum_k s_k^{D,n} - P_g| > \delta_s$ **do**
        for $k = 1 \cdots K$ **do**
            $\{\mathbf{f}_k, \mathbf{b}_k^D, \mathbf{s}_k^D\} \leftarrow$ EXHSRCH($\mathbf{w}$, $\boldsymbol{\lambda}$, k, η)
        end for
        $\lambda_g = \left[\lambda_g + \eta\left(\sum_{n\in g}\sum_k s_k^{D,n} - P_g\right)\right]_0$, $\forall g$
    end while
end function
function EXHSRCH($\mathbf{w}$, $\boldsymbol{\lambda}$, k, η)
    Initialize $\mathcal{L}_k^{opt} \leftarrow -\inf$
    for all $\mathbf{f}_k \in \mathbb{F}$ **do**
        Calculate $s_k^{D,n}$ using (53) for all users n
        Calculate $\mathbf{b}_k^D$ using (47) and (13)
        Calculate $\mathcal{L}_k$ using (51)
        if $\mathcal{L}_k > \mathcal{L}_k^{opt}$ **then**
            $\mathcal{L}_k^{opt} \leftarrow \mathcal{L}_k$, $\mathbf{f}_k^{opt} \leftarrow \mathbf{f}_k$, $\mathbf{b}_k^{D^{opt}} \leftarrow \mathbf{b}_k^D$
        end if
    end for
end function
function OPTIMIZE_w($\mathbf{w}$, B^D, $\mathbf{b}^{D,n}$)
    for $n = 1 \cdots N$ **do**
        $\acute{w}^n = \left[w^n + \mu\left(B^D - \sum_k b_k^{D,n}\right)\right]_0$
    end for
    $w^n = \frac{\acute{w}^n}{\sum_n \acute{w}^n}$ $\forall n$
end function

The precoder matrix for tone k is defined as

$$\mathbf{R}_k^{D,ZF} = \frac{1}{\beta_k}\left(\widetilde{\mathbf{H}}_k^{D^\dagger}\right)^{-1}. \quad (57)$$

Since the precoder matrix for the linear ZF precoder is not unitary, the transmit symbol power for user n on tone k, following (10) is given as

$$\tilde{s}_k^{D,n} = \sum_m \left|\left[\mathfrak{S}_k^{D\intercal}\mathbf{R}_k^{D,ZF}\mathfrak{S}_k^D\right]_{nm}\right|^2 s_k^{D,m}. \quad (58)$$

The sustained rate maximization problem for the DS P2MP defined in (48) can be applied for linear ZF precoder. Considering the optimization problem (48) and (55)–(58), the Lagrangian function for each tone is hence given as

$$\mathcal{L}_k = \sum_n w^n b_k^{D,n} - \sum_g \lambda_g \sum_{m\in g}\sum_l \left|\left[\mathfrak{S}_k^{D\intercal}\mathbf{R}_k^{D,ZF}\mathfrak{S}_k^D\right]_{ml}\right|^2 f_k^l s_k^{D,l}. \quad (59)$$

Again by applying the KKT stationarity condition to the Lagrangian function, the optimal transmit power before precoding is given as

$$s_k^{D,n} = \begin{cases} \left[\dfrac{w^n}{\sum_g \lambda_g \sum_{m \in g} f_k^m \left| \left[\mathfrak{S}_k^{D\top} \mathbf{R}_k^{D,ZF} \mathfrak{S}_k^D \right]_{mn} \right|^2} - \beta_k^2 \Gamma \sigma_k^D \right]_0, & \text{if } f_k^n = 1 \\ 0, & \text{if } f_k^n = 0. \end{cases} \tag{60}$$

The changes can be made in Algorithm 5 accordingly to accommodate the linear ZF precoder with the total group power constraint for each group.

2) INDEPENDENT PER-GROUP FDMA ALLOCATION

The computational complexity of the optimal power allocation and vectoring based per-group FDMA can be further reduced by considering an independent per-group FDMA allocation, that maximizes the sustained rate for each group separately. Similar to Section IV-A2, this low-complexity optimization problem solely optimizes the tone allocation and assumes that the transmit power of all the users is set to the PSD mask defined in the standardisation and that there is no crosstalk (FEXT, i.e., all crosstalk channels are assumed to be zero). Without loss of generality, the optimization problem for group g is then defined as

$$\underset{B^{D,g}, f_k^n}{\text{maximize}} \quad B^{D,g} \tag{61a}$$

$$\text{subject to} \quad B^{D,g} \leq \sum_k f_k^n \cdot \tilde{b}_k^{D,n} \quad \forall n \in g, \tag{61b}$$

$$f_k^n \in \{0, 1\} \quad \forall n \in g, \forall k, \tag{61c}$$

$$\sum_{n \in g} f_k^n \leq 1 \quad \forall k \tag{61d}$$

where $B^{D,g}$ represents the sustained rate for users in group g, $\tilde{b}_k^{D,n}$ represents the precomputed direct data rate for user n on tone k, and f_k^n represents the FDMA decision variable for user n on tone k. Since, the optimization problem in (61) is similar to that of (40), with only $B^{D,g}$ and f_k^n as decision variables instead of $B^{U,g}$ and f_k^n. Algorithm 4 can be modified accordingly by simply replacing the precomputed direct data rates for US ($\tilde{b}_k^{U,n}$) with the precomputed direct data rates for DS ($\tilde{b}_k^{D,n}$).

C. HEURISTIC PER-GROUP FDMA ALLOCATION

Based on the solutions of the per-group FDMA allocation optimization problems for US and DS P2MP, a heuristic FDMA allocation strategy is derived. In contrast to the solutions discussed in Sections IV-A and IV-B, the proposed heuristic has negligible computational complexity. Moreover, the proposed heuristic is common for both US and DS P2MP transmission.

For heuristic per-group FDMA allocation, the sustained rate is maximized by initially defining a target sustained data rate ($R_{target} \in \mathbb{R}_+$). Within each group, the tone allocation is done from highest to lowest frequency tone, with a tone being allocated to the CPE with maximum achieved direct data rate on that tone $b_k^{U/D,n}$. Once the allocation of a tone is done in each group, the actual rate achieved by the active lines on that tone is computed with vectoring in place. If a CPE achieves the target sustained rate, it is excluded from further allocation of tones. At the end of the process, if tones are left unused in all the groups, the target sustained rate is increased, else if all the tones are allocated and some CPEs could not achieve the target sustained rate, it is decreased (bisection method). This process is iteratively repeated until all the tones are allocated and all the CPEs attain the target sustained rate. The procedure is summarized in **Algorithm 6**. The computational complexity of the algorithm is a linear function of the number of tones, within each iteration.

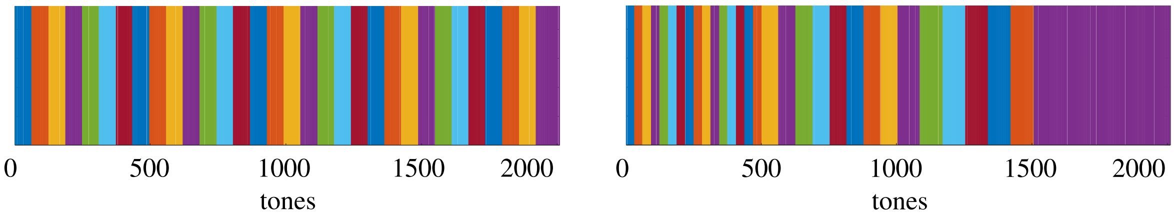
Algorithm 6 Heuristic Per-Group FDMA Allocation for Maximizing the Sustained Rate

Input: $\tilde{\mathbf{b}}_k^{U/D}$, User groups

Initialisation $R_{target}^{max} \in \mathbb{R}_+$, $R_{target}^{min} = 0$
while $|\sum_k b_k^{U/D,n} - R_{target}| > \delta_R$ for any n OR Unallocated tones $> \delta_{tones}$ **do**
  $R_{target} = \frac{\left(R_{target}^{max} + R_{target}^{min}\right)}{2}$
  for $k = K \cdots 1$ **do**
    Within each group allocate tone k to the available CPE with maximum direct data rate at that tone, $\forall$ groups.
    Recalculate actual data rates achieved on that tone for the allocated CPE with vectoring.
    if $|\sum_k b_k^{U/D,n} - R_{target}| < \delta_R$ for any CPE n **then**
      Remove CPE n from further tone allocation
    end if
  end for
  if $|\sum_k b_k^{U/D,n} - R_{target}| < \delta_R$ $\forall n$ & Unallocated tones $> \delta_{tones}$ **then**
    $R_{target}^{min} \leftarrow R_{target}$
  else
    $R_{target}^{max} \leftarrow R_{target}$
  end if
end while

The computational complexity of the proposed heuristic can be further significantly reduced by grouping consecutive tones into bands and then assigning bands to users instead of individual tones. In this paper, we suggest two ways to define bands:

- Uniform bands: The available consecutive tones are uniformly grouped into bands, as shown in Fig. 3(a). Hence, the number of tones within each band is given by (available tones)/(# bands).
- Non-uniform bands: In DSL channels the lower frequency tones offer lower attenuation compared to the higher frequency tones. Hence, a uniform band-rate based tone grouping leads to narrower bands at the lower frequencies and wider bands at higher frequencies (Fig. 3(b)). The band-rate for non-uniform



(a) Uniform tone grouping into bands

(b) Non-uniform tone grouping into bands

FIGURE 3. Grouping of available tones into 32 bands, with each pillar representing a band.

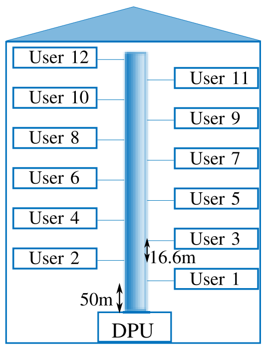


FIGURE 4. A DSL network with FTTB scenario, which is a possible use-case scenario for MGfast.

tone grouping can be heuristically defined based on the direct data rates achieved by a reference line ($\tilde{b}_k^{U/D,ref}$) with length equal to the average physical length of the lines present in the network:

$$\text{band-rate} = \frac{\sum_k \tilde{b}_k^{U/D,ref}}{\#\text{ bands}}. \quad (62)$$

The heuristic for per-group allocation of bands to users follows a similar approach as the allocation of tones, i.e., within each group, the band containing highest tones is allocated to the CPE with maximum achieved direct data rate on that band. Hence, Algorithm 6 can be modified for band allocations by simply replacing tones by bands. Hence, the computational complexity of the heuristic algorithm is reduced by a factor of (# tones)/(# bands).

It is noted that, as a modification, a part of the lower frequency tones (or bands) can be reserved to meet sudden traffic demands by users. In that case, line 4 of Algorithm 6 is modified to run over the unreserved tones or bands. This will allow the network to provide a higher data rate than the sustained rate to a user demanding higher traffic, by temporarily allocating reserved tones to that user, without any need for disturbing the tone or band allocation, and hence vectoring, for the other tones.

V. SIMULATION RESULTS

The simulations performed are based on the scenario shown in Fig. 4. The scenario resembles the typical use-case for P2MP transmission for xDSL networks with $N = 12$ users. The US and DS channel matrices for the scenario in Fig. 4 are based on measurement data provided by a tier-1 DSL operator. MGfast transmission up to a bandwidth of 106 MHz is considered for the simulations, because the channel data was restricted to 106 MHz. However, without loss of generality, the results can be extended to higher frequency ranges suggested for MGfast [6]. Table 1 summarizes the simulation parameters.

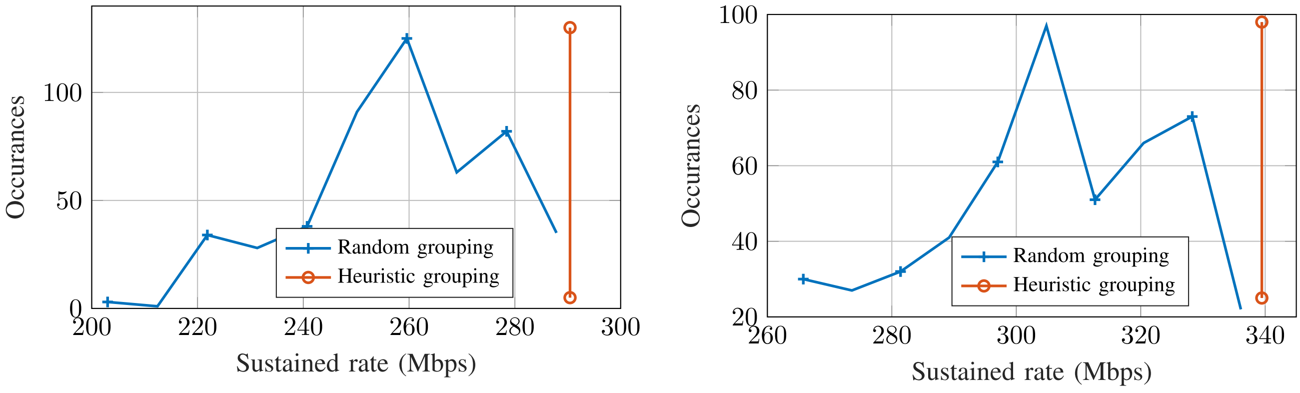
TABLE 1. Simulation parameters.

Parameter	Value
Maximum aggregated transmit power (P_g)	4 dBm
Subcarrier spacing	51.75 kHz
Number of tones (K)	2048
SNR gap (Γ)	10 dB
Applied bandwidth	106 MHz
Noise PSD	-145 dBm/Hz
Number of users (N)	12
Number of groups (G)	3

A. GROUPING STRATEGY

The comparison of the proposed grouping strategy (represented by the vertical bar) to a multitude of feasible grouping combinations, selected randomly (500 different feasible grouping combinations, with the number of groups and the number of CPEs per group kept constant), for both the US and DS scenarios is presented in Fig. 5 as a histogram. The performance metric is the minimal sustained rate achieved by all the users in the network. The sustained rate computation for each grouping combination in Fig. 5 is based on Algorithm 6, which is common for the US and DS scenarios. Similar results have been obtained based on the other sustained rate maximization methods discussed in previous sections. The simulations substantiate that the proposed grouping strategy provides significant improvement in terms of maximizing the sustained rate, compared to random grouping combinations, for both US and DS P2MP transmission scenarios.

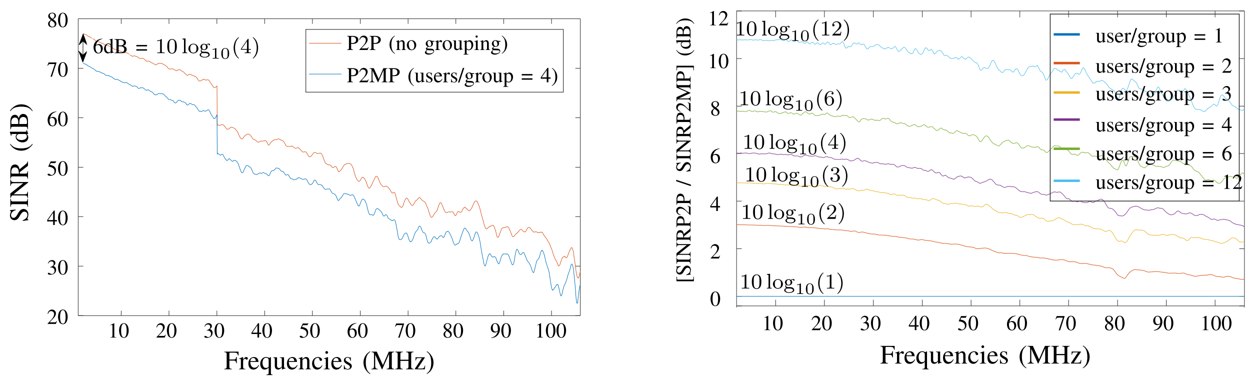
In US P2MP transmission, an addition of signals corresponding to the CPEs belonging to the same group is performed at the receiver, cfr. the grouping matrix $\mathcal{G}_k^U$ for tone k. This addition operation also adds the noises received in each group ($\bar{\mathbf{z}}_k^U = \mathcal{G}_k^U \mathbf{z}_k^U$), thus possibly decreasing the SNR. Fig. 6 shows the impact of grouping on the SINR for a user in US P2MP transmission with vectoring. It can be noticed that the addition operation in US P2MP decreases



(a) Upstream P2MP

(b) Downstream P2MP

FIGURE 5. Histogram for sustained rate vs grouping strategy for US and DS P2MP.



(a) SINR for a users in P2P and P2MP setup

(b) Ratio of P2P SINR and P2MP SINR for different users/group ratios

FIGURE 6. Effect of grouping for US P2MP.

the SNR by $10 \log_{10}(N/G)$ in the presence of vectoring. It is to be noted that the sudden drop in the SINR value at 30 MHz in Fig. 6(a) is due to the shape of the PSD mask defined in [4] for the 106a G.fast profile.

B. PER-GROUP FDMA ALLOCATION STRATEGY

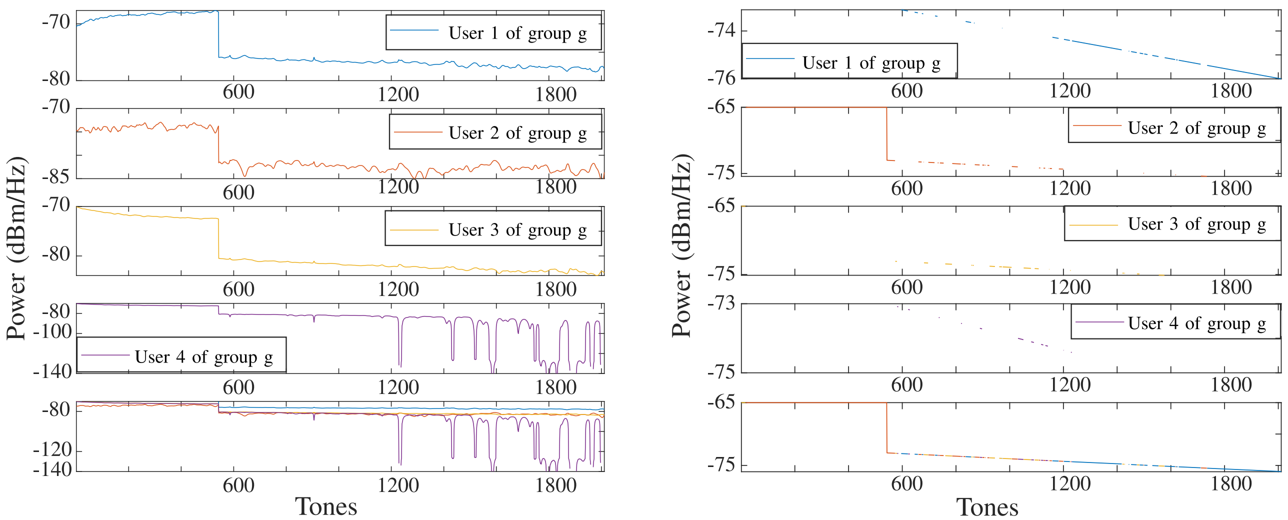
1) UPSTREAM TRANSMISSION

For US P2MP transmission, a sustained rate maximization problem was formulated in (18), for the optimal power allocation and vectoring based per-group FDMA. Fig. 7 shows the resulting optimal power allocation achieved with the Algorithm 2 for a group of 4 users. Fig. 7(a) shows that the optimal power allocation for a non-linear MMSE postcoder is non-FDMA, as discussed in Section IV-A1. This means that in a group, a tone is allocated to more than one user.

For a linear MMSE postcoder, extensive simulations are carried out with a large number of random Lagrange multipliers associated with the sustained rate constraint $\mathbf{w}$ in (18) and (19). For each set of Lagrange multipliers $\mathbf{w}$, the optimal power allocation is computed (Algorithm 2). It is observed that the optimal power allocation in a group is indeed an FDMA allocation. Moreover, Fig. 7(b) shows that the power allocation is equal to the power mask (P^{mask}). This observation lays the foundation for the low complexity exhaustive search based Algorithm 3. It is important to note that the abrupt decline observed at tone 542 (which corresponds to a 30 MHz frequency) in Fig. 7 is attributed, once again, to the PSD mask stipulated for transmission in [4] for the 106a G.fast profile, as utilized in the simulations.

Furthermore, it is noteworthy that from both subfigures 7(a) and 7(b), it can be deduced that the total sum of allocated powers for users within a group is equivalent to the prescribed PSD mask. This serves to illustrate that, irrespective of the power allocation achieved, be it FDMA or non-FDMA, the PSD mask constraint is consistently fulfilled within each group.

Fig. 8 compares the performance of a linear MMSE postcoder and linear ZF postcoder for US P2MP transmission. It can be noticed from Fig. 8 that the difference in data rates obtained for a linear MMSE postcoder and a linear ZF postcoder is marginal. This observation provides a motivation for the formulation of the optimization problem defined in (48). The marginal difference in performance between the linear MMSE postcoder and the linear ZF postcoder in



(a) Optimal power allocation in a group - non-linear MMSE postcoder

(b) Optimal power allocation in a group - linear MMSE postcoder

FIGURE 7. **Optimal power allocation in one group for a set of user weights for US P2MP with linear and non-linear postcoder.**

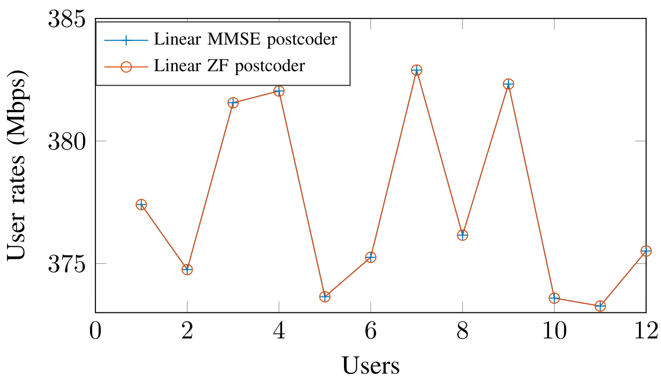


FIGURE 8. **Comparison of data rates achieved by users with linear MMSE postcoder and linear ZF postcoder in US P2MP.**

Fig. 8 can be explained by the ideal use-case scenarios for MGfast, such as FTTB, which are characterized by short loop lengths. The short loops typically have a high SNR associated with them, and hence the ZF postcoder is optimal [53], as postulated in Section IV-B. It can also be observed from the fluctuations in data rates in Fig. 8 that different users in the network achieve different data rates. This disparity is due to the different SINR experienced by different users, which is a function of allocated resources and channel characteristics.

Finally, the performance of the proposed heuristic strategy for the per-group FDMA allocation is compared with the independent per-group FDMA allocation, and the optimal power allocation and vectoring based per-group FDMA in Fig. 9(a). It demonstrates that the achieved sustained rate by the proposed heuristic strategy is very similar to the achieved sustained rates by the latter strategies, despite the significantly reduced computational complexity. In the considered simulation scenario, the sustained rate achieved by the heuristic strategy (Algorithm 6) is only 8% lower than the sustained rate achieved by the optimal power allocation and vectoring based per-group FDMA strategy (Algorithm 3).

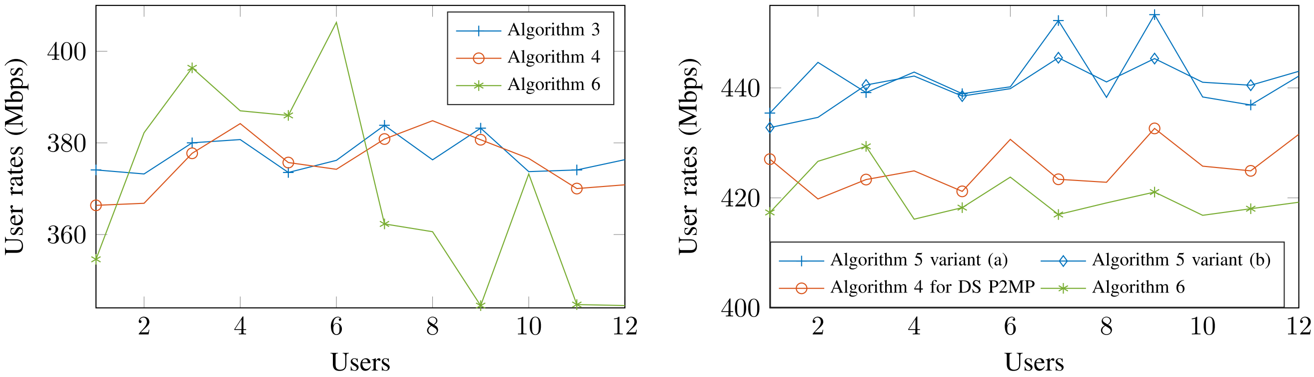
2) DOWNSTREAM TRANSMISSION

In Fig. 9(b), the performance of the proposed heuristic strategy is compared with the optimal per-group FDMA allocation discussed in Section IV-B. It can be observed that, similar to the US case, the heuristic strategy for DS P2MP also performs very close to the optimal solution while offering a significantly reduced computational complexity.

In both US (Fig. 9(a)) and DS (Fig. 9(b)) P2MP transmission, as depicted in Fig. 9, it becomes evident that the achievable data rates by users are not uniform. An optimal solution for the sustained rate maximization problem would ideally lead to a flat curve with the same achievable data rates by all users. However, in practice, the usable set of tones is finite and discrete, and thus, the ideal solution cannot be attained with precision. This can be observed in the achievable data rates by the optimal resource allocation algorithms in US (Algorithm 3) and DS (Algorithm 5) P2MP transmission, where the difference between the maximum and minimum achievable data rates $(\frac{max(rate)-min(rate)}{max(rate)})$ in the network is only 2.76% and 3.95% (2.85% for variant (b)), respectively, which is a small deviation from the ideal scenario (0%).

3) HEURISTIC PER-GROUP FDMA ALLOCATION

The performance of the proposed heuristic per-group FDMA allocation closely follows the performance of the optimal per-group FDMA allocation, for both US and DS P2MP transmission, as illustrated in Fig. 9. Fig. 10 compares the performance of the tone based heuristic allocation with the



(a) Rates achieved by users in US P2MP with Algorithm 4 (independent per-group FDMA allocation), Algorithm 3 (optimal power allocation and vectoring based per-group FDMA) and Algorithm 6 (heuristic per-group FDMA allocation).

(b) Rates achieved by users in DS P2MP with Algorithm 5 (optimal power allocation and vectoring based per-group FDMA with variant (a): non-linear ZF precoder and variant (b): linear ZF precoder), and Algorithm 6 (heuristic per-group FDMA allocation).

FIGURE 9. Comparison of data rates achieved by optimal per-group FDMA allocation algorithms with the heuristic algorithm.

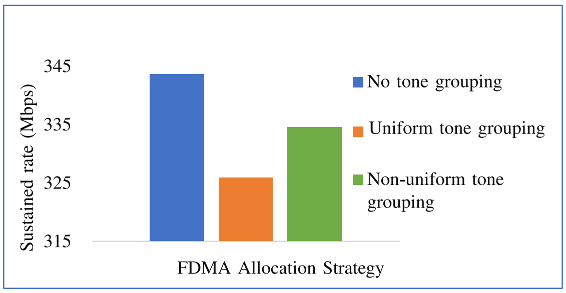


FIGURE 10. Sustained rate achieved by the proposed heuristic per-group FDMA allocation (Algorithm 6) for different tone grouping strategies for US P2MP.

band based heuristic allocation for uniform and non-uniform bands (with user 6 as the reference line). It can be observed that the heuristic with band allocation, especially with non-uniform bands, performs very similar to the heuristic with tone allocation, while offering a significant reduction in the computational complexity.

4) COMPUTATIONAL COMPLEXITY

Based on the simulation results and the discussion presented in this paper, it is evident that the heuristic per group FDMA allocation for maximizing the sustained rate performs comparably to the optimal solutions for P2MP distribution in next-generation DSL networks. Furthermore, to highlight the computational efficiency of the heuristic strategy (Algorithm 6), the runtime computational complexity is compared for the optimal and heuristic resource sharing strategies proposed in this paper, as shown in Table 2. The runtime computational complexity of the proposed algorithms was measured by running simulations in MATLAB on an Apple M2 Pro chip.

TABLE 2. Runtime computational complexity (in seconds) of proposed resource sharing algorithms.

Algorithm	Upstream scenario	Downstream scenario
Algorithm 3	617.48	—
Algorithm 4	20.08	13.06
Algorithm 5 (a) (and (b))	—	216.02 (202.53)
Algorithm 6	2.13	2.18

VI. CONCLUSION

This paper presents resource sharing strategies for P2MP distribution in the latest DSL ITU standard — MGfast. First, optimal solutions are presented for 1) the allocation of CPEs to MTU-Os (grouping) and 2) for the allocation of tones to the CPEs (per-group FDMA allocation). However, the computational complexity of calculating the optimal solutions is excessive. This poses a major challenge, especially given the dynamic behaviour of the network. Furthermore, to ensure QoS, the resource allocation must be done in minimum time, in particular, to have a low bandwidth allocation delay and traffic control convergence time. Thus, based on the analysis of the obtained optimal solutions, low complexity heuristic strategies have been proposed for both grouping and per-group FDMA allocation. The simulations show that, the proposed grouping strategy provides significant improvement in maximizing the sustained rate, compared to random grouping combinations. In terms of the achieved sustained rates, the performance of the proposed heuristic per-group FDMA allocation strategy is similar to the performance achieved by the optimal solutions for the constrained sustained rate maximisation. Finally, the computational complexity of the heuristic per-group FDMA allocation strategy is further reduced by proposing a (non-uniform) band based allocation, with marginal performance loss. Thus, the proposed heuristic solutions allow for a low complexity near-optimal implementation of P2MP in MGfast.

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