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An Innovative Method to Treat Very Small Apertures in Ray-Tracing Simulations

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ABSTRACT This work addresses a specific type of configurations where typical ray-tracing simulations can fail to produce meaningful results, namely when the light intensity transmitted by a very small aperture needs to be determined. In such cases it is indeed computational very expensive, if not prohibitive, to simulate all the rays that are needed in order to get a statistically significant number through the aperture. Our proposed solution exploits a virtual source to intrinsically limit the simulation only to those rays that do cross the aperture thus, massively enhancing the computational efficiency. We lay down the mathematical formalism behind the method and present it also in the form of a simple, generalizable algorithm. Finally, we provide an experimental test that confirms the validity of our proposal across a range of aperture sizes.

INDEX TERMS Ray-tracing, small apertures.

I. INTRODUCTION

THE RAY-TRACING method is a very useful and widespread tool to model the propagation of electromagnetic radiation under the ray-optics approximation that in general holds in the high-frequency limit [1]. It finds applications in a wide range of subjects such as the design of optical instruments [2], [3] and the determination of the signal-strength of wireless systems [4], [5]. In this work, we present a particular method to apply it also in such cases where the geometrical constraints are such that only an extremely small fraction of the available radiation is transmitted through a certain location and said small fraction is of interest in the considered application. This would be the case for instance when the optical system contains a radiation source but is part of a larger environment that shall be kept as much as possible in the dark, like, e.g., in spectroscopic applications, where leakages that can allow radiation from the source to the detector along unwanted paths shall be avoided. The assessment of the amount of stray-light in an optical system is in fact a very common application of ray-tracing in general [6], [7]. Other relevant cases include those where the transmission of small amounts of light is exploited to detect the presence of physical defects in mechanical structures. It is worth mentioning that the treatment of small structures and large differences in the spatial scale of the considered geometry is often challenging within ray-tracing because related techniques very often make use of regular grids [8] that struggle to handle such situations.

In this paper, we will first present the mathematical formulation of the method, discussing its expected limits of validity based on the assumptions and approximations that are employed. In the second part, we will provide experimental validation of the proposed method by means of comparing simulations based on it with a corresponding set of measurements on a physical system.

II. FORMULATION OF THE METHOD

The proposed method targets the situation in which a light source and a detector lie at the opposite sides of an opaque barrier with a very small aperture in between. Under these conditions it is expected that light can in principle reach the detector by means of diffuse and/or multiple reflections but in very small quantities. This leads to the challenge that a simple simulation of this configuration would require the generation of a lot of rays in order to identify also the few that actually hit the detector. To give a magnitude of how severe this issue could be it suffices to notice that a 1-W-source emits approximately 10^{18} photons while modern detectors

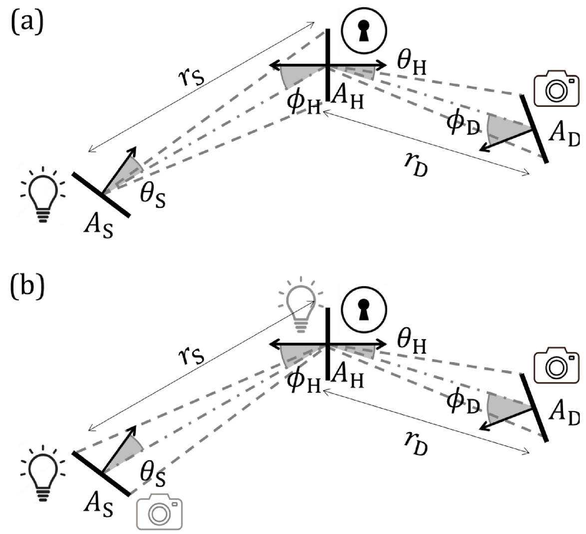

FIGURE 1. **Schematic description of the proposed simulation method with the emission (θ) and incident (ϕ) angles as well distances (r) and areas (A) of the involved elements: source (S), hole (H), and detector (D). Panel (a) represents the situation for the standard direct simulation. Panel (b) represents the newly proposed method.**

can reach sensitivities in the single-photon range. This means that there can be easily 15 to 18 orders of magnitude between the number of emitted and detected photons which in turn results into an unmanageable number of rays that need to be simulated in order to achieve a reasonable chance that at least one of them hits the detector. This is only made worse by the fact that if the number of detected rays is too low the statistical significance of the result is highly questionable. It is clearly not practicable to simulate 10^{15}-10^{18} rays as usually anything above 10^6-10^8 already leads to computational times in the range of hours or days even with reasonably powerful hardware. It can be thus argued that this kind of simulations are impossible with a standard approach.

The method proposed here aims to limit the simulation to only those rays that effectively cross the small aperture and exploits the intrinsic spatial symmetry of the laws of ray-optics to report the result to the actual total number of emitted photons. To this end we exploit the redefinition of the problem described in Fig. 1 where panel (a) depicts the configuration of a hypothetical standard simulation: rays successfully hitting the detector would have to first reach the small aperture (hole, H) from the source (S) and the from there the detector (D). Although the paths are represented with straight lines for the sake of simplicity, in reality they can be deflected numerous times by various optical effects such as reflection, refraction, and diffusion. This fact does not affect the validity of our proposition as we will see in the following. If the hole is very small, only a tiny solid angle (the one around θ_S) within the emission region of the source will pertain to the rays successfully reaching the hole and correspondingly these will amount to a very small fraction of the total emitted rays. Additionally, only a certain fraction of the rays exiting the hole, namely those lying within the solid angle around θ_H will reach the detector. The real bottleneck in this chain of propagation is the solid angle around θ_S because it is expected to be or the order of magnitude of A_H/r_S^2, with A_H area of the hole and r_S distance between source and hole.

Panel (b) in Fig. 1 shows the alternative method we propose in this paper. The main idea is to simulate only rays that successfully reach the hole from the source in order to avoid wasting computational resources in the simulation of rays that have no chance to reach the detector anyway thus effectively eliminating the bottleneck represented by the very small solid angle around θ_S. This is achieved by placing a virtual source covering the area of the hole and emitting in both directions. It is clear that the fraction of rays that reach the detector from this virtual source is the same as the fraction of rays that hit it after successfully clearing from the hole in the standard configuration of Fig. 1(a). Additionally, we count by means of a virtual detector placed where the actual source is the fraction of rays that successfully reach it from the virtual source. This number is significantly larger than the fraction of rays hitting the hole from the actual source but the two are directly related as we will show in this section.

Let us consider first the standard configuration depicted in Fig. 1(a). In the following we will assume that all sources are Lambertian emitters to facilitate the derivation. We will show later in the text that this implies no loss of generality. If the total power emitted by the source S is indicated as P_0 only a fraction η_{SH} will reach the hole and among these only a fraction η_{HD} will hit the detector. The total power P_D at the detector is thus given by:

$$P_D = P_0 \eta_{SH} \eta_{HD}. \tag{1}$$

As mentioned above, the fraction η_{HD} is identical in both configurations. Therefore, relating the configurations amounts to finding a connection between the direct and the reverse ray-paths between source and hole. In the direct configuration, a given ray emitted at an angle θ_{Si} has an intensity

$$I_{Si} = \frac{P_0}{\pi} \cos\theta_{Si}, \tag{2}$$

owing to the assumption of Lambertian emission. If the ray hits the hole, it does so with an irradiance given by

$$E_{Hi} = \frac{I_{Si}}{r_{Si}^2} \cos\phi_H = \frac{P_0}{\pi r_{Si}^2} \cos\theta_{Si} \cos\phi_{Hi}. \tag{3}$$

The total incident power P_H thus hitting the area of the hole reads:

$$P_H = \sum_i E_{Hi} A_H = A_H \sum_i \frac{P_0}{\pi r_{Si}^2} \cos\theta_{Si} \cos\phi_{Hi}, \tag{4}$$

where the sum runs on the rays that do hit the hole. This is, of course, under the implicit assumption that the irradiated surface A_H is much smaller than r_{Si}^2, which is obviously satisfied for the case of small holes. Finally, the fraction η_{SH} of power reaching the hole can be written as.

$$\eta_{SH} = \frac{P_H}{P_0} = A_H \sum_i \frac{1}{\pi r_{Si}^2} \cos\theta_{Si} \cos\phi_{Hi}, \tag{5}$$

We can now compare with the configuration with the virtual source and see that with an analogous reasoning the power hitting the actual source from the virtual one reads:

$$P_S = \sum_i E_{Si} A_S = A_S \sum_i \frac{P'_0}{\pi r_{Si}^2} \cos\phi_{Hi} \cos\theta_{Si}, \tag{6}$$

where the sum runs on the rays that do hit the actual source and P'_0 represents the total power emitted by the virtual source. Similarly, as above we can now compute the fraction η_{HS} of power that reaches the actual source from the virtual one:

$$\eta_{HS} = \frac{P_S}{P'_0} = A_S \sum_i \frac{1}{\pi r_{Si}^2} cos\phi_{Hi} cos\theta_{Si}. \tag{7}$$

This holds under the implicit assumption that the solid angles subtended by source with respect to the hole are small, i.e., $A_S \ll r_{Si}^2$. It is worth noticing that the two sums in (5) and (7) run on exactly the same rays connecting the hole and the source in either direction. Therefore, these two sums are identical which eventually leads to:

$$\eta_{SH} = \frac{A_H}{A_S} \eta_{HS}. \tag{8}$$

This simple but very important result allows us to reconstruct the very small and hard to obtain fraction η_{SH} in the direct configuration from the one η_{HS} that can be instead very easily obtained in the configuration with the virtual source. The fact that the area of the source A_S is much larger than that of the hole A_H confirms indeed that the much smaller value of η_{SH} can be obtained from η_{HS}.

We can now address the assumption of Lambertian emissions and show how the above result can be generalized once this is lifted. Assuming a general form $I_S(\theta)$ for the angle-dependent intensity of the actual source, (5) is modified as follows

$$\eta'_{SH} = \frac{P_H}{P_0} = A_H \sum_i \frac{I_S(\theta_{Di})}{P_0 \pi r_{Si}^2} \cos\phi_{Hi}. \tag{9}$$

In order to reestablish a relationship between η'_{SH} and η'_{HS} we need to add in the computation of the power P_S and thus in η'_{HS} an efficiency factor $f(\phi)$ that is dependent on the incident angle ϕ, something that is possible in most ray-tracing packages:

$$\eta'_{HS} = A_S \sum_i \frac{1}{\pi r_{Si}^2} f(\theta_{Si}) \cos\phi_{Hi} \cos\theta_{Si}. \tag{10}$$

We now only need to choose the efficiency factor as

$$f(\phi) = \frac{I_{S(\phi)}}{P_0 \cos\phi}, \tag{11}$$

So that the sums in (9) and (10) are again identical and the main result (8) is still valid. We can thus claim without loss of generality that a simulation carried out with the virtual source as in Fig. 1(b) can be used to compute both η_{HD} and η_{HS} and thus η_{SH} via (8) and finally P_S.

An additional technical expedient consists of limiting the emission angle of the virtual source to further reduce the number of simulated rays that do not hit the target. This bears for instance a conceptual similarity with the use of narrow-angle propagators in optical simulations [9]. The approach we propose here is to let the virtual source emit with a Lambertian distribution but only within a maximum value α for the emission angle. The rationale is that rays that are oriented parallel to the axis of the hole, to be thought of as a cylinder for simplicity, have more chances of crossing it than those that come with an angle. If we place the virtual source in the middle of the hole oriented orthogonally to its axis it makes sense to exclude from the simulation those rays that form a large angle with the axis because they have significantly lower chances of successfully coming out at either end of the hole. This is achieved by means of limiting the emission angle θ from the source to $0 < \theta < \alpha$. It must be considered though that the result (8) holds for θ values between 0 and $\pi/2$ and that the limiting up to α leads to overestimate of both η_{HS} and η_{HD}. To cure this distortion, we must consider that by limiting θ to α we are effectively skewing the result by the amount of power emitted by a Lambertian source up to an emission angle of α which can be easily shown to amount to α. Therefore, when exploiting this expedient to increase the statistical significance of the simulation we must normalize both η_{HS} and η_{HD} by a factor α to take into account the fact that we are implicitly ignoring emission directions that are unlikely to reach the target. It is additionally necessary to repeat the procedure at different values of α to find the ideal value that maximizes the efficiency. It is in fact expected that for too low values of α the virtual source emits like a very collimated one and relevant angles that would hit the target are not considered. Conversely, when α is too large many angles that carry no information are included in the simulation which results in statistical noise. Under ideal conditions there should be a sweet range for α where all relevant angles are included and only those. This can be achieved by means of iterations for different values of α until the obtained results stabilize. The expectation is that for too low or too high a value the obtained results will drop below the correct outcomes.

Considering the above description, the algorithmic procedure to carry out simulations of a configuration similar to those depicted in Fig. 1 based on this method can be described as follows:

1. Place a Lambertian emitter approximately in the middle of the hole such that it covers its entire cross section and emits in both directions within an angle α. This is the virtual source.
2. Place the actual detector as well as a virtual one covering the entire emitting surface of the actual source. Do not simulate the actual source.
3. This virtual detector shall have an efficiency that is dependent on the incident angle according to (11).
4. Carry out the simulation to obtain the incident power on the actual and virtual detectors, P_D and P_S.
5. Compute the resulting partial pseudo-transmissions $\eta_{HS} = P_S/P_0$ and $\eta_{HD} = P_D/P_0$ and normalize them both with α.

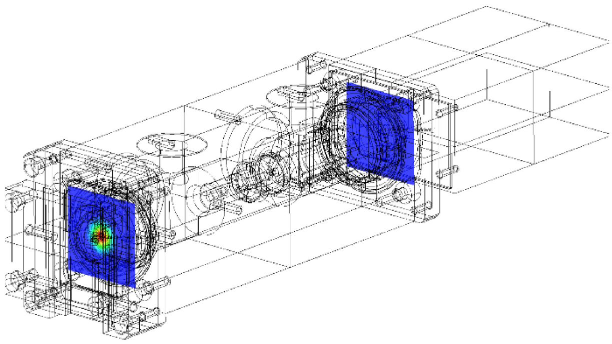

FIGURE 2. Wireframe representation of the measuring chamber indicating the position of actual source and detectors. The position of the cylindrical channel can be seen in the middle. The color maps indicate the simulated light intensity at the location of source and detector with the direct simulation.

6. Get the inverse pseudo-transmission η_{SH} from (8) and use (1) to determine the expected power hitting the actual detector in reality.

Repeat the steps 1-6 for different values of α to determine the ideal value or range of it.

III. EXPERIMENTAL VALIDATION

Although the mathematical formulation in the previous section is carefully laid down and formally correct, it does contain certain approximations and assumptions that, though reasonable, call for empirical verification. In particular, it makes scientifically sense to verify the proposed ansatz experimentally as it grants much more solidity and generalizability. To this end we developed an experimental set-up that reproduces a situation similar to that described in Fig. 1 consisting of two cavities containing each a light-source and a sensor, respectively and separated by a bottleneck where only a small aperture allows minimal passage of light. This consists of a cylindrical channel into which a solid cylinder with a slightly smaller diameter (99% – 100% of outer diameter) can be inserted so that a tiny gap for the light remains. By employing cylinders of different but similar size the overall area of the effective aperture can be varied in a controlled fashion while keeping it small with respect to the remaining involved geometrical sizes. Additionally, the whole cavity was filled with mineral oil in order to add a source of diffusion and absorption processes in the bulk without losing too much transparency. A wireframe representation of the experimental set-up is shown in Fig. 2.

The used source is a 1-W white LED while the detector is a high-sensitivity Silicon Photomultiplier. Considering the size of the apparatus and the typical area of the aperture of $0.4 - 1.0\ mm^2$ it can be expected that less than 10^{-14} of the emitted photons can reach the detector, i.e., a measured photon flux in the range of thousand per second, which is well within the capability of the detector but at the same time represents a significant challenge for a traditional computational treatment because it entails that no less than 10^{12} rays must be simulated in order to have any chance of hitting the detector with any statistical significance.

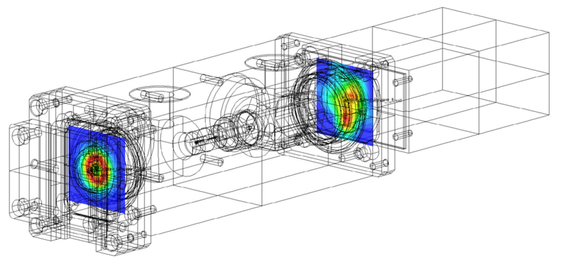

FIGURE 3. Simulation with the virtual source in the middle of the channel. The color maps indicate the simulated light intensity at the location of source and detector.

We discuss first the details of the simulations carried out to reproduce the experimental setup. We used the package LightTools [10] where the CAD model of the measuring chamber can be imported and models for sources and detectors can be implemented. The chamber is made of steel which we characterized separately with goniometric measurements in order to obtain a model of both its specular and diffuse reflectivity and included it in the LightTools simulations. Similarly, the employed mineral oil was also optically characterized experimentally to include its reflection and absorption properties into the model. As a first step we tried the standard approach, namely we modelled a light-source at the location of the physical LED with the same size and power-spectrum as that provided in the source specifications as well as a detector at the same location and with the same geometry of the photomultiplier, whose specified wavelength-dependent quantum efficiency has been included in all subsequent analysis. As expected, even with the maximum aperture, out of 100 million simulated rays, a number that requires 36 h of computational time on a 32 GB – 3.6 GHz personal computer, not even a single one hits the detector, which confirms that configurations of this type are virtually impossible to simulate with a brute-force direct approach (color maps in Fig. 2). Subsequently we applied the method proposed in this paper by means of removing the source at the LED location and replacing it with a virtual detector of the same size as well as adding a virtual light source at the middle point of the cylindrical channel oriented orthogonally to its axis. This source's shape and size was made identical to considered aperture. A representative example of such a simulation is shown in Fig. 3 where it is apparent that both locations of interest (i.e., that of the actual source and detector) receive a statistically significant number of rays thus indicating that the algorithm described in Section II can be successfully applied to estimate the predicted signal measured by the actual detector. For this set of simulations, repeated for different values of the aperture area corresponding the experimentally available ones, we determined that optimal results are achieved when we limit the emission angle of the virtual source to a maximum of $\alpha = 10°$. This is brought about by the fact that the aperture is significantly longer than wide which means that only rays that are very close to parallel to its axis have a chance of passing through.

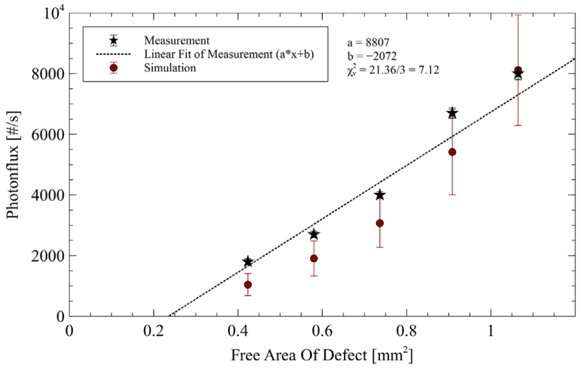

FIGURE 4. **Comparison between the measured (black stars) and simulated (red dots) photon-flux for different areas of the aperture. The dotted line is a linear fit of the measured data that serves as guide to the eye.**

The experimental setup allows us to collect measurements of the photon-flux impinging on the detector for the different available apertures and compare it with the results of our simulations. The results are shown and compared in Fig. 4 where the overall agreement is very well apparent. This finding, combined with the overall formal correctness of the formulation, strongly confirms the validity of our approach and indicates that it could be successfully applied to similar configurations. In particular, the quantitative agreement between experiment and simulation across a wide range of aperture sizes indicates that the result cannot be ascribed to an accidental tuning of the optical parameters.

IV. CONCLUSION

In this work, we presented a novel method to simulate by means of ray-tracing configurations where a very small amount of light passing through a tiny aperture is of interest. This can be useful whenever the transmitted intensity is so small that its determination would require the simulation of an untreatable number of rays. It is worth mentioning that the method can be easily implemented within any standard ray-tracing simulation package as it requires minimal and rather simple manipulation of the simulation setup and results. The steps required to apply it in general have been presented in the form of a simple algorithm. The validity of the proposed approach is argued by means of a mathematical formulation of the concepts underlying it as well as a set of validation measurements across a range of aperture sizes that agree very well with the corresponding simulations. Although the range of potential applications of this approach is limited, we are convinced that its simplicity can be attractive and of interest for potential users even if within a rather small niche.

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