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Adaptive Beamforming With Continuous/Discrete Phase Shifters via Convex Relaxation

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ABSTRACT Adaptive antenna array employing phase shifters only can reduce the hardware cost and power consumption, but the related optimization is well understood to be difficult to solve. In this paper, we examine the use of both continuous phase shifters (CPS) and discrete phase shifters (DPS) in antenna array for adaptive interference suppression. First, we formulate the phase-only tuning problem with CPS via quadratically constrained quadratic program (QCQP), which can be approximated through semidefinite relaxation (SDR) and solved by convex-optimization solvers efficiently. We also demonstrate that quantizing such results directly may not necessarily give acceptable performance. Then, we propose to use binary quadratic program (BQP) for the optimization of the adaptive beamformer utilizing DPS with an arbitrary number of control bits to specify the phase-shift levels. The resultant BQP can be relaxed and solved efficiently too. Moreover, due to the similarity between the CPS and DPS formulation, we can mix them in a single beamforming scheme to provide additional compromises between the output signal-to-interference and noise ratio (SINR) performance and implementation cost. Simulations show that in the considered interference-limited environment, our proposed beamformers with CPS, DPS, and hybrid phase shifters (HPS) give desirable results as expected.

INDEX TERMS Antenna array, continuous phase shifter (CPS), discrete phase shifter (DPS), conventional beamformer (CB), minimum-variance distortionless response (MVDR) beamformer, adaptive beamformer, quadratically constrained quadratic program (QCQP), binary quadratic program (BQP), convex optimization, semidefinite relaxation (SDR).

I. INTRODUCTION

ADAPTIVE antenna array has been widely studied for several decades, which benefits many engineering fields, including radar, communication, sonar, navigation, remote sensing, radio-astronomy, biomedical imaging, automotive application, drone surveillance, and many others [1]–[3]. For the use of beamforming, it is known that the conventional beamformer (CB), also called the delay-and-sum beamformer, is made to let the array response be matched to the array-steering vector of the desired signal. The main lobe of the CB points to the direction of the desired signal, and the received signal power is then maximized. One merit of the CB is its simplicity, in which only phase values of the received signal are adjusted. Nevertheless, the CB does not take notice of the interference from other directions and only optimistically lets the relatively low sidelobe suppress it. Therefore, the output signal-to-interference and noise ratio (SINR) performance of the CB is usually quite away from that of the optimum. It is also known that to obtain the optimal performance of array-pattern synthesis, both the amplitude and phase of the received signal from each antenna should be tuned, e.g., through the use of the minimum-variance distortionless response (MVDR) or minimum mean-squared error (MMSE) beamforming criterion. However, due to the use of additional scaling devices for tuning the amplitude as compared with the beamformer using phase shifters only, the induced hardware cost and power consumption for the MVDR or MMSE beamformer can be significant for many applications, especially when the number of antenna elements is large. Besides, the MVDR or MMSE beamformer is often implemented in the digital domain. For this case, each antenna requires its own radio-frequency (RF) feed, which is very costly and increases the complexity accordingly [4], [5].

In this regard, the design of beamformers using phase shifters only, such as the CB, which can adapt to the interference-limited environment, is very attractive to the research community. The resultant beamformer with the general continuous phase shifters (CPS) can lower the hardware cost compared to the MVDR or MMSE beamformer [3], but it is understood that the related optimization problem is quite challenging to solve. Thanks to the advances in optimization and related techniques, some interesting results exist to approach the solution of the problem [6]–[13]. To name a few, for instance, in [6], [7], different heuristic algorithms were proposed to obtain the phase-only solution for the beamforming weights. Also, with a special formulation, such a result could be obtained via nonlinear programming [8] or combinatorial optimization [9]. It was demonstrated in [10] that the so-called autocorrelation matching method could be used to solve this phase-only problem. In addition, [11]–[13] provided some other new iterative ways to obtain the phase-only weights. Note here that many of them were chiefly based on minimizing the difference between a preset array pattern and that of the proposed phase-only beamformer, or just achieving an array pattern with relatively low sidelobe levels. More recently, convex optimization has been utilized to synthesize the array pattern [14], and to solve the challenging phase-only beamforming problem [15]–[19]. The advantages of these approaches are that the solution can be approached more systematically, setting a more solid theoretical basis for the research.

Additionally, in some practical circumstances, the use of discrete phase shifters (DPS) for beamformers is a promising way to further lower the hardware complexity [20]–[31]. Again, due to the difficulties of the discrete-valued optimization, many of the previous schemes were based on heuristic global optimization algorithms, such as genetic algorithms (GA), particle swarm optimization (PSO), simulated annealing (SA), ant colony optimization (ACO), binary differential evolution (BDE), and so on, to achieve different levels of beamforming performance. And usually, the methods were developed mainly based on some specific numbers of control bits for the phase-shift levels, and were generally not good enough for any interference-limited environment.

In this paper, we examine the use of both CPS and DPS in the antenna array for adaptive interference suppression. First, we formulate the phase-only tuning problem with CPS via quadratically constrained quadratic program (QCQP), which can be approximated through semidefinite relaxation (SDR) and solved by common convex-optimization solvers efficiently. We also demonstrate that quantizing such results directly may not necessarily give acceptable performance. Then, we propose to use binary quadratic program (BQP) for the optimization of the adaptive beamformer utilizing DPS with an arbitrary number of bits to specify the phase-shift levels. The resultant BQP can be relaxed and solved efficiently too. Moreover, due to the similarity between the CPS and DPS formulation, we can mix them in a single beamforming scheme to provide additional compromises between the output-SINR performance and implementation cost. This may fit the case of distributed/cooperative beamforming in which different devices work together to form certain array patterns. For example, some devices or some parts of the device are sophisticated enough to be equipped with CPS, but some are not and only possess DPS. Also, the hybrid approach can be a mix of DPS with a different number of control bits. All these give extra flexibility in the formulation and related design with limited hardware resources. Simulations show that in the considered interference-limited environment, our proposed beamformers with CPS, DPS, and hybrid phase shifters (HPS) give desirable results as expected. In summary, the design and development of the proposed CPS, DPS, and HPS beamformers own the following merits:

1) The proposed adaptive beamforming strategies with CPS, DPS, and HPS are all based on improving the output-SINR performance. In other words, as compared with some state-of-the-art approaches, the interference-limited environment can be treated more intentionally.
2) Since the optimization of CPS, DPS, and HPS beamformers is tackled via convex relaxation, it can then be efficiently solved by convex-optimization solvers such as CVX [32]. This also means that the solution can be obtained systematically with a reasonable amount of computation.
3) The design of the proposed DPS beamformer fits the usage of any number of control bits. Together with the HPS beamformer, which flexibly combines the ways of both CPS and DPS in one beamforming scheme, some good trade-offs among the output SINR, hardware complexity, and power consumption can be obtained for a wide range of purposes. Design guidelines are acquired as well through the discussion of the experimental results given in this work.
4) Our results show that in terms of the output-SINR performance, the beamformer with CPS only can perform consistently close to the optimal MVDR beamformer. Besides, the DPS beamformer with 4 control bits gives excellent performance for most cases. The DPS beamformer with only 1 control bit can still provide acceptable performance, and can even outperform the classical CB in some circumstances.

The remainder of this paper is organized as follows. Section II provides the background of this research. The signal model used in this research and the idea of both the CB and MVDR beamformer are reviewed. In Section III to V, we develop the beamforming schemes with CPS only, DPS only, and a mix of both CPS and DPS, respectively. The design considerations and characteristics of each strategy are explained in detail. Simulation results demonstrating the performance of our proposed methods with various settings and trade-offs are presented in Section VI. Finally, conclusions are drawn in Section VII.

Notation: We adopt the notation of boldface lower-case letters for vectors (e.g., $\mathbf{a}$), and boldface upper-case letters for matrices (e.g., $\mathbf{A}$). The conjugate operator, transpose operator, and conjugate transpose operator are denoted by $(\cdot)^*$, $(\cdot)^T$, and $(\cdot)^H$, respectively. Besides, $\mathbf{1}_{N\times 1}$ denotes an $N \times 1$ vector with all entries being 1; $\mathbf{0}_{N\times 1}$ denotes an $N \times 1$ vector with all entries being 0; $\mathbf{I}_N$ is an $N \times N$ identity matrix. Also, the notation $\mathrm{Tr}(\cdot)$ stands for the trace operation, $\mathrm{Rank}(\cdot)$ represents the rank of the included matrix, and $\mathrm{Diag}(\cdot)$ denotes the diagonal matrix constructed by the included vector. For a complex number/vector/matrix, we use $(\cdot)^{\mathrm{R}}$ and $(\cdot)^{\mathrm{I}}$ to denote respectively the real and imaginary parts. The symbol $\succeq$ is used to denote a generalized inequality, i.e., $\mathbf{A} \succeq \mathbf{0}$ means $\mathbf{A}$ is a symmetric positive semidefinite matrix. In addition, we use $\mathcal{CN}(\mu, \sigma^2)$ to represent the complex Gaussian distribution with mean μ and variance σ^2. Finally, $E[\cdot]$ represents the statistical expectation, and $j = \sqrt{-1}$.

II. SIGNAL MODEL, CONVENTIONAL BEAMFORMER (CB), AND MINIMUM-VARIANCE DISTORTIONLESS RESPONSE (MVDR) BEAMFORMER

In this section, we first describe the signal model used here. Then, we review two benchmark beamforming techniques. One is perhaps the most straightforward beamforming approach which utilizes phase shifters only, and is named the CB. The other is the MVDR beamformer. It adapts the interference-limited environment more sophisticatedly with both amplitude and phase control, and is optimal in terms of providing the largest output-SINR value. Details will be elaborated on in the following.

To not complicate the problem, we consider the use of a classical uniform linear array (ULA) placed in the far field to receive narrowband signals. The array comprises N isotropic antenna elements lying along a straight line with the same spacing d. Weights with amplitude and phase control or phase-only control are employed to tune the array pattern. Besides, assume that $M + 1$ signal sources impinge on the ULA, which include a desired signal plus M uncorrelated interference sources all from different angles of arrival θ_m, with $m = 0, 1, \ldots, M$ and $-90^o \le \theta_m \le 90^o$. The notation θ_0 is used to represent the arrival angle of the desired signal, and θ_m with $m = 1, 2, \ldots, M$ is used to represent the arrival angle of the mth interference. The array-steering vector of the desired signal ($m = 0$) and interference ($m = 1, 2, \ldots, M$) can then be expressed as $\mathbf{v}(\theta_m) = [v_1(\theta_m)\ v_2(\theta_m)\ \cdots\ v_N(\theta_m)]^T$ with $v_n(\theta_m) = \exp(j2\pi \frac{(n-1)d}{\lambda} \sin(\theta_m))$ in which λ is the signal wavelength and $n = 1, 2, \ldots, N$. We assume half-wavelength inter-element spacing, i.e., $d = \frac{\lambda}{2}$, in the following. Hence, the signal received at the considered ULA can be expressed as

$$\mathbf{r} = \mathbf{v}(\theta_0)s + \sum_{m=1}^{M} \mathbf{v}(\theta_m)i_m + \mathbf{z} \tag{1}$$

where s and i_m are the waveforms of the desired signal and the mth interference, respectively, and $\mathbf{z}$ is the additive white Gaussian noise vector generated according to $\mathcal{CN}(\mathbf{0}_{N\times 1}, \sigma_z^2\mathbf{I}_N)$ with σ_z^2 being the noise variance. Accordingly, the signal-to-noise ratio (SNR) and interference-to-signal ratio (ISR) for the environment are defined as $\frac{E[|s|^2]}{\sigma_z^2}$ and $\frac{\sum_{m=1}^{M} E[|i_m|^2]}{E[|s|^2]}$, respectively.

As mentioned, the CB is perhaps the simplest kind of array-processing scheme to enhance the received-signal quality, and only phase shifters are needed in the adjustment. According to the signal model described in (1), the operation and the corresponding outcome of CB can be written as $y = \mathbf{w}_{\mathrm{CB}}^H\mathbf{r}$ with $\mathbf{w}_{\mathrm{CB}} = \frac{1}{N}\mathbf{v}(\theta_0)$. That is, a spatial matched filter is constructed to “capture” the desired signal as much as possible, but the interference may not be probably treated. This can serve as the baseline when comparing the performance of various beamforming methods.

On the other hand, the optimal MVDR beamformer, which adapts to the environment using amplitude-phase weighting, can maximize the output SINR. With $\mathbf{R}_{iz} = E[(\sum_{m=1}^{M} \mathbf{v}(\theta_m)i_m + \mathbf{z})(\sum_{m=1}^{M} \mathbf{v}(\theta_m)i_m + \mathbf{z})^H]$ denoting the covariance matrix of interference-plus-noise, it can be shown that the weight vector of MVDR is $\mathbf{w}_{\mathrm{MVDR}} = (\mathbf{v}(\theta_0)\mathbf{R}_{iz}^{-1}\mathbf{v}(\theta_0))^{-1}\mathbf{R}_{iz}^{-1}\mathbf{v}(\theta_0)$ [2]. Note that matrix inverse is needed to obtain $\mathbf{w}_{\mathrm{MVDR}}$, and both the amplitude and phase of the received signal from each antenna element should be tuned in general. Therefore, the induced computational complexity and hardware cost may be too high for many applications, especially when the size of the ULA is large. Nevertheless, since the MVDR beamformer gives the possibly largest output SINR, this can serve as an upper bound of the output-SINR performance.

III. PROPOSED CONTINUOUS PHASE-SHIFTER (CPS) BEAMFORMING

We propose an adaptive beamforming method using CPS in this section. To seek a good outcome for the beamformer, we formulate a new criterion together with the CPS constraint. It can acquire a considerable value of output SINR according to the interference-limited environment encountered. Here, before going into the details, we first reveal the output-SINR value of a beamformer, i.e.,

$$\gamma = \frac{\mathbf{w}^H\mathbf{R}_s\mathbf{w}}{\mathbf{w}^H\mathbf{R}_{iz}\mathbf{w}} \tag{2}$$

in which $\mathbf{R}_s = E[(\mathbf{v}(\theta_0)s)(\mathbf{v}(\theta_0)s)^H]$ is the covariance matrix of the signal part only, and $\mathbf{w}$ is the weights of the beamformer. For example, $\mathbf{w}$ can be $\mathbf{w}_{\mathrm{CB}}$ or $\mathbf{w}_{\mathrm{MVDR}}$ for the CB or MVDR beamformer as defined and explained in Section II, and this results in γ_{CB} or γ_{MVDR}, respectively.

In the design of the CPS beamformer, we expect a large output-SINR value given in (2). Thus, if we denote the output SINR after the CPS weighting as γ_{CPS}, an optimization problem can be described as

$$\max_{\mathbf{w}_{\text{CPS}}} \ \gamma_{\text{CPS}} \tag{3}$$

$$\text{s.t.} \ |w_{\text{CPS},n}| = 1, \ n = 1, 2, \ldots, N \tag{4}$$

where $\mathbf{w}_{\text{CPS}}$ is the $N \times 1$ weight vector of the CPS beamformer, and $w_{\text{CPS},n}$ is the nth entry in $\mathbf{w}_{\text{CPS}}$ for all possible n. The unit-modulus constraint in (4) restrains the change of the magnitude, and, as a result, only phase values can be adjusted. It is well-understood that the optimization with this unit-modulus constraint is not easy to tackle. We then reformulate the problem as follows. For the objective function in (3), instead of calculating the ratio between the signal power and interference-plus-noise power, we change it to

$$\begin{aligned} &\mathbf{w}_{\text{CPS}}^H \mathbf{R}_s \mathbf{w}_{\text{CPS}} - \alpha \mathbf{w}_{\text{CPS}}^H \mathbf{R}_{iz} \mathbf{w}_{\text{CPS}} \\ &= \mathbf{w}_{\text{CPS}}^H (\mathbf{R}_s - \alpha \mathbf{R}_{iz}) \mathbf{w}_{\text{CPS}} \\ &= \mathbf{w}_{\text{CPS}}^H \mathbf{R} \mathbf{w}_{\text{CPS}} \end{aligned} \tag{5}$$

in which we set $\mathbf{R} = \mathbf{R}_s - \alpha \mathbf{R}_{iz}$ for notational simplicity. That is, the objective function is altered to be the difference between the signal power and interference-plus-noise power. Note that in (5) a new parameter α is set, which is used to provide a balance between the two power values. Practically, since our target is that the output-SINR value of the proposed CPS beamformer can approach that of the MVDR beamformer, we may set α near γ_{MVDR} or just the same order of γ_{MVDR} to make the two included values at a similar level. Consequently, both the signal power and interference-plus-noise power can be suitably treated in the optimization, which will be demonstrated later in the simulation section. Besides, for the constraint in (4), we rewrite it as $w_{\text{CPS},n}^* w_{\text{CPS},n} = 1$, and the whole optimization can then be reformulated to be

$$\begin{aligned} \min_{\mathbf{w}_{\text{CPS}}} \ & -\mathbf{w}_{\text{CPS}}^H \mathbf{R} \mathbf{w}_{\text{CPS}} \\ \text{s.t.} \ & w_{\text{CPS},n}^* w_{\text{CPS},n} = 1, \ n = 1, 2, \ldots, N. \end{aligned} \tag{6}$$

In this way, the previously-mentioned problem in (3) and (4) is changed to become a QCQP problem. However, it is well-known that QCQP is not necessarily convex, and the general case of it is non-deterministic polynomial-time hardness (NP-hard). To approach the solution, we know that one common way is to employ semidefinite relaxation (SDR), which essentially lets the original non-convex optimization problem be approximated by a convex one, and then be solved efficiently by convex-optimization solvers.[1] Specifically, to use the CVX solver to solve the resultant problem by SDR, we first need to change the complex-valued expressions in (6) to contain real values only. To do so, we rewrite the complex-valued matrix $\mathbf{R}$ and complex-valued vector $\mathbf{w}_{\text{CPS}}$ as

$$\widetilde{\mathbf{R}} = \begin{bmatrix} (\mathbf{R})^{\text{R}} & -(\mathbf{R})^{\text{I}} \\ (\mathbf{R})^{\text{I}} & (\mathbf{R})^{\text{R}} \end{bmatrix} \tag{7}$$

$$\widetilde{\mathbf{w}}_{\text{CPS}} = \begin{bmatrix} (\mathbf{w}_{\text{CPS}})^{\text{R}} \\ (\mathbf{w}_{\text{CPS}})^{\text{I}} \end{bmatrix} \tag{8}$$

respectively. The optimization problem in (6) can then be rewritten to be

$$\begin{aligned} \min_{\widetilde{\mathbf{w}}_{\text{CPS}}} \ & -\widetilde{\mathbf{w}}_{\text{CPS}}^T \widetilde{\mathbf{R}} \widetilde{\mathbf{w}}_{\text{CPS}} \\ \text{s.t.} \ & \left((w_{\text{CPS},n})^{\text{R}} \right)^2 + \left((w_{\text{CPS},n})^{\text{I}} \right)^2 = 1, \\ & n = 1, 2, \ldots, N. \end{aligned} \tag{9}$$

Also define a real $2N \times 2N$ symmetric positive semidefinite matrix $\mathbf{W}_{\text{CPS}}$ and a series of $2N \times 2N$ matrices $\mathbf{P}_n$ in which all entries are 0 except the (n, n) entries and the $(n+N, n+N)$ entries being 1, with $n = 1, 2, \ldots, N$. The optimization problem in (6) can then be equivalently stated as

$$\begin{aligned} \min_{\mathbf{W}_{\text{CPS}}} \ & -\text{Tr}\{\widetilde{\mathbf{R}} \mathbf{W}_{\text{CPS}}\} \\ \text{s.t.} \ & \text{Tr}\{\mathbf{P}_n \mathbf{W}_{\text{CPS}}\} = 1, \ n = 1, 2, \ldots, N \\ & \mathbf{W}_{\text{CPS}} \succeq \mathbf{0} \\ & \text{Rank}(\mathbf{W}_{\text{CPS}}) = 1. \end{aligned} \tag{10}$$

The solution $\mathbf{W}_{\text{CPS}}$ is supposed to be the result of $\widetilde{\mathbf{w}}_{\text{CPS}} \widetilde{\mathbf{w}}_{\text{CPS}}^T$. Importantly, to solve this optimization via SDR, the Rank-1 constraint in the constraint set of problem (10) is removed, and (10) is then relaxed to become a semidefinite programming problem, which is convex and can be solved within polynomial time. Based on the solution $\mathbf{W}_{\text{CPS}}$, randomization steps such as those explained in [36] can be employed to find a suitable vector $\widetilde{\mathbf{w}}_{\text{CPS}}$ (or $\mathbf{w}_{\text{CPS}}$) which can probably give not only a good solution for the optimization in (9) (or (6)), but also a large output-SINR value for the original objective function in (3). Note that the computational complexity of randomization steps is generally much less than that of solving the semidefinite programming problem itself; therefore, the computation cost of such randomization can usually be ignored in the whole optimization process.

IV. PROPOSED DISCRETE PHASE-SHIFTER (DPS) BEAMFORMING

In this section, we change our focus to the design of beamformers using DPS to deal with the interference-limited environment adaptively. At first glance, one may say that direct quantization of the outcome of the previously-derived CPS beamformer also results in a beamformer with discrete-valued phase shifts only. However, as shown later, the performance of such an approach may not be good, and this indicates the necessity for the optimization focusing on the usage of DPS developed in the following.

1. In the literature, more advanced methods for solving this kind of QCQP/unit-modulus constrained optimization are available [33]–[35]. To not complicate the matter, we choose the perhaps most well-known technique to solve this problem, i.e., SDR, here. For an N-dimensional SDR problem, it is known that the worst-case complexity of finding the solution is about $O(N^{6.5})$, while it gives a performance bound for the optimization as well as suboptimal results.

As stated, the objective function given in (6) can give good output-SINR performance for a beamformer, and thus we keep utilizing it here as the objective function for the DPS beamformer. The constraint for admitting only discrete-valued phase shifts is set as follows. Assume we employ K control bits to specify the L finite phase-shift levels, with $L = 2^K$. Without loss of generality, the L allowable phase-shift values are denoted by $\phi_1, \phi_2, \ldots, \phi_L$, and then the corresponding weights become $\exp(j\phi_1), \exp(j\phi_2), \ldots, \exp(j\phi_L)$, respectively. Accordingly, the optimization for the DPS beamformer can be altered to be

$$\begin{aligned} \min_{\mathbf{w}_{\mathrm{DPS}}} \quad & -\mathbf{w}_{\mathrm{DPS}}^H \mathbf{R} \mathbf{w}_{\mathrm{DPS}} \\ \text{s.t.} \quad & w_{\mathrm{DPS},n} \in \left\{ e^{j\phi_1}, e^{j\phi_2}, \ldots, e^{j\phi_L} \right\}, \\ & n = 1, 2, \ldots, N \end{aligned} \tag{11}$$

where $\mathbf{w}_{\mathrm{DPS}}$ is the $N \times 1$ DPS weight vector, and $w_{\mathrm{DPS},n}$ is its nth entry. To demonstrate how we solve this problem, we consider a special case that $L = 2$ ($K = 1$), i.e., only two phase-shift values are allowed for the DPS beamformer, and $N = 2$, i.e., only two antenna elements are used to receive the signal for brevity. Moreover, suppose that the resultant DPS weight vector is $\mathbf{w}_{\mathrm{DPS}} = [\exp(j\phi_1) \ \exp(j\phi_2)]^T$. For this case, we may express the resultant objective function (ignoring the minus sign) as that in (12), shown at the bottom of the page, where we let $\mathbf{P}_l = \mathrm{Diag}([\exp(j\phi_l) \ \exp(j\phi_l)])$ with $l = 1, 2$ in this scenario. Also, we define $\mathbf{R}_{uv} = \mathbf{P}_u^H \mathbf{R} \mathbf{P}_v$ with $u, v = 1, 2$, and write

$$\overline{\mathbf{R}} = \begin{bmatrix} \mathbf{R}_{11} & \mathbf{R}_{12} \\ \mathbf{R}_{21} & \mathbf{R}_{22} \end{bmatrix}. \tag{13}$$

With reference to these expressions, we see that the optimization in (11) can be restated to be

$$\min_{\mathbf{x}} \quad -\mathbf{x}^T \overline{\mathbf{R}} \mathbf{x} \tag{14}$$

$$\text{s.t.} \quad \mathbf{x} \in \{0, 1\}^4 \tag{15}$$

$$x_n + x_{n+2} = 1, \ \ n = 1, 2. \tag{16}$$

As indicated in (15), $\mathbf{x}$ is a 4×1 vector with entries being either 0 or 1, and x_n is the nth entry in this vector. The constraint in (16) is used to restrict the phase shifters not being repeatedly assigned. It is not difficult to see that this optimization is a BQP problem.

After understanding the derivation for the special case, we then show that the DPS setting mentioned above for designing an adaptive beamformer can be extended to any number of antenna elements N and any number of phase-shift levels L. To see this, the vector $\mathbf{x}$ in (14) for the general case can be set as

$$\mathbf{x} = \begin{bmatrix} [x_{11} \ x_{21} \cdots x_{N1}]^T \\ [x_{12} \ x_{22} \cdots x_{N2}]^T \\ \vdots \\ [x_{1L} \ x_{2L} \cdots x_{NL}]^T \end{bmatrix} = \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_L \end{bmatrix} \tag{17}$$

which is an $NL \times 1$ vector including L subvectors $\mathbf{x}_l$ with length N for $l = 1, 2, \ldots, L$. Similarly, the matrix $\overline{\mathbf{R}}$ in (14) can be extended to become

$$\overline{\mathbf{R}} = \begin{bmatrix} \mathbf{R}_{11} & \mathbf{R}_{12} & \cdots & \mathbf{R}_{1L} \\ \mathbf{R}_{21} & \mathbf{R}_{22} & \cdots & \mathbf{R}_{2L} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{R}_{L1} & \mathbf{R}_{L2} & \cdots & \mathbf{R}_{LL} \end{bmatrix} \tag{18}$$

which is an $NL \times NL$ matrix including LL submatrices $\mathbf{R}_{uv}$ with size $N \times N$ for $u, v = 1, 2, \ldots, L$. The corresponding constraints in (15) and (16) are changed to be $\mathbf{x} \in \{0, 1\}^{NL}$ and $x_{n1} + x_{n2} + \cdots + x_{nL} = 1$ for $n = 1, 2, \ldots, N$, respectively. Again, it is a BQP problem. One common way to tackle this is to reformulate it as QCQP. To see this, we make use of $\mathbf{x}$ in (17) and $\overline{\mathbf{R}}$ in (18) for the objective function, and rewrite the corresponding two constraints for general values of N and L. Thereupon, the whole optimization becomes

$$\min_{\mathbf{x}} \quad -\mathbf{x}^T \overline{\mathbf{R}} \mathbf{x} \tag{19}$$

$$\begin{aligned} \text{s.t.} \quad & x_{nl}(x_{nl} - 1) = 0, \\ & n = 1, 2, \ldots, N \text{ and } l = 1, 2, \ldots, L \end{aligned} \tag{20}$$

$$\begin{aligned} & \mathbf{w}_{\mathrm{DPS}}^H \mathbf{R} \mathbf{w}_{\mathrm{DPS}} \\ & = \begin{bmatrix} e^{j\phi_1} \\ e^{j\phi_2} \end{bmatrix}^H \mathbf{R} \begin{bmatrix} e^{j\phi_1} \\ e^{j\phi_2} \end{bmatrix} \\ & = \left(\begin{bmatrix} e^{j\phi_1} & 0 \\ 0 & e^{j\phi_1} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} e^{j\phi_2} & 0 \\ 0 & e^{j\phi_2} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right)^H \mathbf{R} \left(\begin{bmatrix} e^{j\phi_1} & 0 \\ 0 & e^{j\phi_1} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} e^{j\phi_2} & 0 \\ 0 & e^{j\phi_2} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right) \\ & = [1 \ 0] \mathbf{P}_1^H \mathbf{R} \mathbf{P}_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + [1 \ 0] \mathbf{P}_1^H \mathbf{R} \mathbf{P}_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} + [0 \ 1] \mathbf{P}_2^H \mathbf{R} \mathbf{P}_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + [0 \ 1] \mathbf{P}_2^H \mathbf{R} \mathbf{P}_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ & = [\,[1 \ 0][0 \ 1]\,] \begin{bmatrix} \mathbf{P}_1^H \mathbf{R} \mathbf{P}_1 & \mathbf{P}_1^H \mathbf{R} \mathbf{P}_2 \\ \mathbf{P}_2^H \mathbf{R} \mathbf{P}_1 & \mathbf{P}_2^H \mathbf{R} \mathbf{P}_2 \end{bmatrix} \begin{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 1 \end{bmatrix} \end{bmatrix} \end{aligned} \tag{12}$$

$$\mathbf{A}\mathbf{x} = \mathbf{1}_{N\times 1} \tag{21}$$

in which $\mathbf{A}$ is an $N \times NL$ matrix with all entries being 0 except the $(n, n), (n, n+N), (n, n+2N), \ldots, (n+LN)$ entries being 1 for $n = 1, 2, \ldots, N$. Similarly, this QCQP can be transformed and relaxed like those in (6)-(10), and then solved via convex optimization. Note here that as L (or K) increases, the dimension of the optimization is unavoidably enlarged, but it may still be manageable with the computation power nowadays. Later, in the simulation section, we will see that 4 control bits are generally enough to let the DPS beamformer provide very good performance for most cases.

V. PROPOSED HYBRID PHASE-SHIFTER (HPS) BEAMFORMING: A MIX OF CPS AND DPS

After deriving both CPS and DPS beamformers, in this section, due to the similarity between the two schemes, we demonstrate that we can mix them to make a new hybrid PS (HPS) beamformer to provide extra compromises between the output-SINR performance and implementation cost. Specifically, we observe that the objective functions in both (6) for CPS and (11) for DPS are in the same quadratic form, and are set toward the same target. At the same time, the constraints for both cases are in quadratic form/linear form too. Therefore, for a specific antenna element, we may set it using either the CPS constraint or the DPS constraint without altering the structure of the optimization. Here, define $\mathcal{I}_{\text{CPS}}$ as the index set of antenna elements applying CPS, and $\mathcal{I}_{\text{DPS}}$ as the index set of antenna elements applying DPS, respectively. The optimization of the proposed HPS beamformer with CPS and L-level DPS can be set to be

$$\min_{\mathbf{w}_{\text{CPS}}, \mathbf{w}_{\text{DPS}}} \quad -\mathbf{w}_{\text{CPS}}^H \mathbf{R} \mathbf{w}_{\text{CPS}} - \mathbf{w}_{\text{DPS}}^H \mathbf{R} \mathbf{w}_{\text{DPS}} \tag{22}$$

$$\text{s.t.} \quad w_{\text{CPS},n}^* w_{\text{CPS},n} = 1, \ n \subset \mathcal{I}_{\text{CPS}} \tag{23}$$

$$w_{\text{CPS},n} = 0, \ n \not\subset \mathcal{I}_{\text{CPS}}$$

$$w_{\text{DPS},n} \in \left\{ e^{j\phi_1}, e^{j\phi_2}, \ldots, e^{j\phi_L} \right\}, \quad n \subset \mathcal{I}_{\text{DPS}} \tag{24}$$

$$w_{\text{DPS},n} = 0, \ n \not\subset \mathcal{I}_{\text{DPS}}$$

in which the first term and the second term in (22) are the objective functions in (6) and (11), respectively, (23) is the constraint for CPS, and (24) is the constraint for L-level DPS. Regarding the previous derivation, we see that it is QCQP too, and can be relaxed and solved efficiently. In fact, it is also possible to employ different levels of DPS in one HPS beamformer. For this case, we need to include the corresponding objective functions and constraints in the optimization. The performance of the proposed CPS, DPS, and HPS beamformers under different settings will be experimentally shown in the next section.

VI. SIMULATION RESULTS

Finally, in this section, some numerical cases that clarify the characteristics of our phase-only beamforming strategies are presented. In the following, we compare the capability of the proposed CPS, DPS, and HPS beamformers, together with the approaches of CB, MVDR beamformer, and the phase-only beamformer presented in [15] for adaptive interference suppression. For our proposed beamformers, MATLAB-based CVX is used to acquire the results of SDR, and C language is used for the randomization. The program is run on a general-purpose computer (CPU: Intel Core i7-9Gen., RAM: 16GB). For all experiments, results can be obtained within a reasonable time.

A. TYPICAL ARRAY PATTERNS OF PROPOSED CPS AND DPS BEAMFORMERS

In the first set of figures, we inspect the behavior of our CPS and DPS beamformers and compare their performance with the optimal MVDR beamformer. To explore the superiority of our methods, we also simulate the state-of-the-art approach proposed in [15], in which the phase-only beamformer is optimized by convex relaxation too, for comparison. The same environment as those of [15, Sec. 3] is set, i.e., $N = 32$, a notch for interference suppression is required in the arrival angles from $-33°$ to $-30°$, and the main beam is pointed to $+10°$. To reveal the features of all considered methods, we set different SNR and ISR values for the environment in the experiments. In Fig. 1, we present the array patterns of the proposed CPS beamformer, the optimal MVDR beamformer, and the beamformer in [15]. Fig. 1(a) shows the results of SNR = 0dB and ISR = 20dB. We see that the optimal MVDR beamformer performs the best beyond question since it places the deepest notch in the interference direction and owns the lowest sidelobe level in the array patterns. Both CPS and the method in [15] have similar results in terms of the resultant array patterns. However, we see that CPS owns a lower sidelobe level than the method in [15]. Since the SNR value is relatively small, the CPS beamformer tries to reduce the sidelobe level for better output-SINR performance. Actually, when we examine their output SINR, the MVDR beamformer, CPS, and the method in [15] give the values of 15.02dB, 14.99dB, and 14.97dB, respectively. The difference among them is not large. Therefore, we may say that they are equally good even though both CPS and the method in [15] use phase shifters only without any amplitude control of the received signal. Then, we increase the SNR value to 30dB; different outcomes can be observed in Fig. 1(b). Again, MVDR is still the best, and the array patterns of both CPS and the method in [15] look alike as well. Nevertheless, in this scenario, the sidelobe of the CPS beamformer is higher than that of the method in [15]. CPS acts like this because when the SNR value is large, the sidelobe level does not affect the output SINR much. The key to good performance is the depth of the notch. The actual output-SINR values of MVDR, CPS, and the method in [15] are 45.00dB, 44.77dB, and 43.31dB, respectively. This time, the difference among them becomes large, and importantly, the proposed CPS beamformer performs 1.46dB better than the method in [15]. This indicates that the CPS beamformer can adapt itself to the environment better than the method in [15], and can provide an

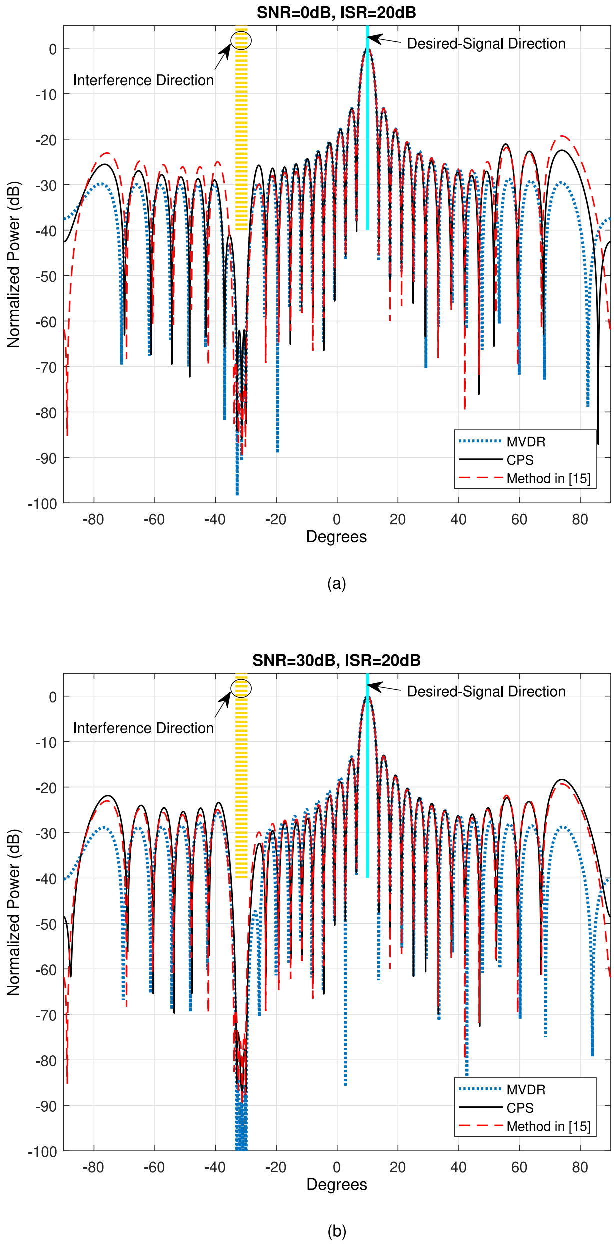

FIGURE 1. Array patterns of proposed CPS beamformer as compared with MVDR and the method in [15]. (a) SNR=0dB and ISR=20dB, (b) SNR=30dB and ISR=20dB.

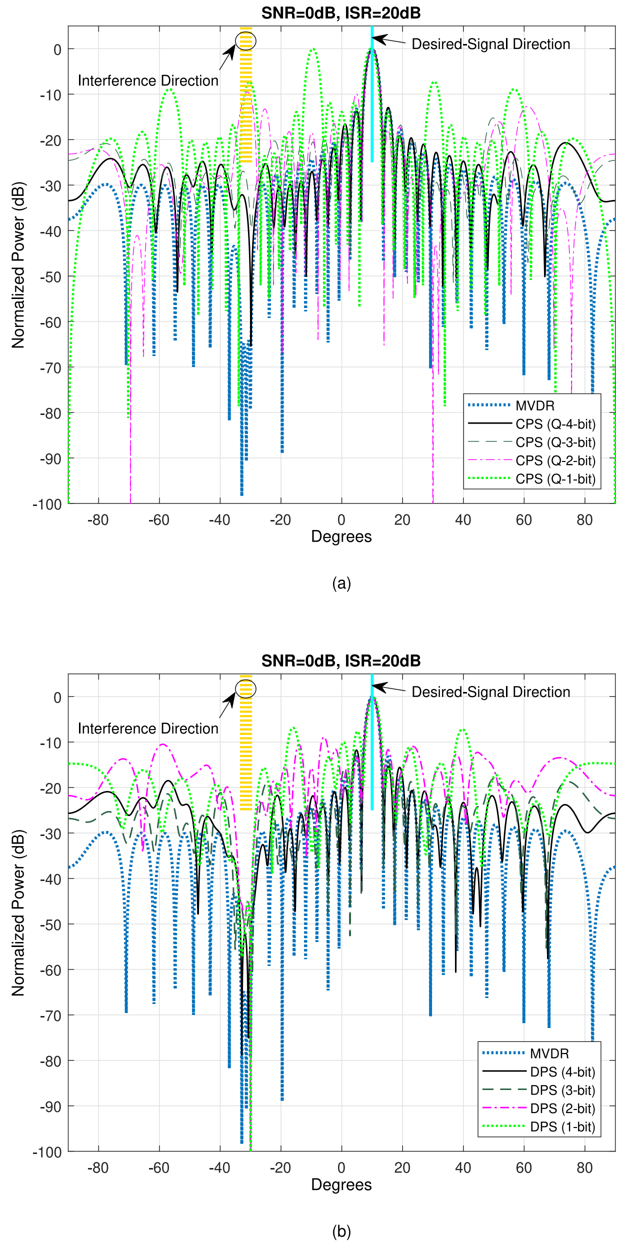

FIGURE 2. Array patterns of different beamformers using discrete phase shfiters only as compared with MVDR. (a) Direct quantization of CPS, (b) Proposed DPS beamformer.

output-SINR value much near that of the optimal MVDR beamformer.

After that, in Fig. 2, we mainly demonstrate the performance of the proposed DPS scheme with various settings. Fig. 2(a) shows the results of directly quantizing the outcomes of the CPS beamforming with 4, 3, 2, and 1 control bit(s) for the phase-shift levels, respectively. As revealed, it is intuitively true that the smaller the number of control bits is, the worse the interference suppression capability becomes. Note here that when the number of control bits decreases to 2 or 1, no notch is formed in the interference direction. While the output-SINR value of MVDR is 15.02dB for this case, those of CPS with 4, 3, 2, and 1 control bit(s) for quantization are 7.87dB, 2.89dB, −15.42dB, and −17.01dB, respectively. We see that the degradation is quite severe as the number of control bits reduces. Interestingly, Fig. 2(b) show the results of the proposed DPS beamformer with 4, 3, 2, and 1 control bit(s). It is not difficult to see that much deeper notches can be formed this time, even when the number of control bits decreases to 1. To be more specific, the output-SINR values of the DPS beamformer with 4, 3, 2, and 1 control bit(s) are 14.66dB, 14.13dB, 12.25dB, and 9.25dB, respectively. Hence, much better output-SINR performance is obtained, and this also means the optimization for DPS is worth doing.

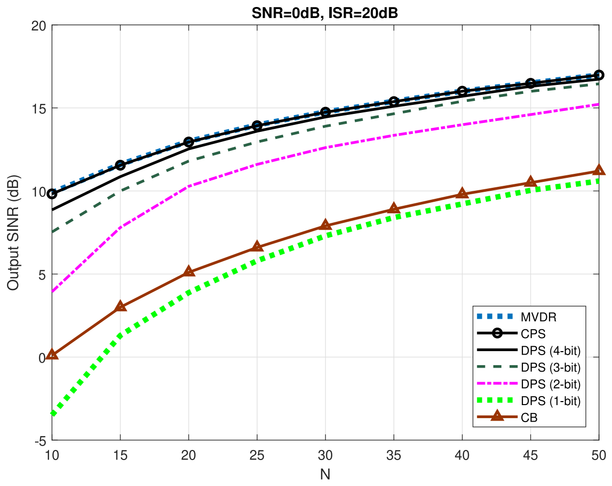

FIGURE 3. Average output-SINR performance of various approaches in the random environment with SNR=0dB and ISR=20dB vs. *N*.

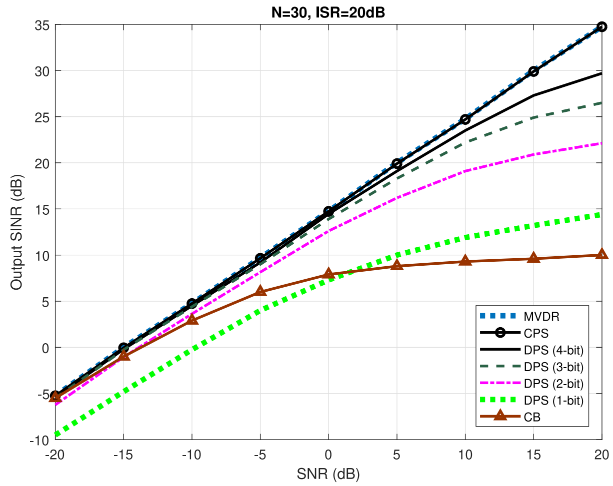

FIGURE 5. Average output-SINR performance of various approaches in the random environment with $N = 30$ and ISR=20dB vs. SNR.

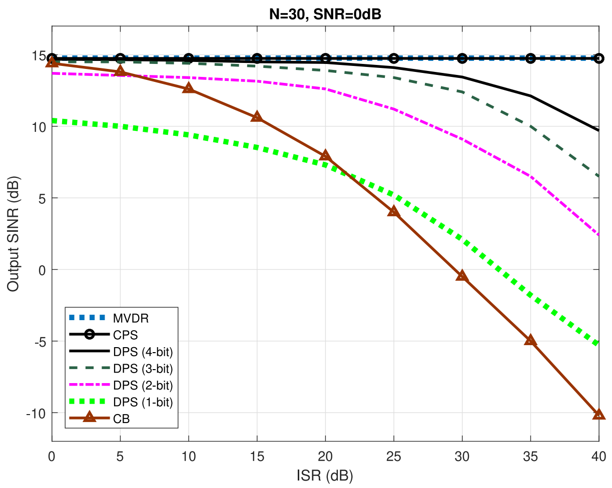

FIGURE 4. Average output-SINR performance of various approaches in the random environment with $N = 30$ and SNR=0dB vs. ISR.

Appendix gives the weight vectors $\mathbf{w}_{\text{CPS}}$ for plotting the array patterns in Figs. 1(a) and 1(b), and the weight vectors $\mathbf{w}_{\text{DPS}}$ for plotting the array patterns in Fig. 2(b), respectively.

B. AVERAGE OUTPUT-SINR PERFORMANCE OF PROPOSED CPS AND DPS BEAMFORMERS

After checking the typical characteristics of our CPS and DPS beamformers, we change to examine their average output-SINR performance, together with the CB and MVDR beamformer for comparison. To test these beamformers, we virtually set three zones in the environment, i.e., zone one is from $-70°$ to $-40°$, zone two is from $-15°$ to $-15°$, and zone three is from $40°$ to $70°$. The desired signal comes randomly from one of the three zones with a random angle of arrival generated within that zone, and likewise, two interference sources appear in the other two zones (one for each) at random. Fig. 3 shows the average output-SINR of different methods with SNR $=$ 0dB, ISR $=$ 20dB, and an increasing N. Intuitively, for all methods, we see a larger value of output SINR as N becomes large. The MVDR beamformer is the best, and the CPS beamformer performs almost as well as MVDR. Besides, more control bits for the DPS beamformer lead to better performance, and the output-SINR value of DPS with 4 control bits can be consistently close to the optimum, especially when N is large. When the number of control bits decreases, the output-SINR performance degrades unavoidably. We observe that when N is larger than 25, using even 1 control bit for the phase-shift levels can bring about the performance quite close to that of the CB. Recall that the CB still requires continuous-valued phase shifters for implementation; the proposed DPS with 1 control bit seems very attractive in actual usage for the case that the performance of the CB is acceptable.

Next, in Fig. 4, based on similar settings, we check the average output-SINR performance with SNR $=$ 0dB, $N = 30$, and an increasing value of ISR. Again, MVDR is optimal, and the CPS beamformer performs almost the same as MVDR. The performance of the DPS beamformer and CB degrades inevitably as ISR becomes large. Notwithstanding, we observe that the DPS beamformer can provide different levels of good performance with a reasonable hardware cost. In the high ISR region, the DPS beamformer with just 1 control bit can notably outperform the CB.

In addition, Fig. 5 reveals the average output-SINR performance with ISR $=$ 20dB, $N = 30$, and different SNR values. We see that as the SNR value increases, all methods give better output SINR. Again, the CPS beamformer is almost as good as the MVDR beamformer, and the output-SINR values of both schemes can linearly increase. Other approaches, including the DPS beamformer and the CB, may not be as good as the optimal MVDR beamformer in the high SNR region since very high precision of weighting is required for good performance in that region. However, DPS is still desirable in the low to medium SNR region for providing various levels of adaptive beamforming performance.

TABLE 1. $\mathbf{w}_{CPS}$ Found by the CPS Beamformer for Use in Figs. 1(a) and 1(b), and $\mathbf{w}_{DPS}$ Found by the DPS Beamformer for Use in Fig. 2(b)

$\mathbf{w}_{\mathrm{CPS}}$ in Fig. 1(a)	$\mathbf{w}_{\mathrm{CPS}}$ in Fig. 1(b)	$\mathbf{w}_{\mathrm{DPS}}$, 4 control bits	$\mathbf{w}_{\mathrm{DPS}}$, 3 control bits	$\mathbf{w}_{\mathrm{DPS}}$, 2 control bits	$\mathbf{w}_{\mathrm{DPS}}$, 1 control bit
-0.6076 - 0.7942j	0.0213 + 0.9998j	1.0000 + 0.0000j	1.0000 + 0.0000j	1.0000 + 0.0000j	1.0000 + 0.0000j
0.2112 - 0.9774j	-0.9121 + 0.4101j	0.7071 + 0.7071j	0.7071 + 0.7071j	1.0000 + 0.0000j	1.0000 + 0.0000j
0.2823 - 0.9593j	-0.8724 + 0.4889j	0.7071 + 0.7071j	0.7071 + 0.7071j	1.0000 + 0.0000j	1.0000 + 0.0000j
0.8941 - 0.4478j	-0.8999 - 0.4360j	0.0000 + 1.0000j	0.0000 + 1.0000j	1.0000 + 0.0000j	1.0000 + 0.0000j
0.9887 + 0.1499j	-0.5777 - 0.8162j	-0.7071 + 0.7071j	-0.7071 + 0.7071j	0.0000 + 1.0000j	-1.0000 + 0.0000j
0.9050 + 0.4254j	-0.2633 - 0.9647j	-0.9239 + 0.3827j	-0.7071 + 0.7071j	0.0000 + 1.0000j	-1.0000 + 0.0000j
0.3994 + 0.9168j	0.4790 - 0.8778j	-0.9239 - 0.3827j	-1.0000 + 0.0000j	0.0000 + 1.0000j	-1.0000 + 0.0000j
-0.1479 + 0.9890j	0.7708 - 0.6370j	-0.7071 - 0.7071j	-0.7071 - 0.7071j	0.0000 + 1.0000j	-1.0000 + 0.0000j
-0.5603 + 0.8283j	0.9995 - 0.0328j	-0.3827 - 0.9239j	-0.7071 - 0.7071j	-1.0000 + 0.0000j	-1.0000 + 0.0000j
-0.9232 + 0.3843j	0.9090 + 0.4168j	0.0000 - 1.0000j	0.0000 - 1.0000j	0.0000 - 1.0000j	1.0000 + 0.0000j
-0.9796 - 0.2009j	0.6011 + 0.7991j	0.7071 - 0.7071j	0.7071 - 0.7071j	0.0000 - 1.0000j	-1.0000 + 0.0000j
-0.7607 - 0.6492j	-0.0647 + 0.9979j	1.0000 + 0.0000j	1.0000 + 0.0000j	0.0000 - 1.0000j	1.0000 + 0.0000j
-0.3628 - 0.9319j	-0.4200 + 0.9075j	1.0000 + 0.0000j	1.0000 + 0.0000j	0.0000 - 1.0000j	1.0000 + 0.0000j
0.2821 - 0.9594j	-0.8862 + 0.4634j	0.7071 + 0.7071j	0.7071 + 0.7071j	1.0000 + 0.0000j	-1.0000 + 0.0000j
0.7023 - 0.7118j	-0.9955 - 0.0949j	0.3827 + 0.9239j	0.0000 + 1.0000j	0.0000 + 1.0000j	1.0000 + 0.0000j
0.9491 - 0.3151j	-0.8935 - 0.4492j	-0.3827 + 0.9239j	0.0000 + 1.0000j	1.0000 + 0.0000j	-1.0000 + 0.0000j
0.9343 + 0.3565j	-0.8935 - 0.4492j	-0.7071 + 0.7071j	-0.7071 + 0.7071j	1.0000 + 0.0000j	-1.0000 + 0.0000j
0.6709 + 0.7415j	0.0454 - 0.9990j	-1.0000 + 0.0000j	-1.0000 + 0.0000j	0.0000 + 1.0000j	-1.0000 + 0.0000j
0.2412 + 0.9705j	0.5720 - 0.8202j	-0.7071 - 0.7071j	-1.0000 + 0.0000j	-1.0000 + 0.0000j	-1.0000 + 0.0000j
-0.4025 + 0.9154j	0.9563 - 0.2923j	-0.3827 - 0.9239j	-0.7071 - 0.7071j	-1.0000 + 0.0000j	-1.0000 + 0.0000j
-0.7878 + 0.6160j	0.9963 + 0.0860j	0.0000 - 1.0000j	0.0000 - 1.0000j	-1.0000 + 0.0000j	1.0000 + 0.0000j
-0.9874 + 0.1580j	0.7187 + 0.6954j	0.7071 - 0.7071j	0.7071 - 0.7071j	0.0000 - 1.0000j	1.0000 + 0.0000j
-0.9057 - 0.4239j	0.2987 + 0.9544j	0.9239 - 0.3827j	0.7071 - 0.7071j	1.0000 + 0.0000j	1.0000 + 0.0000j
-0.5236 - 0.8519j	-0.1818 + 0.9833j	0.9239 + 0.3827j	1.0000 + 0.0000j	1.0000 + 0.0000j	1.0000 + 0.0000j
-0.1056 - 0.9944j	-0.7669 + 0.6417j	0.9239 + 0.3827j	0.7071 + 0.7071j	1.0000 + 0.0000j	1.0000 + 0.0000j
0.4385 - 0.8987j	-0.9453 + 0.3261j	0.0000 + 1.0000j	0.7071 + 0.7071j	1.0000 + 0.0000j	1.0000 + 0.0000j
0.9224 - 0.3863j	-0.9121 - 0.4099j	0.0000 + 1.0000j	0.0000 + 1.0000j	0.0000 + 1.0000j	1.0000 + 0.0000j
0.9943 - 0.1067j	-0.6937 - 0.7203j	-0.7071 + 0.7071j	-0.7071 + 0.7071j	0.0000 + 1.0000j	1.0000 + 0.0000j
0.8739 + 0.4860j	-0.2844 - 0.9587j	-0.7071 + 0.7071j	-0.7071 + 0.7071j	0.0000 + 1.0000j	-1.0000 + 0.0000j
0.2409 + 0.9706j	0.6268 - 0.7792j	-0.9239 - 0.3827j	-1.0000 + 0.0000j	0.0000 + 1.0000j	-1.0000 + 0.0000j
0.1689 + 0.9856j	0.5496 - 0.8355j	-0.9239 - 0.3827j	-1.0000 + 0.0000j	-1.0000 + 0.0000j	-1.0000 + 0.0000j
-0.6412 + 0.7674j	0.9794 + 0.2019j	0.0000 - 1.0000j	0.0000 - 1.0000j	0.0000 - 1.0000j	-1.0000 + 0.0000j

C. AVERAGE OUTPUT-SINR PERFORMANCE OF PROPOSED HPS BEAMFORMER

Finally, mainly based on the random environment defined in the previous part, we demonstrate the characteristics of the proposed HPS beamformer with various settings in Fig. 6. It is known that among the proposed phase-only beamforming approaches, in terms of the hardware needed, the CPS beamformer is the most complicated one while the DPS beamformer with only 1 control bit is the simplest one. Thus, we consider mixing these two schemes here to examine the performance of the proposed HPS beamformer. In Fig. 6(a), with $N = 30$, INR $= 20$dB and SNR $= 0$dB, we show the average output SINR of the HPS beamformer via different numbers of CPS connected to the antennas. The number of DSP with 1 control bit connected to the antennas is then equal to N $(= 30)$ minus the number of CPS. That is to say, in the figure, when the number of CPS equals 0, it is the DPS beamformer with 1 control bit. When the number of CPS equals 30, it becomes the pure CPS beamformer. The index sets $\mathcal{I}_{\mathrm{CPS}}$ and $\mathcal{I}_{\mathrm{DPS}}$ are randomly generated according

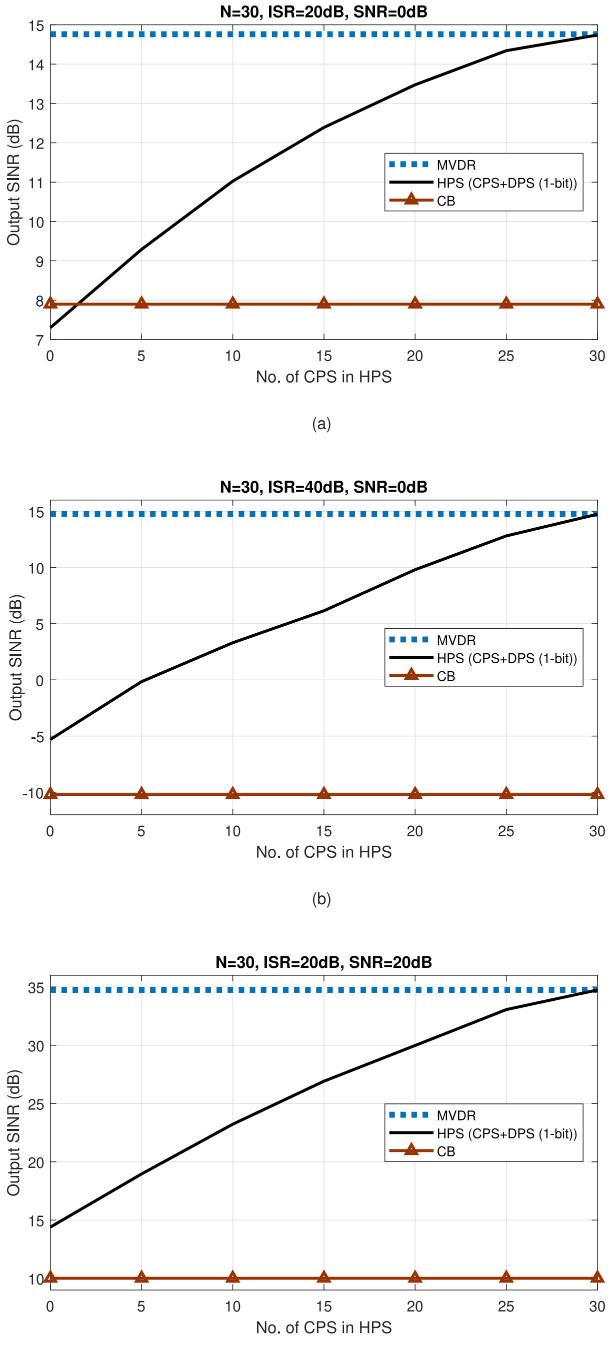

FIGURE 6. Average output-SINR performance of the proposed HPS beamformer with CPS and DPS (1-bit) vs. the number of CPS. (a) ISR=20dB, SNR=0dB, (b) ISR=40dB, SNR=0dB, (c) ISR=20dB, SNR=20dB.

to the numbers of CPS and DPS set in the experiments. The results of the CB and the MVDR beamformer are also shown in the figure for reference. As expected, as the number of CPS increases, the output-SINR values of the HPS beamformer increase quite smoothly. This offers an additional way to give different trade-offs between the output-SINR performance and the hardware cost. Last but not least, we change the ISR to 40dB and the SNR to 20dB, respectively, and the results are shown in Fig. 6(b) and 6(c). We see a similar trend in both figures too. These confirm the robustness of our HPS beamformer in different environments.

In summary, from the information provided in the figures, we may set the adequate phase-only adaptive beamformer according to the performance we need and the hardware complexity the system can tolerate.

VII. CONCLUSIONS

The concept of phase-only beamformers is a crucial way to lower the implementation cost, especially for large-scale antenna arrays. In this paper, we propose various phase-only beamforming strategies, including the CPS beamformer, the DPS beamformer, and the mix of them, named the HPS beamformer. We show that they can successfully adapt to the random interference-limited environment. Essentially, thanks to the advances in convex-optimization techniques, all the formulated problems can be solved efficiently to achieve different levels of performance and hardware complexity. Experimental results give valuable guidelines for the design and development of these phase-only adaptive beamformers. For example, the output SINR of the CPS beamformer can be consistently close to that of the optimal MVDR beamformer. Furthermore, the DPS beamformer with only 1 control bit can be comparable to the CB in terms of the output-SINR performance for most cases. All these benefit the usage of adaptive beamformers for real-world applications.

APPENDIX
WEIGHT VECTORS USED IN FIGS. 1 AND 2

Table 1 shows the weight vectors $\mathbf{w}_{\text{CPS}}$ found by the optimization of CPS for plotting the array patterns in Figs. 1(a) and 1(b), and the weight vectors $\mathbf{w}_{\text{DPS}}$ found by the optimization of DPS for plotting the array patterns in Fig. 2(b), respectively.

REFERENCES

[1] M. Chryssomallis, "Smart antennas," *IEEE Antennas Propag. Mag.*, vol. 42, no. 3, pp. 129–136, Jun. 2000.
[2] H. L. Van Trees, *Optimum Array Processing: Part IV of Detection, Estimation, and Modulation Theory*, 1st ed. New York, NY, USA: Wiley-Intersci., 2002.
[3] L. C. Godara, *Smart Antennas*, 1st ed. Boca Raton, FL, USA: CRC Press, 2004.
[4] R. J. Mailloux, *Phased Array Antenna Handbook*, 3rd ed. Boston, MA, USA: Artech House, 2017.
[5] J. Volakis, *Antenna Engineering Handbook*, 5th ed. New York, NY, USA: McGraw Hill, 2018.
[6] R. L. Haupt, "Phase-only adaptive nulling with a genetic algorithm," *IEEE Trans. Antennas Propag.*, vol. 45, no. 6, pp. 1009–1015, Jun. 1997.
[7] T. V. Luyen and T. V. B. Giang, "Interference suppression of ULA antennas by phase-only control using bat algorithm," *IEEE Antennas Wireless Propag. Lett.*, vol. 16, pp. 3038–3042, 2017.
[8] R. Shore, "Nulling a symmetric pattern location with phase-only weight control," *IEEE Trans. Antennas Propag.*, vol. 32, no. 5, pp. 530–533, May 1984.
[9] A. F. Morabito, A. Massa, P. Rocca, and T. Isernia, "An effective approach to the synthesis of phase-only reconfigurable linear arrays," *IEEE Trans. Antennas Propag.*, vol. 60, no. 8, pp. 3622–3631, Aug. 2012.

[10] M. Khalaj-Amirhosseini, "Phase-only power pattern synthesis of linear arrays using autocorrelation matching method," *IEEE Antennas Wireless Propag. Lett.*, vol. 18, pp. 1487–1491, 2019.

[11] S. T. Smith, "Optimum phase-only adaptive nulling," *IEEE Trans. Signal Process.*, vol. 47, no. 7, pp. 1835–1843, Jul. 1999.

[12] J. Liang, X. Fan, W. Fan, D. Zhou, and J. Li, "Phase-only pattern synthesis for linear antenna arrays," *IEEE Antennas Wireless Propag. Lett.*, vol. 16, pp. 3232–3235, 2017.

[13] Y. Aslan, J. Puskely, A. Roederer, and A. Yarovoy, "Phase-only control of peak sidelobe level and pattern nulls using iterative phase perturbations," *IEEE Antennas Wireless Propag. Lett.*, vol. 18, pp. 2081–2085, 2019.

[14] H. Lebret and S. Boyd, "Antenna array pattern synthesis via convex optimization," *IEEE Trans. Signal Process.*, vol. 45, no. 3, pp. 526–532, Mar. 1997.

[15] P. J. Kajenski, "Phase only antenna pattern notching via a semidefinite programming relaxation," *IEEE Trans. Antennas Propag.*, vol. 60, no. 5, pp. 2562–2565, May 2012.

[16] B. Fuchs, "Application of convex relaxation to array synthesis problems," *IEEE Trans. Antennas Propag.*, vol. 62, no. 2, pp. 634–640, Feb. 2014.

[17] P. Cao, J. S. Thompson, and H. Hass, "Constant modulus shaped beam synthesis via convex relaxation," *IEEE Antennas Wireless Propag. Lett.*, vol. 16, pp. 617–620, 2017.

[18] T. Hong, X.-P. Shi, and X.-S. Liang, "Synthesis of sparse linear array for directional modulation via convex optimization," *IEEE Trans. Antennas Propag.*, vol. 66, no. 8, pp. 3959–3972, Aug. 2018.

[19] Y. Lee, "Adaptive interference suppression of phase-only thinned arrays via convex optimization," *IEEE Trans. Antennas Propag.*, vol. 68, no. 6, pp. 4583–4592, Jun. 2020.

[20] C. Baird and G. Rassweiler, "Adaptive sidelobe nulling using digitally controlled phase-shifters," *IEEE Trans. Antennas Propag.*, vol. 24, no. 5, pp. 638–647, Sep. 1976.

[21] N. Goto and D. Cheng, "Sidelobe-reduction techniques for phased arrays using digital phase shifters," *IEEE Trans. Antennas Propag.*, vol. 18, no. 6, pp. 769–773, Nov. 1970.

[22] D. S. Weile and E. Michielssen, "The control of adaptive antenna arrays with genetic algorithms using dominance and diploidy," *IEEE Trans. Antennas Propag.*, vol. 49, no. 10, pp. 1424–1433, Oct. 2001.

[23] M. Clenet and G. A. Morin, "Visualization of radiation-pattern characteristics of phased arrays using digital phase shifters," *IEEE Antennas Propag. Mag.*, vol. 45, no. 2, pp. 20–35, Apr. 2003.

[24] H. Kamoda, T. Iwasaki, J. Tsumochi, T. Kuki, and O. Hashimoto, "60-GHz electronically reconfigurable large reflectarray using single-bit phase shifters," *IEEE Trans. Antennas Propag.*, vol. 59, no. 7, pp. 2524–2531, Jul. 2011.

[25] S. K. Goudos, "Antenna design using binary differential evolution: Application to discrete-valued design problems," *IEEE Antennas Propag. Mag.*, vol. 59, no. 1, pp. 74–93, Feb. 2017.

[26] T. H. Ismail and Z. M. Hamici, "Array pattern synthesis using digital phase control by quantized particle swarm optimization," *IEEE Trans. Antennas Propag.*, vol. 58, no. 6, pp. 2142–2145, Jun. 2010.

[27] P. Liu, Y. Li, and Z. Zhang, "Circularly polarized 2 bit reconfigurable beam-steering antenna array," *IEEE Trans. Antennas Propag.*, vol. 68, no. 3, pp. 2416–2421, Mar. 2020.

[28] R. Vescovo, "Reconfigurability and beam scanning with phase-only control for antenna arrays," *IEEE Trans. Antennas Propag.*, vol. 56, no. 6, pp. 1555–1565, Jun. 2008.

[29] G. Buttazzoni and R. Vescovo, "Power synthesis for reconfigurable arrays by phase-only control with simultaneous dynamic range ratio and near-field reduction," *IEEE Trans. Antennas Propag.*, vol. 60, no. 2, pp. 1161–1165, Feb. 2012.

[30] A. D. Khzmalyan and A. S. Kondratiev, "The phase-only shaping and adaptive nulling of an amplitude pattern," *IEEE Trans. Antennas Propag.*, vol. 51, no. 2, pp. 264–272, Feb. 2003.

[31] B. S. Yarman, *Design of Digital Phase Shifters for Multipurpose Communication Systems*, 1st ed. Gistrup, Denmark: River Publ., 2019.

[32] M. Grant and S. Boyd. "CVX: MATLAB Software for Disciplined Convex Programming, Version 2.0 Beta." Sep. 2013. [Online]. Available: http://cvxr.com/cvx

[33] O. Aldayel, V. Monga, and M. Rangaswamy, "Successive QCQP refinement for MIMO radar waveform design under practical constraints," *IEEE Trans. Signal Process.*, vol. 64, no. 14, pp. 3760–3774, Jul. 2016.

[34] O. Aldayel, V. Monga, and M. Rangaswamy, "Tractable transmit MIMO beampattern design under a constant modulus constraint," *IEEE Trans. Signal Process.*, vol. 65, no. 10, pp. 2588–2599, May 2017.

[35] M. A. ElMossallamy, K. G. Seddik, W. Chen, L. Wang, G. Ye Li, and Z. Han, "RIS optimization on the complex circle manifold for interference mitigation in interference channels," *IEEE Trans. Veh. Technol.*, vol. 70, no. 6, pp. 6184–6189, Jun. 2021.

[36] Z.-Q. Luo, W.-K. Ma, A. M.-C. So, Y. Ye, and S. Zhang, "Semidefinite relaxation of quadratic optimization problems: From its practical deployments and scope of applicability to key theoretical results," *IEEE Signal Process. Mag.*, vol. 27, no. 3, pp. 20–34, May 2010.

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