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Vacuum Arc Cathode Spot Theory. History and Evolution of the Mechanisms

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***Abstract*—Vacuum arc cathode spot theories are reviewed, beginning from the primary studies in second half of 19th century and in the beginning of 20th century up to the present. The evolution of the main ideas is presented, starting from those which developed for separate phenomena, up to systematic inclusive models. Different approaches were considered for generating a plasma comprised of cathode material, based on cathode vaporization and local explosions. Models with closed systems of equations were developed. A kinetic model advantageously allows calculating the cathode potential drop. These closed models explain various phenomena and spot types, based on two principals: (1) impeded plasma flow enables cathode spot operation, and (2) the heat loss in the cathode must be smaller than the energy input to the cathode spot.**

***Keywords*—** vacuum arc, cathode spot, plasma jet, spot motion

I. Introduction

The electrical arc is a low voltage and high current discharge. The vacuum arc is supported by the cathode material generated from a small "cathode spot". Vacuum arc characteristics related to the cathode spot were determined. Some were detected and studied first in the second half of the 19^{th} century and in the beginning of the 20^{th} century. The details of these investigations are interesting and important in understanding the evolution of modern cathode spot theory. Initially the cathode spot mechanisms were mysterious, and researchers sought to understand how the spots exists, how they support high current density, their behavior including their motion in a magnetic field and the nature of the plasma jets which they emit.

The objective of this paper is to review the main ideas developed to explain the cathode phenomena beginning from the early separate hypotheses of charge particles generation by the cathode up to the present ideas. Of course, this review paper does not present a complete theory of cathode mechanisms but only describes its current state, which now allows understanding the observed and measured characteristics and thus solving the past mystery.

II. Brief Experimental Summary

A. *Early observations*

Early investigators studied the force on the cathode, expansion of cathodic vapor and crater formation. Dewar in 1879 [1] and 1881 [2] first measured hydrostatic pressure within the arc generated in hollow carbon electrodes. He suggested considering "the transit of material from electrode to electrode". Aron [3] indicated that there is an electromotive force at a Hg cathode. He determined the voltage of the Hg arc initiated in a glass tube. Also Aron observed a Hg cathode spot with area of $0.02 cm^2$ for current of 3 A that correspond to current density of 750 A/cm^2. This result was discussed by Stark [4], using data from Aron's work [5], to measure the vapor pressure and vapor flux from electrodes in a glass chamber. Schuster and Hemsalech in 1898 [6] investigated macroscopic expansion of electrode vapor into air. According to this work, the temperature of cadmium vapor was calculated as 2700 K using the measured the velocity of 560 m/s. Duddell in 1904 [7] experimentally studied the arc resistance and a electromotive force indicating a statistical character of these arc parameters: for constant current, electrode material and gap and other conditions, the size and configuration of the vapor column and electrode craters differed. Also the repulsive effects, electrode force and crater sizes on the carbon electrodes in an atmospheric electrical arc were studied experimentally by Tyndall in 1920 [8], Beer & Tyndall in 1921 [9], and Sellerio in 1916 [10] and 1922 [11]. Gunter-Schulze [12] observed cathode spots in a Hg arc and measured the spot size using fast (<0.01 s) photography. He obtained that for currents 5.0, 11 and 41 A, the current density was of 4.1×, 4.1× and 3.7×10^3 A/cm^2 respectively. Tonks [13] studied traces produced by anchored Hg spots on Hg films, and found current densities ranging from 3×10^3 to 18×10^3 A/cm^2. Thus, the early measurements were conducted mainly on Hg cathodes.

B. *Later measurements of spot characteristics*

Improved experimental techniques were developed including profilometry of the craters and traces, and optical methods. Registration of the luminous spot area allowed characterizing the spot dynamics including the time of life, the type and speed of movement, and also the temporal variation of the cathode region.

Cobine & Gallagher in 1948 [14] investigated an arc in atmospheric pressure air. The current density was determined measuring the width of tracks left by a moving spots on oxidized Cu, Al and W cathodes. Straight linear motion was induced by applying an external magnetic field. The densities obtained with 2.6 A were: 1.24×10^5 A/cm^2 on Cu, 2.95×10^4 A/cm^2 on Al, 7.4×10^4 A/cm^2 on W, and 2.10×10^4 A/cm^2 on Hg films on a Cu substrate. Larger current densities of 10^6 A/cm^2 for Hg [15] and between 2.5× and 5×10^4 A/cm^2 for liquid sodium-potassium alloy cathode [16] were detected from photographs obtained using a Kerr cell shutter with an exposure time of 100 ns. The spots either expanded radially with a velocity up to 10^4 cm/s or formed a line, depending on the rate of current rise. Kesaev [17] reported for Hg a current density of 5×10^4 A/cm^2. Later investigations using modern high speed imaging equipment were conducted by Rakhovsy [18], Djakov and Holmes [19, 20], Siemroth *et al* [21], Juttner *et al* [22, 23, 24], Anders *et al* [25, 26, 27,] and others. According to these works the current density was reported in width range of values from 5×10^4 to 10^8 A/cm^2.

Different types of spots were defined according to life time t, and spot velocity v_s. The prevalence of these different types of spots depended on arc current, cathode material, and cathode surface characteristics (e.g. oxide or other film, roughness). It has been commonly accepted that for relatively low current (100-300 A) in the initial stage of an arc, fast moving (up to 10^3-10^4 cm/s) and short life time (<1 µs) cathode spots appear on all metals. With higher current, higher vapor or gas pressure and longer arc pulse duration, the spot velocity is less, and the spot life time is larger. Under some arc conditions, a few spots can be located close one another, producing a "group spot".

While some of the literature labels the spot types by the numbers 1, 2, 3, this characterization confusingly groups together spots with very different characteristics, and the numerical names do not convey any meaning. For example, by "type 1" often named the different spot behaviour (on clean and contaminated surfaces) as fast spot [18]. Also "type 2" includes spots with significantly different v_s and t in works of Ref. [28] and of Ref. [22]. An alternative labeling scheme is proposed in Table I (for cathode materials having Cu-like thermo-physical characteristics) in which the name of each spot type is chosen from a typical characteristics of that spot type.

TABLE I. CLASSIFICATION OF THE SPOT TYPES BY THEIR CHARACTERISTICS AND APPEARANCE CONDITIONS.

Abbr Name	Full type Name	Motion	Conditions: current; velocity; life time
SFS	Super fast spot,	Fast	Contaminated surface with impurity, oxide and other films. ≤10 A; 10^3-10^4 cm/s; <1-10 µs
MFS	Moderate fast spot		Preliminary cleaning the cathode surface by the arc. Non-uniform surface. ≤10 A; 10^3 cm/s; <10 µs
FCS	Film cathode spot		Hg cathode or metal film deposit on glass or metal substrates. ~0.1 A; ≥10^3 cm/s; 1-100 µs
SGS	Slow group spot	Slow	In a vacuum at relatively high arc current. 100-300 A; ≤10 cm/s; ≤1 ms
SIS	Slow individual spot		Gap filled by electrode vapor or low pressure gas. 10 A; 10-100 cm/s; ≤100µs

At low arc currents (≤10 A), the arc voltage oscillated at high-frequency [17] with peaks above the cathode potential drop, considered to be the minimum arc voltage of 15 V (on Cu). The oscillation amplitude decreased with arc current.

According to the original works [17, 29, 30, 31, 32, 33], to reviews [18, 34, 35], the typical parameters for bulk cathodes are: threshold arc current 1.6 A, spot current from about 10 A in a single spot to about 100-300 A in a group spot consisting of several fragments, arc voltage 20–23 V, jet velocity 10^6 cm/s, ion energy/unit charge 30–37 eV, ion current fraction in the plasma jet of 0.1 [36], erosion coefficient 0.05-0.1 mg/C including the mass loss in form of ions and macroparticles. On film and Hg cathodes, the spot current is<1 A.

III. EVOLUTION OF CATHODE SPOT THEORIES

An electrical discharge can be supported only if enough charge particles were generated in the electrode gap to carry the current there. The problem is transfer of the current between the highly conductive metallic cathode and gap plasma with relatively low conductivity. Nnumerous theoretical studies were conducted by modeling cathode spot functioning. Below spot models in from the past century up to the present time are analyzed.

A. Primary hypotheses and Models

Stark [37] in 1901 first attempted to understand the cathode phenomena, attributed electron emission. According to Stark [38], electrons emitted thermionically according to the Richardson law. Richardson studied thermionic emission from platinum and graphite [39]. However, arcs on various cold cathodes cannot be explained by thermionic electron emission. Langmuir 1913 [40] developed a theory of a positive space charge produced by the ions concentrated at the cathode surface. He [41] has suggested that the electric field can be strong enough for sufficient field emission (F-emission) from the cathode. The Langmuir F-emission theory, together with the electric field equation derived by Mackeown [42] in 1929, also meets difficulties to satisfactorily explain the cathode spot in this period, because the current densities measured then were lower than the values we now have.

Further models of electron emission mechanisms include Slepian's 1926 thermal model of the cathode spot [43] and Smith's 1942 hot electron hypotheses of bombardment of the cathode by the energetic electrons from the Maxwellian distribution "tail" for electrons near the surface [44] (model of hot electrons). Other groups assumed that the current density in the cathode spot can be sufficiently large to cause the explosive plasma generation from Joule heating of a local cathode volume. Nekrashevich and Bakuto [45] developed a "migration theory" in 1955, Rotshtein [46] suggested in 1948 a "dense vapor cathode" model. Engel & Robson [47] proposed in 1957 that electrons were emitted by the energy of excited atoms moving from the plasma to the cathode. The above approaches, however, were based on unfounded assumptions and free parameters.

B. Mathematical approaches using a system of equations

An attempt to analyze together a number of cathode processes based on evaporation of the cathode material and electron emission was modeled by Lee in 1959 [48] and Lee and Greenwood in 1961 [49]. They formulated a system of equations describing the cathode heating and electron emission. The five dependent variables were 1) cathode spot temperature T, 2) electric field E_c at the cathode surface, 3) current density j, 4) electrons current fraction s and 5) spot radius r_s.

These variables were treated as constants over the spot area. Four equations were used: 1) Mackeown's equation [42] determined the relation between E_c, j, and s at the cathode; 2) equation of cathode of thermal-field (T-F) electron emission in general form; 3) energy balance equation; 4) equation of total spot current I expressed that current density multiplied by spot area. To determine possible values for the mentioned above the fifth unknown, two limiting conditions were used: a) atom-ion balance, i.e. the ion flux cannot exceed the evaporated atom flux; b) radius r_s was determined by equating the magnetic and plasma pressures. A minimum current of the order of 2-10 A was calculated to be required to support the spot processes. The current density obtained for Cu was 10^5-10^6 A/cm^2 with s=0.5-0.7. However, this work lacked any mechanism for ion flux formation, the system was steady-state, it was not closed and the solutions were multiple-valued.

Another model using limiting conditions was published by Ecker in 1971 [50, 51]. He noted the lack of exact information about physical processes, and felt that the spot parameters could not be precisely determined. Accordingly, he added to Mackeown's equation [42] and the equation of cathode electron emission in general form the following limitations: a) flux from the plasma produced, when all energy of the emitted electrons acquired in the sheath is spent on the vapor ionization, b) the atom-ion balance [49], c) a simple form of the cathode energy balance, and the calculated temperature assuming that the upper limited of its values can be obtained.

Using these three limiting relations, three dependencies of cathode temperature T on current density j were calculated. These dependencies were presented in an "existence diagram" which presented an area in the T-j coordinate plane that included all another possible values of j and T. This area was small in early Ecker's works and, as was assumed, this method effectively solved most of the complicated spot problems.

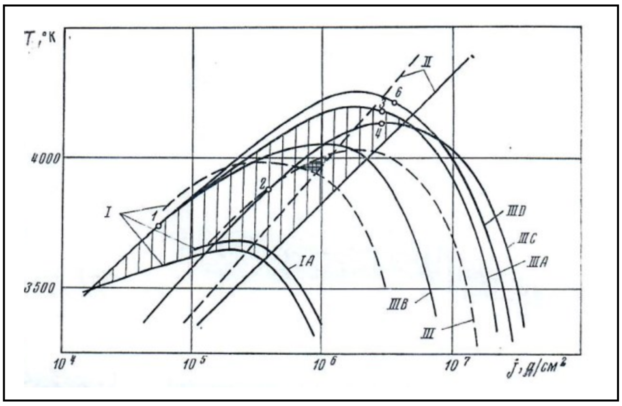

Fig. 1. E-diagram calculated for Cu cathode at I=200 A. Densely hatched region with dotted lines presents the Ecker's early calculations. Sparsely hatched region with solid lines presents Beilis and Lyubimov's calculation [54]. Curves I and II present the solutions of equations together with using atom-ion balance and limited plasma balance respectively. Curve IA was obtained using lee's result [49]. Curves III were obtained from the solution together with relations for cathode energy balance in different forms: IIIA with measured cathode eroson rate; IIIB with $\varphi_{ef}=\varphi$=4.5 eV; IIIC- with $\varphi_{ef}=\varphi-(e^3E)^{1/2}$ (Schottky effect); IIID with taking into account the energy by returned plasma electrons back to the cathode [52]. The (◦) points indicate the exact solution according to the model [52] which will be described below.

However, Ecker later noted in 1980 that the small area was obtained erroneously [53]. A corrected analysis was conducted in 1975 [54] and the results are presented in Fig.1 (see figure caption, dotted curves-Ecker's solution). It can be seen that the curve I is close to the curve IA obtained using Lee's data [49] while both these curves significantly differ from the Ecker's curve (dotted) extending the area in comparison with the Ecker's area of "E-diagram". Dependencies II are weakly different between one to other. The cathode energy balance was similar to that used by Lee [49]. An important question is validity of the Langmuir-Dushman equation [55] for temperature dependent rate of metal evaporation W(T) in order to calculate the cathode mass loss or erosion rate in a vacuum arc. This equation expressed the evaporation in vacuum and not considered the returned particle flux due to rarefied collisions. Therefore, even not go into details, as a large particle flux is returned to the cathode, at least by the ions, this equation cannot be justified. This can be seen by analyzing the difference between the dotted curve III calculated using Langmuir-Dushman equation and solid curve IIIA calculated using measured rate of cathode erosion.

Ecker corrected his calculation in 1976-78 [56, 57] while retaining his earlier point of view [53] about the "E-diagram". Comparison of curves III in Fig. 1, calculated for different forms of the cathode energy balance, shows that these curves are not only different from each other, but it is impossible to state *a priori* how the dependence can be changed. This means that it is impossible to state how the E-Diagram will be changed using an improved limiting energy balance. Given these difficulties with E-Diagrams (especially curve III), the benefit of Ecker's later investigation [58, 59] is doubtful. This conclusion also related to extension of the method taking into account the non-stationarity of spot operation by calculation of the time dependent cathode energy balance and the spot appearing at a roughness cathode surface using effective coefficient of enhancement of cathode electric field [53].

C. Double-valued current density in a structured spot

Some of authors' aassumed in their works two values of current densities modelling in order to understand the electron emission mechanism and the cathode energy fluxes. Typical of this approach was Hull's 1962 work [60]. On one hand, he assumed that at the cathode a relatively large cathode spot with an average current density of 10^5 A/cm^2 should satisfy the cathode energy balance. On the other hand, this spot consisted of a number of small randomly moving spots with a current density of j=10^7 A/cm^2 emitted by field emission (assuming a roughness factor of 2.5). The ion current fraction was arbitrarily set at f=0.05. Thus, the number of assumptions casts uncertainty on the results although the model reflects the observed spot fragments.

Fursey and Vorontsov-Vel'yaminov 1968 [61] studied the cathode processes after local explosion of a protrusion. A new spot initiation vas assumed under an expanding "plasmoid" plasma sphere, which appeared after the simulated explosion due to electron F-emission from the cathode. They claim that this can be realized where the local field at the protrusions was enhance the average cathode field by the large β-factor (enhancement coefficient). However, the space charge sheath thickness around it is much lower than size of the protrusion and thus there will not be any significant enhancement.

Osadin [62] developed a cathode spot model using crater sizes, diameter d and depth h. To determine the mass loss he used the experimental finding that the cathode spot consists of a number of craters in which $d>>h$. Using an assumptions the analysis indicated that the measured current density only agreed with a mechanism based on local fluctuation of the electric field described by Ecker and Muller [63, 64].

Mitterauer in 1972-73 [65, 66, 67,] developed a theory of Dynamic Field Emission (DF-emission) to model the origin of the electrical current in the cathode spot of a vacuum arc. He suggested explosive evaporation and electron emission from protrusions. This process would be supported during the spot life by a number of protrusions from a large cathode area. A plasma cloud was generated due to a "macrospot" – an explosion of a protrusion inside the cloud, in which the atom ionization, space charge sheath and ion bombardment supported the spot current. A similar explosive model was reported by Beilis, Kantsel, Rakhovsky 1973 [68]. This idea was proposed to explain the low current density ($\approx 10^4$ A/cm^2) observed optically [18] by relatively large size of luminous of exploding plasma size reached already after about 10 ns.

Goloveiko in 1968 [69, 70] and Kulyapin 1971 [71] further developed the spot model from Lee's work [49]. Solutions were found by varying the electric field E as a free parameter. Because the solutions were obtained not self-consistently and due to other deficiencies, the indicated agreement between the calculations and the measurements cannot be considered sufficient. Kozlov and Khvesyuk [72] described a spot model which analyzed the cathode plasma structure which consist of two collisionless and collision dominated zones in the cathode region. However there was a contradiction between the assumption of a strong electric field in the second zone and the reality that this is a collisional region with low field. Furthermore, they calculated a very large current density of $j=7\times 10^7$ A/cm^2, with an electron current fraction of $s=0.96$ for a Hg cathode. At this s and $u_c=10$ V, the heat flux due to ion current density is 3×10^7 W/cm^2. At such large power density the mercury will be at a critical state and which must be further explained to understand it.

Hantzsche 1972 [73] studied the thermal regime of the cathode with the heat conduction equation, taking into account Joule energy dissipation and heating by ion energy and the Nottingam effect. Different analytical solutions were obtained when a surface temperature was given. The maximal spot current was calculated for a given spot current density of 10^7 A/cm^2. It was obtained [74, 75] that the heat conduction, as a linear process, cannot compensate the resistive heating if the mean current density exceeds a critical value. However this thermal runway is not possible if there is electron emission cooling or if the transient spot has life time shorter than that necessity for thermal runway.

Summarizing these works, published before and around 1970, most of the spot models attempted to understand the mechanism of electron emission considering the phenomena in the cathode body, at its surface and in the electrical sheath. Most used a steady-state approximation. The plasma in the cathode region was considered using limiting conditions, free spot characteristic values, uncertain experimental values, or arbitrary parameters.

However, the role of the cathode plasma was not determined, including ion motion to the cathode surface, ion flux formation, and a quantitative calculation of plasma electron density and temperature. Therefore these models were not complete and the system of equations was not closed. The nature of ion generation and ion motion was clarified by Beilis and Rakhovsky [76] who first developed a spot model describing ion flux formation by ion diffusion towards the cathode based on *charge-exchange* of ion-atom collisions.

D. *Diffusion model of the cathode spot.*

(a) Thermal model of the cathode

In order to understand the thermal regime in the spot, the time dependent 3-D heat conduction equation was solved for a Gaussian distributed heat influx at the cathode surface with the characteristic Gaussian radius equal to the effective spot radius r_s. The nonlinear heat losses due to evaporation and radiation from the surface were taken in account. The first result of calculation was presented for W in Ref. [77].

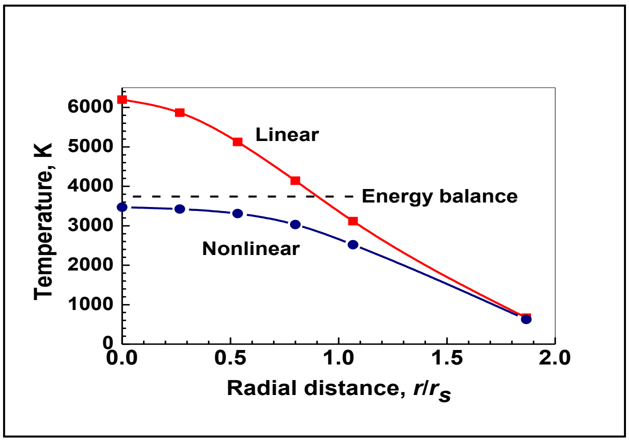

Fig. 2. Transient radial temperature distribution on a Cu cathode under a heat source with spot power 1.7 kW obtained using a linear and non-linear approximation. The dotted line indicates the temperature obtained from the cathode energy balance written for the spot center for t=150 μs, r_s =150 μm.

The result of the calculations for a Cu cathode is presented in Fig. 2 [78]. As can be seen, there is a nearly flat portion of the temperature profile (in nonlinear approach) near the center, which is close to the temperature calculated from the time dependent energy balance written for the spot center. The temperature profile was flatter when r_s and spot life time t were smaller. This profile is flat due to the evaporative cooling, which varies exponentially with the temperature. This flattening will be enhanced by returned heavy particles in Knudsen layer (see below). The simplification of assuming a uniform temperature thus appears profitable, in view of the complexity of the problem with non-uniform temperature and embraces the simplicity of Lee & Greenwood approach [49].

In our spot model, the Gaussian distributed heat source was used to determine the temperature at the spot center in the energy balance. The effective Gaussian radius obtained from the self-consistent solution was equated to r_s. The time-dependent temperature was determined by heat conduction and the transient spot parameters, since the gasdynamic processes have much smaller characteristic times.

(b) Direct electron impact atom ionization

Atom ionization determines ion generation. A model with characteristic plasma zones near the cathode which considered electron impact ionization was developed in 1969 [79] and in 1970 [80, 81]. Analysis of elementary collisions showed that the ion-atom mean free path is significantly shorter than the mean free path of the energetic electron beam emitted from the cathode and accelerated in the space charge sheath. This enabled a model in which diffusion produced an ion flux Γ_i to the cathode due to the ion density gradient near the surface. This approach was mathematically formulated into the following equations. The ion diffusion equation, with ion generation F and N_{e0}- density of emitted electron flux, is:

$$\frac{dn_i}{dt}=\frac{d\Gamma_i}{dx}+F_i(x) \quad (1)$$

$$\Gamma_i=\mu_i n_i E_{pl}-D_i\frac{dn_i}{dx};\quad D_i\sim\frac{v_{iT}}{n_a\sigma_{ia}} \quad (2)$$

$$F_i(x)=n_a\sigma_i N_{e0}Exp(-n_a\sigma_i x) \quad (3)$$

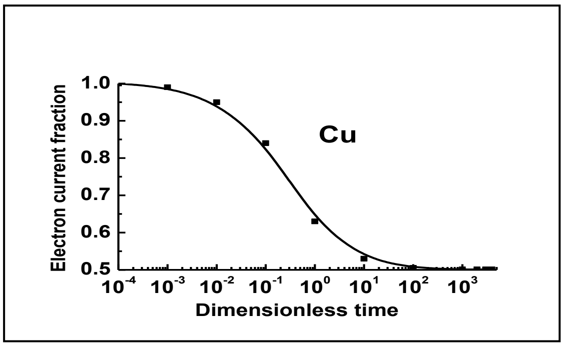

Fig. 3. Electron current fraction as function of dimensionless time for Cu.

Here n_i and n_a are the ion and atom densities respectively, D_i and μ_i are the diffusion coefficient and mobility of the ions respectively, σ_{ia} and σ_i are the charge exchange and atom ionization cross sections respectively and v_{iT} is the ion thermal velocity. Taking into account the ion temperature T_i, defining $s=N_{e0}/[N_{e0}+\Gamma_i(0)]$ and considering that the plasma field satisfies the inequality $eE_{pl}<<n_a\sigma_{ia}kT_i$, the solution of system (1)-(3) is:

$$s(\tau_d)=\left[2-\frac{1-\Phi(\sqrt{\tau_d})}{1-A_d}Exp(\tau_d)+\frac{1-\Phi(\sqrt{\tau_d/A_d^2})}{1-A_d}Exp(\frac{\tau_d}{A_d^2})\right]^{-1} \quad (4)$$

$$\tau_d=\frac{nv\sigma_i^2}{3\sigma_{ia}}t;\qquad A_d=\frac{2\sigma_i}{3\sigma_{ia}}$$

Here τ_d is dimensionless time, and $\Phi(\tau_d)$ is the error function. The solution of eq. (4), presented in Fig. 3, show that s decreased with time to a maximum value at steady state of 0.5.

(c) Diffusion model of the cathode spot - general case [82]

The model in the general case considered an electron beam relaxation zone taking into account ion generation due to direct impact and thermal ionization and the electron scattering due to Coulomb collisions with the charged plasma particles. The following diffusion equations were derived by studying the mass and energy flow in a multi component (electron, ion, atom), partially ionized plasma [82].

$$\Gamma_i=-(1+\theta)D_{ia}\frac{dn_i}{dx}+n_i v+\alpha D_{ia}\Psi;\quad D_{ia}\sim\frac{0.4v_{iT}}{(n_a+n_i)\sigma_{ia}} \quad (5)$$

$$\frac{d\Gamma_i}{dx}=n_a\sigma_i N_{e0}Exp\left[-\int_0^x(n_a\sigma_i+2n_i\sigma_c)dx\right]+\beta n_0^2 n_i(1-\frac{n_i^2}{n_0^2}) \quad (6)$$

Here v is the plasma velocity, where the plasma particles originate from erosion of the cathode, n_0 is the equilibrium plasma density, σ_c is the Coulomb collision cross section, α is the degree of ionization, β is the recombination coefficient, Ψ is a term describing the particle flow due to the pressure and temperature gradients of electrons and ions. The details can be find in Ref. [82].

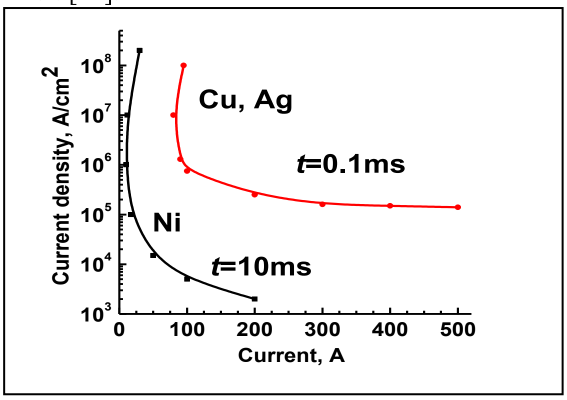

Fig. 4. Current density as function on group spot current u_c=15 V (Cu), 13 V (Ag), and 18 V (Ni).

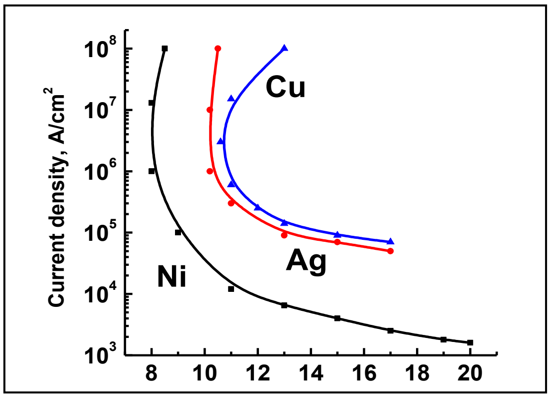

Fig. 5. Steady state current density as function on cathode potential drop.

The total system of equations describing the phenomena in the cathode, sheath and plasma was summarized in Ref. [52]. The plasma parameters including the *s* were directly calculated. This system was mathematically closed as no free parameters were used, but not physically. The cathode potential drop, spot current, erosion rate were given from the experiment for some spot life time. The first calculated results were presented in Ref. [83] and detail analysis was conducted in Ref. [52, 84]. As an example, the current density *j* for a group spot is calculated as a function of spot current (Fig.4) and of cathode potential drop u_c (Fig. 5, *I*=200 A) for erosion rates measured by Kantsel *et al.*[85]. It can be seen that minimal values of *I* and of u_c were obtained. The current density increases when the spot life time decreased (Fig. 6).

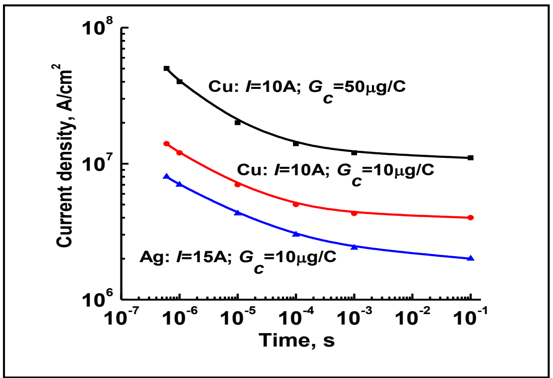

Fig. 6. Current density as function on spot life time.

(d) Applications to low melting and refractory cathodes

Analysis showed that there is no mathematical solution of the system of equations using the diffusion approach (which below will be called the Gasdynamic Model (GDM)) for cathode materials like Hg or W. For relatively low temperature possible Hg, the electron emission current density is insufficient, while the vaporized atom flux is too much to reach a concordance between plasma particle and energy balances. Refractory metals (e.g. W) have the opposite problem. With increasing surface temperature, the electron emission current density increased, while the vaporized atom flux was too low. There also the particle and energy balances had some contradictions.

To overcome these contradictions, new ideas were developed for the near-cathode sheath that determines the spot mechanism. A double sheath model was developed for Hg cathodes [86]. An adjacent layer to the cathode was hypothesized, which served as a "plasma cathode" supplying the required electron flux. For the refractory materials, the electron space charge near the cathode produced an electric field in the direction that reduced the large thermionic electron emission, thus acting as a virtual cathode [87].

IV. Other Spot Models

Below spot phenomena developed in the last decades are described.

A. Models of explosive electron emission and cathode vaprization

Bugaev *et al* 1975 [88] specified the phenomena of explosive electron emission (EEE) as an existence mechanism for cathode spots. Their conclusion was based on comparing their EEE results with experimental cathodic arc findings, including the presence of a dense plasma near the cathode indicating high (10^7-10^8 A/cm^2) current density, arc voltage fluctuation, craters, plasma expansion velocity and the presence of multiply charged ions.

Litvinov *et al* in 1983 [89] were discussed crater formation and spot motion in framework of EEE. An emission center (EC) model due to EEE was summarized in 2000 Ref. [90], using conical protrusions and spherical craters on plane cathode surfaces, created by a previous adjacent microprotrusion explosion. In both papers, the initial EC radius was assumed to be sufficiently small (0.1 µm) so that the Joule heating should reach a very large value which was dominated as a heat source. Note, the EC model demands a current density of 10^9 A/cm before spot initiation and a rate of current rise of 10^9 A/s during the EEE lifetime of about 1–10 ns, which can be realized under relatively high electrical field that occurs at the electrical breakdown.

The EEE phenomenon under interaction of a W-fuzz with relatively hot plasma was considered in recent work 2011 [91]. In essence the vaporization model using Mackeown and electron emission equations etc. (like above gasdynamic model) was used. As the plasma interaction was considered with W-fuzz nanowires, it is not surprising that strong electron emission was induced, reaching an explosive level. Similarly found the explosion conditions through given small sizes of microprotrusion on a W surface and by varying the plasma parameters [92]. These EEE model investigators asserted that *only the explosive process* can characterize the transient behavior of the spot and that other models are stationary and ignore their real transient essence.

Harris and Lau [93] considered the plasma processes taking into account different zones of the near-cathode plasma using a vaporization approach. An approximation for ion charge multiplicity, based on a separate calculation, which does not depend on density of the neutral, was used in the system of equations. They also arbitrarily assumed that ion fluxes towards the cathode and anode were each equal to half of the atom flux evaporated from the cathode. The ion velocity at the anode and cathode side of the ionization zone was arbitrarily set at the sound velocity.

The cathode spot parameters were calculated by Nemchinsky in 1979 [94] and 1983 [95], in essence, using the gasdynamic model of the spot and the vaporization approach [52, 82]. An additional relation that expressed the voltage in the quasineutral plasma was used together with a "minimal principle" condition determined by the total arc voltage dependence on the cathode temperature. Spot motion was explained by the increase of the cathode voltage drop in the direction of motion due to the difference of the plasma column length at the rear and front sides of the spot [96].

Taking into account cathode material phase changes, Joule heating and ionic heating, cathode crater formation was considered by Prock [97] and similarly by Klein *et al* [98]. The thermal runaway phenomenon was analyzed as the origin of micro-explosions and a possible mechanism for crater formation. Thermal mechanism of spot formation free varying *E &j* considered in Ref. [99]. Instability in a vacuum arc was analyzed by solving the system of equations, using the effective cathode voltage and *s* in the plasma jet as input parameters [100], assuming that the cathode plasma is fully ionized and an arbitrary equation of particle conservation.

Thus, these models consisted of a system of equations with arbitrary assumptions and arbitrary parameters, such as ion current density, electric field, surface temperature, and/or electron temperature [101, 102]. An idea to modelling the spot development from primary plasma was considered in Refs [103,104]. This idea was taken into account to study the cathode spot numerically using standard commercial software in Ref. [105, 106]. The primary conditions were described by four free parameters, which determined the time dependent spot parameters.

B. Kinetic spot theory. Closed system of equations

The *kinetic* model describes metal vaporization *into dense plasma near the cathode surface with non-equilibrium plasma flow* [107]. This model is based on an approach developed for metallic atom evaporation into vacuum in work [108]. The kinetic model considers a highly ionized cathode vapor plasma, whose structure consists of the few partially overlapping regions starting from the cathode surface 1 (Fig. 7): the **ballistic zone** comprising a space charge sheath, from the surface 1 to boundary 2, the **Knudsen layer**, having a non-equilibrium particle velocity function distribution from boundary 1 to a boundary 3, the **electron relaxation zone** where the plasma electrons are heated, from boundary 1 to boundary 4, and a **plasma acceleration region** beyond boundary 4.

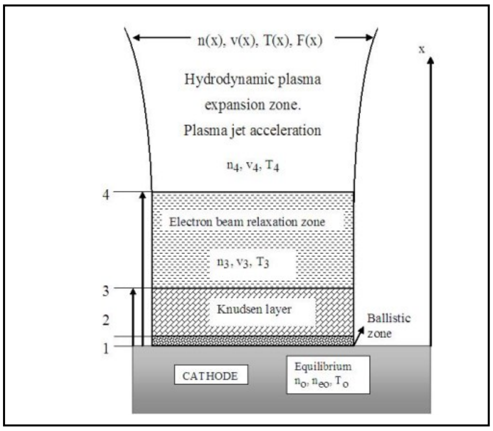

Fig. 7. Schematic presentetion of the kinetic model.

The particle parameters at these boundaries are denoted as $n_{\alpha j}$ $v_{\alpha j}$, $T_{\alpha j}$, where indices α - *e, i, a* denote electrons, ions, atoms respectively and *j=1, 2, 3, 4* denotes the boundary numbers and n_{eo} and n_o are the equilibrium electron and heavy particle densities determined as saturated values by the cathode surface temperature T_0 [109]. The problem of the vacuum arc cathode spot is reduced to determining the parameters at these boundaries i.e. to integrating the equations expressing the conservation laws using the velocity distribution functions.

The mathematical description consists of a system of equations that includes the equations of electron emission in general form, equations of total spot current *I*, ion and electron back fluxes, electric field at the cathode surface E_c, the kinetics of emitted and back flowing particle fluxes and plasma energy. The cathode energy balance includes Ohmic heating and the energy flux from incident ions and electrons from the plasma. Cooling is from heat conduction into the body of the cathode, radiation, evaporation and electron emission. The incoming heat flux to the cathode needed for self-consistent spot operation varies with time and depends on the current and plasma parameters during spot evolution.

A system of Saha equations in the quasi-neutral plasma determines the fraction of neutral atoms and ions in various charge states. The jet velocity *V* is determined by the equations of momentum and energy conservation in the plasma jet. The gasdynamic plasma acceleration is due to the ion and electron pressures and by electron-ion friction, which are determined by the electron beam energy. In the Knudsen layer between boundaries 1 and 3, evaporated and returning heavy particle fluxes are formed; their difference is the net cathode mass loss flux and the cathode erosion rate E_i(g/C)=$m(n_{a3}+n_{i3})v_3/j$, where $m=m_i=m_a$. The spot current *I* was calculated using the equation for cathode plasma, taking into account the spot radius and the general form of Ohm's law. For spot at protrusion its size determined the value of *I*.

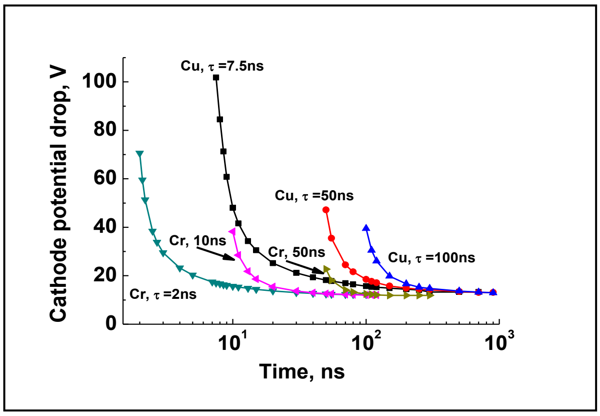

Fig. 8. Cathode potential drop as a function of time, with τ and cathode material (Cu, Cr) as parameters.

The heavy particle and electron kinetic flow considered together allows calculating *the cathode potential drop using the condition of quasi-neutrality* at boundary 3 & 4 [110].

The calculated parameters are the electron temperature $\boldsymbol{T_e}$, heavy particle density ***n***, degree of ionization α_i, cathode temperature ***T***, erosion rate ***G***, cathode electric field $\boldsymbol{E_c}$, current density ***j***, and electron current fraction ***s*** for a continuously developing spot on a bulk cathode. An important calculated parameter is the fraction of the evaporated cathode material $\boldsymbol{K_{er}}$, which is the ratio of the net evaporation rate into the adjacent dense plasma, to the Langmuir evaporation rate.

The system of equations for the cathode plasma is complex and has been described in detail previously [109]. Using the kinetic model, time dependent of several spot parameters for Cu & Cr were calculated after time of spot initiation τ [104]. For simplicity, and to illustrate the main characteristics of the spot, calculations for a I=10 A are presented in Figs. 8-10.

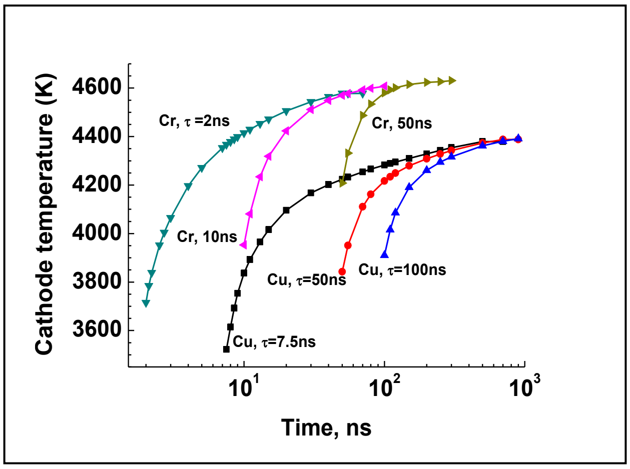

Fig. 9. Spot temperature as a function of time with τ and cathode material (Cu, Cr) as parameters

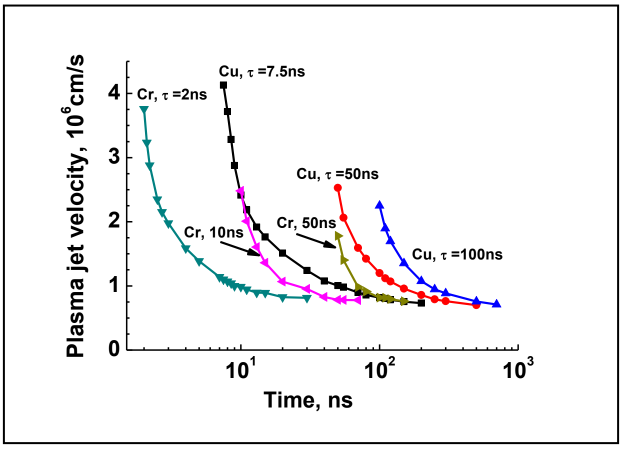

Fig. 10. Plasma jet velocity as a function of time with τ and cathode material (Cu, Cr) as parameters

The analysis showed that he evaporated atom flux is comparable with the flux of the returned atoms and ions. The evaporation fraction K_{er} decreased with time reaching value of ~0.4. The calculated velocity of the dense plasma is significantly smaller than the sound velocity. Therefore, the plasma flow in the Knudsen layer is not free, but rather is impeded. So, the main two principles of the kinetic model were derived [111] and discussed [109, 112]:

i) ***The impeded plasma flow in the plasma adjacent to the cathode is one of the important mechanisms for self-consistent support of the cathode spot ignition and operation***;

ii) The second principle is that the ***cathode heat loss, at least, should be smaller than the inflow power at a low spot current.*** This is equivalent to the effective cathode voltage being, ***at least,*** smaller than the cathode potential drop [112]. Basing on the principles the spot dynamics was obtained. With time, the cathode spot, localized at the cathode surface on a protrusion (or tip) at the cathode surface) with a low heat conduction loss, erodes it and forms a crater so that the cathode spot becomes embedded deeper in the body and 3D heat losses increase. The voltage increases to compensate the heat loss. This explains a cause of the spot motion due to the exhaustion of a small tip under the spot, i.e., cathode spots in deep craters extinguish, and new cathode spots form in adjacent locations on protrusions which have lower heat loss and consequently lower voltage. The finite lifetime of the spot produces discrete random motion on bulk cathodes and voltage fluctuations. But this non stationary spot operation is due to thermal vaporization and without any explosions.

According to calculations (not shown here), for a Cu cathode between 8 and 200 ns, the erosion rate increased from ~50 to ~200 μg/C, current density j from ~1 to 3 MA/cm^2 and the electron current fraction s from 0.7 to 0.8. The larger u_c~100 V is calculated at initial plasma life time of τ=7.5 ns (Cu) and ~70 V at ~2 ns (Cr) for which the solutions are obtained. It can be seen that the maximal u_c significantly decreased with τ (Fig. 8). This result can be used to explain the relatively large voltage of the power supply (at least 70-100 V) requested to the vacuum arc initiation. As the time life of triggered initial low dense plasma can be short, the large voltage of the supply is needed for further arc development.

The cathode plasma is fully ionized at the stage of spot initiating (<30–40 ns), and then α_i decreased to ~0.5–0.6 with time. Charge states 0 through +4 were present (mainly +2 and +3) at τ=7.5 ns for Cu and at τ=2 ns for Cr. At τ=100 ns, the plasma mostly consisted of +1 to +3 ions. Cathode and plasma temperatures, plasma density, u_c, and other parameters changed with time, reaching steady-state levels that did not depend on the initial plasma parameters [104]. These dependencies only show the possibility of spot function up to steady state which in realty can be not reached due to different causes (see below). A plasma jet expanded in the direction of the anode, and could be described using a 2D free boundary approximation [113, 114].

C. Physics of different types of cathode spots

(a) Spot types and motion without magnetic field

The two above mentioned principals were used to understand different spot behavior. Theory indicates that the spot type and characteristics are determined by the arc current, heat loss in the cathode and ambient vapor or gas. Different spot types occur on bulk and film cathodes (Table 1).

When a bulk cathode surface is contaminated by an oxide film and other impurities, super-fast spots (SFS) with velocities up to 10^4 cm/s were observed at relatively low arc current (~100 A) [22, 26]. SFS spots appeared due to intense vaporization of the relatively thin oxides or small contamination in short time from few tens of nanosecond up to about 1 μs under the spot size. This time determined the local spot life time and the mentioned large spot velocity. An estimation show that the thermal wave expands at time ~1ns into film thickness of 0.1 μm. The low spot current is due to low heat loss in the thin surface layer which was calculated in Ref. [115]. Usually the SFS spot types were observed until the cathode was cleaned by the arc [22].

The nonstationary heat conduction in the cathode body determines the transient cathode spot operation and its cyclic extinguishing and reigniting. The spot lifetime is associated with a characteristic thermal time. The thermal time and subsequently the velocity of the spot on clean surfaces depends by the cathode roughness. The spot life time is determined by the thermal time constant and exhaustion of relatively larger metal irregularities in range from 0.1 to ~1 μm in comparison with the small contaminations (<0.1 μm). Due to this difference in condition of the cyclic spot operation, the experiment with a cleaned cathodes detected non-stationary moderate fast spots (MFS) [22, 26].

At higher spot currents (<1 kA) on bulk cathodes in vacuum, the heat loss from the individual spots is compensated by the heat supplied by the large current and nearest the other individual spots. Therefore in vacuum, these individual spots tend to group, thus creating a common vapor cloud that satisfies the first principle. This group spots is favorable and new spots can be ignited under the same plasma cloud under impeded plasma flow condition. In this case, slow group spots (SGS) with a relatively long life time (≤1 ms) were observed [19, 20, 23, 28]. In low pressure ambient gas the slow individual spots (SIS) can exist separately [18, 28], because the first principle is fulfilled due to gas. In very high current arcs (>>1 kA) with intense vaporization the SIS type occurs because the first principle is fulfilled automatic [34].

(b) Transverse magnetic field parallel to the cathode surface

The cathode spots move in the “retrograde” (**-j×B**) motion. This motion was explained by considering the relation between magnetic and gas kinetic pressures in the light of the above mentioned first principle [116]. For a single spot with no transverse magnetic field, the self-magnetic field and the gas kinetic pressure of the plasma are distributed axially symmetric. The spot “moves” randomly, by extinguishing and a new cathode spot ignites on a suitable a nearby protrusion in order to satisfy the second principle (low cathode heat loss) as it was described above. When an external magnetic field B_{em} is applied, the resulting magnetic pressure (which depends on the square of the vector sum of the B_{em} and the self-magnetic field B_{sm}) is distributed asymmetrically, and the larger pressure is on the side opposite to the Ampere direction (Fig. 11).

According to the first principle, ignition of a new spot is more probable under the larger pressure, i.e. cathode spots were formed preferentially where the flow is impeded and therefore, the spot appears to move in the retrograde direction.

For a single spot the pressure difference between the two sides is $\Delta P=2B_{em}B_{sm}/\mu$. Assuming that probability of the new spot ignition is proportional to following magnetic pressure gradient across the spot, i.e. $\Delta P/2r$. The new spot is ignited at a distance of about one or few spot radii r_s during life time Δt including ignition time in some plasma of mass density ρ_s. As the ratio $r_s/\Delta t$ is the spot velocity v_s then according to probability of spot ignition the value of v_s^2 should be proportional a force determined by $\Delta P/2r_s$.

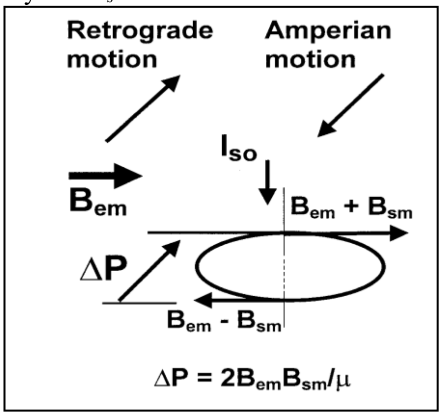

Fig. 11. Schematic presentation of the magnetic field and pressure difference ΔP around the spot with self-magnetic B_{sm} and external magnetic field B_{em}.

Taking in account this fact the spot velocity increase can be obtained formally using an equation of motion. The details can be found in Ref. [116]. Further mathematical analysis shows linear dependence between the spot velocity and the magnetic field by the following expression:

$$v_s = K_B B \qquad K_B = \left(\frac{4}{\mu\rho_s}\right)^{1/2} \tag{7}$$

and between spot velocity and spot current I by

$$v_s = K_I I \qquad K_I = \frac{I}{\pi r_{ef}}\left(\frac{\mu}{2\rho_s}\right)^{1/2} \tag{8}$$

where μ is the magnetic permeability. K_B and K_I are the constants. This indicates that the spot velocity behaves in accordance with previous measurements [117].

Previously Kesaev [17] postulated that the maximum of the magnetic field caused the retrograde direction, assuming that the plasma density increases because the electron diffusion coefficient decreases by a Hall factor of β. However, β is very small and the density will be flattened due to radial particle flow. Drouet [118] also proposed an increase in plasma density at the retrograde side of spot due to anisotropy of the plasma confinement. However he assumed that the adjacent plasma decreases the work function φ in proportion to the plasma density, and thus the retrograde side will have a higher density and lower work function than the Amperian side, and that a new cathode spot will form preferentially where the work function is lower, and hence the electron emission is higher. This qualitative thing is difficult to understand for the highly heated cathode surface with the large temperature.

(c) Spot motion in an oblique magnetic field to the cathode surface

Smith [119] and then Kesaev [120] reported that when magnetic field lines obliquely intersected a cathode surface, the cathode motion had an additional “drift” component in the direction of the opening of the acute angle between the magnetic field lines and the cathode surface, besides the retrograde motion described in (*b*). This additional motion is known as the "acute angle" effect, and the deviation from the retrograde direction is erroneously (as it was for first observed by Smith) called Robson's angle.

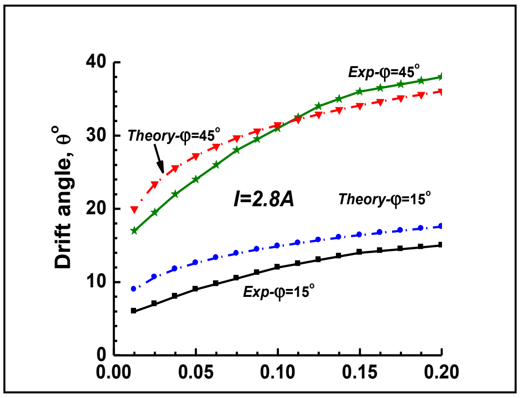

Fig. 12. Drift angle θ as a function of the strength of oblique magnetic field with an acute angle φ of the oblique magnetic field as a parameter. Experiments [121]—solid lines, calculations—dotted lines [122].

Robson just measured the dependence of the drift angle θ on the acute angle φ [121]. We developed a model that takes in account the force under an electric field caused by retrograde spot motion across the normal component of the magnetic field, producing a drift velocity component in the direction of the acute angle between the magnetic field and the cathode surface [122]. Fig. 12 demonstrates the agreement between Robson's data and our calculations.

(d) Spot spliting in a magnetic field.

In general the applied magnetic field is a vector having parallel and normal components to the cathode surface. Let us consider the effect of cathode spot splitting according to the published studies in parallel and oblique magnetic field separately.

Parallel field. It was observed [19, 28, 123] that the spot current, and therefore, the current per single jet, is limited to a material dependent value. The self-magnetic field in a single spot increased with spot current. As is mentioned in (*b*), the relation between the self-magnetic and kinetic pressures along the jet depends on the plasma parameter distribution in the axially expanding cathode plasma. A model of calculation the current per group spot was developed assuming that spots split when the plasma kinetic pressure is comparable to the self-magnetic pressure in the acceleration region of cathode plasma jet [124]. In an external magnetic field, the pressure produced from this field was compared with pressure from self-magnetic field. Calculated and experimental results [123] are presented in Fig. 13 for a Cu cathode. Both the theoretical and measured currents of group spots increase linearly with the magnetic field, and their values agree.

Oblique field. An experiment [125] with a vacuum arc under an oblique magnetic field showed that the group spot current depends on the tangential and normal components of the oblique magnetic field. It was found that the normal component of the magnetic field influence on spot current when the tangential magnetic field is constant.

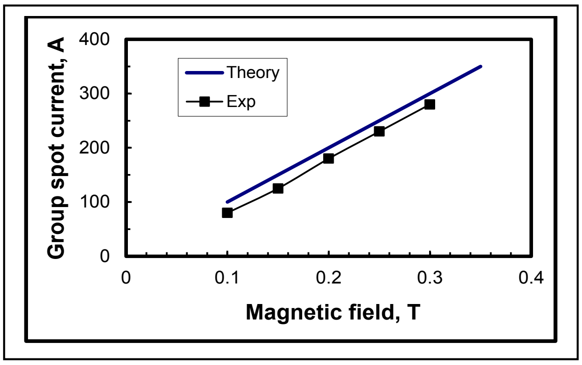

Fig. 13. Dependence of the group spot current on the magnetic field. The points are from the experiment [123]. The solid line is the calculated dependence [124]

Cathode spot splitting in an oblique magnetic field was modeled [126], based on the relations between kinetic pressure of the plasma, and the magnetic pressures produced by the self-magnetic field and by the vector of the applied external magnetic field. These calculations showed that the spot current I_s linearly increases with the transverse component of the magnetic field, B_t, for a constant normal component B_n. While this linear dependence shifted to larger ranges of B_t with B_n, the slope of the theoretical I_s-B_t curves agrees well with measured dependencies (see Fig.1 in Ref.126).

V. Conclusions

The many publications on cathode spots emphasize the importance of this topic and illustrate the progress over the years in obtaining better experimental data and theoretical understanding. Although that the early models considered various processes and structures in the cathode spot separately, they have been important steps, leading to the more recent works. Thus, Lee and Greenwood developed the first systematic approximate spot description. New spot data stimulated further research, including the author's. The main difference between various models is that in some, the cathode potential drop u_c was assumed, whereas in later kinetic models u_c was calculated as part of a self-consistent set of equations. These last models represents a new glance regarding to role of the arc voltage at the moment of arc initiation and spot development, In this case the spot temperature and the current density not rise unlimited with time as in case of given constant u_c. While the author's models are perhaps not complete, it is hoped that they have increased our understanding of arc spot mechanisms and reduced the mystery surrounding them.

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