Principle of Self-Complementarity and Application to Broadband Antennas

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***Abstract*— Theory of self-complementarity which was derived through an investigation of the input impedance of slot antennas with arbitrary shape is described. Examples of self-complementary antennas, modified self-complementary antennas, and their application are presented. It is pointed out that the self-complementarity is very important to obtain the wide frequency band.**

Keywords*— *self-complementary antenna; broadband antenna; wireless broadband; constant impedance antenna

I. Introduction

Many antenna elements such as the linear antennas and the slot antennas use the resonance of the input impedance and have strong frequency-dependent properties, which yield narrow bandwidth. Self-complementary antennas have a remarkable feature of a constant input impedance independent of frequency and have been originated by Mushiake in 1948 [1]-[3]. Self-complementary antennas are based on the Mushiake relationship. Since there is an infinite variety of self-complementary structures, many self-complementary and modified self-complementary antennas have been proposed. So-called log-periodic antennas are considered to be derivatives of the self-complementary antennas.

In his paper, theory of self-complementarity is briefly described. Then, two-terminal planar self-complementary antenna and multi-terminal self-complementary antennas are presented. Modified self-complementary antennas having log-periodic shape developed by researchers in USA, which yield ultra-wide bandwidth, are also discussed. Finally, developmental studies and application of self-complementary antennas are presented.

II. Theory of Self-Complemenarity

There is a duality property between electromagnetic fields $(\boldsymbol{E}_1, \boldsymbol{H}_1)$ in structure #1 and $(\boldsymbol{E}_2, \boldsymbol{H}_2)$ in structure #2 if all electric walls, magnetic walls, electric currents, and magnetic currents in structure #1 are respectively interchanged for magnetic walls, electric walls, magnetic currents and electric currents in structure #2. The Babinet's principle in electromagnetic fields is derived from the duality property and has been used for diffraction problems.

Mushiake applied Babinet's principle to the analysis of a slot antenna having arbitrary shape shown in Fig. 1 (a). Let the slot antenna having voltage V_1 and current I_1 at the driving point generate electromagnetic field $\boldsymbol{E}_1$ and $\boldsymbol{H}_1$ as shown in Fig. 1 (a), which is equivalent to a slot antenna fed by magnetic current shown in Fig. 1 (b). The complementary planar antenna with driving voltage V_2 and current I_2 generate electromagnetic field $\boldsymbol{E}_2$ and $\boldsymbol{H}_2$ as shown in Fig. 1 (c). The driving point voltages V_1 and V_2 are expressed in terms of the driving currents I_1 and I_2 as

$$\begin{aligned} V_1 &= -\int_a^b \boldsymbol{E}_1 \cdot d\boldsymbol{s} = \int_a^b \boldsymbol{H}_2 \cdot d\boldsymbol{s} = \frac{I_2}{2} \\ V_2 &= -\int_d^c \boldsymbol{E}_2 \cdot d\boldsymbol{s} = -Z_0^2 \int_d^c \boldsymbol{H}_1 \cdot d\boldsymbol{s} = \frac{Z_0^2 I_1}{2} \end{aligned}, \tag{1}$$

where

$$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \cong 120\pi \ [\Omega], \tag{2}$$

is the intrinsic impedance in vacuum space [1]-[3].

By using eq. (1), the input impedance of the slot antenna Z_s shown in Fig. 1 (a) can be obtained by the following equation [1]-[3]

$$Z_s = \frac{(Z_0/2)^2}{Z_d}, \tag{3}$$

where Z_d is the input impedance of complementary planar antenna shown in Fig. 1 (c).

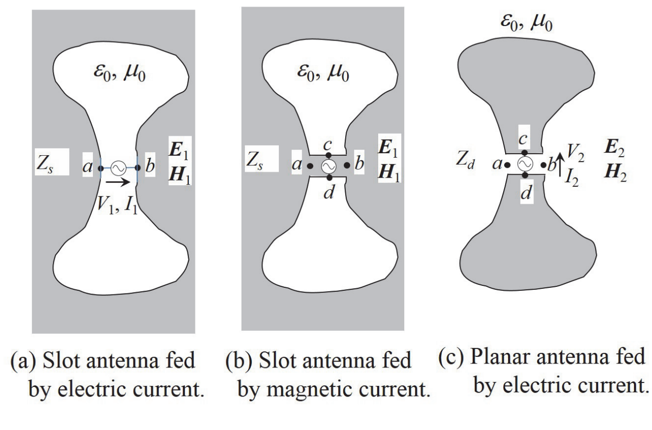


Fig. 1 Slot antenna and complementary planar antenna of arbitrary shape [1]-[3].

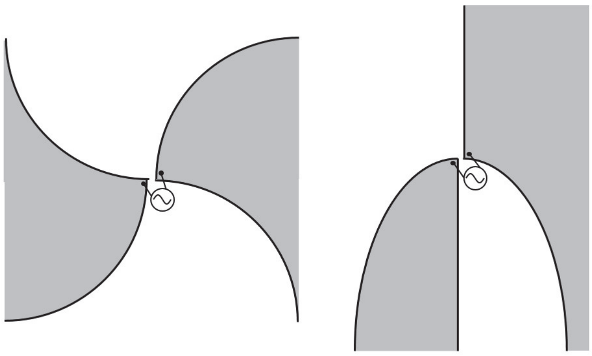
Eq. (3) had been already obtained in Japan to obtain the input impedance of slit antennas earlier than the paper of

Booker [4] as pointed out by [5], and the paper by Asami et al. [6] was a more detailed report of the unpublished earlier paper. (The slot antenna was called the slit antennas in Japan at that time). However, the structures reported in these papers had been restricted for the thin slot antennas and the thin wire antennas. On the other hand, eq. (3) derived by Mushiake does not have such limitation and can be applied for slot antennas and planar antennas with arbitrary shape [1]-[3].

Mushiake also originated the self-complementary structures and discovered that the input impedance of the antennas shown in Fig. 2 is constant independent of the frequency and given by

$$Z_{in} = \frac{Z_0}{2} \cong 60\pi\ \Omega, \qquad (4)$$

since arbitrarily shaped four lines for the boundary of two conducting sheets are exactly identical and the complementary structure in Fig. 2 is exactly the same to the original structure in Fig. 2 [1]-[3]. Eq. (4) is known as “Mushiake relationship” and this antenna is called “self-complementary antenna” [7]-[8]. The principle of the constant impedance of the self-complementary antennas is called the “principle of self-complementarity”.



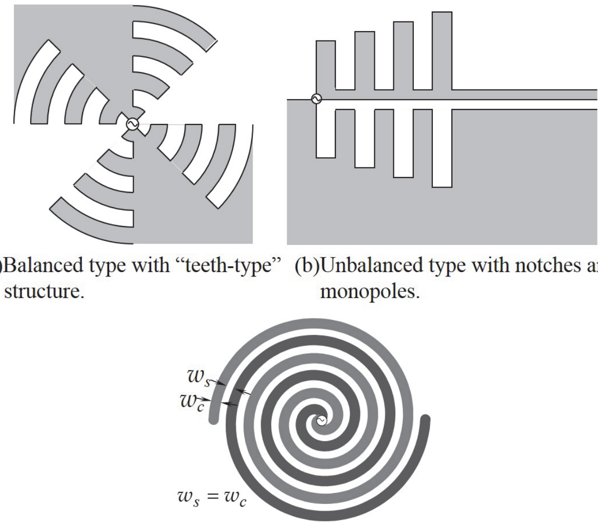
(a)Balanced and rotationally symmetric type. (b)Unbalanced and axially symmetric type.

Fig. 2 Self-complementary antennas.

III. TWO-TERMINAL PLANAR SELF-COMPLEMENTARY ANTENNAS

Self-complementary antennas are very interesting since there is an infinite variety of self-complementary structures as shown in Fig. 3. Although infinite structures are required for the constant impedance of the self-complementary antennas, the structures having teeth or notches and monopoles shown in Fig. 3 (a) and Fig. 3 (b) can realize the constant impedance with a finite conducting plane, because teeth or notches and monopoles radiate electromagnetic power and the truncation effect can be reduced for the finite structures. Archimedean spiral structure shown in Fig. 3 (c) can also reduce the truncation effect because of the radiation from the spiral arms.

Since there is a lot of flexibility of the shape of the self-complementary antennas, it could be possible to obtain antennas having desired gain and/or desired radiation pattern maintaining the constant input impedance, e.g. an antenna having desired frequency dependence of gain.



(a)Balanced type with “teeth-type” structure. (b)Unbalanced type with notches and monopoles.

(c) Archimedean-spiral structure.

Fig. 3 Examples of self-complementary structures.

IV. MULTI-TERMINAL SELF-COMPLEMENTARY ANTENNAS

Deschamps [9] proposed self-complementary multi-terminal planar antennas having the input impedance depending on the structure. Mushiake also developed other types of self-complementary antennas, such as rotationally symmetric multi-terminal planar self-complementary antennas [10]. Fig. 4 shows four terminal self-complementary antennas, where a rotationally symmetric four terminal self-complementary structure is excited in ring connection in Fig. 4 (a) and star connection in Fig. 4 (b). When the exciting voltage is given by

$$\left.\begin{aligned} V_s &= V_1 \exp\left[j\frac{\pi}{2}(s-1) \right] \\ V'_s &= V'_1 \exp\left[j\frac{\pi}{2}(s-1) \right] \end{aligned}\right\}, \quad s = 1,2,3,4 \qquad (5)$$

we obtain the input impedance at each terminal respectively for the cases of ring and star connection as

$$\left.\begin{aligned} Z &= \frac{V_s}{I_s} = Z_0 \sin\frac{\pi}{4} = 60\sqrt{2}\pi\ \Omega \\ Z' &= \frac{V'_s}{I'_s} = \frac{Z_0}{4\sin\frac{\pi}{4}} = 30\sqrt{2}\pi\ \Omega \end{aligned}\right\}. \qquad (6)$$

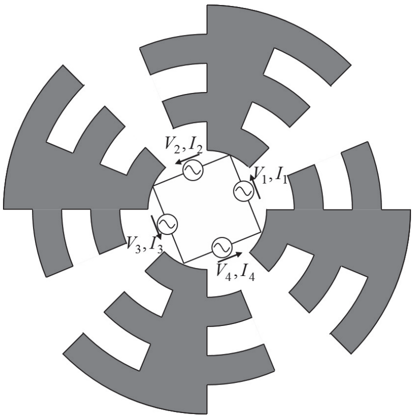
In a general case of a rotationally symmetric n-terminal self-complementary planar structure excited by symmetric n-phase voltages with mth-order rotation, i.e.,

$$\left.\begin{aligned} V_s &= V_1 \exp\left[j\frac{2m\pi}{n}(s-1) \right] \\ V'_s &= V'_1 \exp\left[j\frac{m\pi}{n}(s-1) \right] \end{aligned}\right\}, s = 1,2,\cdots n,\ m = 1,2,\cdots n-1 \qquad (7)$$

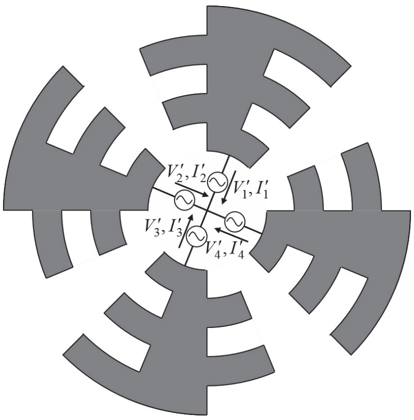
for the cases of ring and star connection, the input impedance at each terminal for ring and star connections is respectively obtained as

$$\left.\begin{aligned} Z &= \frac{V_s}{I_s} = Z_0 \sin\frac{m\pi}{n}\ \Omega \\ Z' &= \frac{V_s'}{I_s'} = \frac{Z_0}{4\sin\frac{m\pi}{n}}\ \Omega \end{aligned}\right\}. \tag{8}$$

More detailed discussion has been given by Deschamps [9].



(a) Ring connection



(b) Star connection

Fig. 4 Four terminal self-complementary antennas ($n = 4, m = 1$). .

V. Modified self-complementary antennas having log-periodic shape

Rumsey and DuHamel found the importance of the principle of self-complementarity [7],[8],[11]. DuHamel and Isbell [11] proposed a self-complementary antenna with log-periodically spaced teeth as shown in Fig. 5 and showed that the input impedance is almost equal to $60\pi \cong 188\Omega$ under the condition that the operating frequency is higher than the frequency at which the length of the longest tooth is a quarter wavelength.

Fig. 5 is the first structure of self-complementary antenna having log-periodic shape. DuHamel and Ore [12] deformed the self-complementary antenna and developed another planar antenna. Since the planar structures radiate bidirectionally and unidirectional antenna was desired, they tried to deform the structure by folding the planar antenna and obtained a modified self-complementary type antenna called “log-periodic antenna” shown in Fig. 6. They also developed a log-periodic antenna composed of thin wires shown in Fig. 10, which is a wind-resistant structure.

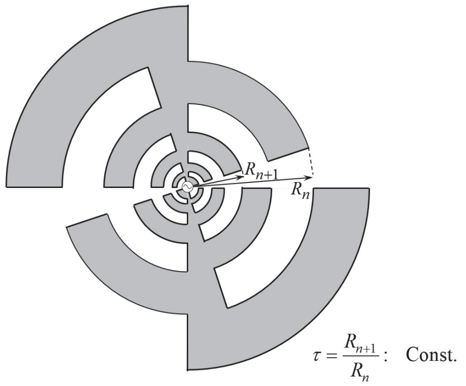


Fig. 5 Self-complementary antenna having log-periodic shape [11].

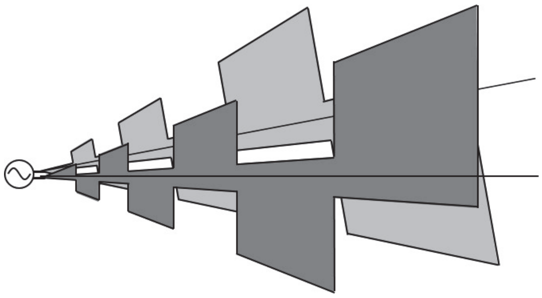


Fig. 6 Modified self-complementary antenna with log-periodic shape having directional radiation pattern [12].

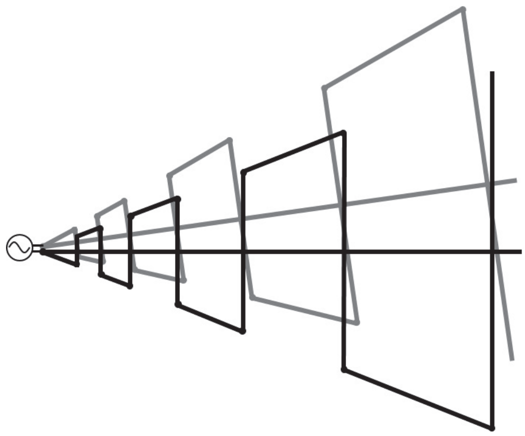


Fig. 7 Modified self-complementary antenna with log-periodic shape composed of thin wires [12].

The log-periodic antennas were further deformed to the log-periodic dipole array (LPDA) antenna by Isbell [13] shown in Figs. 8, and design procedure of the LPDA was proposed by Carrel [14], where theoretical investigation using sinusoidal current distribution on the dipole elements was performed.

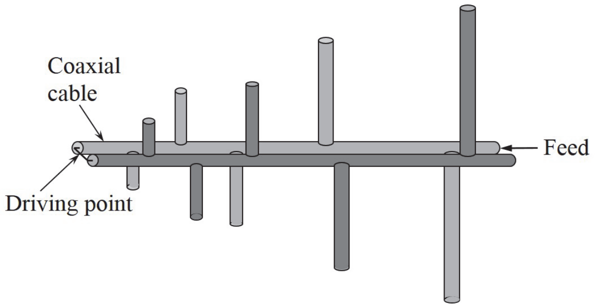


Fig. 8 Log-periodic dipole array (LPDA) [13].

The LPDA has been widely used for ultra-wide frequency operation in communications, TV reception and measurements, especially in the EMC measurements. Fig. 9 shows the numerical and experimental input impedance of LPDA [15], where the numerical results were obtained by the method of moments using piece-wise sinusoidal basis and test functions [16],[17]. As can be seen in Fig. 9, the input impedance of LPDA is approximately $50\ \Omega$ and the broadband characteristics of LPDA are demonstrated.

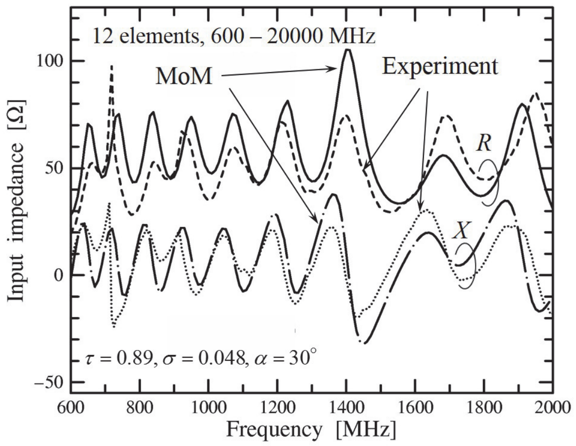


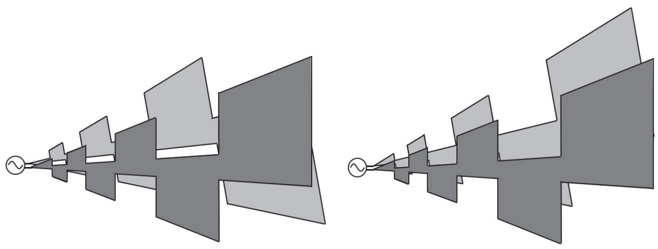
Fig. 9 Input impedance of log-periodic dipole array (LPDA). Comparison of numerical data by method of moments and experimental results [15].

As mentioned above, so-called log-periodic antennas were developed based on the principle of the self-complementarity. The broadband characteristics of the log-periodic antenna are obtained by the principle of self-complementarity rather than the log-periodic structure [18]-[20]. The most important structure in these broadband antennas is the transposed excitation.

Fig. 11 shows a comparison of measured input resistance of self-complementary type log-periodic antenna shown in Fig. 10 (a) and anti-complementary type log-periodic antenna shown in Fig. 10 (b) [10],[20]. In the case of anti-complementary type, frequency-independent characteristics are not obtained, whereas broadband property can be observed in the case of self-complementary type log-periodic antenna.

Nakano [21] also presented a numerical analysis of LPDA shown in Fig. 12, where Fig. 12 (a) shows LPDA with transposed excitation and Fig. 12 (b) is that with non-transposed excitation. The input impedance of the LPDA with transposed excitation shows almost constant impedance of about $100\ \Omega$, but LPDA with non-transposed excitation shows strongly oscillatory curves as can be seen in Fig. 13.

The broadband property of LPDA comes from transposed excitation of the folded up self-complementary antenna. The transposed excitation is inevitable outcome of the modification, and it is essential for the broadband LPDA. However, the log-periodic shape does not provide broadband property to antennas. Therefore, LPDA is actually a modified self-complementary dipole array with log-periodic shape. Fractal antenna is another type of log-periodic antenna but it has periodically oscillating characteristics with respect to the logarithm of frequency rather than the broadband properties [22].



(a) Self-complementary type. (b) Anti-complementary type.

Fig. 10 Self-complementary type and anti-complementary type log-periodic antennas (folded trapezoidal shape) [10],[20].

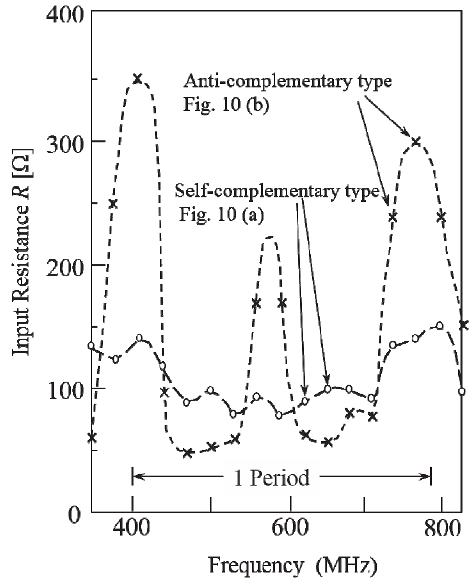
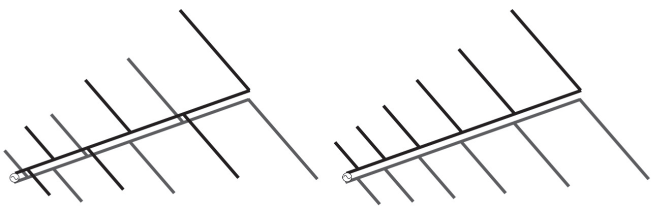


Fig. 11 Comparison of measured input resistance of log-periodic antennas [10],[20].



(a) Transposed excitation. (b) Non-transposed excitation.

Fig. 12 Log-periodic dipole antennas. Comparison of transposed and non-transposed antennas [21].

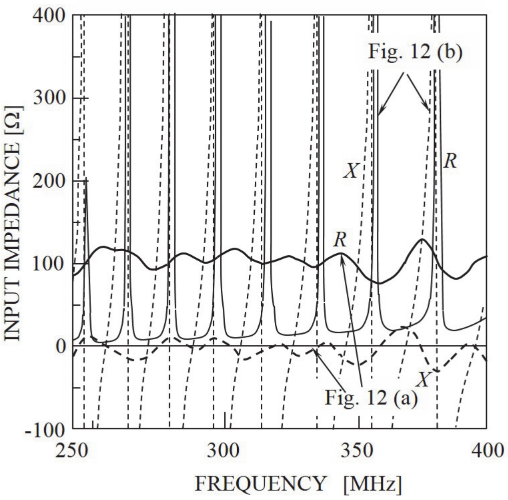


Fig. 13 Input Impedance of log-periodic dipole array antenna. Comparison of transposed and non-transposed antennas [21].

VI. Developmental studies of self-complementary antennas

Mushiake and his colleagues developed other types of self-complementary antennas, such as rotationally symmetric multi-terminal planar self-complementary antennas [10] described in Sec IV, three-dimensional multi-planar self-complementary antennas, stacked self-complementary antennas, and monopole-slot antennas [18],[19].

Fig. 14 is alternate-leaved self-complementary antenna [23], which has not only the broadband input impedance but also frequency independent radiation pattern. Fig. 15 shows an example of three-dimensional self-complementary antenna. Since the vertical and horizontal parts are mutually complementary conducting planes, the input impedance of the antenna is obtained as $Z_{in} \cong 30\pi\ \Omega$ [10],[18]. Fig. 16 shows the monopole-slot array antenna, whose input impedance is obtained theoretically as $Z_{in} = \sqrt{2}30\pi \cong 133\,\Omega$ by Ishizone et al. [24]. The details of these antennas are presented in the paper and book by Mushiake [18],[19]. Also, the numerical investigation of the monopole-notch antennas shown in Fig. 3 (b) was performed by Yamamoto et al. [25] to obtain the relation between the structure parameters and the characteristics such as input impedance and the gain of the array.

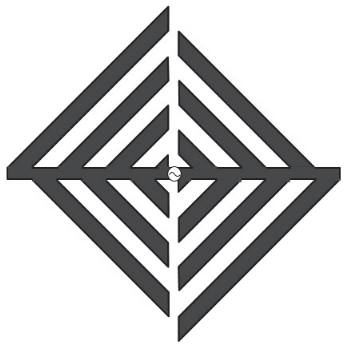


Fig. 14 alternate-leaved self-complementary antenna [23].

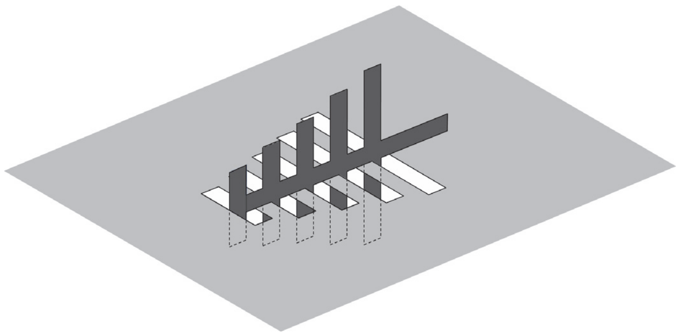


Fig. 15 Example of three-dimensional self-complementary antenna ($n = 2$, $Z_{in} \cong 30\pi\ \Omega$) [18].

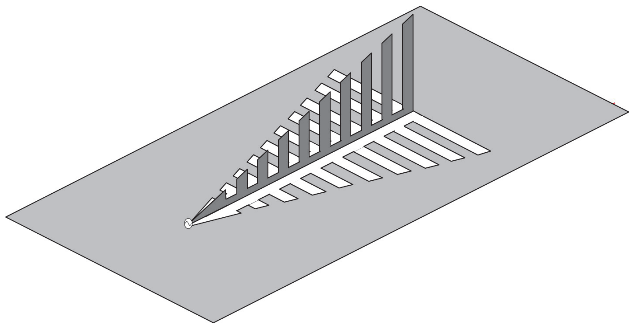


Fig. 16 Monopole-slot antenna ($Z_{in} \cong \sqrt{2}\ 30\pi\ \Omega$) [24].

VII. Applications of original and modified self-complementary antennas

So-called log-periodic antennas were developed based on the principle of the self-complementarity and have been widely used for ultra-wide frequency operation in communications, TV reception and measurements as described in Sec. V. Modified self-complemental antennas are used for the radio astronomy and the application of superconductivity. Cortés-Medellín [26] applied non-planar multi-terminal quasi-self-complemental antenna to ultra-wideband feed antenna for radio astronomy. Kang et.al [27] used modified planar self-complementary antenna with

Josephson junction using high-Tc YBCO superconducting film for wide-band microwave radiation and sensing as shown in Fig. 17.

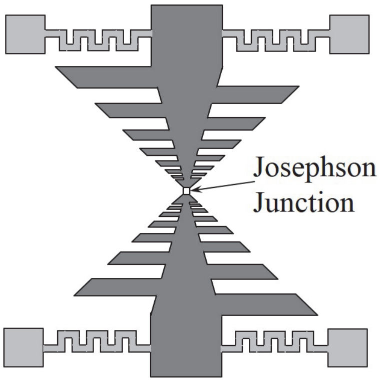


Fig. 17 Josephson junction with a self-complementary antenna for ultra-wideband microwave radiation and sensing [27].

VIII. CONCLUSION

In this paper, theory of self-complementarity which was derived through an investigation of the input impedance of slot antennas with arbitrary shape are described. Examples of self-complementary antennas, modified self-complementary antennas, and their applications are presented. It was pointed out that the self-complementarity is very important to obtain the ultra-wide frequency band.

Discovery of the self-complementary antennas was proposed as the candidate of IEEE Milestone entitled "The Discovery of the Principle of Self-Complementarity in Antennas and the Mushiake Relationship, 1948" and the proposal was approved by IEEE Historical Committee in December 2016 and IEEE Board of Directors in February 2017.

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