EDITOR: Alexander B. Magoun, a.b.magoun@ieee.org

DEPARTMENT: ANECDOTES

Easy Reciprocals

Stanley Mazor, *Fairchild Semiconductor and Intel, Los Altos, CA, 94024, USA*

Prior to the availability of electronic calculators, people used voluminous published data tables to quickly provide transcendental functions, square roots, and logarithms. One would look up a value; if the exact entry was not there, then interpolation between two neighboring values gave an estimate. Such tables are also useful for getting a first-guess value when using iteration to get a better result, as with Newton's method for obtaining a square root. In 1909, Percy Ludgate invented a programmable mechanical computer [1], [2] and suggested doing division, a/b, with multiplication hardware for $a \times b^{-1}$, but taking the reciprocal of b without a division. To get a reciprocal he proposed a lookup table for the first estimate and then a novel iterative method explained here. Each successive estimate for b^{-1} uses a multiply and then a subtract. Nearly 90 years later, Intel's Pentium CPUs and other microprocessors also began using an on-chip table for their divide operations.[1] To get the reciprocal N^{-1} (1/N) when division is not available or practical, the reciprocal can be calculated using an iterative scheme. Note that the product of $N^{-1} \times N$ should equal a value very close to 1, after which the estimated N^{-1} can be improved proportionally in successive steps.

NORMALIZED NUMBERS

Scientific notation is used for real numbers (N) (e.g., 1.2×10^5), and in computer floating point form with a normalized mantissa (e.g. 0.12×06). It is noteworthy that there are only nine possible first digits in the mantissa for N (1–9), and the mantissa range is 0.100000–0.999999.

[1]David W. Deley, "The Pentium Division Flaw," 1995. Accessed: Sep. 8, 2022. [Online]. Available: https://daviddeley.com/pentbug/pentbug.htm

1058-6180 © 2024 IEEE
Digital Object Identifier 10.1109/MAHC.2024.3364901
Date of current version 2 April 2024.

TABLE 1. Estimation of the reciprocal N^{-1} through the first digit N.

N	N^{-1}	$N \times N^{-1}$
.9	1.1	0.99
.8	1.2	0.96
.7	1.4	0.98
.6	1.6	0.96
.5	1.9	0.95
.4	2.4	0.96
.3	3.3	0.99
.2	4.9	0.98
.1	9.9	0.99

A table using this first digit of N for an estimate of the reciprocal N^{-1} is given in Table 1.

Consider the two functions graphed in Figure 1: one uses a division (1/N), and the other is just a subtraction (2-N). Although these are both arithmetic functions, in some environments division might not be available, or subtraction might be considered either a quicker or cheaper operator. Viewing the graphs that these functions make shows almost no similarity to each other except they match in a very narrow range near N = 1. Table 2 tabulates the delta (difference) in

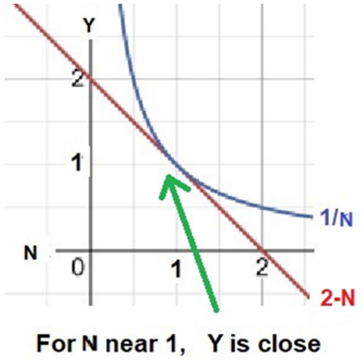

FIGURE 1. Comparison of 1/N and 2-N near N = 1.

TABLE 2. Select delta values for 1/N and 2-N in Figure 1.

N	Delta
0.90	0.0111111
0.92	0.0069565
0.94	0.0038297
0.96	0.0016666
0.98	0.0004081
0.99	0.0001010

value for these two functions for values of N near 1 where the line is tangent to the curve.

SUCCESSIVE APPROXIMATIONS

In this note, we have examined when and how we can use the simpler subtract function to compute a reciprocal in an iterative manner after having a first estimate. Starting with a first estimate of a reciprocal, the results can be improved by using successive approximations, since $N \times N^{-1}$ should equal 1.0 (.999999). Call the first estimate b^{-1} and suppose ($b^{-1} \times b$) is low (.995). Then, b^{-1} should be proportionally larger; get the next estimate (ironically using the reciprocal of .995) by multiplying b^{-1} by $1.005 = (1.0/.995)$. For reciprocals near .9, the trick is to subtract from 2 instead of dividing by $1.005 = (2.0 - .995)$ to get the next value of b^{-1}. The number of iterations depends upon the floating-point arithmetic used.

BIBLIOGRAPHY

[1] B. Coghlan, B. Randell, P. Hockie, T. Gonzalez, D. McQuillan, and R. O'Regan, "Investigating the work and life of Percy Ludgate," *IEEE Ann. Hist. Comput.*, vol. 43, no. 1, pp. 19–37, Jan.–Mar. 2021.

[2] B. Coghlan, B. Randell, P. Hockie, T. Gonzalez, D. McQuillan, and R. O'Regan, "Percy Ludgate (1883–1922), Ireland's first computer designer," *Proc. Roy. Ir. Acad.: Archaeol., Culture, Hist., Literature*, vol. 121C, pp. 303–332, 2021.