OPTIMAL POWER CONTROL OF A THREE-SHAFT BRAYTON CYCLE BASED POWER CONVERSION UNIT

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Abstract: This paper discusses the development of a control system that optimally controls the power output of a Brayton-cycle based power conversion unit. The original three shaft design of the Pebble Bed Modular Reactor (PBMR) power plant is considered. The power output of the system can be manipulated by changing the helium inventory to the gas cycle. The helium inventory can be manipulated in four ways: Injecting helium at the high-pressure side of the system by means of a booster tank; extracting helium at the high-pressure side of the system; injecting helium at the low-pressure side of the system and lastly opening and closing the bypass control valve. The control system has to intelligently generate set point values for each of the four helium manipulation mechanisms to eventually control the power output. In this paper two control strategies are investigated namely PID control and Fuzzy PID (FPID) control. The FPID control strategy is a linear type Fuzzy controller, but can progressively be made nonlinear if nonlinearities exist in the system. An optimal control system is derived by applying an optimisation technique to the gain constants of the controllers. A Genetic Algorithm (GA) is used to optimise the gain constants of both the PID and FPID controllers. The GA uses the ITAE performance index as an objective function.

Key words: Brayton-cycle, PID control, Fuzzy PID control, Genetic Algorithms, Pebble Bed Modular Reactor.

1. INTRODUCTION

In this paper a power generation system will be considered that can produce up to 110 MW of electrical power. This system is called a module and can operate in a stand-alone mode, or as part of a power plant that can have more of these units [HYPERLINK \l "MCN02" 1]. Figure 1 gives a schematic layout of this power generation module. This module contains a graphite-moderated, helium-cooled reactor and uses the Brayton direct gas cycle to convert the heat, which is generated in the core by nuclear fission. The heat is then transferred to the coolant gas (helium), and converted into electrical energy by means of a gas turbo-generator. The ideal Brayton cycle consists of two isentropic and two isobaric processes. In Figure 2 a temperature vs. entropy graph of the ideal Brayton cycle is given. Starting at (1), gas at a low pressure and temperature is compressed in an isentropic process to a higher pressure (2). From (2) to (3), the gas is heated in an isobaric (constant pressure) process to the maximum cycle temperature. From (3) to (4), the hot high-pressure gas is expanded isentropically in a turbine to a lower pressure and temperature. The cycle is completed from (4) to (1) by cooling the gas at constant pressure. By adding to the gas inventory of the cycle, the electrical power generated will be increased and by removing inventory the power generated will be decreased. This is the primary method of controlling power.

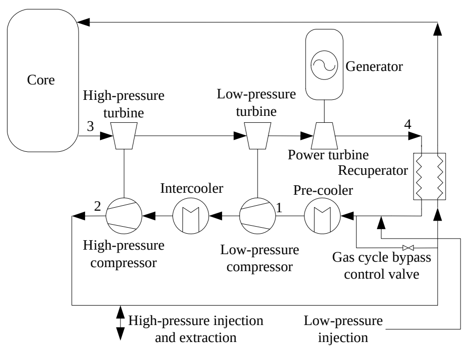

Figure 1: Schematic layout of the Brayton cycle based power conversion unit [1]

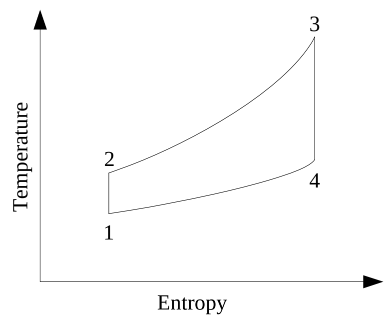

Figure 2: Temperature vs. entropy graph of the ideal Brayton cycle [2]

The power control system constitutes four helium manipulation mechanisms:

- Gas bypass
- Low-pressure injection
- High-pressure extraction
- High-pressure injection (by means of a booster tank)

An existing linear Simulink® model [3,4] of the system shown in Figure 1 is used to illustrate to the reader the effect the four helium manipulation mechanisms have on the power output of the system. The linear model is used as a test platform for the control system. Opening the gas cycle bypass control valve will reduce the power and closing it will increase the power as shown in Figure 3 and Figure 4 respectively. Extraction of gas at the high-pressure side results in an instant decrease in the power of the system. The power response due to extraction is given in Figure 5. A limited amount of helium can be injected at the high-pressure side of the system depending on the pressure in the booster tank. Figure 6 shows the instant increase in power during boosting (high-pressure injection). Injection of gas at the low-pressure side of the system does not result in an instant increase in the power output of the system. The power first decreases and then starts to increase as shown in Figure 7. This phenomenon is called the non-minimum phase effect [1].

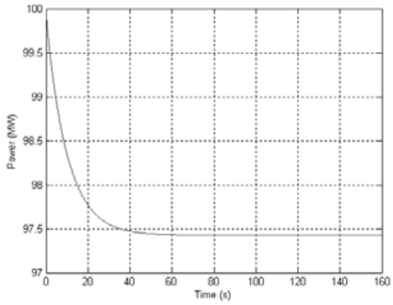

Figure 3: Bypass valve opening

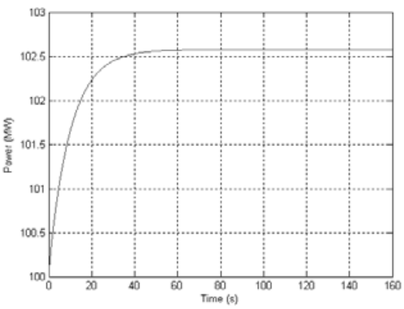

Figure 4: Bypass valve closing

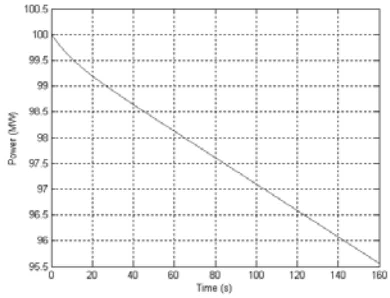

Figure 5: High-pressure extraction

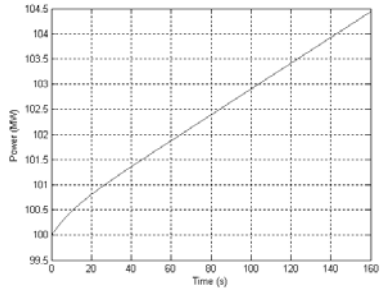

Figure 6: Booster tank high-pressure injection

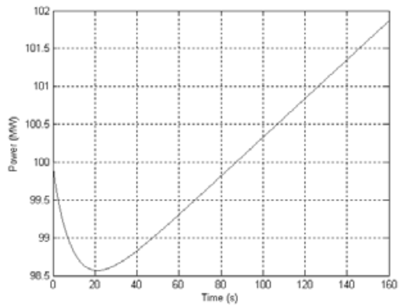

Figure 7: Low-pressure injection

A control system needs to be designed that will intelligently generate set point values for each of the four helium manipulation mechanisms as shown in Figure 8.

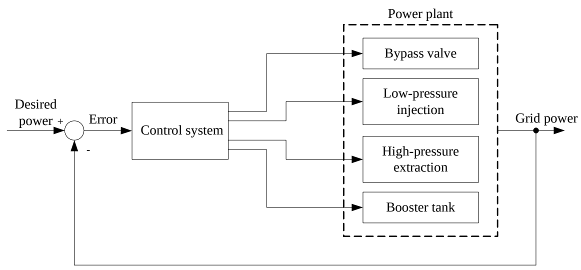

Figure 8: Power control system configuration

2. CONTROL SYSTEM DESIGN

2.1 Control methodology

The control system comprises four individual controllers, each generating a set point value for a specific helium manipulation mechanism. The control system is simulated for both PID and Fuzzy PID control strategies. A schematic layout of the control system is given in Figure 9. The difference between the power reference value, P_{ref}, and the actual electrical power generated, P, called the power error e_p, determines whether helium should be added to, or removed from the cycle. Although helium injection and extraction are normally used as the main control mechanisms, the bypass valve is mainly used to control the power output of the system in this particular case. e_p is the input to a controller that generates a bypass valve set point value, *BPV-sp.*

The efficiency of the system depends greatly on the setting of the bypass valve. If the valve opening it too large, a great amount of helium will be re-circulated through the compressors, rendering the system very inefficient. The non-minimum phase effect can be avoided by closing the bypass valve while injecting helium at the low-pressure side of the system. If the bypass valve opening is too small it would not be possible to avoid the non-minimum phase effect. The bypass valve therefore has to be kept at a predefined reference to allow for a certain amount of reserve capacity without degrading efficiency too much. This predefined reference is called the bypass valve reference, BPV_{ref}. The bypass valve set point value is subtracted from the bypass valve reference value to obtain the bypass valve error, e_{BPV}. e_{BPV} is the input to three other controllers that generate set point values for boosting, low-pressure injection and high-pressure extraction. When the bypass valve operates away from its reference point, these three controllers will generate set point values that will restore the bypass valve to its reference value.

The activation system determines which set point value may be ported to the system. When the power error is positive the low-pressure injection set point is connected to the system and when the power error is negative the high-pressure extraction set point is connected. Boosting is only activated when the power error is positive and above a specified value. The bypass valve set point is always connected. This activation system eliminates conflicting set points among the helium manipulation mechanisms. For example it is not desirable to inject helium and extract helium at the same time.

2.2 Fuzzy PID control

A Fuzzy controller can be regarded as a superset of linear controllers [5-8]. Under certain assumptions it is possible for the Fuzzy controller to emulate a PID controller. In conventional PID controllers, the control variable, $u(t)$ is defined in terms of the error, $e(t)$ between a reference value, y_{ref} and the process output, $y(t)$:

$$u(t) = G_p \cdot e(t) + G_i \int_0^t e(t)dt + G_d \cdot \frac{d}{dt} e(t) \qquad (1)$$

where G_p, G_i and G_d are proportional, integral and derivative gains respectively.

In order to emulate a PID controller by means of a linear Fuzzy controller, the summation in the PID control equation has to be replaced by a Fuzzy rule base acting like a summation [9]. A Fuzzy PID (FPID) controller uses the variables error, *e*, change of error, *ce*, and integral of error, *ie*, in the antecedent of IF-THEN rules and the control variable, *u*, as consequent [10-12].

A Fuzzy controller based on the Mamdani-type Fuzzy inferences would consist of rules having the form:

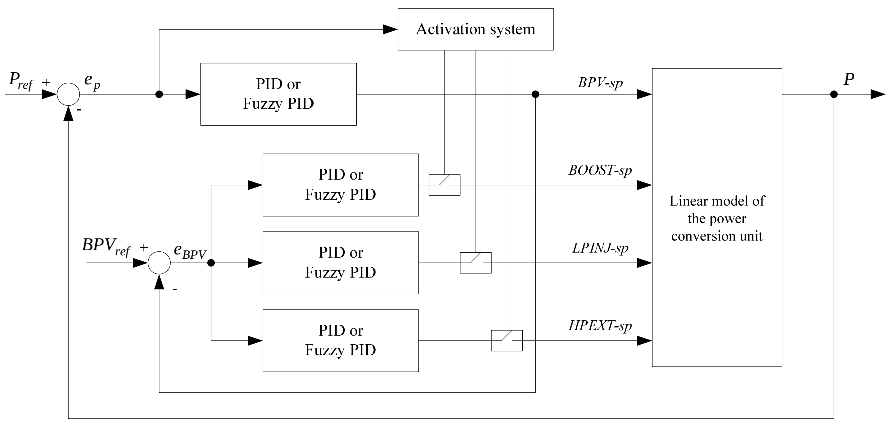

Figure 9: Schematic layout of the power control system

$$R_n : \text{if } (e \text{ is } A_{1,n}) \text{ and } (ce \text{ is } A_{2,n}) \text{ and } (ie \text{ is } A_{3,n}) \text{ then } (u \text{ is } B_n) \tag{2}$$

where n is the rule number and $A_{i,j}$, B_n are Fuzzy sets.

A Fuzzy controller can be represented as an input-output mapping. In the general case it may result in a non-linear shaped control hyper surface. When three inputs e, ce, ie and one output u are considered, this mapping takes the form given in (3).

$$u = f(e, ce, ie) \tag{3}$$

However assumptions need to be made to allow the Fuzzy rule base to act like a summation resulting in a linear mapping given by (4).

$$u = G_p \cdot e + G_i \cdot ie + G_d \cdot ce \tag{4}$$

A Fuzzy controller becomes linear by making the following assumptions with respect to the input universes, rules, membership functions and Fuzzy connectives [9]:

- The input universes of the Fuzzy controller must be large enough for the input to stay within the limits (saturation is not allowed). The input sets must be triangular and cross their neighbouring sets at the membership value $\mu = 0.5$; their peaks thus being equidistant. Any input value can thus be a member of at most two sets; and its membership of each is a linear function of the input value.
- The terms of the rules has to be combined by the **AND** operator (outer product) to ensure completeness. The output sets should preferably be singletons equal to the sum of the peak positions of the input sets. The output sets may also be triangular and symmetric about their peaks, but singletons simplify defuzzification.
- Linearity is also ensured by choosing the algebraic product for the **AND** connective.

The next step in the design process is to derive the Fuzzy gain constants (see Figure 10) from the PID gain constants. The Fuzzy PID controller emulates the PID controller if the following equation holds [5]:

$$\begin{aligned} u &= G_p \cdot e + G_d \cdot ce + G_i \cdot ie \\ &= [FGp \cdot e + FGd \cdot ce + FGi \cdot ie] \cdot FGu \\ &= FGp \cdot FGu \cdot e + FGd \cdot FGu \cdot ce + FGi \cdot FGu \cdot ie \end{aligned} \tag{5}$$

By comparing the gain constants of the FPID controller with the gains of the conventional PID controller in (5), the following relations can be derived [5]:

$$FGp \cdot FGu = G_p \Rightarrow FGu = \frac{1}{FGp} \cdot G_p \tag{6}$$

$$FGd \cdot FGu = G_d \Rightarrow FGd = FGp \cdot \frac{G_d}{G_p} \tag{7}$$

$$FGi \cdot FGu = G_i \Rightarrow FGi = FGp \cdot \frac{G_i}{G_p} \tag{8}$$

If it is assumed that the error is within the range [-E,E] and the input universe of the Fuzzy controller is for example [-100,100], the Fuzzy gain FGp can be derived as follows [5]:

$$\begin{aligned} & e \in [-E, E] \\ & \Rightarrow FGp \cdot e \in [-FGp \cdot E, FGp \cdot E] = [-100, 100] \\ & \therefore FGp = \frac{100}{E} \end{aligned} \tag{9}$$

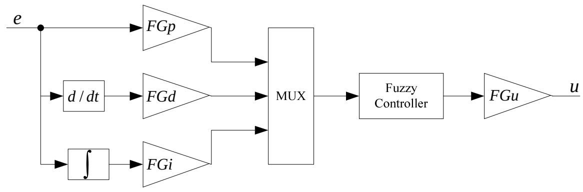

Figure 10: Fuzzy PID controller

A rule base with three inputs, however, easily becomes very large and rules concerning the integral action becomes troublesome. It is therefore common practice to separate the integral action to form a Fuzzy PD+I (FPD+I) controller as shown in Figure 11.

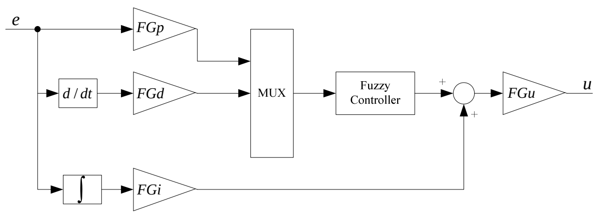

Figure 11: Fuzzy PD+I controller

The controller function is separated into two additive parts as given by (10).

$$u = u_{FPD} + u_I = f_{FPD}(e, ce) + f_I(ie) \tag{10}$$

2.3 Membership function definitions

A Mamdani inference system is used. Consider a universe of discourse, [-E,E]. The membership functions for the inputs and output are defined as shown in Figures 12 and 13. The input membership functions for both the error, e, and change of error, ce, are triangular. The input space is partitioned into three Fuzzy sets called negative (N), about zero (AZ) and positive (P). Singleton membership functions are chosen to define the output control variable u. The output space is partitioned into five Fuzzy sets called Negative big (Neg_Big), Negative small (Neg_Small), Zero (Zero), Positive small (Pos_Small) and Positive big (Pos_Big).

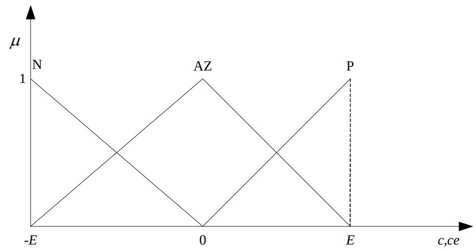

Figure 12: Input membership functions

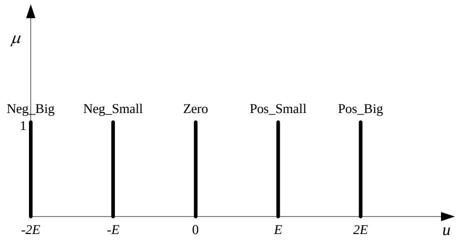

Figure 13: Output membership functions

2.4 Rule base definition

The rule base of the Fuzzy controllers consists of nine rules. These rules link two inputs namely the error and the change of the error to a control output. The rules are defined in the following table.

Table 1: Rule base of Fuzzy controllers

Inputs		Change in error		
		Negative	**About Zero**	**Positive**
Error	**Negative**	**Neg_Big**	**Neg_Small**	**Zero**
	About Zero	**Neg_Small**	**Zero**	**Pos_Small**
	Positive	**Zero**	**Pos_Small**	**Pos_Big**

3. CONTROLLER OPTIMISATION

3.1 Genetic algorithm optimisation

Genetic Algorithms (GAs) are general search algorithms that imitate natural biological evolution. The idea is to evolve populations of individuals that are better adapted to their environment than the individuals from which they are created. GAs operate on a population of potential solutions applying the principle of survival of the fittest to produce successively better approximations to a solution. At each generation of a GA a new set of approximations is created by the process of selecting individuals according to their level of fitness and reproducing them using operators borrowed from natural genetics [13].

The performance of both the PID and FPD+I controller can be improved by adapting the gain values of each controller according to some objective function [14-17]. A strong characteristic of GAs is that they are able to optimise a large amount of parameters simultaneously. In the case of PID control, 4 PID controllers, each having 3 gains, will be optimised. This will result in 12 gain constants that will be optimised simultaneously. In the case of FPD+I control each controller has 4 gains resulting in a total of 16 gains to be optimised simultaneously.

The GA used to optimise the controllers make use of real-valued genes instead of binary encoded genes. Consider for example the four Fuzzy controllers presented in Table 2. As previously stated, all four gain values of each Fuzzy controller will simultaneously be optimised by the GA. Let N_{ind} be the number of individuals in the population and L_{ind} the number parameters that needs to be optimised. In Table 2 the number of individuals used can vary but the number of parameters is fixed at 16. The initial population is therefore an $N_{ind} \times L_{ind}$ matrix shown in Table 2.

3.2 Objective function

The power output response of the system controller by both PID and FPD+I controllers is evaluated by means of the ITAE performance index as given by:

$$ITAE = \int_0^T t\left|e_p(t)\right|dt. \tag{11}$$

The upper limit T is a finite time chosen somewhat arbitrarily so that the integral approaches a steady-state value and $e_p(t)$ is the power error. It is usually convenient to choose T as the settling time, T_s. The lower the value of the performance index the better the performance.

Table 2: Initial GA population

Individuals	Fuzzy controller 1				Fuzzy controller 2				…	Fuzzy controller 4			
	GE	GCE	GIE	GU	GE	GCE	GIE	GU		GE	GCE	GIE	GU
1	40.23	17.7	28.95	15.38	20.45	33.61	10.2	15.56		20.45	33.61	10.2	15.56
	82.06	13.26	13.35	0.09	70.21	8.01	11.61	0.02		70.21	8.01	11.61	0.02
	52.43	25.64	15.20	2.54	30.12	11.3	14.71	0.62		30.12	11.3	14.71	0.62
	47.5	49.10	9.09	10.65	80.33	22.27	1.56	4.32		80.33	22.27	1.56	4.32
	90.50	13.46	25.63	0.89	9.13	2.98	20.88	17.77		9.13	2.98	20.88	17.77
	47.21	25.29	7.89	10.48	43.2	15.21	8.63	18.23		43.2	15.21	8.63	18.23
⋮													
$N_{ind}-1$	90.50	13.46	25.63	0.89	9.13	2.98	20.88	17.77		20.45	33.61	10.2	15.56
N_{ind}	52.43	25.64	15.20	2.54	30.12	11.3	14.71	0.62		43.2	15.21	8.63	18.23

Parameters 1 ⟷ L_{ind}

This specific performance index was chosen because it reduces the contribution of the large initial error to the value of the performance integral, and it emphasizes errors occurring later in time.

4. RESULTS

The optimal gain values for the PID and FPD+I control strategies after 100 generations are summarised in Tables 3 and 4.

Table 3: Optimal gain values of the PID control strategy

PID controller	GE	GCE	GIE
1 (BPV-sp)	47.67	37.75	0
2 (LPINJ-sp)	0	0	0
3 (HPEXT-sp)	134.61	199.61	0
4 (BOOST-sp)	194.69	62.16	157.91

Table 4: Optimal gain values of the FPD+I control strategy

FPD+I controller	GE	GCE	GIE	GU
1 (BPV-sp)	14.51	83.97	0	0.31
2 (LPINJ-sp)	0	0	0	0.17
3 (HPEXT-sp)	0	61.01	0	0.79
4 (BOOST-sp)	19.35	62.34	0	0.69

As can be seen the GA chose the integral gains close to zero. This shows that proportional derivative control is sufficient. The GA penalises low-pressure injection by giving the proportional and derivative gains values of zero. This means that according to the objective function low-pressure injection leads to undesirable responses. Figures 14 and 15 show the plots of the objective function values of the fittest individual in each generation. It can be seen that after approximately 20 generations the objective function value converges.

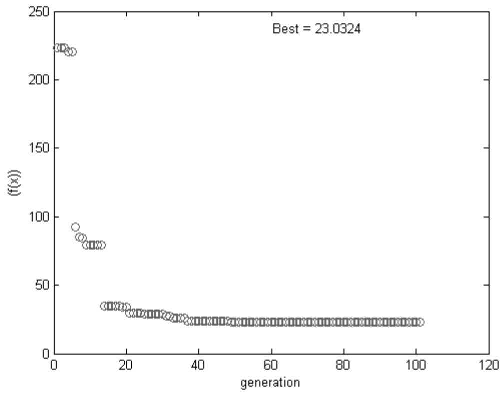

Figure 14: Objective value of the fittest individual in each generation (PID strategy)

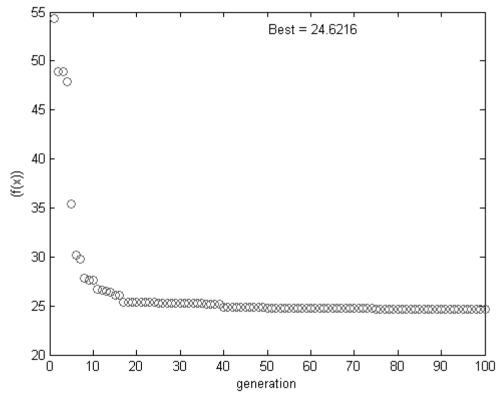

Figure 15: Objective value of the fittest individual in each generation (FPD+I strategy)

The performance improvement of the optimised controllers are now illustrated by testing the system with a specific power reference sequence. The response of a non-optimal system is given in Figure 16 and that of the systems optimised for the PID and FPD+I control strategies are given in Figures 17 and 18 respectively.

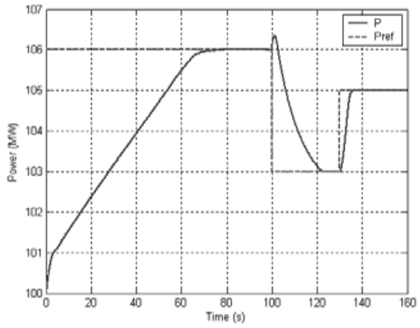

Figure 16: Non-optimal system output (ITAE value of 77.7)

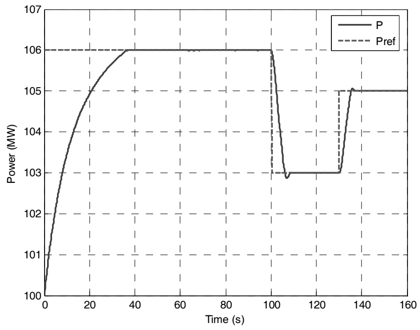

Figure 17 Optimal PID control (ITAE value of 23.03)

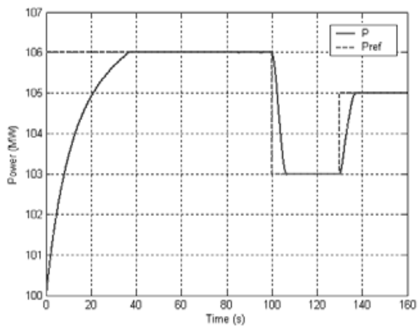

Figure 18 Optimal FPD+I control (ITAE value of 24.62)

The GA was able to derive optimal gain values after 100 generations. The objective function value of 77.7 for a non-optimal system was reduced to values of 23.03 and 24.62 for the optimal PID and FPD+I control strategies respectively. This shows that the GA is an effective parameter optimisation technique.

5. CONCLUSION

In this study both PID and FPD+I control strategies consisting of four controllers each were developed to optimally control the power output of a Brayton cycle based power conversion unit. The performance of these control strategies was optimised by using a GA.

The optimised control systems showed superior performance compared to the non-optimal control system. The ITAE objective function proved to be very effective. However, further work on the objective function is needed to take other constraints into account such as the reserve capacity and system stresses. Fuzzy controllers that simulate PID control were used. These linear Fuzzy controllers can be converted to non-linear Fuzzy controllers by using Gaussian membership functions. Some research can still be done on GAs and their use in relation to Fuzzy systems. Different parts of the Fuzzy system can be optimised by means of a GA. The effect of optimisation of the rule base and the membership function parameter values warrants valuable future work [18,19].

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