TAGUCHI METHOD IN ELECTRICAL MACHINE DESIGN

Albert Sorgdrager*, **Rong-Jie Wang***, **Andre Grobler** †

* *Department of Electrical & Electronic Engineering, Stellenbosch University, Private Bag X1, Matieland 7602, South Africa, E-mail: ajsorgdrager@gmail.com; rwang@sun.ac.za*

† *School of Electrical, Electronic & Computer Engineering, North-West University, Potchefstroom, 2531 South Africa, E-mail: andre.grobler@nwu.ac.za*

Abstract: There has been a considerable amount of research work on using the Taguchi method for electrical machine design in recent years. However, for the large community of electrical machine designers, this method is still less known. The purpose of this paper is to summarise the past and current research work of applying the Taguchi method in electrical machine designs. We attempt to give readers some insight into the advantages and disadvantages, challenges, current status and future perspective of the method.

Key words: Taguchi method, design optimisation, electrical machines, technical review.

1. INTRODUCTION

Design optimisation has become increasingly important in the electrical machine industry, which is largely driven by the market demand for increased performance-to-cost ratio designs [1]. Surrogate model methods (design space reduction, response surface and space mapping) and scalar objective search algorithms (genetic algorithms, particle swarm and differential evolution) are probably the most widely used and favoured methods for electrical machine optimisation in recent years [2]. The use of the Taguchi method in electrical machine design is relatively new. This paper intends to provide a detailed review and discussion of the published work relating to the use of Taguchi method in electrical machine design.

2. TAGUCHI METHOD

The Taguchi method was developed by Dr. Genichi Taguchi during his time at the Japanese Electrical Communications Laboratories in the 1940s. During that time he noticed that through judicious planning, development cost could be significantly reduced [3], which motivated him to develop a method that is now known as the Taguchi method. Taguchi's method was originally used for quality control in manufacturing industries, but has been implemented in various fields including biochemistry, material science, industry process control and some engineering fields [4]. According to Taguchi, quality loss is the cost incurred after the sale of a product whose quality characteristic deviates from the target value [3]. As shown in Figure 1(a), Taguchi's philosophy on quality performance differs from the conventional one in that it aims for a specific target within the conventional no-loss range, which translates into the overall quality improvements (see Figure 1(b)).

In comparison with the traditional Design of Experiments (DOE) method, Taguchi method standardises experimental designs by incorporating following unique features [3, 5]:

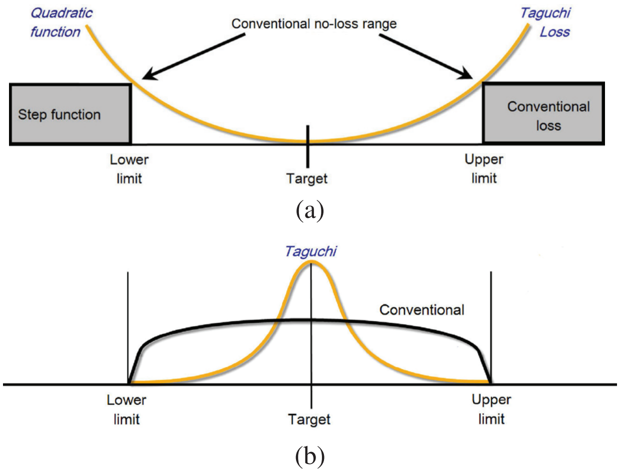


Figure 1: Conventional versus Taguchi's quality philosophy: (a) the view on loss; (b) resultant quality performance.

- *Defining quality*: For the Taguchi method, quality is quantified as being on target and with the lowest variances around the target.
- *Standardised experimental trials:* Taguchi developed a series of orthogonal arrays (OA) for experimental design, which were formulated to reduce the required number of trials through fractional factor analyses, instead of factor analyses as with DOE.
- *Robust design strategy:* For achieving a robust design that is closer to the target with minimum variance, uncontrollable ”noise” factors need to be identified and included in the outer array design.
- *Formulation of quality loss:* By expressing the loss/gain in quality in a simple manner, the quality characteristics can be for a specific target value (nominal-is-best), high as possible (bigger-is-best) or low as possible (smaller-is-best).
- *Signal-to-Noise analysis:* To analyse the performance variance caused by the noise factors, the Taguchi

method employs the Signal-to-Noise (S/N) ratio transformation. This method consolidates multiple data points into a single value reflecting the amount of variance present for the specific quality characteristic selected.

The implementation of the Taguchi method is usually conducted in the following steps:

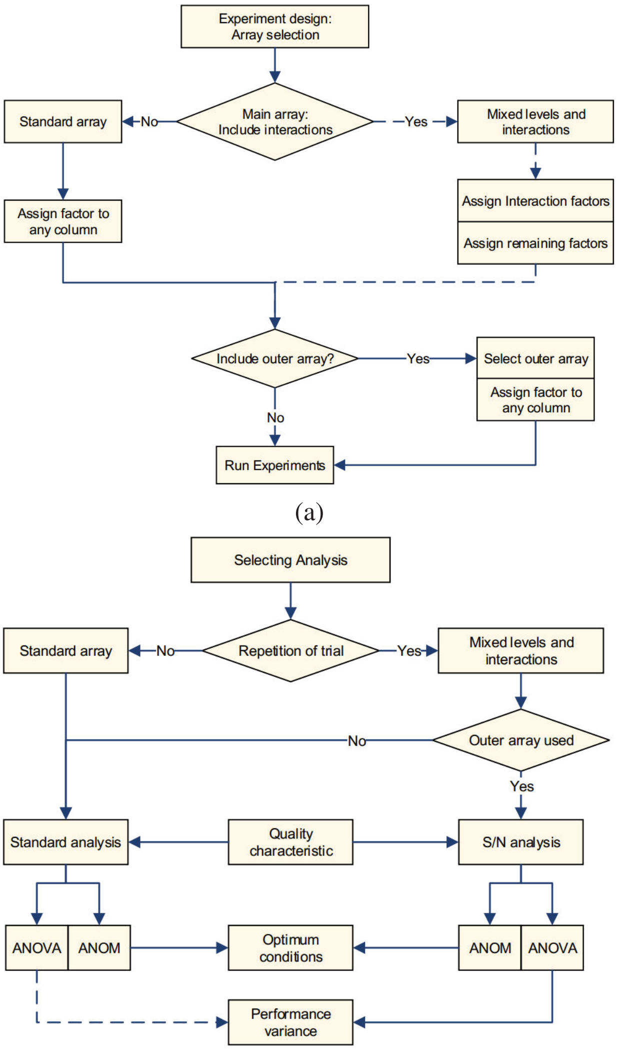
- *Brainstorming and planning* are seen as the most important step in the Taguchi method, which focuses on the goal, problems, attributes of the parameters, experimental trials and the quality characteristics of the design.

- *Experimental trial designs* can be constructed according to Figure 2(a). A key advantage of the Taguchi method is the ability to gain insight into the interactions between two parameters of the main design array. The range of standard OAs are capable of including 2 to 31 design parameters with two-, three-, four- or five-levels for each factor.

- *Conducting experiments* formulated by the main OA should be carried out randomly to avoid the dependency on the experimental setup. Since all the trials are predetermined and can be executed concurrently, the benefits of parallel computing can be exploited.

- *Analyse results to determine optimum conditions:* The experimental framework used can affect how the results of each trial are analysed. The type of analysis required can be selected with the aid of Figure 2(b). If the outer array design was not included, the standard analysis is then used, whereby the output response is used as that of the Analysis of Mean (ANOM) and the Analysis of Variance (ANOVA). In the case that the outer array design was used, the output response is analysed using S/N ratio for the specific quality characteristics. The ANOM is used to identify the optimum conditions of each parameter by studying the main effects of each level, which gives an indication of the performance trend over the parameter range. The ANOVA is a statistical tool used to determine the influence each parameter has on the performance outcome. Once the optimum level conditions for each parameter are determined, the performance of the optimum design can be predicted.

- *Run confirmation test using optimum conditions:* The optimum level conditions determined by the ANOM analysis must be used to confirm the predicted optimal design's performance. The same trial exposure conditions should be applied when confirming the predicted optimum. The actual optimum performance is then compared to the predicted one.



(a)

(b)

Figure 2: Flowcharts describing: (a) experimental design framework [3]; (b) data analysis framework [3].

2.1 *Orthogonal Array Methodology*

Taguchi formulated 18 standard OA that can be used as part of his method in experimental design [4]. Each OA is named as L_n with n representing the number of trials required by the specific OA. A specific OA is also linked to a parameter combination, (l^p), as originally defined for a standard DOE analysis. Table 1 can be used to aid in selecting an OA for up to 15 parameters. Some OAs include multi-level parameters, for example, L_{18} has both two- and three-level parameters. Regardless of the array's design, they are analysed using the same approach. When selecting an OA, it is not always necessary to fully populate all the parameter slots. In some designs, an open parameters slot can be used to investigate the interaction between two parameters.

Table 2 shows the design of the L_8 array, which has 7 columns and 8 rows representing the parameters (A to G) and number of trials (T1 to T8), respectively. The numbers beneath each parameter indicate the state or value selected

Table 1: Selecting standard orthogonal arrays.

		Number of parameters													
		2	3	4	5	6	7	8	9	10	11	12	13	14	15
Levels	2	L_4	L_4	L_8	L_8	L_8	L_8	L_{12}	L_{12}	L_{12}	L_{12}	L_{16}	L_{16}	L_{16}	L_{16}
	3	L_9	L_9	L_9	L_{18}	L_{18}	L_{18}	L_{18}	L_{27}	L_{27}	L_{27}	L_{27}	L_{27}	L_{36}	L_{36}
	4	L'_{16}	L'_{16}	L'_{16}	L'_{16}	L'_{32}	L'_{32}	L'_{32}	L'_{32}	L'_{32}					
	5	L_{25}	L_{25}	L_{25}	L_{25}	L_{50}	L_{50}	L_{50}	L_{50}	L_{50}	L_{50}	L_{50}			

Table 2: Orthogonal array L_8.

		Parameters							
L8		A	B	C	D	E	F	G	Output
Trials	T1	1	1	1	1	1	1	1	Y_1
	T2	1	1	1	2	2	2	2	Y_2
	T3	1	2	2	1	1	2	2	Y_3
	T4	1	2	2	2	2	1	1	Y_4
	T5	2	1	2	1	2	1	2	Y_5
	T6	2	1	2	2	1	2	1	Y_6
	T7	2	2	1	1	2	2	1	Y_7
	T8	2	2	1	2	1	1	2	Y_8

to be investigated, e.g. 1 or 2 states. For any given trial, all the factors are present with their level for that specific trial indicated in the parameters column. Within a column, the parameter has equal representation over all the trials for each level and is seen as balanced. In the case of an L_8 array, level-1 and level-2 are each represented 4 times. For an array to be orthogonally balanced, there have to be equal occurrences of parameter combinations between any two columns. For the L_8 there are 4 possible combinations (1,1), (1,2), (2,1) and (2,2), each occurring twice between two columns. For an orthogonally balanced array, any parameter can be placed in any column and the analysis of the results will not be affected.

Experimental design using OAs are attractive because it reduces the number of experiments and can thus reduce total time spent. If a full factorial analysis is conducted for the 7 two-level parameters of an L_8, a total of 128 experiments are required, whereas with the OA only 7 experiments are needed. It should also be noted that an OA based analysis works best when there is minimum parameter interaction or inter-parameter dependency. If there exists interaction between parameters, the OA still possesses the capability to accurately identify the optimum parameter combination. However, depending on the degree and complexity of the parameter dependency, there might be a difference between the predicted and actual optimum performance. Thus, the use of a confirmation test is highly recommended under such circumstances.

3. TAGUCHI METHOD IN ELECTRICAL MACHINE DESIGN: AN OVERVIEW

3.1 Publication Overview

The Taguchi method has been used widely in engineering design [3–7]. The first use of the method for electrical machine design was reported in [8], in which the method was used to reduce cogging torque of a brushless DC Permanent Magnet (PM) CD-ROM spindle motor. This work stimulated more research interest in applying Taguchi method to different brushless PM motors [9–15]. Figure 3(a) is a histogram of research publications using the Taguchi method in electrical machine designs, where the annual number of relevant published journals and conferences are summarised. Evidently, there has been growing interest in applying the Taguchi method in electrical machines in past decade.

An interesting observation can be made if these publications are grouped geographically as shown in Figure 3(b), i.e., the Taguchi method is mostly used in Eastern Asia with Taiwan producing almost a third of the total publications.[*] The related publications from the rest of the world is rather limited, which is likely because the Taguchi method is less known outside the Asia. Most of the published work outside Asia was published in last 5 years.

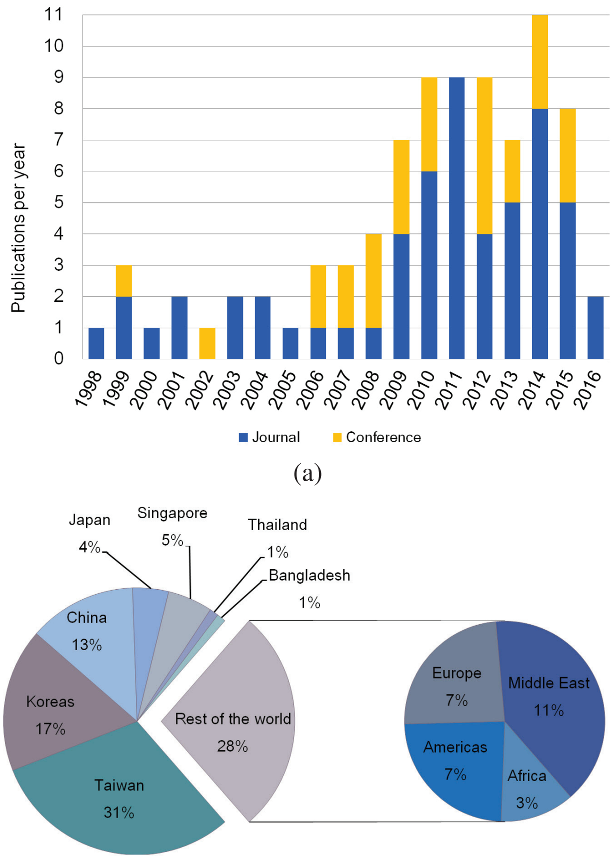
3.2 Scope of Taguchi Method in Electrical Machine Design

In literature, the Taguchi method has been applied to various types of electrical machines such as brushless DC PM motors [8, 9, 11, 13–20], PM Synchronous Machines (PMSM) [10, 12, 21–61], line-start PMSM [62–66], induction machine (IM) [67–71], Reluctance Synchronous Machines (RSM) or Switch Reluctance Machines (SRM) [72–81], axial flux PMSM [82, 83], linear or tubular type machines [84–88], piezoelectric motor [89], active magnetic bearing [90], Halbach array PM machine [91], and superconducting wind generator [92].

As illustrated in Figure 4(a), the large percentage of the published work using the Taguchi method focused

---

[*] Although the literature search was also conducted in Asian languages, it may not be exhaustive. Furthermore, the research carried out in Japanese industry is not always in public domain.



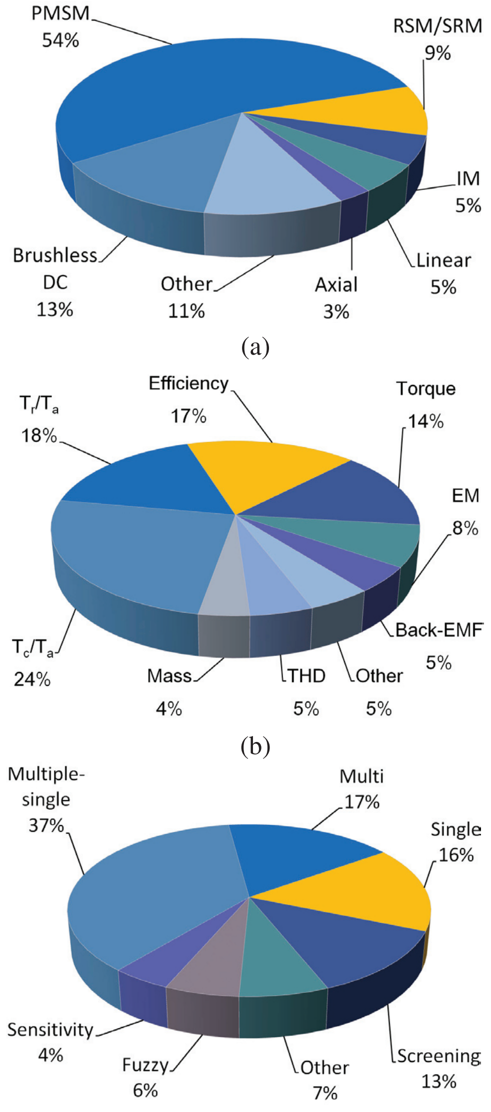
(a)

(b)

Figure 3: (a) Number of annual publications relating to the use of the Taguchi method in electrical machines; (b) Publications by country or region.

on PM machines. As displayed in Figure 4(b), the Taguchi method has been used in electrical machine designs for reducing cogging (T_c/T_a) and ripple (T_r/T_a) torque, Total Harmonic Distortion (THD) and active machine mass, improving efficiency, torque production, electromagnetic (EM) properties (flux or flux density), back-EMF and power factor, and investigating rotor saliency ratio, temperature distribution and acoustic noise. The most common use of the method is to improve the performance or the robustness of an existing design realised by other design methods. Overall more than 50% of the published work involved the torque performance related design problems.

As with any other optimisation methods, the number of parameters directly influences the total number of simulations and the complexity to analyse and realise the final design. With the Taguchi method the available OAs need to be considered when selecting the number of parameters for the optimisation. As shown in Figure 5(a), the L9 is mostly used for the main OA, followed by the L16 and the L18 in literature. These arrays are used to determine the optimum conditions for 4, 5 and 8 parameters, respectively. In addition to the main design parameters, the use of outer design can also be included, which often requires the use of a second OA [11,14,35,41, 44] leading to an increased total number of trials.



(a)

(b)

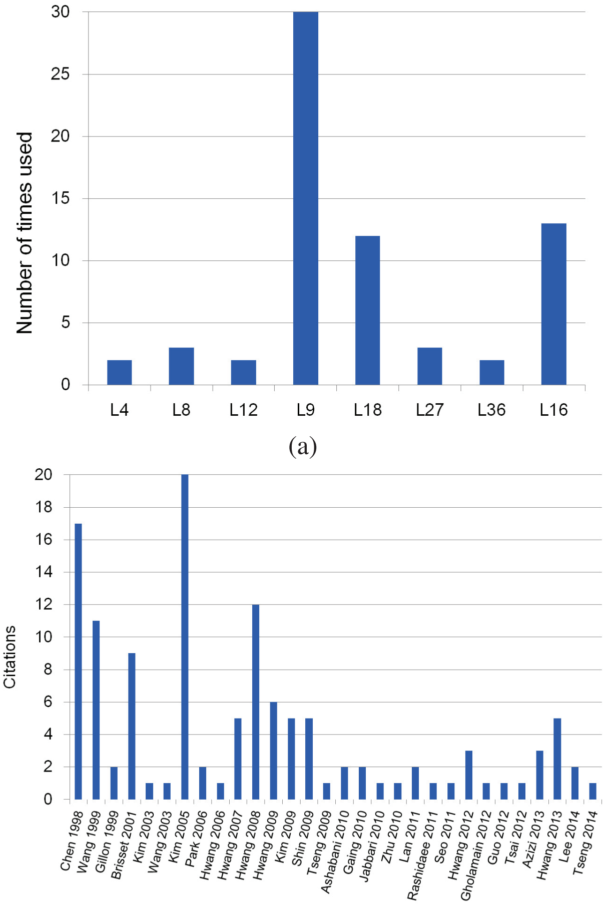
(c)

Figure 4: Percentage breakdown of the Taguchi method related publications in terms of (a) Machine types; (b) Optimisation objectives; (c) Implementation types.

3.3 *Citation Index*

To identify the key publications from the published work and to establish unique contribution made by researchers to advance the use of the Taguchi method in the field of electrical machines, a citation index is compiled in Figure 5(b). The citation index presents the number of citations for each selected paper of significance received from other Taguchi method related publications.

It was found that Kim *et al.* in [22] and Hwang *et al.* in [24, 25, 82, 85] applied practically the same approach as that of Chen *et al.* in [8]. However, Kim *et al.* in [27, 63], Shin *et al.* in [28] and Hwang *et al.* in [46] each proposed methods for multi-response optimisation using the Taguchi method.



(a)

(b)

Figure 5: (a) Commonly used Orthogonal Arrays (OA) in publications; (b) Taguchi machine design publication citations index [8–38, 40–68, 71–80, 82–99].

There has not been any published research incorporating the Taguchi method into an iterative design procedure.

4. REVIEW OF TAGUCHI METHOD IN ELECTRICAL MACHINE DESIGN

Comparing with common optimisation methods, Taguchi method has two inherent limitations, i.e. *single response optimisation* and *relative optimum parameter conditions*. The majority of publications focused on methods/techniques to overcome the single response limitation. There is also some work where Taguchi method was used in conjunction with other optimisation algorithms such as Genetic Algorithm (GA) [93], Particle Swarm optimisation (PSO) [99] and Response Surface (RS) [12] methodology.

After careful review and synthesis of the available literature, it was found that the ways of implementing the Taguchi method for electrical machine design in literature were similar (as in Figure 2(a) and Figure 2(b)). However, there are many different approaches when it comes to determining optimum conditions with multiple responses. To facilitate further discussion, these relevant publications are classified into the six categories. Figure 4(c) illustrates these categories and their percentage representation of the total relevant publications:

- *Single response*: a standard Taguchi implementation where only one objective is investigated. The optimum conditions of each parameter could be determined using the ANOM and/or ANOVA. The latter provides the percentage contribution towards performance variance.

- *Multiple single-response using ANOVA*: each objective is treated as a single stand-alone Taguchi optimisation. The optimum conditions for each objective are determined as with the single response one. The final optimum conditions are selected based on the percentage contribution provided by the ANOVA and the designer's discretion.

- *Multi-response using weighted function*: the responses are combined into a single weighted optimisation function. To ensure equal representation, each response needs to be normalised, after which a weighted value is assigned to each response. Once combined the problem is treated as a standard Taguchi implementation.

- *Multi-response using fuzzy logic*: each of the responses are combined into a single function using fuzzy logic. The fuzzy conversion ensures a more accurate representation without assigning artificial weights to each response. Once combined the problem is treated as a standard Taguchi implementation.

- *Parameter screening for a secondary method*: by reducing the number of initial parameters a more computational expensive method can be used in the second design step. The parameters are screened using ANOVA to select only the parameters with a high contribution towards performance variance. Lower contributing parameters are set using the ANOM. This approach is ideally suited as an extension to the single or multiple responses using ANOVA in providing a non-relative optimum.

- *Sensitivity or robustness analyser*: the Taguchi method is used to investigate the sensitivity of a specific response and the influence certain parameters have on the response by using the ANOVA. This, in turn, can be used to increase the performance robustness of the machine or selecting manufacturing tolerances. This is in part an extension of the single or multiple responses using ANOVA.

Each of the above mentioned implementations and their respective advantages and disadvantages will be discussed in more detail in the following sections.

4.1 Single Response

The single response implementations of the Taguchi method can be conducted with or without using outer noise design such as using an outer OA or exposing the main OA to different operating conditions. Since there is only one objective being investigated, the working flow to analyse the data stays the same as that in Figure 2(b).

The single response Taguchi optimisation without outer noise design has been widely used in literature, which is probably the simplest form of applying Taguchi method as it does not require the use of Quality Control (QC), Mean-Square Deviation (MSD) and S/N ratio calculations. The optimum condition of each parameter is determined using the ANOM and the main effect plots. Despite the apparent advantage of requiring fewer simulations due to the use of OA, the robust design feature of the Taguchi method is lost as the variance reduction is not included. The use of the ANOVA is optional as this only provides information regarding a parameter's influence on performance variance. The applications of this approach relating to electrical machine design include electromagnetic design [23, 31, 54, 61, 76, 83], combined electrical machine and drive design [77, 79] and thermal effects of winding potting material [78].

Azizi *et al.* in [76] investigated the effect that geometrical variables have on the saliency ratio of an RSM. The contribution of each geometry parameter on the objective is examined by using the L36 OA. The ANOM and ANOVA are used to determine the optimum shape of the rotor to maximise the saliency ratio. An interesting aspect of the paper is the inclusion and use of the interaction effects of the design variable in the optimisation. This was done by using the interaction investigation columns built into the OA. The interaction plots were also constructed to investigate the interaction with the most influence. The optimum levels were adjusted accordingly.

The first instance of using single response optimisation with outer noise design was reported in [11], which in fact was the first full implementation of Taguchi method in electrical machine design as it applied the method strictly according to Taguchi's methodology. The aim was to minimise the cogging torque as a function of average torque and reduce the variance in performance caused by manufacturing.

The Taguchi method was selected as it requires fewer simulations than DOE and traditional optimisation techniques. Furthermore, the DOE method does not take non-linearity or noise effects into account, thus cannot reduce performance variances. For both the main and outer arrays the L9 OA was selected, thus a total of 81 2D FEM simulations were required to obtain the necessary information for the ANOM and ANOVA. With the use of an outer array, a *smaller-is-better* MSD was calculated for each main trial, which are needed for the S/N ratio calculations. The confirmation simulations showed a reduction in cogging torque to average torque ratio. This study clearly demonstrated the advantages of using the Taguchi method such as the simplicity of use and the ability of reducing variations when using an outer array. However, it should be noted that the use of an outer array increases the number of simulations and in this case to the same number as the DOE method.

The use of the Taguchi method to maximise the efficiency of a 5 MW PM generator was presented by Tsai in [44]. The publication indicated that the Taguchi method can be used in a fully unconstrained machine design using both the stator outer diameter and stack length as design variables. The trial machine analysis was done using 3D FEM simulations, after which the results were converted to S/N ratio to conduct the ANOM and ANOVA. The verification simulation of the optimal parameter combination confirmed the predicted performance with the final design realising an efficiency greater than 95% at rated conditions.

Other interesting applications of the single-response Taguchi method include minimising cogging torque of a brushless PM motor [35], maximising the back-EMF of a PMSM [43, 55], and optimising the air-gap flux density waveform of an outer-rotor PMSM [56]. In [43] and [55], the back-EMF was maximised by using the design parameters in the main L18 OA and the variance minimised through the use of the L4 outer OA containing the PM noise parameters. Both the average and variance of the back-EMF between the Taguchi robust optimisation model and initial optimisation model are compared in Figure 6. A 12.87% performance improvement and a 11.32% variance reduction were achieved in terms of the back-EMF.

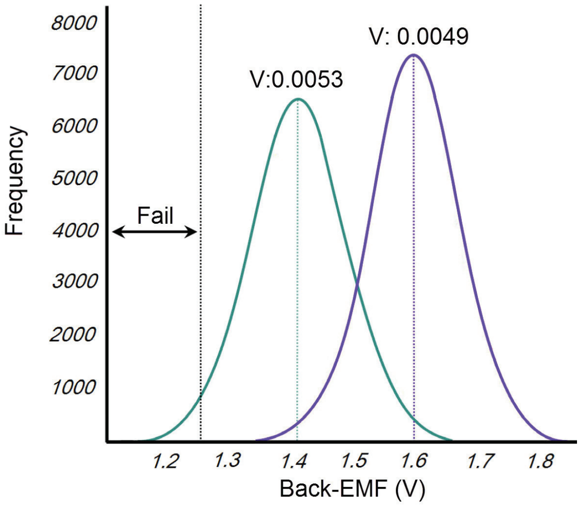


Figure 6: Comparison between the Taguchi robust design and original optimum design. Reconstructed from [43, 55].

4.2 *Multiple Single-Response*

The multiple single-response implementation is by far the most popular way of applying the Taguchi method for machine design as seen in Figure 4(c). It is well suited for both outer and non-outer noise design cases, although the non-outer design is mostly used. For the multiple single-response implementation, each response is obtained from the same OA trial framework, but analysed

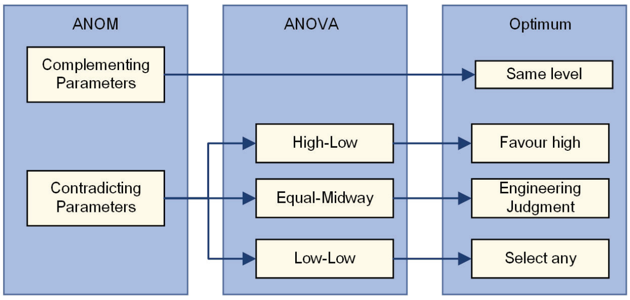


Figure 7: Identification steps for the final optimum of the multiple single-response implementation.

separately. Thus, all the performance objectives under investigation are extracted from the same simulation. For each performance response both the ANOM and ANOVA are required. The ANOM is used to determine the optimum condition for each parameter while the ANOVA is used to add weight to a response when there is no clear overall optimum.

Once the optimum conditions of each response have been identified, the overall optimum can be found using the ANOM and ANOVA results as shown in Fig. 7. The selection of the overall optimum of the parameters may lead to two possibilities: complementing or contradicting combinations. A complementing optimum occurs when a parameter has the same optimum condition for all the responses, thus the optimum is set using the parameter level state identified by the ANOM. A contradicting optimum occurs when a parameter does not satisfy the optimum conditions of all the responses. In such cases, the final optimum can be determined by using the percentage contribution to variance information obtained from ANOVA, which leads to one of following three possible outcomes:

- *High-low contribution:* The optimum condition of the parameter is selected to favour the objective with the highest contribution towards performance variance.
- *Equal-midway contribution:* Here designers have to use their own discretion in selecting the optimum conditions. They can either favour a specific objective or set the optimum so that neither of the two objectives are favoured nor penalised.
- *Low-low contribution:* Neither of the two parameters has a high contribution towards performance variance. The probability in selecting a combination resulting in a poor optimum design is relatively low. For this case, the ANOM's main effect plot has a near horizontal line.

For the above three contradicting cases, it is more challenging to determine the optimum condition for an equal-midway contribution, where the risk of selecting a combination that results in a sub-optimum performance for all the responses is relatively high. Furthermore, designers have to interpret both the ANOM's and ANOVA's results to ensure they select the best option. For this case the ANOM's main effect plots can also be helpful in making selections.

One concern regarding this implementation is the increased complexity in determining the overall optimum as the number of responses increases. The same is also true if there is an increase in parameters (e.g. with the L18 OA that uses 8 parameters) or an increase in parameter levels (e.g. with the L16 that uses 4 levels for each of the 5 parameters).

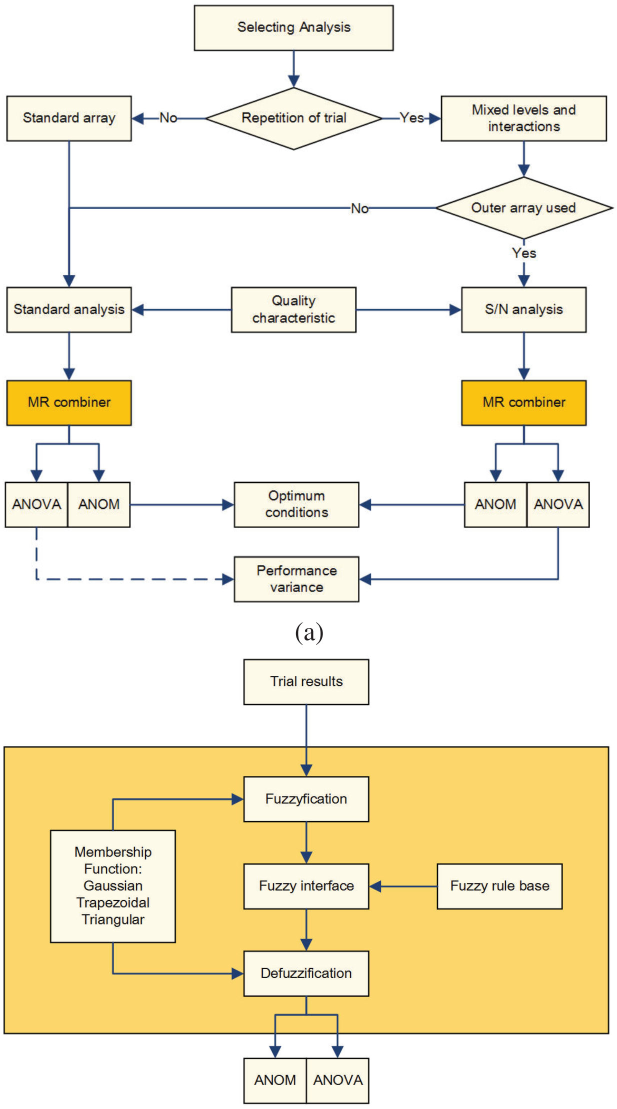
In literature, this approach was mostly used for electrical machine design with two single responses. Several cases were presented where average torque was maximised (or maintained) while minimising torque ripple [8, 21, 22, 26, 58, 75], cogging torque/force [9, 14, 34, 48, 85] or harmonic components in the machine [47, 49, 73, 74]. Hwang *et al.* applied this approach to maximise efficiency while minimising torque ripple [23, 52] or cogging torque [24–26] while Zhu *et al.* used the same approach for structural design [20].

Another interesting multiple single-response implementation was reported in [15] as part of a comparison study between the use of S/N ratio [9] and Grey relational analysis. The objective of the study was to minimise the cogging torque whilst reducing the variance of a brushless DC spindle motor. To reduce the torque variance the same main and outer OA were used to provide a relative comparison between S/N ratio and Grey relational. For both cases, the ANOM and ANOVA results agreed well with only a slight difference between their optimum designs. The advantage of using the Grey relation rather than S/N ratio is that the use of a structured outer design array is not as important as with S/N ratio. The Grey system can provide relatively accurate results with partial data while this is not always applicable to S/N ratio. This can be beneficial in a design that has both a large number of design parameters and noise factors. Thus, instead of using a large OA for the noise factors, a small number of outer designs can be compiled that still cover the whole range. The full comparative study is also presented in [100].

4.3 Multi-Response Using Weighted Function

The first use of a modified Taguchi method for multi-response problems was reported in [18], which was writen in Korean. A similar paper in English was published in 2009 [19]. The main idea in the paper was to improve the weakness of the Taguchi method for multi-response optimisation problems by allocating weighted value to each response function. In order to perform multi-response design by using the OA trial results, it is necessary to normalise the S/N ratio of each response function. The same approach was also used in [19, 27, 28, 63].

To convert a multi-response optimisation problem to a single-response one, as per requirement of the Taguchi



(a)

(b)

Figure 8: Flowcharts describing: (a) data analysis framework for a multi-response Taguchi implementation; (b) Fuzzy logic framework within the MR combiner. Adapted from [29, 95].

method, an additional step is required as indicated by the highlighted blocks in Figure 8(a). The main experimental framework still functions as in Figure 2(a), however, the multiple responses have to be combined into a singular response (a process called Multi-Response (MR) combiner) before the ANOM and ANOVA can be performed. In addition, each response has to be transferred into the same frame of reference to ensure equal representation. There are several ways in which the MR combiner can function for the weighted normalisation.

In literature, there are two variants using the Taguchi method with multi-response using the weighted sum normalisation method, one was proposed by Park [18] *et al.* and the other by Kim [41] *et al.* For both methods, outer noise designs are required, thus, the use of QC and its corresponding MSD formula is required. In Park's method, the more favoured of the two, each response's S/N ratio is first calculated, then normalised and then combined. In Kim's method each response value is first normalised before the S/N ratio of each trial is calculated. The procedure to combine multi-response into a single function using Park's method comprises three steps [19]:

- *Step 1:* Calculate the S/N ratio of each trial using the QC applicable to the specific response. Each response's S/N ratio values are calculated separately.

- *Step 2:* Normalise the S/N ratio of each trial using the z-standardisation method. Each response's S/N ratio normalisation is done separately. The normalisation for a specific trial (n) is done as follows:

$$\mathrm{norm}(S/N_n) = \frac{S/N_n - \mathrm{ave}(S/N)}{\mathrm{sd}(S/N)} \tag{1}$$

  where S/N_n is the S/N ratio of n^{th} trial, $\mathrm{ave}(S/N)$ is the OA trial average and $\mathrm{sd}(S/N)$ is the standard deviation of the OA trial's S/N ratios.

- *Step 3:* Calculate the total S/N ratio (TS) for the n^{th} trial using the multiple response function and the normalised S/N ratio value

$$\mathrm{TS}_n = \sum_{i=1}^{n} w_i \cdot \mathrm{norm}(S/N_n) \tag{2}$$

  where w_i is the weight factor of the i^{th} response that represents the relative importance of the i^{th} response. The sum of the weight value of w is 1.

Once TS has been determined for each of the main OA trial, the ANOM and ANOVA can be conducted as normal. In addition to the work in [27, 28, 63], Park's method was also used in [92], [57] and [80]. The method was successfully used to realise an optimal machine design ranging from two to five responses and was experimentally validated in the majority of the publications. It should be noted that there are various ways to normalise the OA trial data.

Although this method may be relatively easy to implement for the use of optimisations using three or more responses, it has some disadvantages. In Eq. (2) the weights assigned to each response are selected by the designer. This is usually done to favour one response over the other and not to realise a balanced design between each response or to obtain the maximum value of TS by the optimum machine. To overcome this problem Yang [19] additionally proposed the use of a second Taguchi optimisation to maximise TS. For this, each weight is treated as a parameter and the combination of an OA trial is seen as the ratio that must provide the sum of 1. The optimal weight ratio of each experiment is obtained when TS is maximised using the Taguchi method.

4.4 Multi-Response Using Fuzzy-Logic

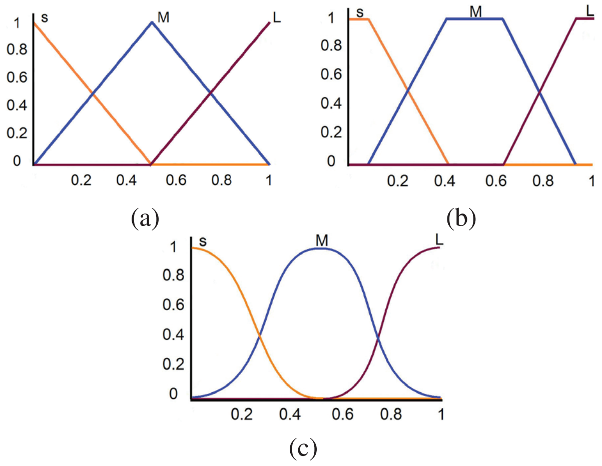
An alternative method when using the Taguchi method for multi-response optimisation is to incorporate the use of fuzzy-logic. A fuzzy-based Taguchi method was first published in [29, 95] by Gaing *et al.*, who later published further work with Chui *et al.* in 2014 [50]. Also in 2010, Hwang used a similar fuzzy base approach in [30]. In 2013, Hwang presented a comparison study between a multiple single-response implementation and a fuzzy logic realised design [46]. The study confirmed the effectiveness of the adapted method to realise a higher performance machine design than the multiple single-response technique. The final design was experimentally validated. It shows that the technique can be applied to solve multi-response machine design optimisation problems.

The incorporation of fuzzy-logic framework within the MR compiler block is described in Figure 8(b). The fuzzy-based method coordinates the multiple responses to obtain the better combination of geometric parameters for achieving multiple targets using the same framework. The method is used primarily for pre-processing of the S/N ratios of each OA trial so that the different attributes of each response can be compared and summed at the same level by using membership functions. This ensures the identification of the best combination of parameters by the ANOM for the selected responses.

A fuzzy rule-based inference system comprises of three basic units: fuzzifier, fuzzy inference engine, and defuzzifier as presented in Figure 8(b). Both the fuzzifier and defuzzifier require the use of a membership function to fuzzify the S/N ratio and OA results and defuzzify them to be summed into a single output. For the membership function either a Gaussian [29, 95], triangular [30, 46, 50] or a trapezoidal [50] function can be selected. The fuzzifier and defuzzifier do not necessarily require the use of the same type of membership function [50]. Typical normalised fuzzifiers are presented in Figure 9. To interpret the fuzzified input data the fuzzy rule base is used. In a case of three states (Small, Medium, Large) with two objectives would result in 9 rules and five defuzzifier curves (Very-Small, Small, Medium, Large, Very-Large). If three objectives are used, 27 rules are formulated with seven defuzzifier curves.

4.5 Parameter Screening

The use of the Taguchi method as a parameter screening tool in complex design optimisation problems with a large number of design variables is an attractive alternative to the DOE method. Comparing with traditional DOE, the advantage of the fractional analyses of the Taguchi method effectively reduces the number of trial simulations required without sacrificing accuracy when conducting the ANOVA.



(a) (b) (c)

Figure 9: Typical normalised fuzzifier membership function: (a) triangular [30, 46, 50] (b) trapezoidal [50] (c) Gaussian [29, 95].

The use of the Taguchi method as a parameter screening method for electrical machine design was first used by Gillion and Brochet in their series of publication [10, 12, 13] between 1999 to 2001. They selected more significant design parameters based on Taguchi screening method to further improve the machine's performance. As the number of design variables is reduced by the Taguchi method, the more complex RS optimisation and objective function can be introduced in the second stage. In addition, constraints or penalty functions can also be included, which are challenging to do in the initial screening. Kim *et al.* in [16] applied this approach to identify the parameters that influence the cogging torque variance due to manufacturing tolerances. Chen in [86] proposed the use of a Genetic Algorithm (GA) along with the Taguchi method to identify the significant parameters influencing the thermal properties of a linear air-cored PMSM. In all cases, the second stage optimisation was done using a numerical optimisation method to maximise the desired objective function.

In [84] Ashabani *et al.* presented the use of the Taguchi screening approach along with an Artificial Neural Network (ANN) for shape optimisation of tubular linear PMSM. A multi-objective optimisation was formulated to reduce force ripple and PM volume while improving thrust development using the following cost function:

$$\mathrm{TS} = \frac{{O_1}^{w1}}{{O_2}^{w2}{\dot{O}_3}^{w3}} \qquad (3)$$

with O_x representing the objective and wx the weight assigned to the objective by the designer. Using the Taguchi method in the first stage, the near optimum values of design variables were obtained by using the ANOM. The two most significant parameters are identified by the ANOVA and used in training of a radial-basis function ANN. The trained ANN then predicts the objective function variations as a surface function (Figure 10) of the two parameters and a maximum is obtained.

Although the Taguchi method reduced the number of simulations required in the first stage, the proposed

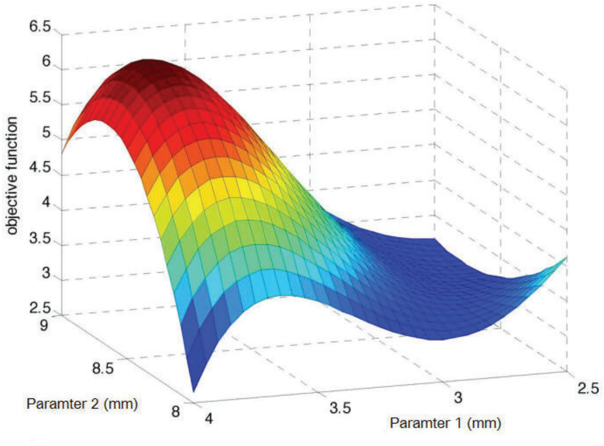


Figure 10: Typical surface response plot for objective function [84].

method introduces additional complexity and requires an additional number of simulations to train the ANN.

The Taguchi screening approach has also been combined with stochastic methods which have gained popularity over the past few years. Hwang in [34] presented the use of the Taguchi screening approach along with the Rosenbrock method to reduce cogging torque in a PMSM. Here Rosenbrock method is employed to determine the optimal settings of the design parameters identified using the Taguchi method as the Taguchi method lacks the ability in finding the global optimum without the use of an alternative method. A main concern of this approach is the high computational costs, especially with a large number of parameters.

5. CONCLUSION

The Taguchi method has been increasingly used in electrical machine designs. Apart from more conventional implementation of the Taguchi method, there has been a significant amount of research efforts addressing the limitations of the Taguchi method to make it more suitable for the use in multi-response optimisations. Many techniques have been developed to facilitate multi-response optimisation with the Taguchi method. However, they essentially all try to transform multiple responses into a single response function.

Both the weighted sum and fuzzy based methods are very attractive options for the use in multi-response problems along with the Taguchi framework. The weighted sum method is easy to implement as it only requires the normalisation of the results before it can be summed together into a single response. The judicious selection of weighted ratio distribution between responses can be challenging. The fuzzy based Taguchi method requires some additional implementation steps and analysis of the results before they can be combined into one response. The use of membership functions ensure an accurate representation of each response in the final response value, thus reducing the total number of required simulations in order to find the domain optimum solution.

The use of the multiple single-response approach is a simple and popular way of using the Taguchi method. However, it relies on the designer's knowledge and experience to select parameter states when dealing with contradicting optimums for the objectives.

When using the Taguchi method with the outer array design, both the average performance and its variance are improved, thus a robust design can be realised. To reduce the number of required simulations, it is important to ensure that only the applicable parameters are included for both the main and outer OA. The use of the Taguchi parameter screening method can be useful in this regard.

Although the use of the Taguchi method as a parameter screening tool in the hybrid optimisation structure is an attractive approach, it is still highly dependent on the second-tier optimisation. Usually mainstream optimisation algorithms are used to realise the final optimum design instead of the Taguchi method itself. Thus, despite all the benefits that Taguchi method screening may offer such as effective selection of design parameters (using ANOVA) and design domain, the robust design benefit of Taguchi method is partially lost.

This paper presents a detailed literature review on the applications of the Taguchi method in electrical machine design. Although extensive research on this subject can be found in literature, there is still ample scope for further development work. There are also no attempts in literature to use the Taguchi method directly as an optimisation tool within an iterative optimisation framework. Future work in this direction should be encouraged.

REFERENCES

[1] J. Gieras, *Advancements in Electric Machines*, Springer, 2008.

[2] Y. Duan and D. Ionel, "A review of recent developments in electrical machine design optimisation methods with a permanent-magnet synchronous motor benchmark study," *IEEE Trans. Ind. Appl.*, vol. 49, no. 3, pp. 1268–1275, May 2013.

[3] R. Roy, *A primer on the Taguchi method*, Society of Manufacturing Engineers, 2010.

[4] G. Taguchi, S. Chowdhury, and Y. Wu, *Taguchi's quality engineering handbook*, Wiley-Interscience, 2005.

[5] P. Ross, *Taguchi techniques for quality engineering*, McGraw-Hill, New York, 1988.

[6] R. Roy, *Design of experiment using the Taguchi approach*, Wiley, New York, 2001.

[7] M. Phadke, *Quality engineering using robust design*, Prentice Hall PTR, 1995.

[8] S. Chen, T. Low, and B. Bruhl, "The robust design approach for reducing cogging torque in permanent magnet motors [for CD-ROM spindles]," *IEEE Trans. Magn.*, vol. 34, no. 4, pp. 2135–2137, July 1998.

[9] X. Gao, Z. Liu, S. Chen, and T. Low, "Optimisation and sensitivity analysis of HDD spindle motors to manufacturing process noises using FEA and Taguchi method," in *IEEE International Magnetics Conference (INTERMAG)*, pp. EQ06–EQ06, May 1999.

[10] F. Gillon and P. Brochet, "Shape optimisation of a permanent magnet motor using the experimental design method," *IEEE Trans. Magn.*, vol. 35, no. 3, pp. 1278–1281, May 1999.

[11] H. Wang, Z. Liu, S. Chen, and J. Yang, "Application of Taguchi method to robust design of BLDC motor performance," *IEEE Trans. Magn.*, vol. 35, no. 5, pp. 3700–3702, Sept. 1999.

[12] F. Gillon and P. Brochet, "Screening and response surface method applied to the numerical optimisation of electromagnetic devices," *IEEE Trans. Magn.*, vol. 36, no. 4, pp. 1163–1167, July 2000.

[13] S. Brisset, F. Gillon, S. Vivier, and P. Brochet, "Optimisation with experimental design: an approach using Taguchi's methodology and finite element simulations," *IEEE Trans. Magn.*, vol. 37, no. 5, pp. 3530–3533, Sept. 2001.

[14] T. Low, S. Chen, and X. Gao, "Robust torque optimisation for BLDC spindle motors," *IEEE Trans. Ind. Electron.*, vol. 48, no. 3, pp. 656–663, June 2001.

[15] X. Gao, Z. Liu, S. Chen, and T. Low, "Grey relational analysis for robust design of BLDC spindle motor," in *Digest of the Asia-Pacific Magnetic Recording Conference*, Aug. 2002.

[16] Y. Kim, J. Hong, and J. Hur, "Torque characteristic analysis considering the manufacturing tolerance for electric machine by stochastic response surface method," *IEEE Trans. Ind. Appl.*, vol. 39, no. 3, pp. 713–719, May 2003.

[17] Y. Kwon, J. Kim, H. Yi, and G. Park, "Optimisation of magnet pole of brushless dc motor by experimental design method," in *2006 12th Biennial IEEE Conference on Electromagnetic Field Computation*, pp. 470–470, 2006.

[18] H. Park, B. Yang, S. Rhee, and B. Kwon, "Novel design of flux barrier in IPM type BLDC motor by considering the multi-response Taguchi method," *The Transactions of The Korean Institute of Electrical Engineers (in Korean)*, vol. 56, no. 3, pp. 498–505, 2007.

[19] B. Yang, K. Hwang, S. Rhee, D. Kim, and B. Kwon, "Optimisation of novel flux barrier in interior permanent magnet-type brushless dc motor based on modified Taguchi method," *Journal of Applied Physics*, vol. 105, no. 7, pp. 07F106, 2009.

[20] W. Zhu, X. Yang, and Z. Lan, "Structure optimisation design of high-speed BLDC motor using Taguchi method," in *International Conference on Electrical and Control Engineering (ICECE)*, pp. 4247–4249, June 2010.

[21] Y. Okada, Y. Kawase, and S. Sano, "Development of optimizing method using quality engineering and multivariate analysis based on finite element method," *COMPEL-The international journal for computation and mathematics in electrical and electronic engineering*, vol. 23, no. 3, pp. 733–739, 2004.

[22] S. Kim, J. Lee, Y. Kim, J. Hong, Y. Hur, and Y. Jung, "Optimisation for reduction of torque ripple in interior permanent magnet motor by using the Taguchi method," *IEEE Trans. Magn.*, vol. 41, no. 5, pp. 1796–1799, May 2005.

[23] C. Hwang and T. Lin, "Design optimisation for cogging torque minimization of a high-speed 2-pole IPM machine," in *12th Biennial IEEE Conference on Electromagnetic Field Computation*, pp. 237–237, 2006.

[24] C. Hwang, S. Cheng, and P. Li, "Design optimisation for cogging torque minimization and efficiency maximization of an SPM motor," in *2007 IEEE International Electric Machines Drives Conference*, vol. 1, pp. 642–646, May 2007.

[25] C. Hwang, L. Lyu, C. Liu, and P. Li, "Optimal design of an SPM motor using genetic algorithms and Taguchi method," *IEEE Trans. Magn.*, vol. 44, no. 11, pp. 4325–4328, Nov. 2008.

[26] C. Hwang, C. Chang, and P. Li, "Design optimisation for cogging torque minimization and efficiency maximization of a high-speed PM motor," in *International Conference on Power Electronics and Drive Systems (PEDS)*, pp. 938–943, Nov. 2009.

[27] K. Kim, J. Lee, H. Kim, and D. Koo, "Multiobjective optimal design for interior permanent magnet synchronous motor," *IEEE Trans. Magn.*, vol. 45, no. 3, pp. 1780–1783, March 2009.

[28] D. Shin and B. Kwon, "Multi-objective optimal design for in-wheel permanent magnet synchronous motor," in *International Conference on Electrical Machines and Systems (ICEMS)*, pp. 1–5, Nov. 2009.

[29] Z. Gaing and J. Chiang, "Robust design of in-wheel pm motor by fuzzy-based Taguchi method," in *IEEE PES General Meeting*, pp. 1–7, July 2010.

[30] C. Hwang, C. Huang, and J. Jang, “Optimal design and analysis of an SPM motor using fuzzy inference and Taguchi method,” in *Republic of China 31st Power Engineering Symposium (in Chinese)*, pp. 1208–1212, Dec. 2010.

[31] A. Jabbari, M. Shakeri, and A. Niaki, “Torque ripple minimization in PM synchronous motors using tooth shape optimisation,” *International Journal of Advanced Design and Manufacturing Technology*, vol. 3, no. 2, pp. 27–31, 2011.

[32] P. Srikomkham and S. Ruangsinchaiwanich, “Optimal rotor design of a PSC motor using Taguchi method and FEM,” in *International Conference on Electrical Machines and Systems (ICEMS)*, pp. 1341–1346, Oct. 2010.

[33] K. Abbaszadeh, F. Alam, and S. Saied, “Cogging torque optimisation in surface-mounted permanent-magnet motors by using design of experiment,” *Energy Conversion and Management*, vol. 52, no. 10, pp. 3075–3082, 2011.

[34] C. Hwang, P. Li, C. Chang, and C. Liu, “Optimal design of an IPM motor using Taguchi and Rosenbrock’s methods,” *Journal of Physics: Conference Series*, vol. 266, no. 1, pp. 012069, 2011.

[35] M. Islam, R. Islam, T. Sebastian, A. Chandy, and S. Ozsoylu, “Cogging torque minimization in pm motors using robust design approach,” *IEEE Trans. Ind. Appl.*, vol. 47, no. 4, pp. 1661–1669, July 2011.

[36] Z. Lan, X. Yang, F. Wang, and C. Zheng, “Application for optimal designing of sinusoidal interior permanent magnet synchronous motors by using the Taguchi method,” *Transactions of China Electrotechnical Society*, vol. 26, no. 12, pp. 37–42, 2011.

[37] S. Rashidaee and S. Gholamian, “Reduction of cogging torque in ipm motors by using the Taguchi and finite element method,” *International Journal of Computer Science and Engineering Survey*, vol. 2, no. 2, pp. 1–10, May 2011.

[38] S. Gholamian and S. Rashidaee, “Cogging torque reduction in surface permanent magnet motors using Taguchi experiment design and finite element method,” *International Journal of Intelligent Systems and Applications*, vol. 4, no. 11, pp. 33–39, 2012.

[39] J. Hwang, C. Liu, and P. Chen, “Design of permanent-magnet synchronous gear motor with high efficiency for elevators,” in *IEEE Third International Conference on Sustainable Energy Technologies (ICSET)*, pp. 205–210, Sept. 2012.

[40] M. Khan, I. Husain, R. Islam, and J. Klass, “Design of experiments to address manufacturing tolerances and process variation influencing cogging torque and back EMF in the mass production of the permanent magnet synchronous motors,” in *IEEE Energy Conversion Congress and Exposition (ECCE)*, pp. 3032–3039, Sept. 2012.

[41] K. Kim, S. Lee, S. Cho, J. Jang, T. Lee, J. Hong, and S. Kim, “Multi-response Taguchi robust design of back electromotive force and cogging torque considering the manufacturing tolerance for electric machine,” in *International Conference on optimisation of Electrical and Electronic Equipment (OPTIM)*, pp. 379–387, May 2012.

[42] M. Kimura, D. Kori, A. Komura, H. Mikami, K. Ide, T. Fujigaki, M. Iizuka, and M. Fukaya, “A study of permanent magnet rotor for large scale wind turbine generator system,” in *International Conference on Electrical Machines (ICEM)*, pp. 1161–1171, Sept. 2012.

[43] S. Lee, K. Kim, S. Cho, J. Jang, T. Lee, and J. Hong, “Taguchi robust design of back electromotive force considering the manufacturing tolerances in IPMSM,” in *International Conference on Electromagnetic Field Problems and Applications (ICEF)*, pp. 1–4, June 2012.

[44] W. Tsai, “Robust design of a 5mw permanent magnet synchronous generator using Taguchi method,” in *International Conference on Computing and Convergence Technology (ICCCT)*, pp. 1328–1334, Dec. 2012.

[45] M. Chowdhury, M. Islam, A. Gebregergis, and T. Sebastian, “Robust design optimisation of permanent magnet synchronous machine utilizing genetic and Taguchi’s algorithm,” in *IEEE Energy Conversion Congress and Exposition*, pp. 5006–5012, Sept. 2013.

[46] C. Hwang, C. Chang, and C. Liu, “A fuzzy-based Taguchi method for multiobjective design of PM motors,” *IEEE Trans. Magn.*, vol. 49, no. 5, pp. 2153–2156, May 2013.

[47] M. Liu, Z. Han, Y. Pei, and P. Shi, “optimisation of permanent magnet motor air-gap flux density based on the non-uniform air gap,” in *International Conference on Mechatronic Sciences, Electric Engineering and Computer*, pp. 3422–3426, Dec. 2013.

[48] A. Shirazi1, B. Yousefi, S. Gholamian, and S. Rashidaee, “Application of Taguchi experiment design for decrease of cogging torque in permanent magnet motors,” *International Journal on Computational Sciences and Applications*, vol. 3, no. 2, pp. 31–38, 2013.

[49] M. Beniakar, P. Kakosimos, C. Krasopoulos, A. Sarigiannidis, and A. Kladas, “Comparison of in-wheel permanent magnet motors for electric traction,” in *Electrical Machines (ICEM), 2014 International Conference on*, pp. 2472–2478, Sept. 2014.

[50] M. Chui, J. Chiang, J. Lee, and Z. Gaing, "Multi-objective optimisation design of interior permanent-magnet synchronous motors for improving the effectiveness of field weakening control," in *Int. Conf. on Electrical Machines and Systems (ICEMS)*, pp. 517–521, Oct. 2014.

[51] J. Cui, W. Xiao, L. Wang, H. Feng, J. Zhao, and H. Wang, "optimisation design of low-speed surface-mounted PMSM for pumping unit," *International Journal of Applied Electromagnetics and Mechanics*, vol. 46, no. 1, pp. 217–228, 2014.

[52] C. Hwang, S. Hung, C. Liu, and S. Cheng, "Optimal design of a high speed SPM motor for machine tool applications," *IEEE Trans. Magn.*, vol. 50, no. 1, pp. 1–4, Jan. 2014.

[53] M. Khan, I. Husain, M. Islam, and J. Klass, "Design of experiments to address manufacturing tolerances and process variations influencing cogging torque and back EMF in the mass production of the permanent-magnet synchronous motors," *IEEE Trans. Ind. Appl.*, vol. 50, no. 1, pp. 346–355, Jan. 2014.

[54] K. Kim, "A novel method for minimization of cogging torque and torque ripple for interior permanent magnet synchronous motor," *IEEE Trans. Magn.*, vol. 50, no. 2, pp. 793–796, Feb. 2014.

[55] S. Lee, K. Kim, S. Cho, J. Jang, T. Lee, and J. Hong, "Optimal design of interior permanent magnet synchronous motor considering the manufacturing tolerances using Taguchi robust design," *IET Electric Power Applications*, vol. 8, no. 1, pp. 23–28, Jan. 2014.

[56] S. Lee, K. Kim, S. Cho, J. Jang, T. Lee, and J. Hong, "Optimal design of the magnetizing fixture using Taguchi robust design in the ring-type PMSM," in *IET International Conference on Power Electronics, Machines and Drives (PEMD)*, pp. 1–6, April 2014.

[57] D. Misu, M. Matsushita, K. Takeuchi, K. Oishi, and M. Kawamura, "Consideration of optimal number of poles and frequency for high-efficiency permanent magnet motor," in *International Power Electronics Conference (IPEC)*, pp. 3012–3017, May 2014.

[58] W. Ren, Q. Xu, and Q. Li, "Asymmetrical V-shape rotor configuration of an interior permanent magnet machine for improving torque characteristics," *IEEE Trans. Magn.*, vol. 51, no. 11, pp. 1–4, Nov. 2015.

[59] C. Xia, L. Guo, Z. Zhang, T. Shi, and H. Wang, "Optimal designing of permanent magnet cavity to reduce iron loss of interior permanent magnet machine," *IEEE Trans. Magn.*, vol. 51, no. 12, pp. 1–9, Dec. 2015.

[60] Z. Xu, S. Wang, Z. Zhang, T. Chin, and C. Sung, "optimisation of magnetizing parameters for multipole magnetic scales using the Taguchi method," *IEEE Trans. Magn.*, vol. 51, no. 11, pp. 1–4, Nov. 2015.

[61] M. Vukotic and D. Miljavec, "Design of a permanent-magnet flux-modulated machine with a high torque density and high power factor," *IET Electric Power Applications*, vol. 10, no. 1, pp. 36–44, 2016.

[62] B. Ahn, W. Kim, B. Kim, K. Ko, and J. Lee, "A study on the optimal barrier design for high efficiency of LSPM," in *International Conference on Electrical Machines and Systems (ICEMS)*, pp. 3427–3429, Oct 2008.

[63] W. Kim, K. Kim, S. Kim, D. Kang, S. Go, H. Lee, Y. Chun, and J. Lee, "A study on the optimal rotor design of LSPM considering the starting torque and efficiency," *IEEE Trans. Magn.*, vol. 45, no. 3, pp. 1808–1811, March 2009.

[64] A. Sorgdrager, R. Wang, and A. Grobler, "Retrofit design of a line-start PMSM using the Taguchi method," in *IEEE Int. Electric Machines Drives Conf. (IEMDC)*, pp. 489–495, May 2015.

[65] A. Sorgdrager, R. Wang, and A. Grobler, "Transient performance investigation and Taguchi optimisation of a line-start PMSM," in *IEEE Int. Electric Machines Drives Conf. (IEMDC)*, pp. 590–595, May 2015.

[66] A. Sorgdrager, R. Smith, and R. Wang, "Rotor design of a line-start permanent magnet synchronous machine using the Taguchi method," in *Proc. of the 23rd Southern African Universities Power Engineering Conference*, pp. 227-232, Jan. 2015.

[67] M. Gracia and K. Hameyer, "Multi-objective optimisation of an induction machine using design of experiments," in *IEEE Conference on Electromagnetic Field Computation*, pp. 327-327, May 2008.

[68] K. Park, K. Kim, S. Lee, D. Koo, K. Ko, and J. Lee, "Optimal design of rotor slot of three phase induction motor with die-cast copper rotor cage," in *Int. Conf. on Electrical Machines and Systems (ICEMS)*, Oct. 2008, pp. 61–63.

[69] T. Minari, Y. Fujino, Y. Takeshima, and K. Mizutani, "Development of high efficiency induction motor by using Taguchi method," Sumitomo Heavy Industries Technical Review (in Japanese), Tech. Rep., 2012.

[70] K. Mizutani, T. Minari, and T. Mineo, "Development of IE3 premium efficiency motor," Sumitomo Heavy Industries Technical Review (in Japanese), Tech. Rep., 2014.

[71] C. Lin and C. Hwang, "Multiobjective optimisation design for a six-phase copper rotor induction motor mounted with a scroll compressor," *IEEE Trans. Magn.*, vol. 52, no. 7, pp. 9401604, 2016.

[72] A. Omekanda, “Robust torque and torque-per-inertia optimisation of a switched reluctance motor using the Taguchi methods,” *IEEE Trans. Ind. Appl.*, vol. 42, no. 2, pp. 473–478, March 2006.

[73] L. Ji and X. Yang, “Optimisation design on salient pole rotor of BDFRM using the Taguchi method,” in *Power Electronics Systems and Applications (PESA), 2011 4th International Conference on*, pp. 1–4, June 2011.

[74] S. Gong, X. Yang, and L. Ji, “Parameters calculation and optimized design for brushless doubly-fed reluctance machine,” in *Int. Conf. on Power Electronics Systems and Applications (PESA)*, pp. 1–4, June 2011.

[75] M. Tavakkoli and M. Moallem, “Torque ripple mitigation of double stator switched reluctance motor (DSSRM) using a novel rotor shape optimisation,” in *2012 IEEE Energy Conversion Congress and Exposition (ECCE)*, pp. 848–852, Sept. 2012.

[76] H. Azizi and A. Vahedi, “Rotor geometry parameter optimisation of synchronous reluctance motor using Taguchi method,” *Przeglad Elektrotechniczny*, vol. 89, no. 12, pp. 197–201, 2013.

[77] Z. Gaing, Y. Hsiah, M. Tsai, M. Hsieh, and M. Tsai, “Hybrid design model for optimal designing of a switched reluctance motor,” in *Int. Conf. on Electrical Machines and Systems (ICEMS)*, pp. 505–510, Oct. 2013.

[78] J. Wang, S. Kim, and N. Kim, “A study on the bearingless switched reluctance rotation motor with improved motor performance,” *Journal of Mechanical Science and Technology*, vol. 27, no. 5, pp. 1407–1414, 2013.

[79] Z. Gaing, K. Kuo, J. Hu, M. Hsieh, and M. Tsai, “Design and optimisation of high-speed switched reluctance motor using soft magnetic composite material,” in *International Power Electronics Conference (IPEC)*, pp. 278–282, May 2014.

[80] H. Azizi and A. Vahedi, “Sensitivity analysis and optimum design for the stator of synchronous reluctance machines using the coupled finite element and Taguchi methods,” *Turkish Journal of Electrical Engineering and Computer Sciences*, vol. 23, no. 1, pp. 38–51, 2015.

[81] K. Deguchi, S. Sumita, and Y. Enomoto, “Analytical method applying a mathematical model for axial-gap-switched reluctance motor,” *Electrical Engineering in Japan*, vol. 196, no. 3, pp. 30–38, 2016.

[82] C. Hwang, P. Li, F. Chuang, C. Liu, and K. Huang, “Optimisation for reduction of torque ripple in an axial flux permanent magnet machine,” *IEEE Trans. Magn.*, vol. 45, no. 3, pp. 1760–1763, March 2009.

[83] Ü. Kurt, G. Önbilgin, “Design and optimisation of axial flux permanent magnet synchronous machines using Taguchi approach,” in *International Conference on Electrical and Electronics Engineering*, pp. 202–205, Nov. 2009.

[84] M. Ashabani, Y. Mohamed, and J. Milimonfared, “Optimum design of tubular permanent-magnet motors for thrust characteristics improvement by combined Taguchi neural network approach,” *IEEE Trans. Magn.*, vol. 46, no. 12, pp. 4092–4100, Dec. 2010.

[85] C. Hwang, P. Li, and C. Liu, “Optimal design of a permanent magnet linear synchronous motor with low cogging force,” *IEEE Trans. Magn.*, vol. 48, no. 2, pp. 1039–1042, Feb. 2012.

[86] W. Chen, J. Lin, D. Chen, and F. Nian, “Optimisation design of a Taguchi-based real-code genetic algorithm for thermal reducing of air-core linear brushless permanent magnet motor,” *International Journal of Modeling and optimisation*, vol. 4, no. 5, pp. 402–409, Oct. 2014.

[87] C. Hsiao, S. Yeh, and J. Hwang, “Design of high performance permanent-magnet synchronous wind generators,” *Energies*, vol. 7, no. 11, pp. 7105–7124, 2014.

[88] W. Tseng, C. Kuo, and L. Su, “Optimizing design parameters of a novel PM transverse flux linear motor,” *Transactions of the Canadian Society for Mechanical Engineering*, vol. 39, no. 3, pp. 443–454, 2015.

[89] Y. Ting, J. Huang, F. Chuang, and C. Li, “Dynamic analysis and optimal design of a piezoelectric motor,” *IEEE Trans. Ultrason., Ferroelect., Freq. Control*, vol. 50, no. 6, pp. 601–613, June 2003.

[90] S. Seo, M. Lee, S. Kim, and N. Kim, “Robust optimum design of a bearing-less rotation motor using the Kriging model,” *International Journal of Precision Engineering and Manufacturing*, vol. 12, no. 6, pp. 1043–1050, 2011.

[91] T. Shi, Z. Qiao, C. Xia, H. Li, and Z. Song, “Modeling, analyzing, and parameter design of the magnetic field of a segmented Halbach cylinder,” *IEEE Trans. Magn.*, vol. 48, no. 5, pp. 1890–1898, May 2012.

[92] H. Sung, G. Kim, K. Kim, S. Jung, M. Park, I. Yu, Y. Kim, H. Lee, and A. Kim, “Practical design of a 10 MW superconducting wind power generator considering weight issue,” *IEEE Trans. Appl. Supercond.*, vol. 23, no. 3, pp. 5201805, June 2013.

[93] J. Tsai, T. Liu, and J. Chou, “Hybrid Taguchi-genetic algorithm for global numerical optimisation,” *IEEE Trans. Evolutionary Computation*, vol. 8, no. 4, pp. 365–377, Aug. 2004.

[94] P. Chatsirirungruang, “Application of genetic algorithm and Taguchi method in dynamic robust parameter design for unknown problems,” *The Int. Journal of Advanced Manufacturing Technology*, vol. 47, no. 9-12, pp. 993–1002, 2010.

[95] Z. Gaing, Q. Wang, and J. Chiang, “optimisation of in-wheel pm motor by fuzzy-based Taguchi method,” in *Int. Power Electronics Conf. (IPEC)*, pp. 1312–1316, June 2010.

[96] X. Wu and G. Zhou, “Application of improved Taguchi method to the multi-response optimisation,” in *IEEE Int. Conf. on Industrial Engineering & Engineering Management*, Part 3, pp. 1829–1832, Sept. 2011.

[97] S. Wang and T. Chueh, “Research of permanent magnetic brushless motor on epoxy resin with Taguchi method,” in *Int. Conf. on Consumer Electronics, Communications and Networks (CECNet)*, pp. 3423–3424, April 2012,

[98] J. Kuo and M. Chang, “Multiobjective design of turbo injection mode for axial flux motor in plastic injection molding machine by particle swarm optimisation,” *Mathematical Problems in Engineering*, 11 pages, 2015.

[99] H. Wang, Q. Geng, and Z. Qiao, “Parameter tuning of particle swarm optimisation by using Taguchi method and its application to motor design,” in *IEEE Int. Conf. on Information Science & Technology*, pp. 722–726, April 2014.

[100] X. Gao, “Robust design of miniaturised spindle motors for hard disk drive,” Ph.D. dissertation, Electrical and Computer Engineering at the National University of Singapore, 2002.