STATE SPACE MODEL EXTRACTION OF A NATURAL CIRCULATION U-TUBE: A NETWORK APPROACH

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Abstract: The Reactor Cavity Cooling System (RCCS) in a High Temperature Gas-cooled Reactor (HTGR) provides protection to the concrete structures surrounding the reactor pressure vessel. The RCCS comprises stand pipes circulating water. These pipes may fundamentally be considered as U-tubes. Since the RCCS is critical in case of a Loss of Flow Accident (LOFA), it is very important to characterise the dynamics of such a system both for systems and control engineering purposes. Detailed Computational Fluid Dynamic (CFD) models do exist, but these models are too complex for the purposes mentioned. The majority of thermohydraulic simulation codes utilise a network approach towards representing thermohydraulic systems. This concept is used to extract state space models from graphic representations of such systems. The purpose of this paper is to illustrate the application of a State Space Model Extraction (SSME) method applied to a fundamental thermohydraulic system, namely a U-tube. The solution of the extracted state space model is compared with a validated systems CFD code called Flownex® and shows good correlation.

Key words: state space model extraction, gas-cooled reactor, U-tube, thermohydraulic systems, linear graph

1. INTRODUCTION

Due to the high temperature operation of HTGRs a challenge related to the characterisation and control of an RCCS is presented. Dynamic models of the RCCS capturing the dominant dynamics play a major role in the temperature control system design of the reactor vessel. Detailed thermohydraulic models for analysing the characteristics of an RCCS do exist [1–4]. However, although these models present valuable answers during the structural design process, they do not present insight regarding the dominant dynamics. Kazeminejad [5] states that over the past 30 years there has been great effort on the part of the power utilities to develop simpler models and modelling tools for the thermohydraulic simulation of reactor dynamics. The use of reduced order models for the study of dynamic behaviour of nuclear reactors is valuable since they allow faster calculation speeds and qualitative understanding of the physical phenomena involved.

A vast number of simulation software on the market use model libraries and graphical interfaces to develop dynamic system models. Only a small number of software packages are able to automatically extract mathematical models in state space format from a graphically designed system [6, 7]. This paper demonstrates a novel method developed by Uren [8] that utilises a linear graph, also called a network, representation of a dynamic system and combines it with a lumped parameter modelling approach to eventually extract a state space model of a thermohydraulic system. This method should not be confused with the standard Finite Element Methods (FEMs). FEMs are normally used in the solution of differential equations representing a thermohydraulic system. The SSME method is focused on extracting reduced order differential equations in state space format, which is ideal for control system design.

Section 2 of this paper will describe the physical system under consideration followed by section 3 explaining the network modelling approach towards thermohydraulic systems. In section 4 a network representation of the U-tube is derived. The state space model extraction of the U-tube is explained in section 5. Finally the state space model validation results are discussed in section 6.

2. PHYSICAL SYSTEM DESCRIPTION

The primary function of the RCCS is maintaining the reactor cavity temperatures within required temperature limits. Fig. 1 shows a section view of the reactor and reactor cavity. The RCCS is a network of pipes surrounding a nuclear Reactor Pressure Vessel (RPV) wall. During normal operation water is circulated through the pipe network by means of forced convection (pumps), but provision is made in case of a LOFA. Such a scenario may be due to a catastrophic mechanical failure of the pumps or due to the loss of power. This scenario is also called the passive operation mode of the RCCS, during which water will still be able to circulate through the pipes due to natural convection [9–11]. Two standpipes of the RCCS are shown in Fig. 2. Cold water enters the standpipe through an inlet manifold. The riser pipe is exposed to the heat radiated from the RPV which causes the water in the riser pipe to be less dense. The cold, more dense water in the downcomer pushes the water in the riser up into the outlet manifold. The water circulates due to natural convection. A standpipe may be viewed as a U-tube where

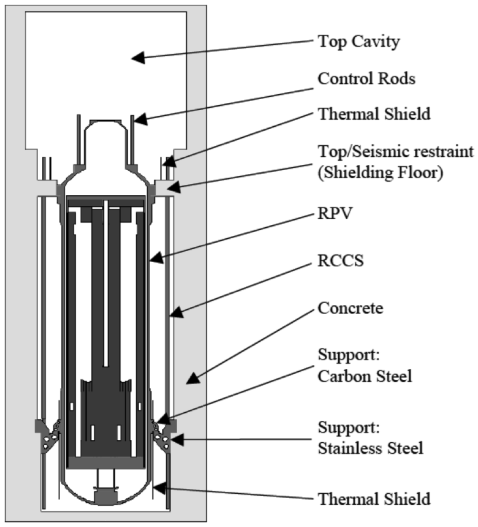

Figure 1: Section view of the reactor unit and reactor cavity [11]

one leg has a fixed temperature and the other leg is exposed to a heat source and both ends of the U-tube have the same pressure reference. It is this part that is of particular interest in terms of state space model extraction and will be the focus of this paper.

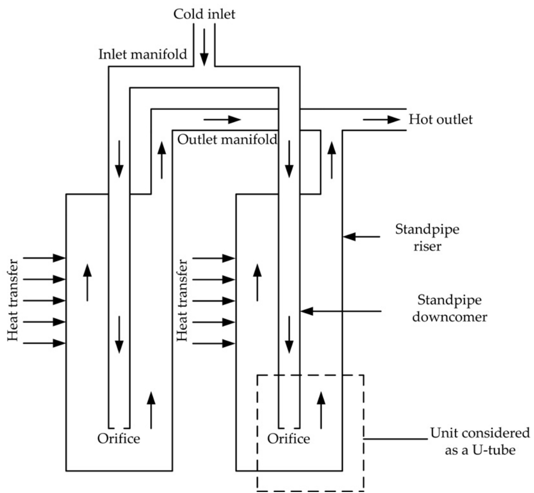

Figure 2: Two standpipes of the RCCS [9]

3. NETWORK MODEL REPRESENTATION OF THERMOHYDRAULIC SYSTEMS

Generalised system elements will be derived that can be used to describe the key system phenomena namely energy generation, storage and dissipation. Following a network approach, a thermohydraulic system can be represented by a network consisting of nodes (squares) connecting components (circles) as shown in Fig. 3. Each component can be broken up into energy networks of generalised elements and variables representing the key dynamics of each physical domain present [12]. The unifying link between different physical domains can be regarded as energy transmission. Energy transmission is associated with one through variable (e.g. mass flow) giving the flux of energy flow, and an across-variable (e.g. pressure) giving the pitch of flow. The lines representing each element are oriented to indicate the reference directions of the element effort and flow variables. The convention that is used in this paper is that the line will be oriented in the reference direction of positive flow and decreasing effort. The five generalised elements are summarised in Table 1 and their network representations are shown in Fig. 4.

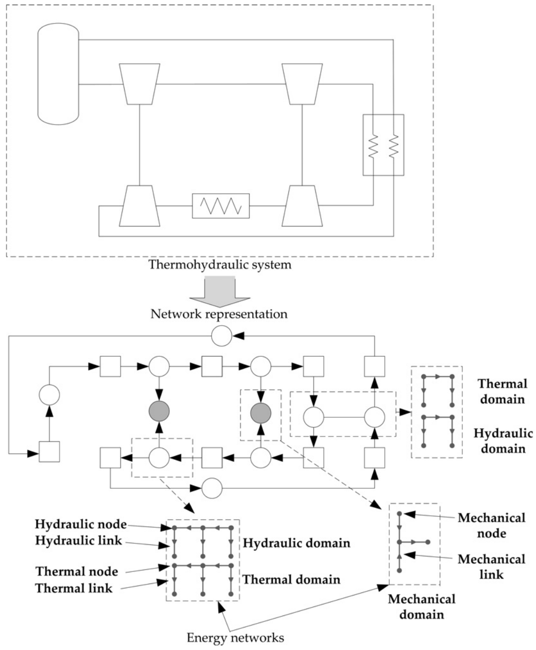

Figure 3: Thermohydraulic system

Table 1: The five generalised system elements [13, 14]

Element category	Elements	Symbol
Energy source	Effort source Flow source	S_e S_f
Energy storage	Effort store (inductive element) Flow store (capacitive element)	L C
Energy dissipation	Energy dissipator (resistive element)	R

The directions of the arrows should not be confused with mass flow direction. The equations that govern fluid flow and heat transfer in networks are the continuity, momentum and energy equations. These equations can be described by first order partial differential equations. However, by using finite volume discretisation these

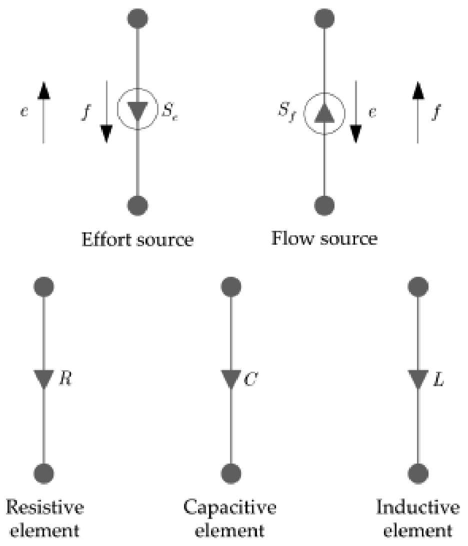

Figure 4: Generalised elements

equations can be transformed into first order ordinary differential equations. The discretisation is used to obtain simpler ordinary differential equations from which the appropriate generalised elements can be derived. The basic building block of a network approach is the control volume (CV), or node, which can represent a certain volume of fluid or solid. Inside this control volume scalar values are assumed to be representative of the average conditions inside the control volume. The fluid and energy flow inside the pipe is assumed to be one-dimensional. This means that only the flow velocity component normal to the cross-sectional area of the pipe is taken into account.

3.1 Hydraulic domain network model

Consider an example of a simple hydraulic network representation of a pipe section shown in Fig. 5. The pressure source elements at the end points represent boundary pressure variables. There are two types of source elements in the hydraulic domain, namely a pressure source S_{eh} and a mass flow source S_{fh}, where the subscripts e and f indicate the generalised variables (effort and flow) and the subscript h indicates the hydraulic domain. k is the source index. Sources may also represent elements such as a pump (flow source) or an elevation head (effort source). The hydraulic capacitance element, C_h, models mass storage. The hydraulic resistance and inductance elements (R_h, L_h) model the friction and momentum phenomena between the control volumes. The conservation of mass equation for a CV is given by

$$\frac{dM_i}{dt} = \sum(Av\rho)_{j-1} - \sum(Av\rho)_j = \dot{m}_{j-1} - \dot{m}_j, \quad (1)$$

where A is the area of the CV, M_i is the mass in the i-th control volume and $\dot{m}_{j-1}$, and $\dot{m}_j$ represent the mass flow entering and leaving the control volume. The variables most often used in the hydraulic domain and which also form the state variables are the pressure, P, and mass flow

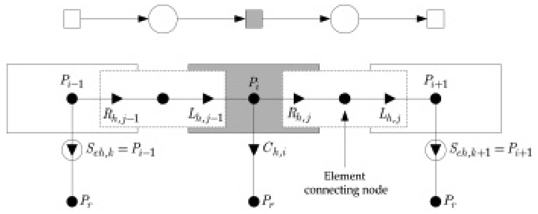

Figure 5: Hydraulic network of a pipe section

rate, $\dot{m}$. It is therefore desirable to introduce the pressure variable into (1) so that

$$\frac{dM_i}{dP_i}\frac{dP_i}{dt} = \dot{m}_{j-1} - \dot{m}_j, \quad (2)$$

and

$$\frac{dP_i}{dt} = \frac{1}{C_h}(\dot{m}_{j-1} - \dot{m}_j), \quad (3)$$

where

$$C_h = dM_i/dP_i = V_i(d\rho_i/dP_i) \quad (4)$$

is called the hydraulic capacitance element. It is an element that describes the compressibility of the fluid.

The momentum equation is given by

$$\ell_j \frac{d\dot{m}_j}{dt} = P_i A_j - P_{i+1} A_j - A_j\, \kappa_j\, |\dot{m}_j| \dot{m}_j + A_j\, \rho_j\, g(z_i - z_{i+1}), \quad (5)$$

where ℓ_j is the length between the nodes i and $i+1$ and κ_j is a friction factor. The gravitational constant is given by g and $(z_i - z_{i+1})$ is the elevation difference between two nodes. Equation (5) may then be written as:

$$\frac{d\dot{m}_j}{dt} = \frac{1}{L_h}(P_i - P_{i+1}) - \frac{R_h}{L_h}\dot{m}_j + \frac{1}{L_h}S_{eh}, \quad (6)$$

where

$$R_h = \kappa_j\, |\dot{m}_j| = \frac{f\,\ell_j/D_j + K}{2\,A_j^2\,\rho_j}|\dot{m}_j|, \quad (7)$$

$$L_h = \frac{\ell_j}{A_j}, \quad (8)$$

and

$$S_{eh} = \rho_j g(z_i - z_{i+1}) = \rho_j g \Delta z_j. \quad (9)$$

R_h is a hydraulic resistance element and L_h is a hydraulic inductance element, where f is the Darcy-Weisbach friction factor, K is the secondary loss factor and D_j is the diameter.

3.2 Thermal domain network model

The energy conservation equation can be written in terms of temperature values and the different heat transfer processes as follows:

$$V_i\rho_i\frac{dh_i}{dt} = \sum(\dot{m}_{j-1}h_{i-1}) - \sum(\dot{m}_j h_i) + \dot{Q}_{\text{cond}} + \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} + \dot{Q}_{\text{gen}}, \quad (10)$$

where $\dot{Q}_{\text{cond}}$, $\dot{Q}_{\text{conv}}$ and $\dot{Q}_{\text{rad}}$ represent heat transfer by means of conduction, convection and radiation. $\dot{Q}_{\text{gen}}$ represents heat generation inside the control volume and $\sum(\dot{m}_{j-1}h_{i-1})$ and $\sum(\dot{m}_j h_i)$ represent the flow streams in and out of the control volume, each representing a loss or gain of enthalpy (h) for the control volume. When an ideal gas is considered, (10) may be written in terms of temperature and a specific heat constant c_p so that

$$V_i\rho_i c_p \frac{dT_i}{dt} = \sum(\dot{m}_{j-1}c_pT_{i-1}) - \sum(\dot{m}_j c_p T_i) + \dot{Q}_{\text{cond}} + \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}} + \dot{Q}_{\text{gen}}. \quad (11)$$

The state variable pair for the thermal domain is enthalpy h or temperature T and energy flow, $\dot{w}$. Energy flow can be defined as

$$\dot{w} = \dot{m}h = \dot{m}c_pT. \quad (12)$$

Energy flow is also equivalent to energy heat transfer rate $\dot{Q}$ and therefore $\dot{w} = \dot{Q}$. Equation (11) may be written in terms of temperature and energy flow as follows

$$\frac{dT_i}{dt} = \frac{1}{C_t}\dot{w}_{j-1} - \frac{1}{R_tC_t}T_i + \frac{1}{C_t}(\dot{w}_{\text{cond}} + \dot{w}_{\text{conv}} + \dot{w}_{\text{rad}} + \dot{w}_{\text{gen}}), \quad (13)$$

where

$$C_t = V_i\rho_i c_p \quad (14)$$

and

$$R_t = 1/\dot{m}_j c_p \quad (15)$$

The conduction, convection and radiation terms in (13) can further be expanded in terms of generalised elements [8]. Consider an example of a thermal network of a pipe section in Fig. 6. The resistance elements, R_t, can

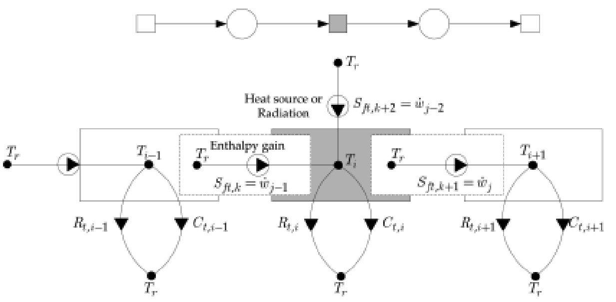

Figure 6: Thermal network of a pipe section

model enthalpy losses, conduction and convection. These elements are connected in parallel with the capacitance elements. Enthalpy gains, energy generation and radiation can be modelled with generalised thermal flow sources, S_{ft}. T_r is a reference temperature.

4. THERMOHYDRAULIC NETWORK MODEL OF THE U-TUBE

Consider a representation of a U-tube in Fig. 7. In the actual system the hot outlet is fed to heat exchangers and storage tanks. Taking this into account it will be assumed that the inlet and outlet of the U-tube are connected to an infinite heat sink. The inlet temperature can therefore be fixed at 15 °C. The RCCS system is open to the atmosphere and therefore the inlet and outlet pressures of the U-tube are fixed at 100 kPa. A constant heat transfer rate of 1 kW will be applied to the riser of the U-tube, simulating the heat radiated from the RPV wall. For illustrative purposes the U-tube will be discretised into four control volumes as shown in Fig. 7. Only a small part of the actual system is considered and hence the length and the diameter of the U-tube are chosen to be 0.6 m and 0.02 m respectively. The hydraulic and thermal behaviours of the U-tube will

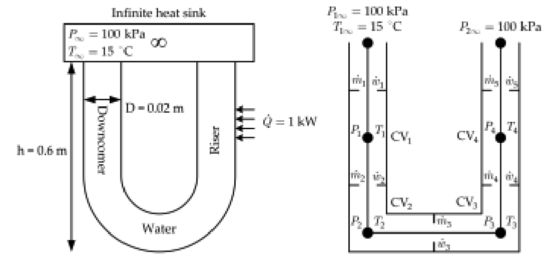

Figure 7: Representation of the U-tube and a conceptual view of the control volumes

be represented by two state space models extracted from hydraulic and thermal networks respectively. These two state space models are coupled by the mass flow rate through the U-tube.

4.1 Hydraulic network model of the U-tube

The hydraulic network of the U-tube is shown in Fig. 8. The atmospheric pressures at the input and output of the U-tube are modelled by hydraulic pressure sources ($S_{eh,1}, S_{eh,2}$). Natural convection is initiated when a body force acts on the fluid in which there are density gradients. This body force is due to gravitation and is modelled as a hydraulic effort source given by

$$S_{eh,k} = \rho_j g \Delta z_j, \quad (16)$$

where

- k is the index of the sources modelling the body forces,
- ρ_j is the mean density between two nodes,
- g is the gravitational constant, and
- Δz_j is the height difference between two nodes.

In Fig. 8 it can be seen that the energy flow direction of the sources are in the opposite direction of the other elements. This indicates that these sources are effort sources.

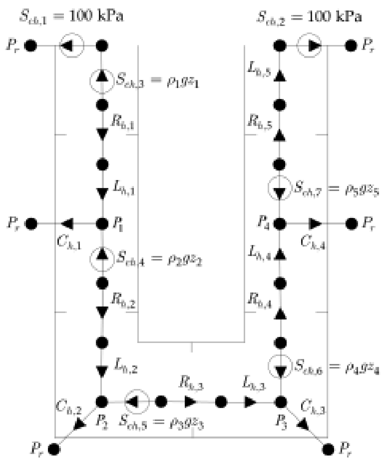

Figure 8: Hydraulic network of the U-tube

4.2 *Thermal network model of the U-tube*

The thermal network of the U-tube is shown in Fig. 9. The energy flows between control volumes are represented by the energy flow sources $S_{ft,1},\ldots,S_{ft,4}$. These energy flow sources as well as the thermal resistances are functions of the mass flow rates that are calculated from the hydraulic model. The energy flow sources $S_{ft,5},\ldots,S_{ft,8}$ represent the external energy transfer.

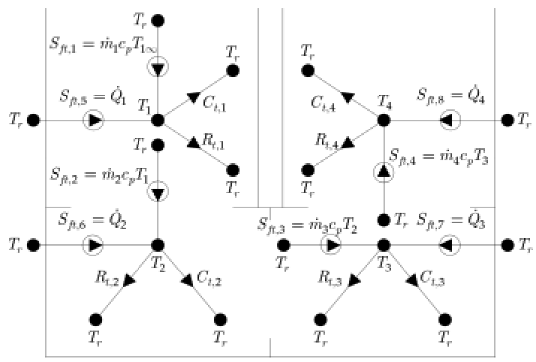

Figure 9: Thermal network of the U-tube

5. STATE SPACE MODEL EXTRACTION

In the previous section a thermohydraulic system, a U-tube, is partitioned into hydraulic and thermal network representations. These networks contain the structural and elemental information about the particular thermohydraulic network. This information can be used to extract a state space model of such a system. A method was developed by Uren [8] based on linear graphs [12] that converts this information algebraically into an incidence and element matrices respectively. These matrices are used along with a graph-theoretic selection of trees, co-trees as well as input and state variables to extract state space representations of the thermohydraulic system [15–17]. The symbolic parameters are then substituted with the relevant numerical values. The state space model can then be solved and validated to make sure it captures the dominant system dynamics. The approach followed from the physical system description up to a final state space model is portrayed in Fig. 10. The hydraulic network

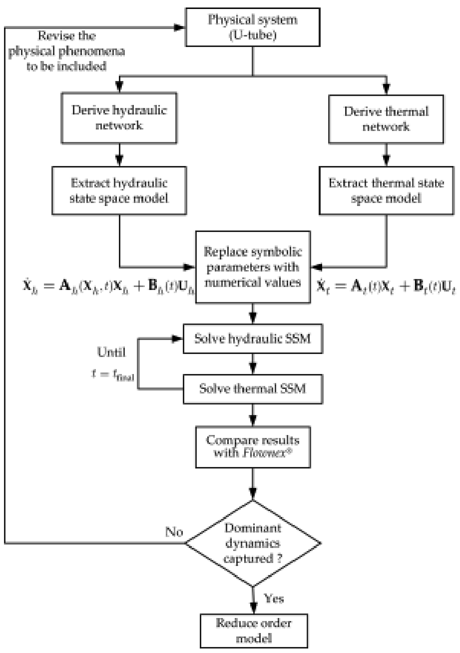

Figure 10: Methodology for reduced order model extraction and validation (U-tube)

is used to construct an incidence matrix representing the structure of the hydraulic domain. Each hydraulic element has a specific elemental equation. All the elemental equations are also represented in matrix form. By applying the state space model extraction algorithm to the incidence and elemental matrices, a symbolic hydraulic state space model is extracted. The hydraulic state space model is given by

$$\dot{\mathbf{X}}_h = \mathbf{A}_h(\mathbf{X}_h,t)\mathbf{X}_h + \mathbf{B}_h(t)\mathbf{U}_h \tag{17}$$

where

$$\mathbf{X}_h = \begin{bmatrix} P_1 & \ldots & P_4 & \dot{m}_1 & \ldots & \dot{m}_5 \end{bmatrix}' \tag{18}$$

$$\mathbf{U}_h = \begin{bmatrix} S_{eh,1} & \ldots & S_{eh,7} \end{bmatrix}' \tag{19}$$

$$\mathbf{A}_h = \begin{bmatrix} 0 & \ldots & 1/C_{h,1} & \ldots \\ 0 & \ldots & 0 & \ldots \\ 0 & \ldots & 0 & \ldots \\ 0 & \ldots & 0 & \ldots \\ -1/L_{h,1} & \ldots & -R_{h,1}/L_{h,1} & \ldots \\ 1/L_{h,2} & \ldots & 0 & \ldots \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix} \tag{20}$$

$$\mathbf{B}_h = \begin{bmatrix} \vdots & \vdots & \vdots & \vdots \\ 1/L_{h,1} & 0 & 1/L_{h,1} & \dots \\ 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & \dots \\ 0 & -1/L_{h,5} & 0 & \dots \end{bmatrix} \quad (21)$$

The state vector, $\mathbf{X}_h \in \Re^{9\times1}$, contains four internal pressures and five mass flow rates. The input vector, $\mathbf{U}_h \in \Re^{7\times1}$, contains the two effort sources representing the boundary pressures and five effort sources representing the body forces due to gravitation. The fragmented representations of $\mathbf{A}_h \in \Re^{9\times9}$ and $\mathbf{B}_h \in \Re^{9\times7}$ given in (20) and (21) contain resistive, capacitive and inductive elements that are time varying (due to densities that vary with time) and nonlinear (the resistive elements are dependent on the mass flow rate as given in (7)).

By applying the state space model extraction algorithm a symbolic thermal state space model is extracted. The thermal state space model is given by

$$\dot{\mathbf{X}}_t = \mathbf{A}_t(t)\mathbf{X}_t + \mathbf{B}_t(t)\mathbf{U}_t \quad (22)$$

where

$$\mathbf{X}_t = \begin{bmatrix} T_1 & T_2 & T_3 & T4 \end{bmatrix}' \quad (23)$$

$$\mathbf{U}_t = \begin{bmatrix} S_{ft,1} & \dots & S_{ft,8} \end{bmatrix}' \quad (24)$$

$$\mathbf{A}_t = \begin{bmatrix} -1/C_{t,1}/R_{t,1} & 0 & \dots \\ 0 & -1/C_{t,2}/R_{t,2} & \dots \\ 0 & 0 & \dots \\ 0 & 0 & \dots \end{bmatrix} \quad (25)$$

$$\mathbf{B}_t = \begin{bmatrix} 1/C_{t,1} & 0 & \dots & 1/C_{t,1} & \dots \\ 0 & 1/C_{t,2} & \dots & 0\dots & \\ 0 & 0 & \dots & 0 & \dots \\ 0 & 0 & \dots & 0 & \dots \end{bmatrix} \quad (26)$$

The state vector, $\mathbf{X}_t \in \Re^{4\times1}$, contains the four internal temperatures. The input vector, $\mathbf{U}_t \in \Re^{8\times1}$, contains four energy flow sources representing the internal heat transfer and another four representing external heat transfer. The matrices $\mathbf{A}_t \in \Re^{4\times4}$ and $\mathbf{B}_t \in \Re^{4\times8}$ are time-varying due to the fact that the thermal resistance, $R_{t,i}$, depends on the mass flow rate and the thermal capacitances, $C_{t,i}$, on the density.

6. RESULTS

Once the state space models for the different domains have been extracted in symbolic form, the symbolic parameters are substituted with numerical values. The state space models are solved in a specific order. The hydraulic state space model is solved first to calculate the mass flow rate used in the thermal state space model to calculate the temperatures. The solution is compared with results obtained from *Flownex*®. *Flownex*® is an advanced and extensively validated commercial thermohydraulic simulation package and is therefore used for validation of the state space models. The accuracy of the state space model is quantified by using the Integral of the Absolute magnitude of the Error (IAE) performance index. This particular index was used since it is widely used in modelling and computer simulation studies [18].

For natural convection to take place, density gradients have to exist. The density profiles of the downcomer and riser pipes are shown in Fig. 11 and were obtained from a *Flownex*® simulation. These density profiles are used when solving the state space model of the U-tube. It can be seen that the density of the water in the downcomer stays constant while the density of the water in the riser decreases as heat transfer takes place. The heat source was activated at time $t = 5$ s. Fig. 12 shows the temperature of the water in the riser pipe with respect to time. The temperature values calculated by solving the state space model compares well with the temperature values obtained from *Flownex*®. Fig. 13 shows good correlation between the flow rate values obtained from the state space model and *Flownex*®. The calculated internal pressures are summarised in Table 2 and also show good correlation. The smaller the value of the IAE performance index, the better the correlation between the state space model and *Flownex*®. The performance index also gives an indication whether the state space model does portray the dominant dynamics of the actual system.

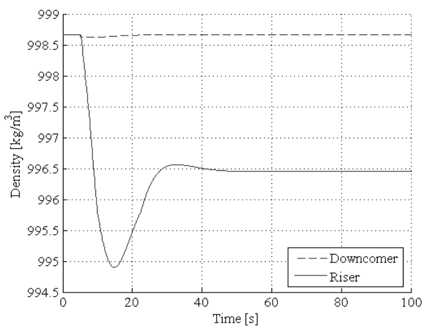

Figure 11: Downcomer and riser densities

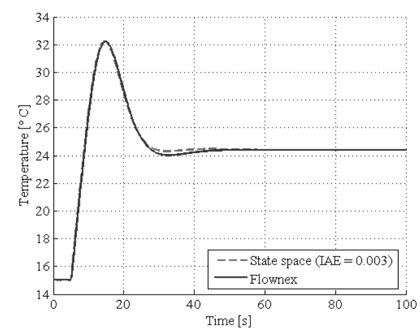

Figure 12: Temperature in the riser pipe

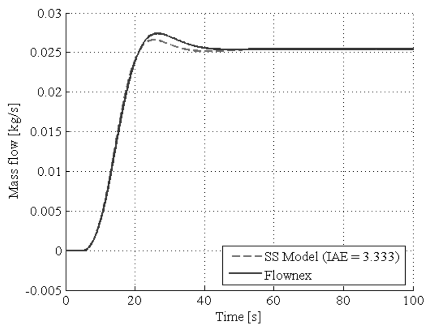

Figure 13: Mass flow rate due to natural convection

Table 2: Calculated internal pressures

Internal pressure	*Flownex®* (kPa)	State space model (kPa)
P_1	102.936	102.937
P_2	105.871	105.874
P_3	105.870	105.873
P_4	102.933	102.934

7. CONCLUSIONS

In this paper generalised thermohydraulic elements were derived from governing equations. These elements model the key dynamics in a thermohydraulic system namely energy storage, dissipation and generation. These elements were used to develop multi-domain network representations of a U-tube. These network representations describe the key dynamics of the system represented. The state space model extraction method converts these networks to symbolic state space models in symbolic form. The symbolic values are then substituted with numeric values and the state equations can be solved using standard differential equation solvers. The solution results showed remarkable correlation with Flownex® simulations. Future work will focus on integrating the state space extraction method with system CFD codes to automatically extract reduced order state space models useful for control systems design.

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