

Multi-objective DSTATCOM Placement Based on Sensitivity Analysis and Genetic Algorithm in Unbalanced MV Distribution Networks

Xiangjing Su, Huanhuan Liu, Yang Fu
School of Electrical Engineering
Shanghai University of Electric Power
Shanghai, China
xiangjing_su@126.com

Peter Wolfs
School of Electrical and Technology
Central Queensland University
Rockhampton, Australia
p.wolfs@cqu.edu.au

Abstract—As a new type of reactive compensation device, distribution static compensator (DSTATCOM) is gradually utilized to mitigate voltage drop and power losses caused by increasing loads. Considering load variations, this study proposes a multi-objective placement model based on sensitivity analysis and genetic algorithm (GA) for the sequential and optimal allocation of DSTATCOM in unbalanced MV distribution networks. Weighted sum method is adopted to reflect the preferences on conflicting placement objectives, including network losses, voltage magnitude and balance profiles, as well as economic costs. To efficiently solve the multi-objective placement problem defined above, an improved direct distribution load flow approach integrating DSTATCOM is presented, and a comprehensive loss sensitivity index is defined to reasonably select the installation site. Finally the proposed placement model and strategy are simulated and validated on an unbalanced Australian MV network with MATLAB.

Index Terms - DSTATCOM; genetic algorithm; multi-objective optimization; sensitivity analysis; unbalanced distribution network;

I. INTRODUCTION

Electric distribution networks play a critical role in the power delivery process, connecting the high-voltage transmission with low-voltage customers. Unlike transmission systems, the R/X ratio of distribution networks is higher, causing serious power losses and voltage drop [1]. Therefore, it is necessary to utilize reactive compensation devices to reduce power losses and improve voltage profile. Conventional methods to keep the distribution voltages within normal range are by adjusting voltage regulators, shunt capacitors and transformers with online tap changer. However these traditional devices have obvious disadvantages, e.g. slow response, step-by-step mechanical operation and limited daily switching times. Also they fail to supply continuously variable reactive power and may oscillate with inductive devices [2].

In the past decades, various distribution electronic devices have been rapidly developed to solve the problems above. Among them, distribution static compensator (DSTATCOM) has shown great advantages. DSTATCOM is a shunt connected voltage source converter which can achieve fast and variable reactive compensation by injecting reactive current into networks. However, the benefits DSTATCOM can bring highly depend on its placement, i.e. locating and sizing.

By far, lots of studies have been carried out for the optimal placement of DSTATCOM, which can be broadly divided into two categories. The first is simultaneous placement which

simultaneously determines the location and size of multiple DSTATCOMs in an optimization. This strategy is usually implemented by taking both site and size as variables simultaneously. The incurred optimization problems are complicated with heavy computation burden and are usually solved by advanced heuristic algorithms e.g. particle swarm optimization [3], genetic algorithm (GA) [4], harmony search algorithm [5], and immune algorithm [6]. GA is one of the most popular for its effectiveness and simple theory. Based on principles of natural genetics and selection, GA is different in many aspects from classical optimization techniques, e.g. in the beginning it uses a population of initial points rather than a single point, actual objective function instead of the derivative value, which makes it a powerful tool for placement problems. Thus GA is adopted for the optimal DSTATCOM placement in this study. The other kind of DSTATCOM placement is sequential placement where the locating and sizing are completed sequentially. Specifically, the locating is firstly carried out based on sensitivity analysis which significantly reduces the search space and therefore speeds up the optimization process of sizing afterwards. By far, a great variety of sensitivity indexes have been proposed for the placement purpose, e.g. voltage stability index [7], fast voltage stability index [8], and power loss sensitivity [9]. However, current practice of sensitivity analyses tend to generate a group of potential installation sites which are geographically close to each other and cause local over compensation. In this study, a more reasonable loss index is defined to sequentially determine the optimal buses for DSTATCOM placement [10].

Although many studies have been conducted for the placement of DSTATCOM, there are still some technical challenges, mainly: (1) existing placements mostly involve a single objective, either loss reduction or economic cost minimization while practical problems usually involve multiple mutually conflicting objectives; (2) all the existing DSTATCOM placement studies are based on the assumption of balanced networks while distribution networks are significantly unbalanced in practice due to uneven load connections, unsymmetrical line arrangements and random renewable integrations; (3) majority of current DSTATCOM placement models simply consider a constant load profile through the project lifetime, which cannot exactly describe the great load variations in operation, leading to unreasonable and even infeasible solutions.

To address the technical challenges above, this study proposes a comprehensive placement strategy based on loss sensitivity analysis and GA for the sequential and optimal

allocation of DSTATCOMs in MV unbalanced distribution networks. Firstly, a multi-objective placement model is formulated considering network loss reduction, voltage magnitude and balance profiles improvement, as well as economic costs. Then, the popular weighted sum method is utilized to accurately reflect the preferences on the conflicting placement objectives. Also, an improved direct distribution load flow approach and GA are used to solve the multi-objective optimal DSTATCOM placement problem above. For a reasonable and efficient DSTATCOM placement in unbalanced distribution networks, a sequential strategy is proposed based on a comprehensive loss sensitivity index to identify the optimal placement sites iteratively. Finally, the proposed placement model and strategy are tested on an Australian MV distribution network by MATLAB.

II. MULTI-OBJECTIVE DSTATCOM PLACEMENT MODEL

A. Optimization Objectives

1) *Voltage Magnitude Profile*: Optimal DSTATCOM placement enhances the three-phase voltage level throughout the network, which can be measured by the deviations from the rated value as follows:

$$f_1 = \sum_{i=1}^n \sum_{p=1}^3 (V_N - V_i^p) \quad (1)$$

where V_N is the rated network voltage, i is the bus number and P represents the phase number, both of which are needed to describe an unbalance network.

2) *Network Energy Losses*: To ensure reasonable network losses after DSTATCOM placement, three typical load levels are considered for the annual energy loss calculation by:

$$f_2 = \sum_{k=1}^3 T_k PL_k^D \quad (2)$$

where k represents three typical load levels i.e. peak, medium and light. Accordingly T_k is the duration time for load level k with 8760 hours in total. Besides, PL_k^D is the average power loss in the duration of load level k , and can be calculated by:

$$PL_k^D = \sum_{l=1}^{n-1} \sum_{p=1}^3 PL_{lk}^D, k = 1, 2, 3 \quad (3)$$

where PL_{lk}^D is the power loss at phase P branch l of level k , which can be calculated as follows, considering both self and mutual branch resistances:

$$\begin{bmatrix} PL_{lk}^a \\ PL_{lk}^b \\ PL_{lk}^c \end{bmatrix} = \begin{bmatrix} R_{aa} & R_{ab} & R_{ac} \\ R_{ba} & R_{bb} & R_{bc} \\ R_{ca} & R_{cb} & R_{cc} \end{bmatrix} \times \begin{bmatrix} I_{lk}^{a^2} \\ I_{lk}^{b^2} \\ I_{lk}^{c^2} \end{bmatrix} \quad (4)$$

3) *Economic Cost*: Cost is a vital factor for DSTATCOM placement. Typically DSTATCOM placement costs involve the initial purchase and installation cost, as well as the operation and maintenance cost, which is defined as [11]:

$$f_3 = \sum_{i=1}^n (C_{PI} \times \frac{[1+B]^t \times B}{[1+B]^t - 1} + C_{OM}) \times Q_{iD} \quad (5)$$

where C_{PI} is the initial purchase and installation cost, taken as 50\$/kVar while C_{OM} is the annual cost of operation and maintenance, often given at 3\$/kVar. In this study, the annual

purchase and installation cost is estimated based on the lifetime of DSTATCOM and its annual return rate B with typical values of 30 years and 0.1 respectively. Additionally, Q_{iD} is the installation size of DSTATCOM.

4) *Voltage Unbalance Profile*: To alleviate the network unbalance after the DSTATCOM placement, an objective of voltage unbalance profile is defined as the deviations from the desired value (e.g. 0) by:

$$f_4 = \sum_{i=1}^n (VUF_i - VUF_{desired})^2 \quad (6)$$

and voltage unbalance factor (VUF) is defined by [12]:

$$VUF = \frac{82\sqrt{V_{abe}^2 + V_{bce}^2 + V_{cae}^2}}{\bar{V}_L} \quad (7)$$

where \bar{V}_L is the average line-line voltage, and V_{abe} is the difference between \bar{V}_L and the line-line voltage V_{ab} .

B. Weighted Sum Method

To reflect placement preferences, the popular weighted sum method is adopted to convert the multi-objective problem above into a single-objective one [13]. As shown in equations (8) and (9), in the weighted sum method, each objective is multiplied by a weighting factor and then summed up, where the weighting factors should satisfy:

$$\sum_{i=1}^4 w_i = 1, w_i \geq 0 \quad (8)$$

Since the weighting factors are affected not only by their own values but also the magnitudes of objectives. To ensure the assigned weighting factors accurately reflect placement preferences, all the objectives are normalized first for a similar magnitude range. Usually the normalization is performed by dividing each objective by its maxima [14].

$$F = \sum_{i=1}^4 w_i J_i = \sum_{i=1}^4 w_i \frac{f_i}{f_i^{\max}} \quad (9)$$

C. Proposed Multi-Objective Optimization Model

Objective Function:

$$\min F \quad (10)$$

subject to

$$\begin{cases} P_{Li}^p + V_i^p \sum_{j \in I} V_j^p (G_{ij}^p \cos \theta_{ij}^p + B_{ij}^p \sin \theta_{ij}^p) = 0 \\ Q_{Li}^p - Q_{Di}^p + V_i^p \sum_{j \in I} V_j^p (G_{ij}^p \sin \theta_{ij}^p - B_{ij}^p \cos \theta_{ij}^p) = 0 \end{cases} \quad (11)$$

$$V_{\min} \leq V_i^p \leq V_{\max} \quad (12)$$

$$Q_D^{\min} \leq Q_{Di}^p \leq Q_D^{\max} \quad (13)$$

where equation (11) is the equality constraints representing network power balance. Among them, P_{Li}^p and Q_{Li}^p are the active and reactive load power at phase p bus i , and Q_{Di}^p is the reactive injection by DSTATCOM. Besides, inequality constraints are the limits on voltages in equation (12) and DSTATCOM reactive capacity in equation (13). Specifically, V_{\min} and V_{\max} are the voltage lower and upper limits and set as $\pm 6\%$ below and above the rated [15] while Q_D^{\min} and Q_D^{\max} are the minimum and maximum sizes of DSTATCOM respectively.

D. Multi-Objective Optimization Solution

The multi-objective DSTATCOM placement of equations (1-13) is essentially an optimal power flow (OPF) problem. In this study, the well accepted genetic algorithm and an improved direct load flow approach (shown in section III) are adopted to effectively and efficiently solve the DSTATCOM placement OPF problem. All the simulations are performed by coding on MATLAB.

III. DIRECT LOAD FLOW INTEGRATING DSTATCOM

A. Modelling of DSTATCOM

DSTATCOM can be considered as a shunt connected voltage source converter, offering fast and variable reactive power by injecting reactive currents. The static model of DSTATCOM is shown in Fig. 1. According to reference [16], relationship between the current and reactive power injection by DSTATCOM is given by:

$$I_D = \left(\frac{Q_D}{V_n} \right)^* \quad (14)$$

Figure 1. Single line diagram of static DSTATCOM model

It is necessary to point out that the reactive current and power injected by DSTATCOM vary with the voltage at the injection point, until reaching the maximum reactive rating of DSTATCOM. In that case, DSTATCOM is no longer used to regulate voltage, but as a fixed capacitor or a constant reactive injection at its maximum rating. As the installation size of DSTATCOM depends on its maximum rating, it is reasonable to treat DSTATCOM as negative reactive loads.

B. Direct Load Flow Integrating DSTATCOM

To ensure the efficient solution of the OPF problem defined in equations (1-13), a superior distribution load flow approach, i.e. direct load flow (DLF) [17], is applied and extended in this study. Taking advantages of the special topological characteristics of distribution networks, DLF approach solves load flow directly based on two constant matrices, i.e. bus-injection to branch-current (BIBC) matrix and branch-current to bus-voltage (BCBV) matrix.

Specifically, with an initial voltage, the current injection by both load and DSTATCOM is calculated by:

$$I_i^{p(k)} = \left(\frac{P_{Li}^p + j(Q_{Li}^p - Q_{Dj}^p)}{V_i^{p(k)}} \right)^* \quad (15)$$

where P_{Li}^p and Q_{Li}^p are the real and reactive load power while Q_{Dj}^p is the reactive injection by DSTATCOM at phase p and bus i . Then, based on the BIBC and BCBV matrices, voltage update formulations are constructed by equation (16)

$$\begin{cases} [\Delta V^k] = [BCBV][BIBC][I^k] \\ [V^{k+1}] = [V^1] - [\Delta V^k] \end{cases} \quad (16)$$

The solution for distribution load flow can be obtained by solving equations (15-16) iteratively. The whole process is repeated till convergence criteria satisfied. Convergence criteria are usually the synthesis of maximum voltage magnitude difference of consecutive iterations and maximum iteration times, the iteration is terminated as either satisfied.

IV. SEQUENTIAL DSTATCOM PLACEMENT STRATEGY

Considering all the buses as the potential positions for DSTATCOM placement will lead to a heavy computation burden, especially for unbalanced network applications. To apply the proposed DSTATCOM placement model to large and unbalanced distribution networks, sensitivity analysis that can help reduce search space has been adopted and a reasonable sequential placement strategy is also proposed.

A. Proposed Comprehensive Loss Sensitivity Index

DSTATCOM placement not only reduces real power losses, but also has considerable impact on reactive losses by injecting capacitive reactive power. Thus a comprehensive sensitivity index (CSI) considering both real and reactive losses is proposed as follows:

$$\begin{cases} \frac{\partial PL}{\partial Q_D} = \frac{|PL_k - PL_{k-1}|}{|Q_{Dk} - Q_{Dk-1}|}, \quad \frac{\partial QL}{\partial Q_D} = \frac{|QL_k - QL_{k-1}|}{|Q_{Dk} - Q_{Dk-1}|} \\ CSI = \sqrt{\left(\frac{\partial PL}{\partial Q_D} \right)^2 + \left(\frac{\partial QL}{\partial Q_D} \right)^2} \end{cases} \quad (17)$$

Where PL_k , QL_k and PL_{k-1} , QL_{k-1} are the network real power and reactive power losses after and before the change of DSTATCOM from Q_{Dk-1} to Q_{Dk} .

B. Sequential Placement Strategy

Current locating methods often do the sensitivity analysis only once and then select a desired number of buses with highest sensitivities at one time. However, the selected buses are usually geographically close to each other, leading to a potential over-compensation [18]. To address this issue, a sequential DSTATCOM placement strategy is proposed:

Step 1: Based on equation (17), with the previously placed DSTATCOM kept, conduct the network loss sensitivity analysis for each phase-phase bridge of all buses.

Step 2: Average the loss sensitivities of the three phase-phase bridges as the value of the corresponding bus in unbalanced networks and then locate the bus with the highest sensitivity.

Step 3: For the bus located in step 2, perform the proposed comprehensive optimization of equations (10-13) for the optimal DSTATCOM sizes in unbalanced phase-phase bridges. As commercial DSTATCOMs have the same sizes for all three bridges, the final size at the bus is set as the maximum of the three bridge values.

V. SIMULATION AND DISCUSSION

A. Test network and Parameters Setting

The proposed DSTATCOM placement model and strategy are verified by simulations on a 60-bus 12.7/22 kV unbalanced Australian distribution network shown in Fig.2. Network load profiles under the peak, medium and light

conditions are given in Table I while all parameters used by objectives, weighed sum method and GA are set in Table II.

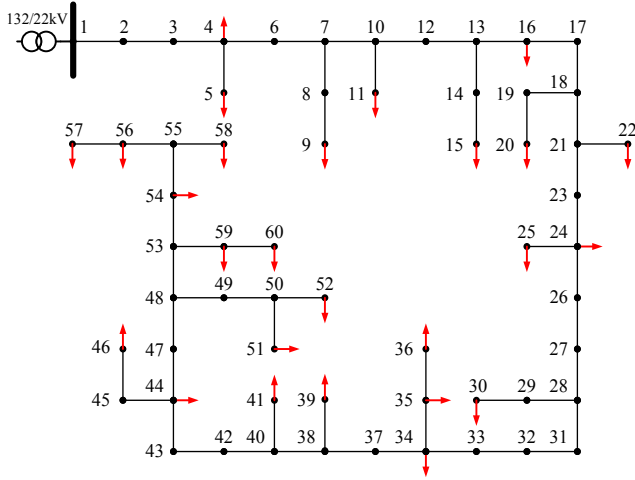


Figure 2. Unbalanced MV distribution network in Western Australia

TABLE I. LOAD PROFILES OF PEAK, MEDIUM AND LIGHT CONDITIONS

Load (MW)	Phase A	Phase B	Phase C	Total
Peak	6.34	5.46	6.05	17.85
Medium	4.74	4.11	4.61	13.46
Light	3.31	2.88	3.25	9.44

TABLE II. PARAMETERS SETTING IN SIMULATION

Objectives	Weighted Sum	GA
V_N (kV)	1.27	w_1 0.3
n	60	w_2 0.2
T_1 (h)	1000	w_3 0.2
T_2 (h)	6760	w_4 0.3
T_3 (h)	1000	-
		Population size 200
		Individual length 10
		Cross probability 0.6
		Mutation probability 0.2
		Iteration number 20

B. Results Analysis

1) Overall performance after placement

Fig. 3 describes the total objective function and individual objectives before and after the multi-objective and sequential DSTATCOM placement, and Table III demonstrates the corresponding optimal placement sites and sizes. Specifically, based on the proposed multi-objective placement model in section II and sequential placement strategy in section IV, the optimal placement happens at buses 55, 38 and 56 with phase installation capacity of 862.2kVar, 616.8kVar and 5.86kVar respectively. Besides, after the first three placements, the total objective function (i.e. ‘objvalue’ in Fig.3) as well as the objectives J1(f_1), J2(f_2) and J4(f_4) decrease obviously representing an improved network operational performance, although at a higher economic cost of J3(f_3). However, after the fourth DSTATCOM placement, the objective function increases with most objectives showing similar trends as a result of overcompensation. It means the fourth placement is not cost effective and the optimal placement times are three.

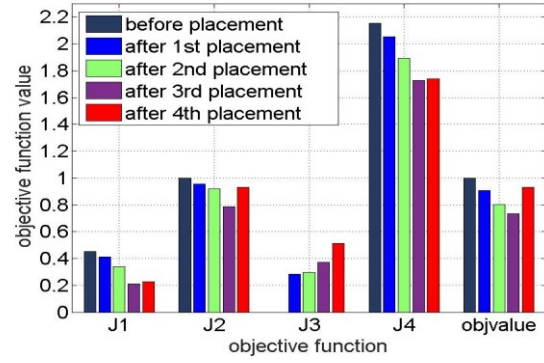


Figure 3. Objective values before and after sequential placement

TABLE III. SELECTED SITE AND INSTALLED SIZE OF EACH PLACEMENT

Items	Sequential DSTATCOM Placement			
	Base case	1st	2nd	3rd
Selected sites	—	55	38	56
Installed sizes per phase (kVar)	—	862.2	616.8	5.86

2) Individual objectives after placement

With the overall objective function minimized (Fig.3) after the sequential DSTATCOM placement, each operational objective has also been improved. As illustrated in Table IV, after the first three optimal placements, the lowest network voltage has increased to 12.30kV, 12.41kV and 12.52kV comparing against the base case without DSTATCOM at 12.10kV, 12.20kV and 12.30kV, for the peak, medium and light load conditions respectively. The voltages improvement across three phases can be further supported by Figures 4-6.

TABLE IV. VOLTAGE PROFILE AFTER EACH PLACEMENT

Minimum voltage(kV)	Sequential DSTATCOM Placement			
	Base case	1st	2nd	3rd
Peak	12.10	12.19	12.30	12.30
Medium	12.20	12.31	12.41	12.41
Light	12.30	12.42	12.52	12.52

As shown in Table V, before DSTATCOM placement, network suffered serious power losses due to heavy loads at 688.9 kW, 395.2kW and 200.3 kW for the peak, medium and light load conditions respectively. After the sequential placement of three DSTATCOMs, the corresponding power loss has been reduced to 643.8 kW, 363.8 kW and 175.3 kW. It should be note that, comparing with voltage magnitude improvements in Fig.3, power loss reduction is less significant. This is because network energy loss f_2 is assigned with a smaller weighting factor at 0.2, as given in Table II. To ensure the overall minimization of objective function, more efforts will be made on the objectives with higher weights e.g. voltage magnitude profile f_1 at 0.3.

As given in Table VI, after the sequential placement of three DSTATCOMs, the network balance profile f_4 has also been significantly improved with the minimum VUF declined from 7.26%, 5.02% and 3.31% before placement to only 5.61%, 4.47% and 2.58% after placement for the peak, medium and light load conditions respectively.

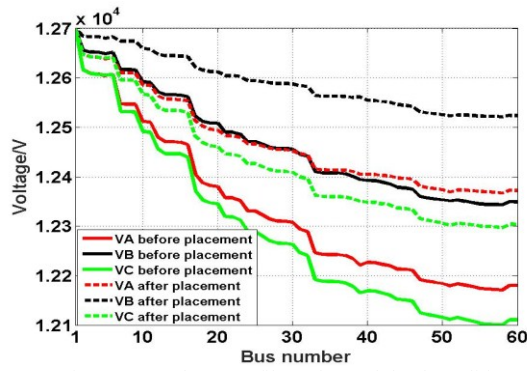


Figure 4. Voltage profile under peak load condition

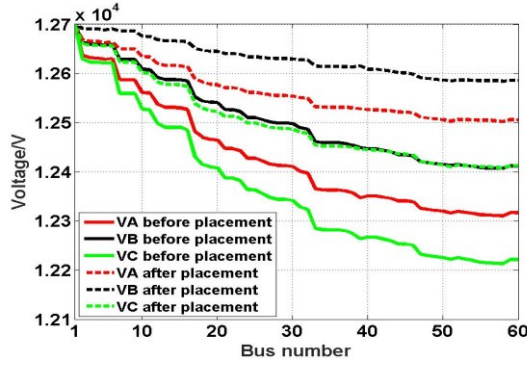


Figure 5. Voltage profile under medium load condition

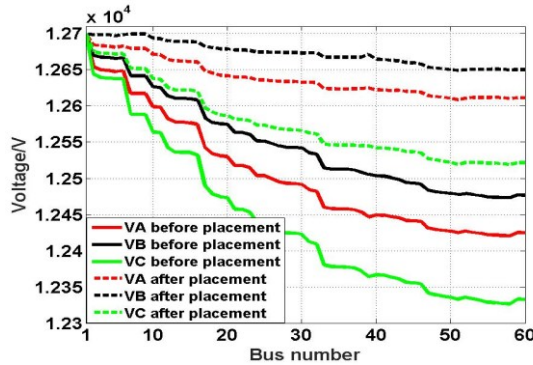


Figure 6. Voltage profile under light load condition

TABLE V. POWER LOSS AFTER EACH PLACEMENT

Power Loss (kW)	Sequential DSTATCOM Placement			
	Base case	1st	2nd	3rd
Peak	688.9	652.8	645.4	643.8
Medium	395.2	367.1	366.1	363.8
Light	200.3	179.3	178.5	175.3

TABLE VI. VOLTAGE UNBALANCE AFTER EACH PLACEMENT

Maximum VUF(%)	Sequential DSTATCOM Placement			
	Base case	1st	2nd	3rd
Peak	7.26	6.32	5.98	5.61
Medium	5.02	4.94	4.62	4.47
Light	3.31	2.89	2.61	2.58

VI. CONCLUSION

In this study, a multi-objective model and a sequential strategy are proposed for the optimal DSTATCOM placement in unbalanced MV distribution networks. To accurately

reflect placement preferences, the popular weighted sum method is utilized to convert the multi-objective problem into a single-objective one. Also to effectively and efficiently solve the optimal load flow problem of DSTATCOM placement, both heuristic genetic algorithm and an improved direct load flow approach are adopted. Detailed simulations on an Australian MV distribution network demonstrate that the proposed multi-objective model and sequential strategy for optimal DSTATCOM placement are feasible and effective. It provides a reasonable and practical alternative for the traditional simultaneous placement with a promising prospect to unbalanced devices placement such as DGs.

REFERENCES

- [1] H. N. Ng, M. A. Salama and A. Y. Chikhani, "Classification of capacitor allocation techniques". *IEEE Trans. Power Delivery*, vol. 15, pp. 387-392, Apr. 2000.
- [2] S. M. Ramsay, P.E. Cronin, R.J. Nelson and F.E. Menendez, "Using distribution static compensators (DSTATCOMs) to extend the capability of voltage-limited distribution feeders," 40th Annual Conference Rural Electric Power Conference.
- [3] Z. Devi and M. Geethanjali, "Optimal location and sizing determination of distributed generation and DSTATCOM using particle swarm optimization algorithm. *Electr Power Energy Syst*, vol. 62, pp. 70-562, 2014.
- [4] S., Deependra, and K. S. Verma. "GA based Optimal Sizing & Placement of Distributed Generation for Loss Minimization" *proceedings of world academy of science*, vol. 26, Dec. 2007
- [5] T. Yuvaraj, K.R. Devabalaji and K. Ravi. "Optimal placement and sizing of DSTATCOM using Harmony Search algorithm", *International Conference on Alternative Energy in Developing Countries and Emerging Economies*, 2015
- [6] S. A. Taher and S. A. Afsari. "Optimal location and sizing of DSTATCOM in distribution systems by immune algorithm", *Electrical Power and Energy Systems*. vol. 60, pp. 34-44, 2014.
- [7] A. Chakrabarti , D. P. Kothari , A. K. Mukhopadhyay and De Abhinandan, "An introduction to reactive power control and voltage stability in power transmission". *PHI Learning Private Limited*, 2000.
- [8] A. Parizad, A. Khazali and M. Kalantar. "Optimal placement of distributed generation with sensitivity factors considering voltage stability and losses indices". *In: Proceedings of IEEE 18th Iranian conference on electrical engineering* 2010.
- [9] Murthy VVSN and Kumar Ashwani. "Comparison of optimal DG allocation methods in radial distribution systems based on sensitivity approaches". *Int J Electr Power Energy Syst*. vol. 53, pp. 67-450, 2013.
- [10] A. R. Gupta and A. Kumar "Energy savings using D-STATCOM placement in radial distribution system". *Procedia Comput Sci* vol. 70, pp. 64-558, 2014.
- [11] A. R. Gupta and A. Kumar, "Optimal placement of D-STATCOM using sensitivity approaches in mesh distribution system with time variant load models under load growth". *International Conference on Recent Advances in Engineering and Computational Sciences 2015*.
- [12] P. Pillay and M. Manyage, "Definitions of voltage unbalance", *IEEE Power Eng. Rev.*, vol. 21, pp. 49-51, May. 2001.
- [13] H. A. Abbass and R. Sarker, "A Pareto frontier differential evolution approach for multi-objective optimization problems," *In Proceedings of the Congress on Evolutionary Computation*, vol. 2, pp. 971-978, 2001
- [14] R. T. Marler and J. S. Arora, "The weighted sum method for multi-Objective optimization: New sights," *Struct. Multidisc. Optim.*, vol. 41, pp. 853-862, Jun. 2010.
- [15] *Standard Voltages*, Australia Std. AS 60038-2012, Dec. 2012
- [16] S. A. Taher, S. A. Afsari . "Optimal location and sizing of DSTATCOM in distribution systems by immune algorithm," *Electr Power Energy Syst*, vol. 60, pp. 34-44, Jun. 2010.
- [17] Teng Jen-Hao. "A direct approach for distribution system load flow solutions," *IEEE Trans Power Delivery*, vol. 18, pp. 882-887, Jun. 2010.
- [18] V. Farahani, S. H. H. Sadeghi, H. A. Abyaneh, S. M. M. Agah and K. Mazlumi, "Energy loss reduction by conductor replacement and capacitor placement in distribution systems," *IEEE Trans. Power Syst.*, vol. 28, pp. 2077-2085, Aug. 2013.