Optimal PMU Placement for Distribution Networks

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***Abstract*—This paper presents the optimal phasor measurement (PMU) placement for distribution networks. The optimal PMU placement (OPP) formulation from related literatures is presented in integer linear programming (ILP) framework where zero injection buses (ZIBs), PMU channel limitation, PMU outages and line outages were considered. Existing methods for OPP search space reduction (OPPSSR) such as the predetermined buses, ZIBs and leaf buses are also presented. To further reduce the search space for OPP, a method network reduction and deterministic PMU placement were proposed in dealing with non-branching bus series (NBS) that may be present in radial systems such as large distribution networks. Results have shown that the proposed OPPSSR methods was able to give the same optimal solutions with reduced amount of optimization.**

***Index Terms*—Phasor Measurement Unit (PMU), Optimal PMU Placement (OPP), Integer Linear Programming (ILP), Non-branching Bus Series (NBS), OPP Search Space Reduction (OPPSSR)**

I. INTRODUCTION

POWER system operation, protection and control as well as its system analytics require accurate monitoring of its system states. Monitoring of system states is traditionally done through supervisory control and data acquisition (SCADA), where measurements might be unsynchronized and have low sampling rate. These issues were addressed by the development of phasor measurement unit (PMU) by A. G. Phadke and J. S. Thorp at Virginia Tech introduced in [1] and [2]. It provides real-time and synchronized measurements by the use of global positioning system (GPS) [3]. PMUs are later improved to micro-synchrophasors (μPMU) by University of California in conjunction with Power Standards Lab (PSL) and Lawrence Berkeley National Lab (LBNL) [4].

A PMU can measure directly both the magnitude and phase angle of both the phase voltages of the bus where it is installed and the phase currents of lines incident to that bus. Note that the number of incident lines that can be measured by a PMU depends on the number of current channels available on the device. With the assumption that line impedances are known, values of other bus voltages and line currents that cannot be measured directly through an installed PMU may be calculated through Ohm's law and Kirchhoff's current law (KCL). These measurements are used for the graph-theoretic approach of topological observability of the system [5].

Since some measurements may be calculated, it is not necessary to install PMUs in all the buses of the power system. The cost for the device, installation, telemetry and maintenance may then be reduced. This is addressed through optimal PMU placement (OPP) [6] which ensures complete system observability with minimal number of PMUs installed. A power system is completely observable if the measurements made on it allow determination of bus voltage magnitude and angle at every bus of the network [5].

For decades, different methods have been used in solving the OPP problem. It is generally presented as an integer linear programming (ILP) optimization where the size of the variables and constraints are that of the number of buses in the network [6]. The complexity and size of the problem increases if zero injection buses (ZIBs) [7] and PMU channel limitation [8] are to be considered. This is computationally expensive for large systems, i.e. sytems with large number of buses like distribution networks.

To alleviate the tedious optimization, some buses may be predetermined to have or not have a PMU installation for OPP search space reduction (OPPSSR). This predetermined locations may be of preferred buses, ZIBs or leaf buses and leaf parents. The effect of not installing PMUs at zero injection buses were presented in [9]. Predetermining leaf buses and leaf parents have proven to be essential before performing an OPP [10]. Its effectiveness for distribution networks were also presented in [11] and [12]. Moreover, literature [13] presented a two stage OPP of first placing PMUs on the longest bus series of a distribution network then followed by an optimization for the remaining buses.

Given the radial topological structure of distribution networks, there may exist non-branching bus series (NBS) on large spanning networks. This paper then proposes methods for network reduction and deterministic PMU placement for NBS with the general ILP framework for OPP and may also be used with the OPPSSR of the above stated literatures.

The remainder of this paper is organized as follows. Section II presents the OPP formulation as well as its extension for other practical considerations. Section III presents effective methods in OPPSSR as well as the formulation for the proposed methods. Section IV are the results and discussions for the proposed and existing methods for OPPSSR. Finally, Section V for the conclusions and findings.

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II. PRELIMINARIES

This section presents the general formulation of OPP through the ILP framework of [6]. Modifications of the general formulation is also presented for several important considerations such as the inclusion of ZIBs [14] [15], n-PMU outage [16], line outages [10] and PMU channel limit [8].

A. Optimal PMU Placement Problem Formulation

For a n-bus system, the objective function for OPP is the minimization of the cost of PMUs to be installed.

$$\min \sum_{i}^{n} w_i x_i \tag{1}$$

where w_i is the cost and x_i is the decision binary variable (1 to install, 0 otherwise) of installing a PMU at bus i. For simplicity, the cost w_i is assumed to be uniform in this paper.

For complete system observability, measurements made on the system allows the determination of bus voltage magnitude and angle at every bus of the network [5]. This means that every bus of the network should be observable by at least once. Hence, a n-bus system with minimum bus observability $o_{min} = 1$ will have n number of constraints that are formulated as

$$o_i = \sum_{j}^{n} a_{ij} x_i \qquad i = 1, \ldots, n \tag{2}$$

$$a_{ij} = \begin{cases} 1, & \text{if Bus}_i = \text{Bus}_j \\ 1, & \text{if Bus}_i \text{ is adjacent with Bus}_j \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

where $o_i \geq o_{min}$ is the number of PMUs observing bus i and a_{ij} is the binary connectivity coefficient between bus i and j. Assuming that the line impedances are known, a_{ij} is defined by the PMU's capability of measuring the bus voltage from bus i and bus j by Ohm's law.

The set of constraints may also be written in terms of vectors and matrices as

$$\mathbf{Ax} \geq \mathbf{b} \tag{4}$$

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}_{n \times n} \tag{5}$$

$$\mathbf{x} = \begin{bmatrix} x_1 \cdots x_n \end{bmatrix}_{1 \times n}^{\top} \tag{6}$$

$$\mathbf{b} = \begin{bmatrix} o_{min} \cdots o_{min} \end{bmatrix}_{1 \times n}^{\top} \tag{7}$$

where $\mathbf{A}$ is the binary connectivity matrix, vector $\mathbf{x}$ is the binary decision variable in installing PMUs and vector $\mathbf{b}$ is the minimum bus observability required for each bus.

B. OPP Considering Zero Injection Buses

A bus is said to be a ZIB if all of the following conditions are satisfied:

- There is no connected load or generator.
- The current injection of the bus is zero.
- The active and reactive power measurements are zero.

Since the sum of currents is zero and line impedances are known, the voltage of one bus adjacent to a ZIB may be known by KCL if currents of all lines of other adjacent buses are known. This reduces the number of PMUs needed for complete system observability.

A way of incorporating ZIBs to the OPP calculation is by the modification of the general binary connectivity matrix $\mathbf{A}$ through (3) [14].

$$a_{ij} = \begin{cases} 1, & \text{if Bus}_i = \text{Bus}_j \\ 1, & \text{if Bus}_i \text{ is adjacent with Bus}_j \\ 1, & \text{if Bus}_i \text{ \& Bus}_j \text{ are connected by ZIB} \\ 0, & \text{otherwise} \end{cases} \tag{8}$$

C. OPP Considering PMU Outages

PMU outages leads to unavailability of the affected measurements. To increase the reliability of measurements, redundancy is introduced by increasing the observability of each bus. This is done by setting $o_{min} = n+1$ for n outage contingency. It will then increase the required number of PMUs installed.

D. OPP Considering Line Outages

Line outages, like PMU outages, may lead to unavailability of the affected measurements. To account for such contingency, the binary connectivity matrix $\mathbf{A}$ is modified through (3). The modified cases considering possible line outages of lines given by set L_{out} are as follows

$$L_{out} = \{L_{ij} \mid \text{considering line outage at } L_{ij}\} \tag{9}$$

$$a_{ij} = \begin{cases} 1, & \text{if Bus}_i = \text{Bus}_j \\ 1, & \text{if Bus}_i \text{ is adjacent with Bus}_j \\ & \quad \text{and Line}_{ij} \notin L_{out} \\ 1, & \text{if Bus}_i \text{ \& Bus}_j \text{ are connected by ZIB} \\ & \quad \text{and Path}_{ij} \text{ contains lines } \notin L_{out} \\ 0, & \text{otherwise} \end{cases} \tag{10}$$

The same applies for lines that contain switches such as those used for curtailment and for on-line network reconfiguration.

E. OPP Considering Channel Limitation

Most OPP studies often assumes that PMUs have sufficient number of current channels for all incident lines for every bus of the network. This may not always be the case in practial application since actual PMUs have finite number of channels for current measurements.

For a n-bus system that uses PMUs with N number of current channels, let r_i be the number of possible line combinations for bus i with L_i number of incident lines and r_0 be the sum of all r_i.

$$r_i = \begin{cases} {}^{L_i}C_N, & \text{if } L_i > N \\ 1, & \text{otherwise} \end{cases} \tag{11}$$

The number of possible combinations of N out of L_i lines is

$${}^{L_i}C_N = \frac{L_i!}{N!(L_i - N)!} \tag{12}$$

Let R_i be the set of all r_i possible line combinations for bus i from its L_i incident lines. To consider PMU channel limitation, (3) can be redefined for each kth element of R_i as

$$a_{ijk} = \begin{cases} 1, & \text{if Bus}_i = \text{Bus}_j \\ 1, & \text{if Line}_{ij} \in R_{ik} \\ 1, & \text{if Bus}_i \text{ \& Bus}_j \text{ are connected by ZIB} \\ & \quad \text{and Path}_{ij} \text{ contains lines } \notin L_{out} \\ & \quad \text{and there exists a ZIB Bus}_l \\ & \quad\quad \text{where Line}_{il} \in R_{ik} \text{ and } \in \text{Path}_{ij} \\ 0, & \text{otherwise} \end{cases} \tag{13}$$
$$i, j, l \in \{1, \ldots, n\}; \quad k \in \{1, \ldots, r_i\}$$

Accordingly, (5) and (6) are redefined as

$$\mathbf{A}^\top = \begin{array}{c} \begin{array}{cccc} \text{Bus}_1 & \text{Bus}_2 & \cdots & \text{Bus}_n \end{array} \\ \left[\begin{array}{cccc} a_{111} & a_{121} & \cdots & a_{1n1} \\ \vdots & \vdots & \vdots & \vdots \\ a_{11r_1} & a_{12r_1} & \cdots & a_{1nr_1} \\ a_{21(r_1+1)} & a_{22(r_1+1)} & \cdots & a_{2n(r_1+1)} \\ \vdots & \vdots & \vdots & \vdots \\ a_{21(r_1+r_2)} & a_{22(r_1+r_2)} & \cdots & a_{2n(r_1+r_2)} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1r_0} & a_{n2r_0} & \cdots & a_{nnr_0} \end{array}\right] \end{array} \begin{array}{c} \\ \text{Bus}_1 \\ \vdots \\ \text{Bus}_1 \\ \text{Bus}_2 \\ \vdots \\ \text{Bus}_2 \\ \vdots \\ \text{Bus}_n \end{array} \tag{14}$$

$$\mathbf{x} = \Big[x_1 \cdots x_{r_0} \Big]^\top_{1 \times r_0} \tag{15}$$

where $\mathbf{x}$ is now the bus-channel binary decision vector indicating what buses should the PMUs be installed and its corresponding lines that are to be monitored.

There are cases where PMUs may be installed on one bus to observe more lines. Measuring the bus voltage more than once from the same source using multiple PMUs is not effective since these may be affected at the same time if a problem occurs within that bus. Thus, it is better to have redundant measurements from different bus sources. A way of limiting the number of PMUs at the same bus is by adding PMU count limit PMU_{max} constraints to (14) and (7) for each bus.

$$\mathbf{A}^\top_{max} = \begin{array}{c} \begin{array}{cccc} \text{Bus}_1 & \text{Bus}_2 & \cdots & \text{Bus}_n \end{array} \\ \left[\begin{array}{cccc} -1_{111} & 0_{121} & \cdots & 0_{1n1} \\ \vdots & \vdots & \vdots & \vdots \\ -1_{11r_1} & 0_{12r_1} & \cdots & 0_{1nr_1} \\ 0_{21(r_1+1)} & -1_{22(r_1+1)} & \cdots & 0_{2n(r_1+1)} \\ \vdots & \vdots & \vdots & \vdots \\ 0_{21(r_1+r_2)} & -1_{22(r_1+r_2)} & \cdots & 0_{2n(r_1+r_2)} \\ \vdots & \vdots & \vdots & \vdots \\ 0_{n1r_0} & 0_{n2r_0} & \cdots & -1_{nnr_0} \end{array}\right] \end{array} \begin{array}{c} \\ \text{Bus}_1 \\ \vdots \\ \text{Bus}_1 \\ \text{Bus}_2 \\ \vdots \\ \text{Bus}_2 \\ \vdots \\ \text{Bus}_n \end{array} \tag{16}$$

$$\mathbf{b}_{max} = \Big[(-PMU_{max}) \cdots (-PMU_{max}) \Big]^\top_{1 \times n} \tag{17}$$

$$\mathbf{A}_{new} = \Big[\mathbf{A}_{old} \ \ \mathbf{A}_{max} \Big]^\top_{r_0 \times 2n} \tag{18}$$

$$\mathbf{b}_{new} = \Big[\mathbf{b}_{old} \ \ \mathbf{b}_{max} \Big]^\top_{1 \times 2n} \tag{19}$$

III. OPP SEARCH SPACE REDUCTION

Solving the OPP of large systems such as distribution networks is computationally expensive. This could be alleviated by reducing the search space in finding solutions. This section presents methods from related literatures and proposed methods for reducing the search space for the OPP problem.

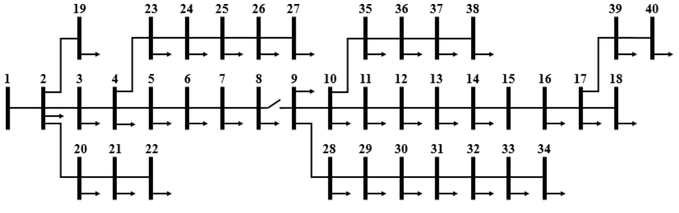

Fig. 1. 40-bus Test System

The test system shown in Figure 1 is used for illustration in the following discussions. A ZIB is located at bus 15 and a switch is located between buses 8 and 9. Let us also define a term called non-branching bus series (NBS) for consecutive and non-branching connection of buses, one after the other, forming a single straight line series of buses. We denote these set of buses as $NBS_{a,b}$ where $\{a, b\}$ are the end buses of the set. Hence, for the system in Figure 1, it contains $NBS_{1,1}$, $NBS_{3,3}$, $NBS_{18,18}$, $NBS_{19,19}$, $NBS_{5,8}$, $NBS_{11,16}$, $NBS_{20,22}$, $NBS_{23,27}$, $NBS_{28,34}$, $NBS_{35,38}$, $NBS_{39,40}$.

A. No PMUs in Zero Injection Buses

PMUs installed at ZIBs measure current phasors of incident lines; thus, applying KCL at a ZIB provides no additional information [9]. Exclusion of ZIBs from the OPP can be achieved by making the decision variable x_i of (6) equal 0 for all ZIB i. This reduces the number of optimization variables by number of ZIBs. For the given system in Figure 1, the number of optimization variables is reduced by one (Bus_{15}).

For ZIBs connecting two buses, currents going in and out from ZIBs remain the same. This implies that for any number of PMU channels, measurements can be propagated through ZIBs by Ohm's law or by KCL.

B. Predetermined PMU Locations

1) Considering Preferred Buses: In actual power system, there may be certain buses that are of great importance where PMUs are preferred to be installed. Such action will reduce the number of optimization variables. This is implemented by setting all decision variable x_i of (6), where PMUs are preferred to be installed, to 1 [10].

2) Considering Channel Limitation: For cases where there is a channel limit for the number of lines a PMU can observe, it will be easier if the lines are to be predetermined as well for preferred buses which will reduce eq. (11) to one. If the line-channel assignment is to be optimized as well, similar process of eqs. (16) to (19) is done by adding minimum PMU count PMU_{min} constraints for each preferred bus i where elements of $\mathbf{A}_{min}$ are set to 0 except for those corresponding to bus i which are set to one.

$$\mathbf{A}_{min} = \begin{array}{cccccc} & \cdots & \text{Bus}_i & \cdots & \text{Bus}_i & \cdots \\ & [0\cdots & 1_{i11} & \cdots & 1_{iir_i} & \cdots 0] & \text{Bus}_i \end{array} \tag{20}$$

$$\mathbf{b}_{min} = \Big[PMU_{min}\Big]_{1\times 1} \tag{21}$$

$$\mathbf{A}_{new} = \Big[\mathbf{A}_{old}\ \ \mathbf{A}_{min}\Big]^{\top}_{r_0\times(|\mathbf{A}_{old}|+1)} \tag{22}$$

$$\mathbf{b}_{new} = \Big[\mathbf{b}_{old}\ \ \mathbf{b}_{min}\Big]^{\top}_{1\times(|\mathbf{b}_{old}|+1)} \tag{23}$$

3) Considering Leaf Buses: In addition to the predetermined PMU locations at preferred buses, there may also be an existense of leaf buses in a given network. These are the buses that has only one incident line. For the given system in Figure 1, buses $\{1, 8, 18, 19, 22, 27, 34, 38, 40\}$ are the radial buses. Note that bus 8 is included since the the network switch between buses 8 and 9 may not be closed all the time.

A Leaf bus may only be observed if a PMU is placed on it, placed on its adjacent bus or on a bus where the path to that bus is composed of ZIBs. With this, method of [10] is extended in predetermining the decision on some buses by defining f_{pre} as a function that predetrmines the decision variable x_i of bus i with minimum bus observability of o_{min} such that

$$f_{pre}(x_i, o_{min}) = \begin{cases} 1, & \text{if Bus}_i \text{ is adjacent to a leaf} \\ 1, & \text{if Bus}_i \text{ is a leaf \& } o_{min} = 2 \\ 0, & \text{if Bus}_i \text{ is a leaf \& } o_{min} = 1 \\ ?, & \text{otherwise} \end{cases} \tag{24}$$

Only two possibilities for bus observability $\{1, 2\}$ since a leaf bus, for complete system observability, may only be observed by installing a PMU on it and/or by installing a PMU on its adjacent bus. The buses that can be reached through a series of ZIBs from a leaf bus might be more than one; thus, ZIBs are not included in the definition of (24).

For the given system in Figure 1 with $o_{min} = 1$, leaf buses $\{1, 8, 18, 19, 22, 27, 34, 38, 40\}$ will require no PMUs while leaf parents $\{2, 7, 17, 21, 26, 33, 37, 39\}$ are predetermined to have PMUs installed. On the other hand, for minimum bus observability of $o_{min} = 2$, both leaf buses and leaf parents will require installation of PMUs.

C. Network Reduction

Distribution networks, being topologically radial, mostly consists of extending and branching series of connected buses. Some of these series of buses may be grouped by NBS. Since NBS has a definite structure of non-branching series of variable number of buses with two end buses for connection, possible observability of these buses are finite and can be generalized. With the concept of leaf buses, NBS subsets that can be optimized deterministically are reduced recursively.

The size of NBS subsets for recursive reduction is determined by the minimum NBS where distance between PMUs is maximized but is still completely observable for a given bus observability o_{min} and PMU channels available ch_{max}.

$$o_{min} = 1,\ ch_{max} \geq 2 : \Big[1\ \ 0\ \ 0\ \ 1\Big]_{\alpha=4} \Rightarrow \Big[1\Big]_{\beta=1} \tag{25}$$

$$o_{min} = 2,\ ch_{max} \geq 2 : \Big[1\ \ 1\ \ 0\ \ 1\ \ 1\Big]_{\alpha=5} \Rightarrow \Big[11\Big]_{\beta=2} \tag{26}$$

$$o_{min} = 1,\ ch_{max} = 1 : \Big[1\ \ 0\ \ 1\Big]_{\alpha=3} \Rightarrow \Big[1\Big]_{\beta=1} \tag{27}$$

$$o_{min} = 2,\ ch_{max} = 1 : \Big[1\ \ 1\Big]_{\alpha=2} \Rightarrow \Big[11\Big]_{\beta=2} \tag{28}$$

$$o_{min} = 1,\ ch_{max} = 0 : \Big[1\Big]_{\alpha=1} \Rightarrow \Big[1\Big]_{\beta=1} \tag{29}$$

$$o_{min} = 3,\ ch_{max} \geq 2 : \Big[1\ \ 1\ \ 1\Big]_{\alpha=3} \Rightarrow \Big[111\Big]_{\beta=3} \tag{30}$$

Vector elements that are set to one are buses where the PMUs are installed while zero otherwise. PMU channels greater than two is not considered since any bus in a NBS may only have a maximum of two incident lines. Moreover, minimum bus observability may not be greater than three since a bus can only be observed by an installed PMU at that bus or calculated from its adjacent buses of which may only be one or two.

Only the cases (25), (26) and (27) are to be considered since the other cases cannot be reduced further and would require PMUs installed at every bus. For reference, let (25) be as *Case 1*, (26) as *Case 2* and (27) as *Case 3*. The recursive reduction of NBS subsets of size α to size β from a n size NBS can then be formulated as

$$f_{rcount}(\alpha, \beta, n) = \begin{cases} \frac{(n-\beta)-f_{rem}(n-\beta,\alpha-\beta)}{\alpha-\beta}, & n \geq \alpha \\ 0, & \text{otherwise} \end{cases} \tag{31}$$

$$f_{rrem}(\alpha, \beta, n) = \begin{cases} f_{rem}(n-\beta, \alpha-\beta) + \beta, n \geq \alpha \\ n, & \text{otherwise} \end{cases} \tag{32}$$

$$f_{rem}(a, b) = \begin{cases} f_{rem}(a-b, b), & a \geq b \\ a, & \text{otherwise} \end{cases} \tag{33}$$

where (31), (32) and (33) are respectively the fuction that counts the number of possible recursive reduction, remainder buses after the reduction and a function that finds the remainder of dividing a by b for $\alpha, \beta, n, a, b \in \mathbb{Z}_{\geq 0} = \{0, 1, 2, \dots\}$.

For the given system in Figure 1, performing network reduction for *Case 1* results to a reduced network shown in Figure 2. The $NBS_{5,8}$, $NBS_{11,16}$, $NBS_{23,27}$ and $NBS_{35,38}$ are reduced once the size of $\alpha = 4$ subsets to $\beta = 1$ that are represented as $5'$, $12'$, $24'$, and $35'$ respectively. Note that bus 15, being a ZIB, is ignored in the reduction. Moreover, $NBS_{28,34}$ is reduced twice recursively and is replaced by $28''$. This reduces the number of variables and constraints from 40 down to 21 for *Case 1* and down to 4 (Bus_3,Bus_{11},$Bus_{12'}$,Bus_{20}) if the leaf buses and leaf parents are predetermined.

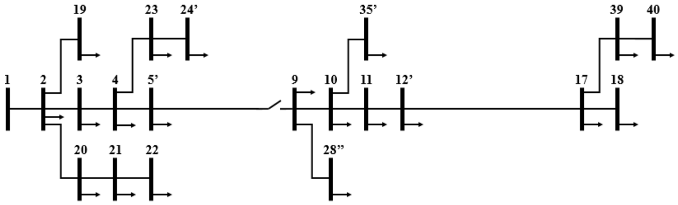

Fig. 2. 40-bus Test System Reduction for Case 1

D. Deterministic PMU Placement for Reduced NBS

As discussed in III-B3, predetermining the PMU locations by considering leaf buses reduces the search space for optimization. This holds true for equivalent buses of reduced networks described in III-C. For the reduced network of Figure 2, leaf buses $\{1, 5', 18, 19, 22, 24', 28'', 35', 40\}$ are predetermined to have no PMUs while leaf parents $\{2, 4, 9, 10, 17, 21, 23, 39\}$ are predetermined to have PMUs installed. Optimization variables corresponding to leaf buses may be omitted and variables corresponding to adjacent buses are set to constants of value 1. Constraints for both set of buses are also omitted from the optimization since those are already presatisfied.

OPP for reduced NBSs, after an initial OPP or by predetermined leaf buses, can be found by direct and deterministic manner. Because of its definite structure, reduced NBSs can easily be expanded back to its original form recursively as well as propagating its OPP. This could be explicitly formulated as

$$f_{Case1}(a, b, c) = [b, (c(1-b)), (a(1-b)(1-c)), b] \tag{34}$$

$$f_{Case2}(a, b) = [a, b, (a-b)^2, a, b] \tag{35}$$

$$f_{Case3}(a) = [a, (1-a), a] \tag{36}$$

Function (34) takes input b for binary decision of installing a PMU while a and c are binary inputs indicating if the bus is observed by another bus for each side. Note that second and third vector elements of (34) can be interchanged for different placements with the same redundancy. Function (35)'s and (36)'s inputs are the binary decisions of installing PMUs.

For example, consider the reduced NBS $28''$ of Figure 2. As a leaf bus, it is predetermined not to have a PMU for *Case 1*. Using (34), it is observed by a predetermined PMU installed at bus 9 ($a = 1$), it has no PMU installed ($b = 0$) and is not observed from the other end ($c = 0$). This initial expansion of bus $28''$ back to series of buses $[28\ 29\ 30\ 31']$, with OPP knowledge at bus $28''$, propagates the OPP deterministically to $[0\ 0\ 1\ 0]$. Recursively, expanding bus $31'$ will yield to series of buses $[31\ 32\ 33\ 34]$ with an OPP of $[0\ 0\ 1\ 0]$.

E. Network Decomposition

Predetermining PMU locations as well as network reduction are done to reduce the search space of optimization. Network decomposition may also be done to create smaller optimization processes that are easier to solve.

Distribution systems may have network switches used for curtailment, on-line reconfiguration and allowed-islanding for subnetworks with distributed generators. As discussed in II-D, buses that are connected with switches are assumed to be open when performing OPP. This may create subnetworks that are disjoint to each other. These subnetworks may independently perform OPP which will give the same result if the entire network is used. These networks may be identified visually or in terms of the ILP framework such that

$$\begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_0 \\ \mathbf{A}_0 & \mathbf{A}_2 \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} \geq \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \end{bmatrix} \tag{37}$$

where $\mathbf{A}_0$ is a zero matrix. Networks denoted by $\{\mathbf{A}_1, \mathbf{x}_1, \mathbf{b}_1\}$ and $\{\mathbf{A}_2, \mathbf{x}_2, \mathbf{b}_2\}$ are said to be disjoint if it satisfies eq. (37). OPP can then be performed on these networks independently. As for Figure 1, the location of the network switch partitions the network in to two disjoint subnetworks; hence, OPP may be applied independently to each subnetwork.

IV. RESULTS AND DISCUSSIONS

The 40-bus test system of Figure 1 and an ILP solver were used for comparing the application of OPPSSR to that of traditional OPP of the entire network. Each of the three cases of Section III-C are simulated with $ch_{max} = 2$ for *Case 1* and *Case 2* to better illustrate the effect of PMU channel limitation.

The OPPSSR process starts with the reduction of NBSs (Table I-col5 & Table II-col3). PMU locations for ZIBs, leaf buses, leaf parents and leaf converted NBSs (Table I-col2,3,4) is then predetermined. This results to only few buses to optimize as shown in Table II-col2. Lastly, the observability settings for the non-leaf reduced NBSs (Table II-col3) is recursively propagated as post-determined buses.

Table III shows the OPP of applying OPPSSR and that of the traditiona way of performing OPP directly using the entire network. Since a given network may have multiple solutions, this study uses the indices introduced in [16] to select the ones with the highest redundancy in bus observability. These indices are the bus observability index (BOI) and the system observability redundancy index (SORI) where BOI is the number of PMUs observing a given bus and SORI is the sum of all BOI. For PMUs at buses that have to select incident lines to observe, the buses in square brackets indicate the lines that are assigned wih current channels.

TABLE I
PREDETERMINED BUSES FOR THE 40-BUS TEST SYSTEM

Case	ZIB	Leaf	Leaf Parent	Leaf NBS
1	15	1,5’,18,19,22, 24’,28”,35’,40	2,4,9,10,17, 21,23,39	5-8,24-27, 28-34,35-38
2	15	1,8,18,19,22, 23’,30’,38,40	2,4,7,17,21, 29,37,39	23-27,30-34
3	15	1,6’,18,19,20’, 23”,28”’,36’,40	2,4,5,9,17, 35,39	6-8,20-22, 23-27,28-34, 36-38

TABLE II
OPTIMIZATION FOR THE 40-BUS TEST SYSTEM

Case	Optimize	Post-determined NBS
1	3,11,12’,20	13,14,16
2	3,5,6,9,10, 11’,20,28,35,36	12,13,14,16
3	3,10,11”	12,13,14,16

TABLE III
OPTIMAL PMU PLACEMENT FOR THE 40-BUS TEST SYSTEM

	ILP$_{\text{OPPSSR}}$		ILP$_{\text{Traditional}}$	
Case	OPP	SORI	OPP	SORI
1	2[1,19],4[3,5], 7,9,10[11,35], 13,17[16,18], 21,23,26,30, 33,37,39	42	2[1,19],4[3,5], 7,9,10[11,35], 13,17[16,18], 21,23,26,30, 33,37,39	42
2	1,2[1,19],3,4[3,5], 5,7,8,9,10[11,35], 12,13,16,17[16,18], 18,19,20,21,22, 23,24,26,27,28, 30,31,33,34,35, 37,38,39,40	88	1,2[1,19],3,4[3,5], 5,7,8,9,10[11,35], 12,13,16,17[16,18], 18,19,20,21,22, 23,24,26,27,28, 30,31,33,34,35, 37,38,39,40	88
3	2[1],3[4],4[23], 5[6],7[8],9[28], 11[10],13[12], 16[15],17[18],19, 20[21],22,24[25], 26[27],29[30],31[32], 33[34],35[36], 37[38],39[40]	43	2[1],3[4],4[23], 5[6],7[8],9[28], 11[10],13[12], 16[15],17[18],19, 20[21],22,24[25], 26[27],29[30],31[32], 33[34],35[36], 37[38],39[40]	43

V. CONCLUSION

OPP for distribution networks is presented in this paper. Practical considerations such as the effect of ZIBs, PMU channel limitation, PMU outages and line outages were taken in to consideration. Methods from related literatures and proposed methods were presented by using the ILP formulation for OPP.

The proposed method for network reduction for distribution networks were shown to be effective in reducing the 40-bus test system of Figure 2 to an equivalent network of 21-bus, 27-bus and 19-bus for *Case 1*, *Case 2* and *Case 3* respectively. With the application of predetermined buses, the optimization is further reduced to 4-bus, 10-bus and 3-bus as shown in column 2 of Table II minimizing the need for optimization.

Another proposed method for deterministic PMU placement was built from the first method to avoid the need for optimization on some parts of the network. This method have successfully reverted the NBSs from network reduction back to its original form with optimal PMU settings. Finally, results (Table III) shows that the proposed method for network reduction and deterministic PMU placement gives identical results as that of the traditional way of performing OPP using directly the entire network with less need for optimization.

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