Network topology identification algorithm based on adjacency matrix

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***Abstract*—Power grid topology identification is not only an important part of advanced software for the distribution network management system, but also the basis for a variety of analytical calculations. The paper presents a new method to speed up the power grid topology identification, which solving the disadvantages of the matrix large computation and slow computation speed. With the node-branch incidence matrix representation of the topological structure of the network is the basic method, through the definition of matrix "or" or "and" operation and switch state vector, and achieve rapid dynamic changes in the network topology by using the symmetric elimination to reduce the order number of the adjacency matrix and the apply symmetry to the process of finding the connected matrix. Compared with the traditional algorithm, the algorithm can reduce the amount of calculation and accelerate the computation speed, suitable for topology analysis of complex power grid.**

***Index Terms*-- network topology identification, adjacency matrix, node elimination, symmetry**

I. INTRODUCTION

Power network topology identification is providing the network structure of data for the analysis for power flow calculation, power outage range and load transfer, is the basic analysis of advanced application software of power system[1],[2]. Its function is to convert the physical model of the power system into the network topology analysis need of the mathematical model.

The essence of power network topology identification is to judge the connectivity between components in the power grid, according to the state of the switch will be divided into the analysis of bus and electrical island. At present, the main analysis methods are tree search method[3]-[6] and matrix method[7]-[13]. The tree search method is to select a certain point as a starting point, according to the defined method to search the connecting bus and branch, and finally analyze the network. The matrix method is based on the adjacency matrix. Through a variety of methods to obtain the fully connected matrix of the network. Literature [7] and [8] uses the method of calculating the square of the matrix to obtain the fully connected matrix, however literature [9] put forward the method of quasi square on the basis of the square method. Although these methods with a certain extent have accelerated the calculation and reduced the amount of calculation, but has not reduced the order of the matrix, and the matrix is still very large. The literature [10] is proposed to reduce the order of the matrix using the node elimination technique, thus reducing the computational times, but also needs a lot of computing in the process of the nodes elimination and the operation.

For the characteristics of matrix computation, this paper on the basis of reference [8] proposed a speed up the network analysis algorithm, using switch's overturn later simplified node-branch incidence matrix, and apply symmetry section of the process of the node elimination and the adjacency matrix operation to reduce the amount of redundant computation and improve the calculation speed.

II. INCIDENCE MATRIX AND ITS SIMPLIFICATION

A. Incidence matrix

According to the knowledge of graph theory, we can use node-branch association matrix to describe the topological structure of a topological network. As shown in Figure 1 of the main wiring diagram, when the physical model is abstracted as mathematical model used for calculation, the node of the main wiring as node topology, the switching element as the branch of topology graph, the outlet and inlet line of every bus are also used as the node, can the formation of incidence matrix $A=[a_{ij}]$]. The row of the incidence matrix A is a node, the column is a branch, A_{ij} represents the node i and branch j relevance, when the node i and branch j is connected, $a_{ij}=1$, otherwise $a_{ij}=0$.

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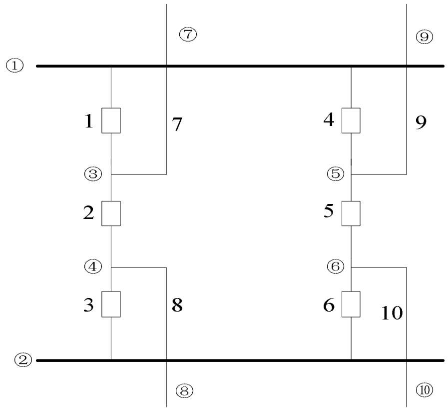

Fig.1 Main wiring diagram

The correlation matrix is A_0 when all the switches are closed in Figure 1, the A_0 is

$$\boldsymbol{A}_0=\begin{bmatrix}1&0&0&1&0&0&0&0&0&0\\0&0&1&0&0&1&0&0&0&0\\1&1&0&0&0&0&1&0&0&0\\0&1&1&0&0&0&0&1&0&0\\0&0&0&1&1&0&0&0&1&0\\0&0&0&0&1&1&0&0&0&1\\0&0&0&0&0&0&1&0&0&0\\0&0&0&0&0&0&0&1&0&0\\0&0&0&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&0&0&1\end{bmatrix}$$

B. *Simplify the incidence matrix*

By defining the following vector operations, we can easily express the network topology identification method.

The row vector *a* and the row vector *b* have the same dimension, and the "and" operation between a and b is defined as $a \otimes b$ .It expresses the meaning of the corresponding elements in the row vector to do "and" operation. The "or" operation between a and b is defined as $a \oplus b$, it expresses the meaning of the corresponding elements in the row vector to do "or" operation.

By using the switch vector *S* to modify the node-branch incidence matrix *A* and obtaining the network incidence matrix A' at this moment.

$$A'(i) = A(i) \otimes S \tag{1}$$

$A'(i)$ --Row vector of line i of the incidence matrix after the switch is shifted.

$A(i)$ --Row vector of line i of the correlation matrix without changing the switch.

According to the operation rules of Boolean matrix, the *j* column all elements in the matrix are 0 when the s_j=0 in the switch vector *S*. Indicates that the j branch is not connected to other nodes, which may exist in two cases: (1) The logical branch in the main connection is disconnected; (2) In the topology analysis of the station, the calculation branch is set to off state.

Delete the columns of all elements is 0 in the incidence matrix A', only useful branches form a simple incidence matrix A'', which can reduce the order of the matrix, reduce the amount of storage, accelerate the speed of network topology. For example, when the switch is in the state of S=[0,0,1,1,0,1,1,1,1,1], the correlation matrix at this time is A_0' and the simplified matrix is A_0''.

$$A_0'=\begin{bmatrix}0&0&0&1&0&0&0&0&0&0\\0&0&1&0&0&1&0&0&0&0\\0&0&0&0&0&0&1&0&0&0\\0&0&1&0&0&0&0&1&0&0\\0&0&0&1&0&0&0&0&1&0\\0&0&0&0&0&1&0&0&0&1\\0&0&0&0&0&0&1&0&0&0\\0&0&0&0&0&0&0&1&0&0\\0&0&0&0&0&0&0&0&1&0\\0&0&0&0&0&0&0&0&0&1\end{bmatrix}$$

$$A_0''=\begin{bmatrix}0&1&0&0&0&0&0\\1&0&1&0&0&0&0\\0&0&0&1&0&0&0\\1&0&0&0&1&0&0\\0&1&0&0&0&1&0\\0&0&1&0&0&0&1\\0&0&0&1&0&0&0\\0&0&0&0&1&0&0\\0&0&0&0&0&1&0\\0&0&0&0&0&0&1\end{bmatrix}$$

III. ADJACENCY MATRIX AND ITS SIMPLIFIED TOPOLOGYA

A. *The formation of the adjacency matrix*

Adjacency matrix is the connection between the nodes, and the nodes are connected by the branch, so the node-node adjacency matrix can be formed by the node-branch matrix multiplication the branch-node matrix. Similarly, as the definition of the incidence matrix as the definition of the adjacency matrix of $C=[c_{ij}]$, when the two node are connected, c_{ij}=1, or c_{ij}=0. It is obvious that all the diagonal elements of the adjacency matrix are 1, and C is a symmetric matrix. For the network contain n nodes and m branch, define the multiplication of the matrix:

$$C = A \bullet B \tag{2}$$

$$c_{ij} = \bigcup_{k=1}^{m} (a_{ik} \cap b_{kj}) \tag{3}$$

We can get the adjacency matrix $C^{(1)}$ when we put the simplify correlation matrix A_0'' into formula (2).

$$C^{(1)} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Topology identification of the network is to make clear the relationship between nodes, and $C^{(1)}$ show the two nodes are directly connected with the branch, known as the first-order connectivity, the adjacency matrix is called first-order connectivity matrix. The matrix of $C^{(1)}$ multiplied by $C^{(1)}$ is $C^{(2)}$, which represents the two node is two level connection and the matrix is called two level connection matrix. Repeat this process, which has the maximum n-1 connection for n nodes. When the results of the two matrix are the same, the matrix is the full connectivity matrix of the network, and the topology of the network is the analysis of the fully connected matrix.

B. *The simplification of the adjacency matrix*

Matrix analysis method, although the algorithm is simple and clear, but the number of nodes in the network are numerous, forming the adjacency matrix is very large, and the need for n-1 times square of matrix operations, large amount of calculation and increases the network topology identification of computing time, cannot achieve the demand of the network dynamic topology. Therefore, this paper put forward the method of symmetric node elimination. It reduces the number of nodes in the network and utilizes adjacency matrix symmetry and Boolean operations to simplify the operation process, thereby reducing the amount of computation, speeding up the operation, shortening the time of topology analysis.

a) Use symmetric elimination to eliminate intermediate nodes

Because of the large number of nodes in the distribution network, it is not necessary to know the connection between all nodes. Usually, we just need to know the relationship between some important nodes, so do not need too much concern for the nodes and branches can be eliminated, thereby reducing the number of nodes in the network and reducing the order of adjacency matrix.

Node elimination process is the process of simplifying the adjacency matrix. The order of the adjacency matrix is reduced by one order when a node is eliminated. For example, in the process of eliminating the node k, the two level connection relationship between i and j, which is connected by the node k in the adjacency matrix, must be changed into direct connection, namely c_{ij}=1.The specific algorithm of node elimination is: for each element c_{ij} of the adjacency matrix C and （$c_{ik} \cap c_{kj}$） "or" operation to form a new element of c_{ij}'. The elements in the matrix are updated each time, and the rows and columns corresponding to the nodes in the C are deleted Since the adjacency matrix is symmetric, so

$$c_{ji}' = c_{ij}' = c_{ij} \cup (c_{ik} \cap c_{kj}) \tag{4}$$

b) Using the symmetry of the connected matrix to find the full connected matrix

When calculating the fully connected matrix, each one element requires computing (2n-1) operations and the matrix contains n^2 elements. Therefore, it needs to calculate $n^2(2n-1)$ operations in a squaring operation. By using the symmetry of the adjacency matrix, we can reduce the amount of the calculation by half. The formula for calculating the elements of the connected matrix is calculated by using symmetry:

$$c_{ji} = c_{ij} = \bigcup_{k=1}^{m} (c_{ik} \cap c_{kj}) \tag{5}$$

Figure 2 shows the flow chart of the elements of the connected matrix.

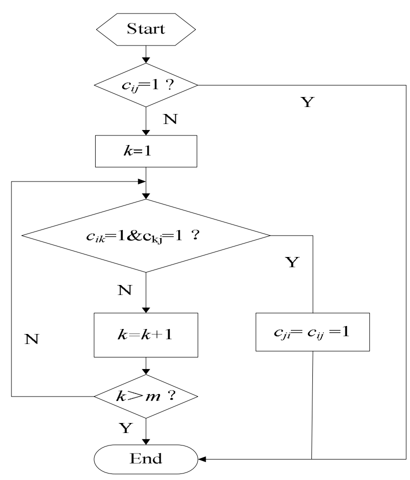

Fig.2 Flow chart of the computation the element of the connectivity matrix

IV. EXAMPLE ANALYSIS

As shown in Figure 4 for the substation wiring diagram. With 38 node wiring diagram as an example to illustrate the process of network topology:

1) Read the state of the switch S at this time to form a simplified incidence matrix.

2) The adjacency matrix of the network containing 38 nodes is obtained by the simplified correlation matrix C_0.

3) The elimination of the nodes in the adjacency matrix C_0 is used by symmetric elimination to form the adjacency matrix C of the nodes that ultimately need to be cared for..

4) The fully connected matrix is obtained, and then the topology of the network is identified by the analysis of the entire connected matrix.

The network of figure 4 is shown below to illustrate the comparison of the amount of calculation under different calculation methods for the number of different nodes that are concerned. The calculation of the adjacency matrix method is 4007100 times. Table 1 lists the number of calculation elimination methods and symmetric elimination method for the different nodes, and the comparison with the adjacency matrix method.

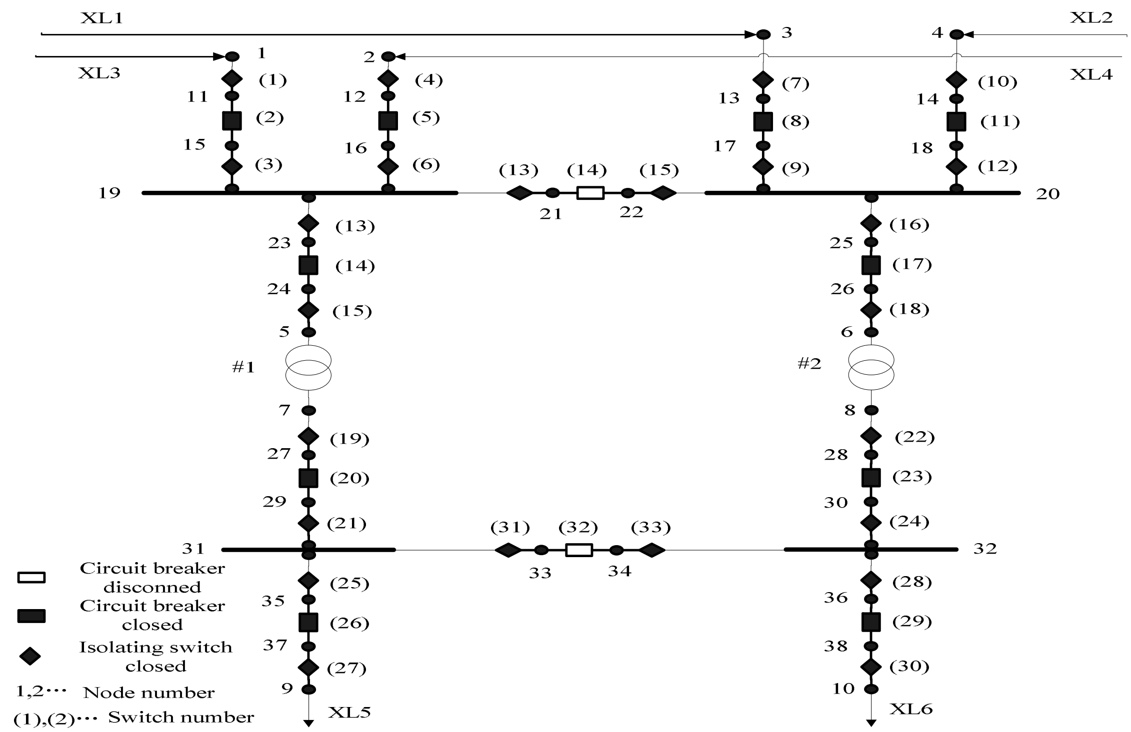

Fig4. Substation connection diagram

TABLE1. The comparison of calculation of adjacency matrix method, node elimination and symmetric node elimination method

Node number	The number Of note elimination *R*	The number of symmetric note elimination *S*	Compare the amount of computation *M*/*R*	Compare the amount of computation *M*/*S*
35	2881630	1399652	1.39	2.86
30	1557940	753037	2.57	5.32
25	760350	365072	5.27	11.00
20	326610	155382	12.3	25.79
15	124470	58592	32.20	68.39
10	51680	24327	77.54	164.72

Note：（1）A "and" and "or" operation as a single operation.

（2）The number of calculations in the table is the maximum number of times.

According to the table, we can see that the more nodes you eliminate, the fewer nodes you keep and the less the network topology analysis computation. The relationship between the 1-10 node is preserved by eliminating the 11-38 nodes in the graph. The adjacent matrix C is formed through the third step of the topology process.

$$C=\begin{bmatrix}1&1&0&0&1&0&0&0&0&0\\1&1&0&0&1&0&0&0&0&0\\0&0&1&1&0&1&0&0&0&0\\0&0&1&1&0&1&0&1&0&0\\1&1&0&0&1&0&0&0&0&0\\0&0&1&1&0&1&0&0&0&0\\0&0&0&0&0&0&1&0&1&0\\0&0&0&0&0&0&0&1&0&1\\0&0&0&0&0&0&1&0&1&0\\0&0&0&0&0&0&0&1&0&1\end{bmatrix}$$

The analysis of the network topology is the analysis of all connected matrix. By using the adjacency matrix *C* to obtain the full connectivity matrix, we can see that C is a fully connected matrix. For the analysis of C, the network is divided into four sections (1,2,5) (3,4,6) (7,9) (8,10). As can be seen from table 1,it is necessary to calculate 4007100 times when the adjacency matrix is used in the analysis of the matrix, and the nodes elimination method is about 25182 times. This shows that the calculation of the adjacency matrix is 77 times the node elimination and 164 times the elimination of the symmetric nodes. While the actual formation of fully connected matrix process does not need to be n-1 times the multiplication operation. After elimination of the symmetric matrix C is the fully connected matrix does not need the calculation, only 16532 times of calculation, the computation is the adjacency matrix method 1/242.

In order to visually and clearly express the calculation of the elimination and the symmetric elimination method of the different nodes. The histogram of Figure 5 is produced by using the calculation of elimination and symmetric elimination method in the table 1. It is clear that the calculation of symmetrical elimination is half the elimination method, which can reflect the superiority of the symmetric elimination method.

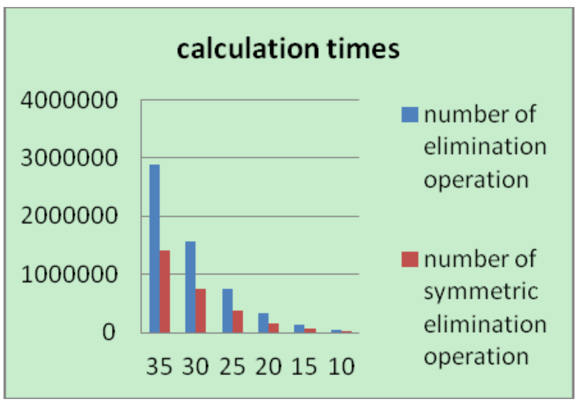

Fig5. The comparison of calculation of node elimination and symmetric node elimination method

V. CONCLUSION

Aiming at the characteristics of the matrix method, such as large number of nodes, large memory consumption and slow computation speed. Firstly, the state matrix of the switch is utilized to simplify the correlation matrix and reduce the order of the matrix. Then we use the method of symmetric node elimination to eliminate the nodes which need not be considered, and further reducing the order of the matrix. Finally, in the calculation of the full connectivity matrix in the process of utilizing symmetry matrix and Boolean operation principle to reduce the amount of computation, accelerate the update speed of elements in the process of calculation, improve the network topology analysis speed. The correctness of the symmetry elimination method is proved by an example.

The proposed method is applied to the distribution automation system, through the application in Zhengzhou power supply company, to ensure the reliability of operation data, provide good data support for the advanced application of system analysis, the distribution network topology identification for power outage analysis, power source, power supply range analysis of topology identification time shortened by 20% than the traditional methods.

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