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An Advanced Uncertainty Measure Using Fuzzy Soft Sets: Application to Decision-Making Problems

Nitin Bhardwaj* and Pallvi Sharma

Abstract: In this paper, uncertainty has been measured in the form of fuzziness which arises due to imprecise boundaries of fuzzy sets. Uncertainty caused due to human's cognition can be decreased by the use of fuzzy soft sets. There are different approaches to deal with the measurement of uncertainty. The method we proposed uses fuzzified evidence theory to calculate total degree of fuzziness of the parameters. It consists of mainly four parts. The first part is to measure uncertainties of parameters using fuzzy soft sets and then to modulate the uncertainties calculated. Afterward, the appropriate basic probability assignments with respect to each parameter are produced. In the last, we use Dempster's rule of combination to fuse independent parameters into integrated one. To validate the proposed method, we perform an experiment and compare our outputs with grey relational analysis method. Also, a medical diagnosis application in reference to COVID-19 has been given to show the effectiveness of advanced method by comparing with other method.

Key words: fuzzy soft sets; Dempster–Shafer theory; grey relational analysis; entropy; belief measures and medical diagnosis

1 Introduction

The fuzzy logics have emerged as a very important and useful topic in past recent years. It has aroused as an important mathematical tool to deal with uncertainties and vagueness of data. Zadeh[1] presented the concept of fuzzy set theory in 1965 as a transformation of classical set theory.

It can solve the problems of decision-making and deal with the problem of vagueness, uncertainty, and imprecision of data. Various theories like classical set theory[2], fuzzy set theory[1], probability theory, possibility theory[3], and Dempster–Shafer evidence theory[4,5] have been given to deal with certain types of uncertainties. Each theory has its own merits and demerits. Soft set theory is one of the theories initiated by Molodstov[6] in 1999 which can give exact solutions to various engineering and computer science problems. Fuzzy soft theory was given by Maji et al.[7] This theory has wider applications which can be easily found in Refs. [8–13]. Fuzzy soft sets can solve the problems of decision-making in real life. It deals with uncertainties and vagueness of data. Uncertainty refers to epistemic situations involving imperfect or unknown information. There are different forms of uncertainty, namely, fuzziness which arises due to imprecise boundaries, non-specificity (imprecision), discord and strife, etc. Measuring uncertainty is an open issue. Many belief entropies like Deng entropy[14], W-entropy[15], Hohel uncertainty measure[16], Dubois and Prade measure[17], Pan and Deng[18] uncertainty measure, etc., are introduced to deal with this open issue. They measure the uncertainty of parameters in different forms. Also, there are different approaches to solve decision making problems using fuzzy soft sets. Hou[19] made use of grey relational analysis to take care of the issues of problems in making

• Nitin Bhardwaj is with Department of Mathematics, Lovely Professional University, Punjab 144411, India. E-mail: nitin.15903@lpu.co.in.
• Pallvi Sharma is with Department of Mathematics, Lovely Professional University, Punjab144411, India. E-mail: pallavi.sharma0303@gmail.com.
∗ To whom correspondence should be addressed.
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decisions. Li et al.[20] proposed grey relational analysis with the utilization of Dempster–Shafer (D–S) evidence hypothesis to settle on choices using fuzzy soft sets. D–S rule of combination can combine multiple evidences to produce an integrated one. As a result of the viability in displaying the vulnerability and imprecision without the earlier data, this hypothesis is broadly utilized in a ton of regions.

In this paper, we have used fuzzified evidence theory[21] along with D–S theory to solve the problem of decision making. Uncertainty in the form of fuzziness is considered to solve the problems of decision making. Also, a medical diagnosis problem in respect of COVID-19 has been solved which helps a doctor to take decision on patient's condition easily. We have also compared our proposed method with the method proposed by Li et al.[20] to show the effectiveness of our method.

The paper is assembled in the following way. Section 2 introduces the prerequisites for further work. Section 3 explains the methodology used for the proposed method. It has four sub parts. The first part involves the measurement of uncertainty of parameters in the form of total degree of fuzziness, the second part is the brief description of steps involved to solve the decision making problem, the third part performs an experiment (Example 3) to solve the problem, and the fourth part is a practical application of our proposed work to handle decision making problem in real-life situation (medical diagnosis). Section 4 is the summary of whole paper which briefly explains the highlights of the paper.

2 Preliminary

2.1 Fuzzy soft set

Definition 1: Fuzzy set[1]. Let $\mathcal{X}$ be a non-empty set and $\mathcal{A} \subseteq \mathcal{X}$. A fuzzy set $\mathcal{A}$ is determined by its membership function $\mu_A : \mathcal{X} \to [0, 1]$ whose value determines "the grade of membership" of point x in $\mathcal{A}$ for x belongs to $\mathcal{X}$.

Definition 2: Fuzzy soft sets[1,7]. Let $\mathcal{X}$ be an initial universe set with $\mathcal{E}$ as the set of parameters. The pair $(\mathcal{F}, \mathcal{A})$ is a fuzzy soft set over $\mathcal{X}$ where $\mathcal{A} \subseteq \mathcal{E}$ and $\mathcal{F}$ is a mapping defined as $\mathcal{F} : \mathcal{A} \to I^{\mathcal{X}}$, where $I^{\mathcal{X}}$ is the power set of $\mathcal{X}$ (Table 1).

It is evident that every soft set can be contemplated as a fuzzy soft set. Also, when both $\mathcal{X}$ and $\mathcal{A}$ are finite, fuzzy soft sets are either represented by matrices or in tabular form.

Table 1 Representation of set $(\mathcal{F}, \mathcal{A})$.

Parameter/subset of $\mathcal{X}$	g_1	g_2	g_3	g_4
a_1	0.5	0.2	0.2	0.1
a_2	0.6	0.1	0.1	0.2
a_3	0.4	0.3	0.2	0.1

Example 1 Let $\mathcal{X} = \{g_1, g_2, g_3, g_4\}$ be the universal set and $\mathcal{A} = \{a_1, a_2, a_3\}$ be the set of parameters. Then, $(\mathcal{F}, \mathcal{A})$ is a fuzzy soft set over $\mathcal{X}$ described as follows:

$$\mathcal{F}(a_1) = g_1/0.5, g_2/0.2, g_3/0.2, g_4/0.1,$$
$$\mathcal{F}(a_2) = g_1/0.6, g_2/0.1, g_3/0.1, g_4/0.2,$$
$$\mathcal{F}(a_3) = g_1/0.4, g_2/0.3, g_3/0.2, g_4/0.1.$$

Definition 3: Fuzzy soft intersection[1,7]. The fuzzy soft intersection of two sets $(\mathcal{F}, \mathcal{A})$ and $(\mathcal{G}, \mathcal{B})$ over a common universe $(\mathcal{X}, \mathcal{E})$ is the fuzzy soft set $(\mathcal{H}, \mathcal{C})$ where $\mathcal{C} = \mathcal{A} \cap \mathcal{B}$ and $\forall a \in \mathcal{C}$, we conclude

$$\mu^a_{(\mathcal{H},\mathcal{C})}(x) = \min\{\mu^a_{(\mathcal{F},\mathcal{A})}(x), \mu^a_{(\mathcal{G},\mathcal{B})}(x)\}, \forall x \in \mathcal{X},$$

where $\mu^a_{(\mathcal{H},\mathcal{C})}$, $\mu^a_{(\mathcal{F},\mathcal{A})}$, and $\mu^a_{(\mathcal{G},\mathcal{B})}$ are the membership values for fuzzy soft sets $(\mathcal{H}, \mathcal{C})$, $(\mathcal{F}, \mathcal{A})$, and $(\mathcal{G}, \mathcal{B})$, respectively.

2.2 Uncertainty measures

This section contains the definitions of different types of entropies used to measure the uncertainty of information.

Definition 4[22]. An entropy measure is a sequence of mappings $E_n : \mathcal{X}_n \times P_n \times W_n \to \mathbb{R}^+$ satisfying several properties (symmetry, monotonicity, additivity, etc.).

Definition 5: Shannon entropy[23]. Shannon in 1948 introduced the concept of Shannon entropy to handle basic probability problem.

Shannon entropy (H) is derived as

$$H = -\sum\nolimits_i^N p_i \log_2 p_i,$$

where p_i is the probability of state i satisfying $\sum\nolimits_i^N p_i = 1$ and N is the number of basic states in a system.

Definition 6: Deng entropy[14]. This novel belief entropy was introduced by Deng in 2016. It also measures the uncertainty conveyed by basic probability assignment. Deng entropy is denoted by E_d. It is defined as

$$E_d = -\sum\nolimits_i m(A_i) \log m(A_i) / 2^{|A_i|} - 1,$$

where m is the mass function and A_i is the hypothesis of belief function. Deng entropy is degenerated into Shannon entropy when the belief value is allocated to one single element.

Definition 7: W-entropy[15]. This type of entropy was given by Wang et al.[15] in 2019. It is the unified form about belief entropy based on Deng entropy[14] which considers the scale of frame of discernment and the relative scale of focal element with respect to Frame Of Discernment (FOD).

W-entropy is calculated as

$$E_w(m) = \sum m(A)\log_2\left(\frac{m(A)}{2^{|A|}-1}(1+\xi)^{f|\mathcal{X}|}\right),$$

where ξ is a constant and $\xi \geqslant 0$, and $f|\mathcal{X}|$ is the function determines the cardinality of $\mathcal{X}$. $f|\mathcal{X}| = \sum_{B\subseteq\mathcal{X},B\neq A}\frac{|A\cap B|}{2^{|\mathcal{X}|}-1}$.

Definition 8: Fuzziness[21]. A measure of fuzziness is a function from the set of all fuzzy subsets of $\mathcal{X}$ to the set of all positive real numbers. The function $f(A)$ expressed the degree that the boundary of A is not sharp.

The measure of fuzziness is calculated as

$$f(A) = \sum_{x\in\mathcal{X}}(1-|2A(x)-1|) \tag{1}$$

The range of function f is $[0, |\mathcal{X}|]$; $f(A) = 0$ if A is a crisp set; $f(A) = |\mathcal{X}|$ when $A(x) = 0.5\forall x \in \mathcal{X}$.

Definition 9: Fuzziness in evidence theory[21]. Total degree of fuzziness $F(m)$ of the body of evidence $\langle m, F\rangle$ is calculated as follows:

$$F(m) = \sum_{A\in F} m(A)f(A),$$

where $f(A)$ is given by Eq. (1).

Definition 10: Performance measure[24,25]. The performance measure of a method satisfies the optimal criteria for resolving decision making problem. It is denoted by Y_S.

Mathematically,

$$Y_S = \frac{1}{\sum_i^{n_c}\sum_j^{n_c}|F(e_i)(O_p)-F(e_j)(O_p)|} + \sum_{i=1}^{n_c}F(e_i)(O_p),$$

here, n_c is the number of choice parameters and $F(e_i)(O_p)$ depicts the membership value of the ideal object O_p for the choice parameter e_i.

If the performance measure of one method is greater than other, then that method is much finer than other, and vice versa.

2.3 Dempster–Shafer evidence theory

Dempster–Shafer theory is proposed by Dempster[4] and Shafer[5]. This theory deals with the uncertain information and is applied to uncertainty modelling[26,27], decision making[28,29], information fusion[30–32], etc. This theory does not need prior information in modelling uncertainty and also is able to fuse multiple evidences into integrated one.

Definition 11: Frame of discernment[5]. A frame of discernment is a finite non-empty set of mutually exclusive and exhaustive hypotheses denoted by $\Theta = \{A_1, A_2, \ldots, A_n\}$ and $A_i \cap A_j = \varnothing$ denoted by Θ and 2^Θ represents the set of all subsets of Θ.

Definition 12: Basic Probability Assignment (BPA)[5]. It is also known as mass function. A mass function is a mapping m from 2^Θ to $[0, 1]$ which satiates the following situations:

$$m(\varnothing) = 0 \text{ and } \sum_{A\in 2^\Theta} m(A) = 1.$$

If $m(A) > 0$, A is called a focal element and its union is known as the core of the mass function.

Definition 13: Belief function[5]. It can be defined as a mapping Bel: $2^\Theta \to [0, 1]$ satisfying following conditions:

$$\mathrm{Bel}(\varnothing) = 0, \mathrm{Bel}(\Theta) = 1,$$

$$\text{and } \mathrm{Bel}(A) = \sum_{B\subseteq A} m(B), \forall A \subseteq \Theta.$$

$\mathrm{Bel}(A)$ exemplifies the imprecision and uncertainty in decision making problems. When there is single element, then, $\mathrm{Bel}(A) = m(A)$.

Definition 14: Dempster's rule of combination[4]. This rule computes an integrated set of combined evidences. Supposed m_1 and m_2 are two independent BPAs in Θ, then rule of combination is defined as

$$m(A) = \begin{cases} \frac{1}{1-K}\sum_{B\cap C=A} m_1(B)m_2(C), & A \neq \varnothing; \\ 0, & A = \varnothing \end{cases} \tag{2}$$

$$K = \sum_{B\cap C=\varnothing} m_1(B)m_2(C) \tag{3}$$

where $B \in 2^\Theta$ and $C \in 2^\Theta$, and $K \in [0, 1]$ represents the coefficient for confliction between two BPAs.

2.4 Grey relational analysis

Li et al.[20] utilized grey relational analysis with Dempster–Shafer theory to solve the problem of decision making. They calculated grey relational degree and then calculated uncertainty degree of various parameters. Further, BPA of each independent alternative can be obtained on the basis of this degree and used Dempster's rule of combination to fuse different alternatives into collective alternative. Finally, the best alternative based on the ranking of these fused alternatives can be obtained.

Definition 15: Grey mean relational degree[20]. The grey means relational degree between d_{ij} and $\widetilde{d_i}$ which can be computed as

$$r_{ij} = \frac{\min_{1\leqslant i\leqslant s}\Delta d_{ij} + 0.5\max_{1\leqslant i\leqslant s}\Delta d_{ij}}{\Delta d_{ij} + 0.5\max_{1\leqslant i\leqslant s}\Delta d_{ij}},$$
$$i = 1, 2, \ldots, s,\ j = 1, 2, \ldots, n \tag{4}$$

where d_{ij} denotes the membership value of x_i with e_j, $\widetilde{d_i}$ is the mean of all parameters with respect to each alternatives, and Δd_{ij} is the difference information between d_{ij} and $\widetilde{d}_i$.

2.5 Fuzzy preference relations

Definition 16: Fuzzy preference relation[33]. Fuzzy preference orderings can be defined as fuzzy binary relations related to reciprocity and maximum and minimum transitivity. Mathematically, it is denoted by

$$P = (p_{jk})_{n\times n},$$

where $p_{jk} \in [0, 1]$ represents the preference value of alternative e_j over e_k.

Also, $p_{jk} + p_{kj} = 1$, $p_{jj} = 0.5$, $1 \leqslant j \leqslant n$, and $1 \leqslant k \leqslant n$.

Definition 17: Consistency matrix[34]. The consistency matrix can be developed on the basis of fuzzy preference relation as follows:

$$p = \overline{(p_{jl})}_{n\times n} = \left(\frac{1}{n}\sum_{k=1}^{n}(p_{jk} + 0.5p_{kl})\right)_{n\times n} \tag{5}$$

3 Our Proposed Methodology

Uncertainty can be exhibited in extraordinary ways. These forms signify distinct types of uncertainty. One of the forms of uncertainty is fuzziness. Fuzziness (vagueness) results from imprecise boundaries of fuzzy sets. In this section, fuzzified evidence theory along with D–S theory and Dempster's rule of combination has been used. First, we measure the uncertainties (fuzziness) of parameters taking the scale of frame of discernment and relative scale of focal element with respect to FOD into consideration. Next, we use the fuzzy preference relation analysis to produce the consistency matrix. At that point, the vulnerabilities of parameters are adjusted and a while later, a suitable fundamental BPA in terms of each parameter is produced. In the last, we utilize the Dempster's rule of combination to blend the independent parameters into integrated one. Inevitably, the best ideal decision can be got dependent on the positioning of choices. The flowchart of the proposed technique has been appeared in Fig. 1.

3.1 Measurement of uncertainty of parameters e_j (j = 1, 2, ..., n)

Total degree of fuzziness of the parameters with respect to alternatives can be calculated as

$$F_d(A) = \sum\nolimits_{A\in f} m(A)\log_2 m(A) f(A)(1+\xi)^{f|\mathcal{X}|} \tag{6}$$

where $f(A)$ is the degree of fuzziness and is calculated by using Eq. (1). The factor $(1+\xi)^{|\mathcal{X}|}$ considers the scale of FOD and the relative scale of focal elements with respect to FOD. Also, ξ is the constant greater than 0 and an appropriate number can be given to it based on practical example and $f\,|\mathcal{X}|$ represents the cardinality of $\mathcal{X}$ defined as

$$f|\mathcal{X}| = \sum\nolimits_{B\subseteq\mathcal{X}, B\neq A}\frac{|A\cap B|}{2^{|\mathcal{X}|}-1}.$$

Example 2 Let us suppose that the frame of discernment is $\mathcal{X} = \{a_1, a_2, \ldots, a_5\}$. A body of evidence $\langle m, F\rangle$ is listed as

$$m_1 : m_1 = \{a_1, a_2, a_3\} = 0.3,\ m_1 = \{a_4, a_5\} = 0.7,$$
$$m_2 : m_2 = \{a_1, a_2, a_3\} = 0.3,$$
$$m_2 = \{a_1, a_2, a_4, a_5\} = 0.7.$$

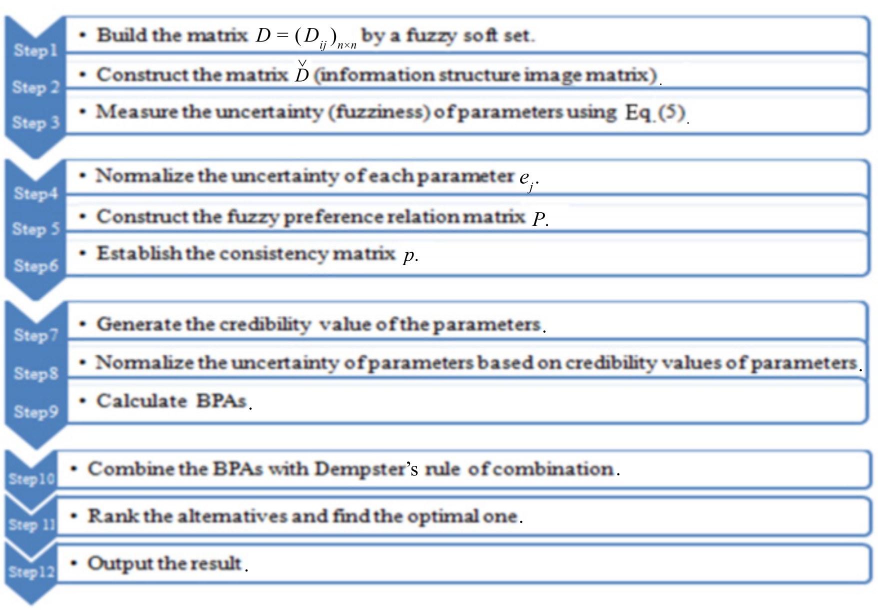

Fig. 1 Flowchart of our proposed method.

The total degree of fuzziness of m_1 and m_2 is calculated as

$$F_d(m_1)=$$

$$\sum_{A\in f} m(m_1)\log_2 m(m_1) f(m_1)(1+\xi)^{\sum_{B\subseteq\mathcal{X},B\neq A}\frac{|A\cap B|}{2^{|\mathcal{X}|}-1}}=$$

$$0.3\times\log_2(0.3)\times1.2\times2^0+0.7\times\log_2(0.7)\times1.2\times2^0=$$

$$-0.625\,31-0.432\,24=-1.057\,55,$$

and

$$F_d(m_2)=$$

$$\sum_{A\in f} m(m_2)\log_2 m(m_2) f(m_2)(1+\xi)^{\sum_{B\subseteq\mathcal{X},B\neq A}\frac{|A\cap B|}{2^{|\mathcal{X}|}-1}}=$$

$$0.3\times\log_2(0.3)\times1.2\times2^{\frac{2}{31}}+0.7\times\log_2(0.7)\times1.2\times2^{\frac{2}{31}}=$$

$$-0.65391-0.45201=-1.10592.$$

3.2 Brief description of steps for the proposed method

Let $\Theta=\{x_1,x_2,\dots,x_i,\dots,x_t\}$ be the FOD and $\mathcal{B}=\{e_1,e_2,\dots,e_j,\dots,e_n\}$ be the set of parameters. $\mathcal{F}$: $\mathcal{B}\to2^{\Theta}$ is defined as $\mathcal{F}(e_j)\,(x_i)=d_{ij}$.

(1) Evolve the matrix $D=(d_{ij})_{n\times n}$ by the use of fuzzy soft set ($\mathcal{F}$, $\mathcal{B}$) over Θ and d_{ij} is the membership value of x_i with respect to e_j.

$$\hat{D}=\begin{bmatrix} d_{11} & \dots & d_{1j} & \dots & d_{1n}\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ d_{i1} & \dots & d_{ij} & \dots & d_{in}\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ d_{t1} & \dots & d_{tj} & \dots & d_{tn}\end{bmatrix} \tag{7}$$

(2) Construct the information structure image sequence with respect to each parameter e_j using formula $\widetilde{d_{ij}}=\dfrac{d_{ij}}{\sum\limits_{i=1}^{t} d_{ij}}$.

$$\hat{D}=\begin{bmatrix} \widetilde{d_{11}} & \dots & \widetilde{d_{1j}} & \dots & \widetilde{d_{1n}}\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ \widetilde{d_{i1}} & \dots & \widetilde{d_{ij}} & \dots & \widetilde{d_{in}}\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ \widetilde{d_{t1}} & \dots & \widetilde{d_{tj}} & \dots & \widetilde{d_{tn}}\end{bmatrix} \tag{8}$$

(3) Total degree of fuzziness of the parameters may be zero in some cases. So the proposed formula is used to measure the uncertainty of the parameter, denoted by $V(e_j)$:

$$V(e_j)=\exp F_d(e_j)=$$

$$\exp\sum_{i=1}^{t} d_{ij}(\log_2 d_{ij}) f(d_{ij})(1+\mathcal{E})^{f|\mathcal{X}|} \tag{9}$$

(4) Normalize the uncertainty of the parameter e_j as follows:

$$\overline{V(e_j)}=\frac{V(e_j)}{\sum\limits_{h=1}^{n} V(e_h)},\quad 1\leqslant j\leqslant n \tag{10}$$

(5) Construct the fuzzy preference relation matrix based on the variance of uncertainties of parameters. The diagonal elements of the matrix are allocated to 0.5 according to Definition 16. When there are only two parameters, the off-diagonal elements are allocated to 0.5 as none other parameters are there to judge which one parameter is preferred to other. When there are more than two parameters, $n>2$, the variance for the parameter $e_j (1\leqslant j\leqslant n)$ is computed as

$$\mathrm{Var}(e_j)=$$

$$\mathrm{Var}(\{(e_1)\bar{V}(e_2),\dots,\bar{V}(e_{j-1}),\bar{V}(e_{j+1}),\dots,\bar{V}(e_n)\}) \tag{11}$$

And the off-diagonal elements p_{jk} and p_{kj} are calculated as follows:

$$p_{jk}=\frac{\mathrm{Var}(e_j)}{\mathrm{Var}(e_j)+\mathrm{Var}(e_k)} \tag{12}$$

$$p_{kj}=\frac{\mathrm{Var}(e_k)}{\mathrm{Var}(e_j)+\mathrm{Var}(e_k)} \tag{13}$$

where $1\leqslant j\leqslant n$ and $1\leqslant k\leqslant n$.

(6) Based on above fuzzy preference matrix obtained, we built the consistency matrix p utilizing Eq. (5).

(7) Based on the consistency matrix p, the credibility value of the parameter e_j is calculated as

$$\mathrm{Cred}(e_j)=\frac{2}{n^2}\sum_{k=1}^{n}\overline{p_{jk}},\quad 1\leqslant j\leqslant n, 1\leqslant k\leqslant n \tag{14}$$

where $\sum\limits_{j=1}^{n}\mathrm{Cred}(e_j)=1$, these values will be taken as the loads to show the relative reliability preference of parameters.

(8) On the basis of credibility values of parameters, normalized uncertainty can be modulated as

$$\mathrm{MV}(e_j)=\mathrm{Cred}(e_j)\times\overline{V(e_j)},\quad 1\leqslant j\leqslant n \tag{15}$$

(9) Now, we normalized the modulated uncertainty of

parameters as the final degree of fuzziness as

$$\overline{\mathrm{MV}}(e_j) = \frac{\mathrm{MV}(e_j)}{\sum\limits_{h=1}^{n} \mathrm{MV}(e_k)},\ 1 \leqslant j \leqslant n \tag{16}$$

(10) The basic probability assignment of the alternative x_i and Θ with respect to e_j is calculated as

$$m_{e_j}(\varnothing) = 0 \tag{17}$$

$$m_{e_j}(x_i) = \widetilde{d_{ij}} \times (1 - \overline{\mathrm{MV}}(e_j)) \tag{18}$$

$$m_{e_j}(\Theta) = 1 - \sum\nolimits_{i=1}^{t} m_{e_j}(x_i) \tag{19}$$

where $1 \leqslant j \leqslant t, 1 \leqslant k \leqslant n$, and $\sum_{A \subseteq \Theta} m_{e_k}(A) = 1$, for $j = 1, 2, \ldots, n$. Hence, m_{e_j} is the basic probability assignment on Θ.

(11) There are independent parameters which we have to fuse into integrated one; we make use of Dempster's rule of combination based on Definition 14. Then, the final BPA of the alternative x_i obtained is viewed as alternative's belief measure. In the end, the candidate alternatives are positioned dependent upon the final BPAs of the alternatives x_i and the ideal one can be acquired.

3.2.1 Experiment

Example 3 Suppose there is a decision-making problem for which (F, D) represents fuzzy soft set and $\Theta = \{x_1, x_2, x_3\}$ is the frame of discernment along with $D = \{e_1, e_2, e_3, e_4, e_5\}$ as the set of parameters. Following steps are followed to solve this experiment.

(1) Forming the matrix $D = (d_{ij})_{n\times n}$ brings about by fuzzy soft set over Θ:

$$D = \begin{bmatrix} 0.85 & 0.73 & 0.26 & 0.32 & 0.75 \\ 0.56 & 0.82 & 0.76 & 0.64 & 0.43 \\ 0.84 & 0.55 & 0.82 & 0.53 & 0.47 \end{bmatrix}.$$

(2) Formulate $\bar{D}$, the information structure image matrix is

$$\bar{D} = \begin{bmatrix} 0.3778 & 0.3476 & 0.1413 & 0.2148 & 0.4545 \\ 0.2489 & 0.3905 & 0.4130 & 0.4295 & 0.2606 \\ 0.3733 & 0.2619 & 0.4457 & 0.3557 & 0.2848 \end{bmatrix}.$$

(3) The uncertainty measurement of the parameters $e_j (j = 1, 2, 3, 4, 5)$ is calculated using Eq. (9) as

$V(e_1) = 0.2675,\ V(e_2) = 0.153,\ V(e_3) = 0.2428,$
$V(e_4) = 0.0378,\ V(e_5) = 0.0452.$

(4) Normalize the above uncertainty of the parameters using Eq. (10): $\bar{V}(e_1) = 0.3582$, $\bar{V}(e_2) = 0.2057$, $\bar{V}(e_3) = 0.3250$, $\bar{V}(e_4) = 0.0507$, $\bar{V}(e_5) = 0.0605$.

(5) Establish $P = (p_{jk})_{n\times n}$, the fuzzy preference relation matrix is

$$P = \begin{bmatrix} 0.5 & 0.3831 & 0.4486 & 0.4839 & 0.4683 \\ 0.6169 & 0.5 & 0.5671 & 0.6016 & 0.5865 \\ 0.5514 & 0.4329 & 0.5 & 0.5355 & 0.5199 \\ 0.5161 & 0.3984 & 0.4645 & 0.5 & 0.4843 \\ 0.5317 & 0.4135 & 0.4801 & 0.5157 & 0.5 \end{bmatrix}.$$

(6) Construct the consistency matrix $p = (\overline{p_{jl}})_{n\times n}$ as

$$p = \begin{bmatrix} 0.5 & 0.3824 & 0.4488 & 0.4841 & 0.4686 \\ 0.6176 & 0.5 & 0.5665 & 0.6017 & 0.5862 \\ 0.5512 & 0.4335 & 0.5 & 0.5353 & 0.5197 \\ 0.5159 & 0.3983 & 0.4647 & 0.5 & 0.4845 \\ 0.5314 & 0.4138 & 0.4803 & 0.5155 & 0.5 \end{bmatrix}.$$

(7) Produce the credibility value of parameter $e_j (j = 1, 2, 3, 4, 5)$ by using Eq. (14) as

$\mathrm{Cred}(e_1) = 0.2173,\ \mathrm{Cred}(e_2) = 0.1702,$
$\mathrm{Cred}(e_3) = 0.1968,\ \mathrm{Cred}(e_4) = 0.2109,$
$\mathrm{Cred}(e_5) = 0.2047.$

(8) On the basis of consistency matrix, we modulated the normalised uncertainty of parameter e_j using Eq. (15) $(j = 1, 2, 3, 4, 5)$ as

$\mathrm{MV}(e_1) = 0.077\,824,\ \mathrm{MV}(e_2) = 0.035\,020,$
$\mathrm{MV}(e_3) = 0.063\,967,\ \mathrm{MV}(e_4) = 0.010\,689,$
$\mathrm{MV}(e_5) = 0.012\,370.$

(9) Normalize the modulated uncertainty calculated above as

$\overline{\mathrm{MV}}(e_1) = 0.389\,300,\ \overline{\mathrm{MV}}(e_2) = 0.175\,209,$
$\overline{\mathrm{MV}}(e_3) = 0.320\,033,\ \overline{\mathrm{MV}}(e_4) = 0.053\,470,$
$\overline{\mathrm{MV}}(e_5) = 0.061\,919.$

(10) Now, compute the basic probability assignments of alternatives with respect to e_j using Eqs. (17)–(19) which can be seen from Table 2.

(11) Merge the BPAs of alternatives by the use of Formula (14) to get the fusing results which are going to be known as the belief measures of alternatives exhibited by Table 3 and Fig. 2.

(12) On the basis of belief values of alternatives, their final ranking can be obtained. It has been observed that $x_2 > x_3 > x_1$. Hence, the maximum value showed that

Table 2 BPA of x_i with respect to e_j.

BPA	e_1	e_2	e_3	e_4	e_5
$m(x_1)$	0.2307	0.2867	0.0961	0.2033	0.4264
$m(x_2)$	0.1520	0.3221	0.2808	0.4065	0.2444
$m(x_3)$	0.2280	0.2160	0.3031	0.3367	0.2672
$m(\Theta)$	0.3893	0.1752	0.3200	0.0535	0.0620

Table 3 Alternatives' belief measures in two unlike ways.

Method	Bel (x_1)	Bel (x_2)	Bel (x_3)
Grey relational analysis method	0.0745	0.1013	0.0990
Proposed method	0.0212	0.0325	0.0275

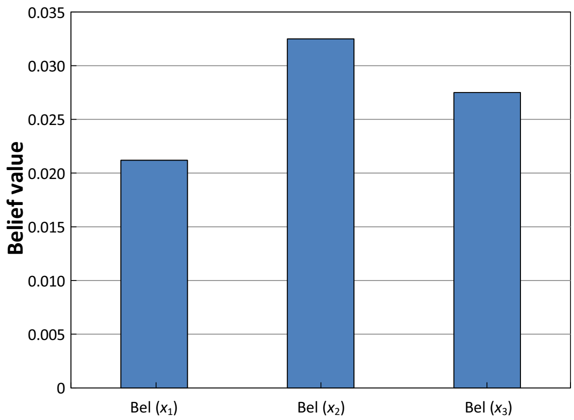

Fig. 2 Interpretation of belief values for experiment.

ideal choice is x_2 which can be easily seen through Table 4. Hence, the maximum value showed that ideal choice is x_2 and it can be easily seen through Fig. 2 as well.

Also, we compare our proposed method with the grey relational approach by comparing the belief values of alternatives along with the performance measure. It has been shown in Table 4. The uncertainty's belief measure fell to 0.000 104 attained from suggested method. It has also been observed that our proposed method can reduce the uncertainty and decision-making level as compared to grey relational method. We likewise compute the measure of performance which indicates that our technique is more exact and efficient than the other method.

3.2.2 Application

As we all know, the concept of uncertainty plays an important role in taking decisions in real-life problems. It is very difficult for human beings to take decisions with accuracy and efficiency in real-life problems. Fuzzy soft sets handle this problem efficiently with more accuracy. Hence, considering the real-life decision making problem, it can easily be shown that the given method is more efficient and accurate. We also compare our experimental result with grey relational analysis method. Fuzzy soft sets are extensively used in medical diagnosis field. Nowadays, the whole world is suffering from severe disease named corona virus. It becomes very difficult for doctors to detect that which type of disease a patient is suffering from. By using this proposed method, the ideal choice can be made out.

Table 4 Comparison of different methods in Example 3.

Method	Ranking	Optimal value	$m(\Theta)$	γ (Performance measure)
Grey relational approach	$x_2>x_3>x_1$	x_2	0.0223	1.631
Proposed method	$x_2>x_3>x_1$	x_2	0.000 103 5	1.832

Example 4 Suppose that the universal set Θ consists of three types of diseases, namely, {dengue, corona virus, cholera} represented as $\{x_1, x_2, x_3\}$ and $G=$ {high fever, cough, shortness of breath, nausea, vomiting, watery diarrhoea, rapid heart rate, physical examination, laboratory, rest} $=\{g_1, g_2, g_3, g_4, g_5, g_6, g_7, h_8, h_9, h_{10}\}$ represents the set of parameters.

Let I_1 and I_2 be the two subsets of G given by $I_1=\{g_1, g_2, g_3, g_4, g_5, g_6, g_7\}$ and $I_2=\{h_8, h_9, h_{10}\}$ where (F, I_1) is the fuzzy soft set representing "symptoms of diseases" and (F, I_2) defines "decision-making tools". Tables 5 and 6 represent these two fuzzy soft sets.

Let us take an example of a patient who puts up with a disease having two symptoms—{high fevers, shortening of breathe}. A doctor needs to make the most suitable diagnosis regarding symptoms, namely, {physical examination, lab investigation, history}. To find out the exact solution, $(F, I_1) \cap (F, I_2)$ is constructed in Table 7. There are three diseases $\{x_1, x_2, x_3\}$, and $k_1=(g_1, h_1), k_2=(g_1, h_2), k_3=(g_1, h_3), k_4=(g_3, h_1)$, $k_5=(g_3, h_2)$, and $k_6=(g_3, h_3)$ represent pair of one symptom and one decision-making tool. Here, Θ is FOD defined by Eq. (11) and $E=\{k_1, k_2, k_3, k_4, k_5, k_6\}$ is the set of parameters.

Following steps are to be followed to solve this numerical problem:

(1) Forming the matrix $D=(d_{ij})_{n\times n}$ bring about by (F, I) over Θ as below:

Table 5 Fuzzy soft set (F, I_1).

Alternative	g_1	g_2	g_3	g_4	g_5	g_6	g_7
x_1	0.50	0.70	0.00	0.30	0.20	0.80	0.90
x_2	0.40	0.60	0.90	0.00	0.90	0.70	0.00
x_3	0.60	0.00	0.10	0.40	0.00	0.70	0.00

Table 6 Fuzzy soft set (F, I_2).

Alternative	h_8	h_9	h_{10}
x_1	0.40	0.70	0.50
x_2	0.20	0.10	0.90
x_3	0.10	0.60	0.30

Table 7 Fuzzy soft set (F, I).

Alternative	k_1	k_2	k_3	k_4	k_5	k_6
x_1	0.40	0.50	0.50	0.00	0.00	0.00
x_2	0.20	0.10	0.40	0.20	0.10	0.90
x_3	0.10	0.60	0.30	0.10	0.10	0.10

$$D=\begin{bmatrix}0.40&0.50&0.50&0.00&0.00&0.00\\0.20&0.10&0.40&0.20&0.10&0.90\\0.10&0.60&0.30&0.10&0.10&0.10\end{bmatrix}.$$

(2) Formulate $\bar{D}$, the information structure image matrix is

$$\bar{D}=\begin{bmatrix}0.5714&0.4167&0.4167&0.00&0.00&0.00\\0.2857&0.0833&0.3333&0.6667&0.5&0.90\\0.1429&0.5&0.25&0.3333&0.5&0.10\end{bmatrix}.$$

(3) The uncertainty measurement of the parameters $k_j(j=1,2,3,4,5,6)$ using Eq. (9) is as below:

$$V(k_1)=0.156\,38,\quad V(k_2)=0.078\,18,$$
$$V(k_3)=0.024\,24,\quad V(k_4)=0.620\,05,$$
$$V(k_5)=0.766\,63,\quad V(k_6)=0.828\,95.$$

(4) Normalize the above uncertainty of the parameters by using Eq. (10) as

$$\bar{V}(k_1)=0.063\,198,\quad \bar{V}(k_2)=0.031\,595,$$
$$\bar{V}(k_3)=0.009\,796,\quad \bar{V}(k_4)=0.2505,$$
$$\bar{V}(k_5)=0.309\,821,\quad \bar{V}(k_6)=0.335\,006.$$

(5) Establish $P=(p_{jk})_{n\times n}$, the fuzzy preference relation matrix is

$$P=\begin{bmatrix}0.5&0.5246&0.5473&0.4889&0.5324&0.5615\\0.4754&0.5&0.5228&0.4643&0.5078&0.5372\\0.4527&0.4772&0.5&0.4417&0.4850&0.5144\\0.5111&0.5357&0.5583&0.5&0.5434&0.5724\\0.4676&0.4922&0.5150&0.4566&0.5&0.5294\\0.4385&0.4628&0.4856&0.4276&0.4706&0.5\end{bmatrix}.$$

(6) Construct the consistency matrix $p=(\overline{p_{jl}})_{n\times n}$ as

$$p=\begin{bmatrix}0.5&0.5245&0.5472&0.4889&0.5323&0.5616\\0.4755&0.5&0.5228&0.4644&0.5078&0.5371\\0.4528&0.4773&0.5&0.4417&0.4851&0.5143\\0.5111&0.5356&0.5583&0.5&0.5434&0.5726\\0.4677&0.4922&0.5149&0.4566&0.5&0.5293\\0.4384&0.4629&0.4857&0.4274&0.4707&0.5\end{bmatrix}.$$

(7) Produce the credibility value of parameter $k_j(j=1,2,3,4,5,6)$ using Eq. (14) as under

$$\mathrm{Cred}(k_1)=0.1581,\quad \mathrm{Cred}(k_2)=0.1663,$$
$$\mathrm{Cred}(k_3)=0.1738,\quad \mathrm{Cred}(k_4)=0.1544,$$
$$\mathrm{Cred}(k_5)=0.1688,\quad \mathrm{Cred}(k_6)=0.1786.$$

(8) On the basis of consistency matrix, we modulated the normalized uncertainty of parameter $k_j(j=1,2,3,4,5,6)$ by using Eq. (15) as

$$\mathrm{MV}(k_1)=0.0099,\quad \mathrm{MV}(k_2)=0.005\,253,$$
$$\mathrm{MV}(k_3)=0.017\,03,\quad \mathrm{MV}(k_4)=0.038\,68,$$
$$\mathrm{MV}(k_5)=0.052\,313,\quad \mathrm{MV}(k_6)=0.059\,834.$$

(9) Normalize the modulated uncertainty calculated above as follows:

$$\overline{\mathrm{MV}}(k_1)=0.059\,544,\quad \overline{\mathrm{MV}}(k_2)=0.031\,306.$$
$$\overline{\mathrm{MV}}(k_3)=0.010\,149,\quad \overline{\mathrm{MV}}(k_4)=0.230\,59,$$
$$\overline{\mathrm{MV}}(k_5)=0.311\,793,\quad \overline{\mathrm{MV}}(k_6)=0.356\,619.$$

(10) Now, compute the basic probability assignments of alternatives with respect to the parameters k_j using Eqs. (17) – (19) which can be seen from Table 8.

(11) By the use of Definition 14, we combine BPAs of alternatives to get the fusing results which are known as the belief measures of alternatives. This is conveyed by Table 9 and Fig. 3.

On the basis of belief values of alternatives, their final ranking can be obtained. It has been observed that $x_2 > x_3 > x_1$. Hence, the maximum value showed that ideal choice is x_2 which can be easily seen through Fig. 3.

Additionally, when we solved this example with grey relational analysis given by Li et al.[20], it has been observed that our method can decrease the uncertainty to greater level which can be seen by comparing the uncertainty's belief measures through Table 10. We also calculated the performance measure γ for both methods. It has been found that our method is more accurate and efficient as compared to grey relational approach.

Table 8 BPA of x_i with respect to k_j.

BPA	k_1	k_2	k_3	k_4	k_5	k_6
$m(x_1)$	0.5374	0.4037	0.4125	0.00	0.00	0.00
$m(x_2)$	0.2687	0.0807	03299	0.5130	0.3441	0.5790
$m(x_3)$	0.1344	0.4843	0.2475	0.2564	0.3441	0.0644
$m(\Theta)$	0.0595	0.0313	0.0101	0.2306	0.3118	0.3566

Table 9 Alternatives' belief measures in two unlike ways.

Method	Bel (x_1)	Bel (x_2)	Bel (x_3)
Grey relational analysis method	0.0295	0.1260	0.0578
Proposed method	0.004 058	0.008 227	0.004 996

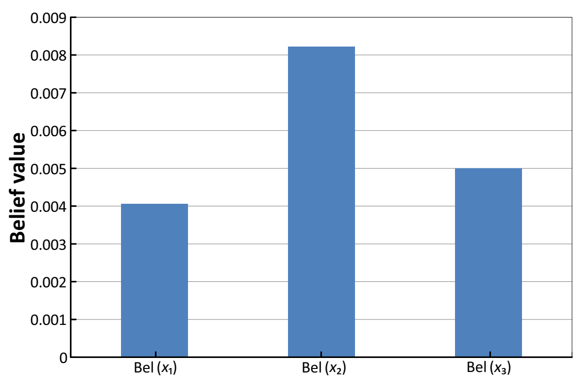

Fig. 3 Belief values of alternatives for the proposed method.

Table 10 Comparison of different methods in Example 4.

Method	Ranking	Optimal value	$m(\Theta)$	γ (Performance measure)
Grey relational approach	$x_2 > x_3 > x_1$	x_2	0.01468	1.5919
Proposed method	$x_2 > x_3 > x_1$	x_2	6.9578×10^{-7}	2.2698

4 Conclusion

In this paper, uncertainty of the parameters is measured in the form of total degree of fuzziness. By using this method, a doctor can easily detect the disease according to their respective symptoms and diagnosis. In the given application, there are three diseases and six pairs of symptoms and decision-making tools. It has been shown that the belief measure of uncertainty fell to 6.9578×10^{-7} from 0.014 683 in our proposed method. Thus, it can easily be deduced that the proposed method was progressively productive and reduced the level of uncertainty of the parameters and it is much more accurate to evaluate the symptoms of corona within a patient. The limitation of this work is that it does not consider other types of uncertainties like non-specificity, discord, strife, etc.

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Pallvi Sharma received the MS degree in mathematics from University of Jammu, India in 2017. She is currently pursuing the PhD degree in mathematics, particularly in topology (fuzzy topology) from Lovely Professional University, Punjab, India. Her area of interest lies in topology and fuzzy topology.

Nitin Bhardwaj is currently working as an associate professor and assistant dean in Lovely Professional University, Punjab, India. He received the PhD degree in mathematics from NIMS University, Jaipur, India in 2016. He has specialization in the field of fuzzy topology.