

Q -Factor for Multiport Antennas and Achievable Bandwidth Estimation

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Abstract—This article studies the use of quality factor Q as a tool to design multiport antenna structures with matching networks. Approximative formulas for computing the Q -factor from the port impedances of multiport antennas are presented. We show that the achievable performance level can be reliably estimated based on the Q -factor without the need for computationally heavy and time-consuming matching network optimization. The efficiency of the proposed method is confirmed with a practical two-port antenna cluster design for smartphones operating in the 0.7–1.0 GHz frequency band. The results show that the proposed port impedance-based Q -factor formulas provide an efficient tool to accelerate the design process of multiport antennas.

Index Terms—Antenna impedance, matching network design, multiport antennas, quality factor.

I. INTRODUCTION

WITH the increasing demands and requirements for antennas in wireless devices, the role of effective design tools and methods is becoming more and more important. A good example of this is antenna systems in modern smartphones. With the constantly increasing screen-to-body ratio, the antennas covering numerous frequency bands need to be realized in smaller and smaller physical volumes. This, as well known, leads to significant challenges, especially in the sub-1 GHz frequencies. As a consequence, many of the recent mobile antenna designs with very small ground clearances, in the range of 2 mm, utilize lumped component matching networks [1]–[4], which complicates the design process.

While multiport antennas have mostly been used, e.g., to realize beam steering, completely new approaches have emerged during the last years. One such approach is the multiport antenna cluster technique that utilizes several coupled active antenna feeds combined with an adaptive excitation scheme [5]. This allows more freedom to control fields in the antenna structure and helps to reach the challenging design

goals [6]. While introducing new opportunities, multiport antennas generally have similar challenges as single-feed antennas, one of the major ones being the bandwidth, its analysis, evaluation, and realization [7]–[9]. As with traditional antennas, matching networks can also be used to improve the performance of multiport antenna clusters [10]. However, the use of antenna clusters introduces also new design challenges. For example, the coupling of the elements in a cluster has an important role in the operation and, unlike in traditional antenna arrays, should not generally be minimized [11]. Rather, coupling introduces another degree of freedom that has to be optimized in the design process.

As a consequence, designing multiport antennas is very challenging when antenna structure, matching networks, and the feeding arrangement all need to be simultaneously taken into account. For example, antennas working well on a narrow band may have poor performance when matched for wider bandwidth. On the other hand, antennas with initially worse performance may be very competitive with proper matching. Especially, the matching network optimization is time-consuming since practically applicable analytical methods for wideband matching do not exist even for single-port antennas. Therefore, numerical optimization methods have to be used. Some tools and methods for determining bandwidth estimations for single-port antennas with simple matching circuits exist [12], [13], but theory and computationally efficient tools for fast evaluation of bandwidth capacity of multiport antenna structures are missing. One of the new challenges related to antenna clusters is that the bandwidth capacity is a function of the frequency-dependent excitation vectors, and this needs to be taken into account properly.

The quality factor Q has been used as a tool to measure bandwidth and its limits for many decades [14]–[16], and new methods for Q -factor calculations have still been developed in the recent years [17]–[21]. Limits for Q -factors for certain simple antennas, such as electric and magnetic dipoles, different methods to find optimal Q -factor, and the corresponding surface current distributions, have been presented [22], [23]. Q -factor is generally used to define an inverse relation between Q and the fractional bandwidth FBW

$$\text{FBW} \propto \frac{1}{Q}. \quad (1)$$

This holds well for electrically small and narrowband antennas, but care must be taken when applying these results for

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wideband and multiresonant antennas [24], [25]. From the two main approaches for calculating the Q -factor, the electromagnetic (EM) fields, or an approximation from the input impedance, the impedance approach is generally known to better describe the bandwidth performance [17]. However, the chosen criteria for determining the bandwidth have a significant effect on these results and how they should be interpreted. For example, the half-power bandwidth may not always be properly predicted from the Q -factor, but the connection between Q and FBW still holds for small enough bandwidth and matching criteria, even for multiple resonances [24]. In practical antenna design problems, however, the bandwidth and matching level cannot be freely chosen, and thus, utilizing the Q -factor can be difficult. There are not many cases where the Q -factor would have been beneficial in designing practical wideband antennas incorporating matching networks. Cases for multiport antennas are even rarer.

In this work, we derive formulas for computing the Q -factor for multiport antennas using the antenna port impedance matrix. Instead of trying to find the analytical connection for the bandwidth and Q -factor or finding counterexamples to disprove this, we study the possibility to utilize the Q -factor as a practical tool to accelerate the design process of multiport antennas for, e.g., smartdevices.

We introduce new formulas to calculate the Q -factor with proper input signals that are taken into account. We show that the proposed multiport Q -factor, Q_{ZM} , gives a reliable estimation of the matching potential of the system, that is, how the system performance will be with realistic matching components and properly fed input signals. With the proposed method, a large number of different antenna structures can be evaluated extremely fast, thus providing a new tool for designers of multiport antennas to accelerate the otherwise laborious and slow design process. The main novelty of this work can be summarized as follows.

- 1) We introduce a port impedance-based Q -factor formula and approximations for this, thus allowing a quick determination of optimal feeding signals and corresponding Q -factor for multiport antennas.
- 2) We verify that these new Q -factor concepts are reliable for realistic multiresonance antennas.
- 3) We demonstrate how these new tools can be used to accelerate practical antenna design.

II. THEORY

A major challenge in modern antenna design is to maximally reach the full performance potential of the available structure. Compared to traditional single-port configurations, the multiport antenna cluster technique with actively fed ports can provide a more efficient approach to obtain that goal. Due to the effect of the coupling between the ports, however, the operation of multiport antennas is not as straightforward as with single-feed antennas. Hence, new advanced design tools are required.

In designing and evaluating the antenna efficiency, as well as calculating the Q -factor, it is important to properly take into account the active input impedances, i.e., the feeding signals,

of the feeds. In this section, we first summarize the required formulas for multiport antenna efficiency calculations and then introduce the multiport-specific Q -factor calculation.

A. Design of Multiport Antennas

The antenna cluster technique is based on the collaborative use of two or more coupled antenna elements or elements with multiple feeds [5]. When these elements are fed with a feeding weight vector \mathbf{a} with proper amplitude and phase elements

$$a_n = A_n e^{j\phi_n} = A_n \angle \phi_n \quad (2)$$

the reflected waves from the ports partly cancel each other, and thus, the radiated power is increased.

The operation of an antenna cluster can be characterized with the total active reflection coefficient (TARC) calculated from the scattering parameter matrix \mathbf{S} [26]

$$\text{TARC} = \sqrt{\frac{\mathbf{a}^H (\mathbf{S}^H \mathbf{S}) \mathbf{a}}{\mathbf{a}^H \mathbf{a}}}, \quad (3)$$

which corresponds to the traditional reflection coefficient of single-port antennas generalized for multiport antennas, or the corresponding efficiency

$$\eta = \frac{\mathbf{a}^H \mathbf{D} \mathbf{a}}{\mathbf{a}^H \mathbf{a}}. \quad (4)$$

Here, \mathbf{D} is the radiation matrix that can be calculated either from the scattering parameters as

$$\mathbf{D} = \mathbf{I} - \mathbf{S}^H \mathbf{S} \quad (5)$$

or from the far-field patterns \mathbf{F} of the elements [27]

$$D_{ij} = \frac{1}{4\pi} \iint_{4\pi} \mathbf{F}_i \cdot \mathbf{F}_j^* d\Omega. \quad (6)$$

For lossless or low-loss structures, these two approaches give practically the same result. For lossy cases, (6) should be used to properly take the losses into account [27]. The maximum efficiency and the corresponding feeding weights \mathbf{a} for each frequency can be solved from (4) as the largest eigenvalue and eigenvector of the radiation matrix [28]

$$\eta_{\max} = \max\{\text{eig}(\mathbf{D})\}. \quad (7)$$

B. Antenna Q -Factor

Next, we introduce formulas for computing the antenna Q -factor using the method-of-moment (MoM) matrix. Since the obtained Q -factor is based on the EM formulation of the problem, we call it a *field-based* Q -factor.

Let S denote the surface of a lossless perfect electric conductor (PEC) antenna. By $\mathbf{Z}^{\text{MoM}} = \mathbf{R}^{\text{MoM}} + j\mathbf{X}^{\text{MoM}}$, we denote MoM matrix due to the electric field integral equation (EFIE) formulation of the structure. The field-based antenna Q -factor can be defined as [23]

$$Q = 2\omega \frac{\max(W_e, W_m)}{P_{\text{rad}}} \quad (8)$$

where the electric and magnetic stored energies are given by

$$W_e = \frac{1}{8} \mathbf{J}^H \left(\frac{\partial \mathbf{X}^{\text{MoM}}}{\partial \omega} - \frac{\mathbf{X}^{\text{MoM}}}{\omega} \right) \mathbf{J} \quad (9)$$

$$W_m = \frac{1}{8} \mathbf{J}^H \left(\frac{\partial \mathbf{X}^{\text{MoM}}}{\partial \omega} + \frac{\mathbf{X}^{\text{MoM}}}{\omega} \right) \mathbf{J} \quad (10)$$

and P^{rad} is the radiated power

$$P^{\text{rad}} = \frac{1}{2} \mathbf{J}^H \mathbf{R}^{\text{MoM}} \mathbf{J}. \quad (11)$$

\mathbf{J} is the vector of the coefficients of the basis function approximation of the current density on the antenna surface, and $(\cdot)^H$ denotes the complex conjugate transpose.

Equation (8) can be used to predict the antenna Q -factor for both single and multiport antennas. In the latter case, it is used as follows. If α_n , $n = 1, \dots, N$, are the complex weighting coefficients of the N ports of the antenna cluster, for example, voltages or currents, the current density \mathbf{J} of the cluster is given by

$$\mathbf{J} = \sum_{n=1}^N \alpha_n \mathbf{J}_n \quad (12)$$

where N is the number of the ports of the cluster. Here, \mathbf{J}_n is the current density on the antenna with unit input at port n and zero input on the other ports.

Since the Q -factor definition (8) contains the MoM matrix and its derivatives in (9) and (10), which are very large for complex antennas structures, the evaluation of this field-based Q -factor can be computationally very intense. This limits the usability of (8) for practical antenna design and optimization tasks in which several antenna geometries have to be tested across a wide frequency band or multiple frequency bands. In Section II-C, we discuss *impedance-based* Q -factor that gives a more efficient formula for computing the antenna Q -factor.

C. Impedance-Based Antenna Q -Factor

As discussed, e.g., in [17], [18], and [20], the Q -factor of an antenna can be approximated using the input impedance of the antenna. For a single-port antenna, the *impedance-based* Q -factor is given by [17], [20]

$$\begin{aligned} Q_Z &= \omega \frac{|R' + j(X' + |X|/\omega)|}{2R} \\ &= \omega \frac{\sqrt{(R')^2 + (X' + |X|/\omega)^2}}{2R} \end{aligned} \quad (13)$$

where R and X are the real and imaginary parts of the input impedance Z , and R' and X' are their derivatives with respect to ω . Following [29], this can be generalized for multiport antennas as:

$$\begin{aligned} Q_{ZM} &= \omega \frac{\max\{|\mathbf{I}^H \mathbf{R}' \mathbf{I} + j\mathbf{I}^H (\mathbf{X}' \pm \mathbf{X}/\omega) \mathbf{I}|\}}{2\mathbf{I}^H \mathbf{R} \mathbf{I}} \\ &= \omega \frac{|\mathbf{I}^H \mathbf{R}' \mathbf{I} + j(\mathbf{I}^H \mathbf{X}' \mathbf{I} + |\mathbf{I}^H \mathbf{X} \mathbf{I}|/\omega)|}{2\mathbf{I}^H \mathbf{R} \mathbf{I}}. \end{aligned} \quad (14)$$

Here, $\mathbf{Z} = \mathbf{R} + j\mathbf{X}$ is the impedance matrix, $\mathbf{I} = [I_1, \dots, I_N]$ contains the input currents of the ports, and $(1/2)\mathbf{I}^H \mathbf{R} \mathbf{I}$ gives radiated power in the lossless case.

We note that formula (14) differs from the one proposed in [29]. In order to have a formula that is consistent with the corresponding formula for single-port antennas, we have added the derivative of the real part of the impedance matrix to the proposed formula (14).

D. Approximations for Q -Factor

As illustrated, e.g., in [11], the proper choice of the feeding weights (port input signals) is crucial for the optimal performance of the antenna cluster. In Section II-A, we reviewed how these weights are determined to maximize the efficiency. This choice, as will be shown later, however, does not necessarily lead to the minimal Q -factor. In this section, we discuss the choice of the feeding weights so that the antenna Q -factor can be minimized.

The ideal way would be to solve the feeding coefficients from an eigenvalue equation similar to (4) and (7) corresponding to a general Rayleigh quotient

$$\frac{\mathbf{I}^H \mathbf{M} \mathbf{I}}{\mathbf{I}^H \mathbf{N} \mathbf{I}}. \quad (15)$$

Due to the form of formula (14), this is not possible. Therefore, we propose some approximations so that the Q -factor can be expressed in the desired form. This would provide us a computationally efficient way to find the optimal weights using a generalized eigenvalue equation.

The first option is to use a similar approximation as has been used in several publications handling single-port antennas, i.e., assume the effect of the term \mathbf{R}' so small that it can be neglected. This can be a fairly accurate approximation at least for antennas with very small clearances in modern smartphones. Using this, the impedance-based Q -factor (14) can then be approximated as [29]

$$Q_{ZM}^{(1)} \approx \frac{\omega}{2} \frac{|\mathbf{I}^H \mathbf{X}' \mathbf{I} + |\mathbf{I}^H \mathbf{X} \mathbf{I}|/\omega|}{\mathbf{I}^H \mathbf{R} \mathbf{I}}. \quad (16)$$

Since this form can be written as a general Rayleigh quotient by separating the input current terms and the impedance matrix terms

$$Q_{ZM}^{(1)} = \frac{\omega \max\{|\mathbf{I}^H (\mathbf{X}' \pm \mathbf{X}/\omega) \mathbf{I}|\}}{2 \mathbf{I}^H \mathbf{R} \mathbf{I}} \quad (17)$$

the optimal feeding weights can be solved as the eigenvector corresponding to

$$\min \left\{ \text{eig} \left(\mathbf{X}' \pm \frac{1}{\omega} \mathbf{X}, \mathbf{R} \right) \right\}. \quad (18)$$

Here, we need to calculate both options for the sign of the second term and then choose the correct one for each frequency point.

The second option is to calculate an upper limit for the impedance-based Q -factor of (14). To do this, we separate all three terms in the original equation as

$$Q_{ZM}^{(2)} \approx \frac{\omega}{2} \frac{|\mathbf{I}^H \mathbf{R}' \mathbf{I}| + |\mathbf{I}^H \mathbf{X}' \mathbf{I}| + |\mathbf{I}^H \mathbf{X} \mathbf{I}|/\omega}{\mathbf{I}^H \mathbf{R} \mathbf{I}}. \quad (19)$$

Similar to the first approximation, the Q -optimal weights can again be solved from a set of general eigenvalue equations

$$\min \left\{ \text{eig} \left(\pm \mathbf{R}' \pm \mathbf{X}' \pm \frac{1}{\omega} \mathbf{X}, \mathbf{R} \right) \right\}. \quad (20)$$

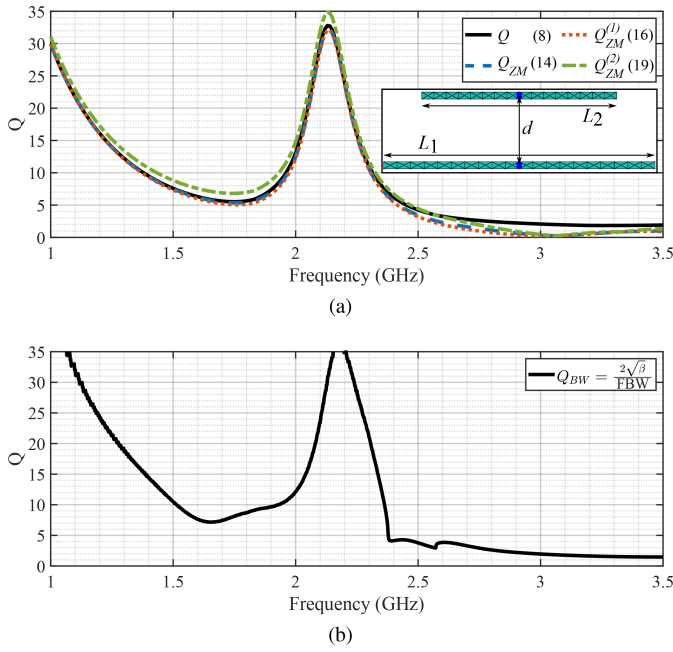


Fig. 1. (a) Field and impedance-based Q -factors for two dipoles at the distance of 20 mm. (b) Q_{BW} calculated from the exact matched bandwidth with $\sqrt{\beta} = 0.18$.

Although both of the proposed approximations require solving multiple eigenvalue equations, this does not significantly increase the computational complexity since the sizes of the matrices agree with the number of ports.

III. MULTI-PORT Q -FACTOR EVALUATION AND ANTENNA PERFORMANCE

In this section, we first compare the proposed Q -factor approximations with the impedance and field-based Q -factors. In order to study the connection between the Q -factor calculated for the antenna only and the efficiencies with proper matching networks applied, we also describe the tools used to design matching networks and compare the results for two-port antenna clusters.

A. Comparison of Different Q -Factor Formulas

To compare the proposed Q -factor approximations (16) and (19) with the impedance-based Q (14) and the field-based Q (8), we first consider a simple numerical example. We have two parallel dipole antennas of lengths $L_1 = 78$ mm and $L_2 = 56$ mm. The distance between the dipoles is $d = 20$ mm, and both dipoles are fed at their centers with voltage-gap generators. For MoM solution, the dipoles are modeled as thin, 2 mm wide PEC strips, and the integral equation is discretized using Galerkin's method with RWG basis and test functions.

Fig. 1(a) shows the field and impedance-based Q -factors for the two dipoles with equal input voltages at the ports. The input voltages are defined with a normalized vector with elements $V_n = 1/\sqrt{N}$, $n = 1, \dots, N$ for N input ports. The required port currents can then be obtained using the admittance matrix \mathbf{Y}

$$\mathbf{I} = \mathbf{Y}\mathbf{V}. \quad (21)$$

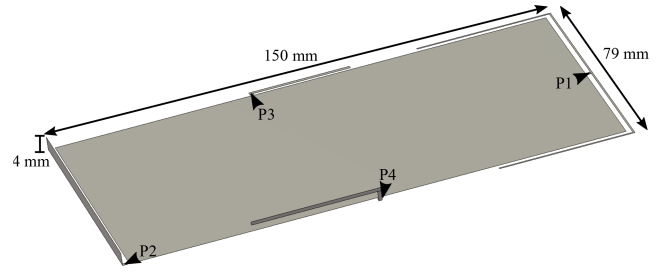


Fig. 2. Geometry and port locations of the four-port antenna system.

Also, the results obtained by the approximative formulas (16) and (19) are shown. We observe that the impedance-based Q -factor agrees fairly well with the field-based Q -factor provided that the Q -factor is large enough, approximately larger than five, as mentioned in [20]. The results also show that (16) agrees very well with (14) and that (19) is very close to the other two and is always above these as it should be.

Q -factors are often compared with equivalent quality factor Q_{BW} calculated from the exact matched bandwidth as

$$Q_{BW} = \frac{2\sqrt{\beta}}{\text{FBW}} \quad (22)$$

with chosen bandwidth criteria $\sqrt{\beta}$ [17], [24]. Fig. 1(b) shows the Q_{BW} for the two dipoles calculated with the method presented in [17, Sec. VI] with the matching level chosen as $\sqrt{\beta} = 0.18$, i.e., reflection coefficient of -15 dB. The results of Fig. 1 confirm that, with properly chosen bandwidth criteria, Q_{BW} agrees well with the other Q -factors also with multiport antennas.

As a more practical example with more complexity and multiple resonances, we consider a four-port antenna system. The antenna dimensions are 150 mm \times 79 mm \times 4 mm, corresponding roughly to the size of current smartphones. The structure consists of a PEC ground plane and four different antenna elements with 2 mm clearance on all four sides. The elements include both planar and 3-D parts, as illustrated in Fig. 2.

Fig. 3 displays the Q -factor of the four-port antenna. Similarly, as in the previous example, the input amplitudes of the ports are equal. Also, in this more complex case, the impedance-based Q -factor agrees fairly well with the impedance-based Q -factors on the frequency band 1.0–3.5 GHz. To better understand these results, Fig. 4 shows the real and imaginary parts of the diagonal elements of the impedance matrix \mathbf{Z} . The small discrepancies between the field- and impedance-based Q -factors appear mainly for small values of Q and high values of the derivatives of the input impedances, especially at higher frequencies where the electrical size of the structure is not anymore small. The most important results from this section are that the proposed approximations (16) and (19) are accurate and that the field- and impedance-based Q -factors of multiport antennas agree well with each other.

Next, our aim is to apply the Q -factor for estimating the achievable bandwidth of multiport antennas with matching

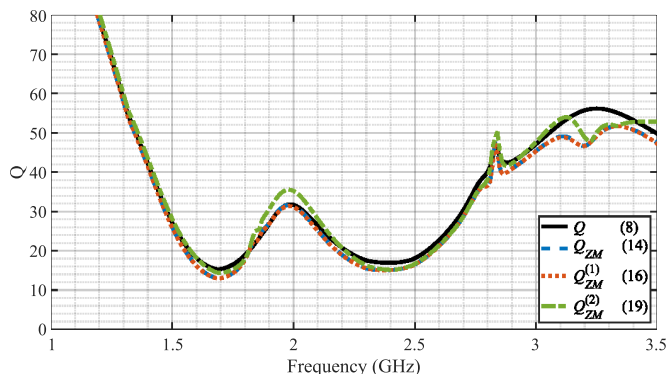


Fig. 3. Field and impedance-based Q -factors for the four-port antenna.

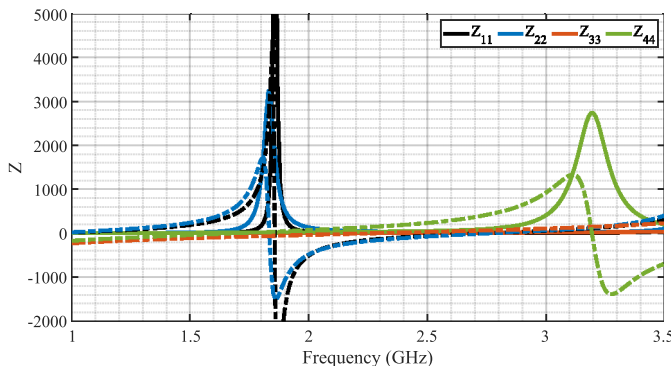


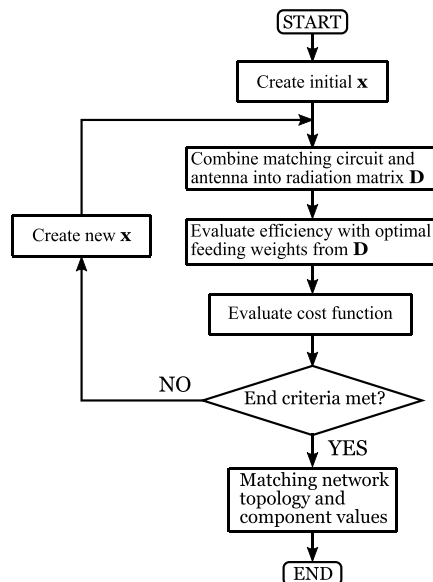
Fig. 4. Real (solid lines) and imaginary (dashed lines) parts of the diagonal elements of the impedance matrix for the four-port antenna.

networks in a realistic design. Since, in real design tasks, the optimization of the matching networks plays an important role, tools to design these networks are described next.

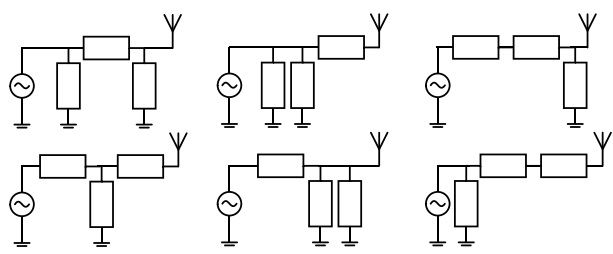
B. Matching Network Optimization With Antenna Clusters

As with traditional antennas, the antenna cluster technique can be used both with self-resonant antennas [26], [27] or antennas utilizing matching networks [10]. Finding matching networks for multiport antennas is a more complicated task than for traditional antennas since we have to simultaneously take into account the effect of the matching components on the efficiency and also their effect on the optimal complex feeding weights. To the best of our knowledge, any of the commercial optimization tools or other published impedance matching techniques cannot be directly applied since they do not properly take into account the effect of the feeding weights of multiport antennas.

The approach used in this work is to numerically optimize the matching networks and the feeding weights simultaneously in the same optimization process. Fig. 5(a) shows the flowchart of the applied optimization process. Vector \mathbf{x} includes the optimization variables, i.e., the component types (inductor/capacitor) and values. In this work, ideal components are used, but the same process can as well be used with realistic component models. In the most general case, also the topologies of the matching networks need to be optimized since they can have a significant effect on the operation



(a)



(b)

Fig. 5. (a) Flowchart of the matching network optimization for multiport antennas. (b) Matching network topologies considered in the optimization.

of the antennas [30]. Three-element topologies are used for all feeding ports, and the six possible topologies are shown in Fig. 5(b).

All of these factors can be handled in the same optimization process by including them into the vector \mathbf{x} so that it determines all the properties of the matching network that are optimized. As an example, \mathbf{x} for a two-port antenna with three-element matching circuits in each feeding port then has two variables for the topologies and six variables for the components. The first two variables define which of the six topologies of Fig. 5(b) are used for each of the feeds, and the rest defines the component values for each of the six matching network elements.

The cost function to be optimized is calculated from the efficiency of the antenna cluster for a given \mathbf{x} , as presented in Fig. 5(a). The efficiency from (4) is evaluated with the optimal feeding weights calculated from the radiation matrix \mathbf{D} and (7) with the effect of the matching networks embedded in \mathbf{D} [31]. Finally, the task is to find such \mathbf{x} , which maximizes the efficiency over the targeted frequency band or frequency bands. Different types of optimization algorithms can be used for this. One potential option, which is also used in this work, is the genetic algorithm. Realization of this algorithm and a more detailed description can be found, for example, from MATLAB [32].

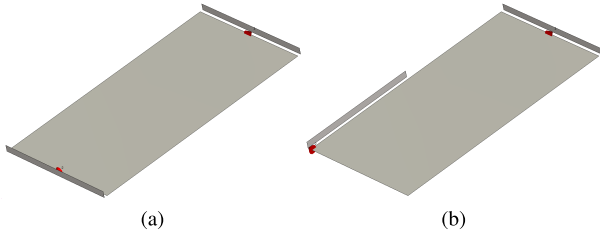


Fig. 6. Test antenna structures for the Q -factor evaluation. (a) Case 1: symmetric. (b) Case 2: nonsymmetric.

C. Q -Factor as a Figure-of-Merit for Multiport Antennas

The problem with the matching network optimization presented in Section III-B is that the iterative process is relatively slow and studying a large number of different antenna structures, even with modern computers, easily becomes too time-consuming. However, as shown in Section III-A, the Q -factor for a multiport antenna can be calculated from the antenna impedances with good accuracy. Therefore, it could potentially be used as a tool to predict the achievable performance level without the need for the time-consuming matching network optimization in each antenna iteration step, but it should only be done for the most potential designs according to the Q -results.

One challenge related to the multiport Q -factor is the effect of the feeding weights. In the results of Section III-A, the input signals were set to equal amplitude with the same phase. This, however, does not necessarily give the minimum Q -factor and properly present the bandwidth potential. To study this, we use two multiport antennas, as shown in Fig. 6, with the same external dimensions as the antenna structure of Fig. 2. In both cases, the first element is in one of the short edges of the ground plane with the feed line in the middle. In case 1, the second element is symmetrically copied to the other end, while case 2 uses a second element nonsymmetrically placed on the long edge of the ground plane.

The natural way to calculate the feeding weights for the Q -factor evaluation would be to use TARC-optimal weights calculated from the S-parameters of the antenna only using (7). In addition to this, we also evaluate the Q -factor with such feeding weights that minimize the Q -factor of (14). This is done by numerically minimizing the Q -factor at each frequency point using MATLAB's `fminunc`-function using the input signal amplitudes and phases as the optimization variables.

The Q -factors for both structures calculated with both methods and the matched efficiencies, i.e., efficiencies with optimized matching networks, are shown in Fig. 7. Fig. 8 presents the corresponding feeding amplitudes and phases for all the cases. These results show that both cases have almost identical efficiency on the 0.7–1.0 GHz band. In case 1, the feeding weights are identical for both methods, and also the resulting Q -factors are identical. In case 2, however, the feeding weights are noticeably different from each other. For the Q -optimal weights, more power is fed for port 1 at 0.7 GHz and roughly equal powers for both ports at 1.0 GHz. For the TARC-optimal weights, the situation is the opposite.

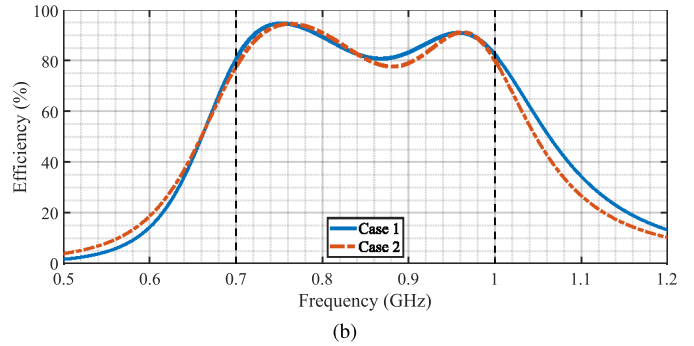
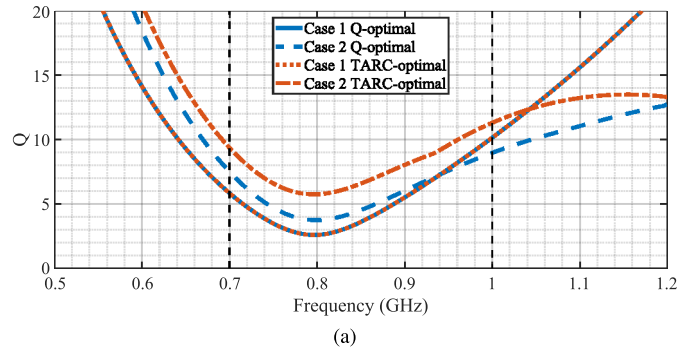


Fig. 7. (a) Q -factors calculated with Q -optimal and TARC-optimal feeding weights and (b) efficiencies with matching circuits for a symmetric (Case 1) and nonsymmetric (Case 2) antenna structures.

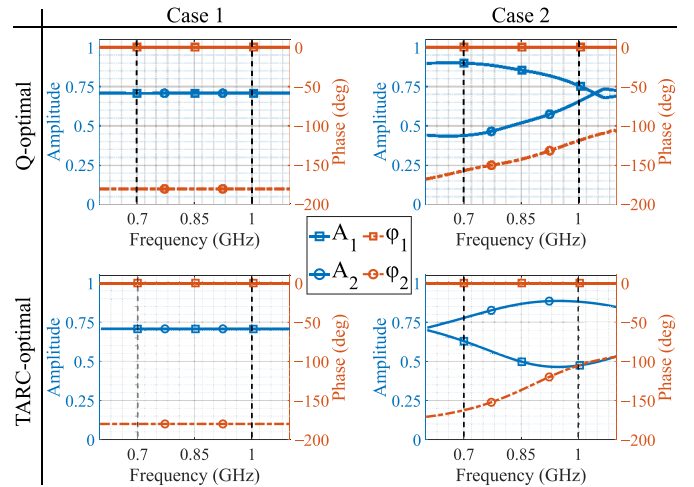


Fig. 8. TARC-optimal and Q -optimal feeding weights for the Q -factor evaluation for the structures of Fig. 6.

From almost equal powers at 0.7 GHz, the power of port 2 increases toward 1.0 GHz. As a result, also the Q -factors differ. With TARC-optimal weights, the Q -factor is somewhat larger than with Q -optimal weights. Because the efficiencies of cases 1 and 2 are almost equal, the Q -factors should also match and this happens only for the Q -optimal weights. Calculating the feeding weights based on TARC only takes into account the unmatched impedances and does not properly take into account how the impedance can be matched with proper matching networks. As a consequence, the resulting

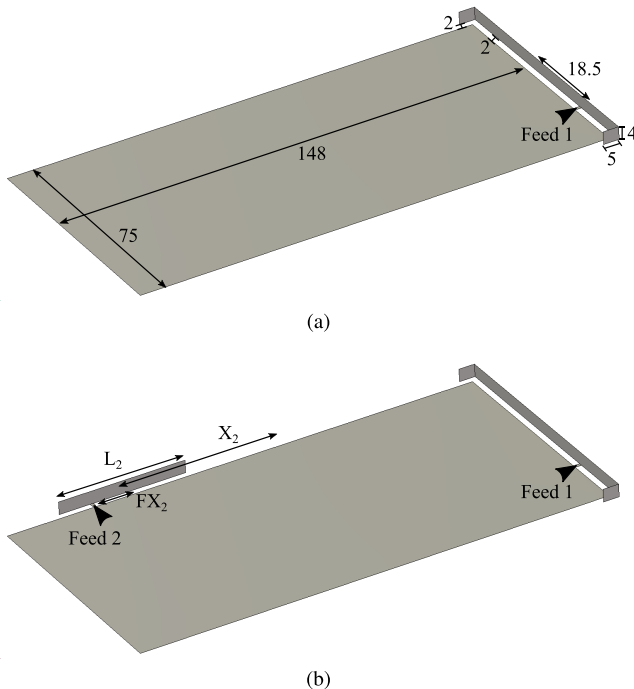


Fig. 9. (a) Reference antenna and (b) two-element antenna cluster structures. All dimensions in mm.

Q -factor does not present the bandwidth potential as well as with Q -optimal feeding weights in this case.

The results of this section show that, for evaluation of Q -factor for multiport antennas, the feeding weights need to be chosen correctly. If the weights are chosen such that, e.g., the TARC of the structure without the matching networks is minimized, the resulting Q -factor is not necessarily any more related to the property that we are finally interested in. This means that the achievable efficiency bandwidth of the structure with proper matching networks and feeding signals, calculated from (4) and (7), cannot be properly predicted from the Q -factor with TARC-optimal weights.

IV. PRACTICAL DESIGN EXAMPLE

Next, we introduce a practical application case for the computation of the Q -factor of a multiport antenna design and demonstrate how the proposed tools can be used in accelerating the design process.

A. Application of the Q -Factor Method

We investigate the proposed Q -factor evaluation and its relation to the achieved efficiencies with optimized matching networks by studying the antenna structures shown in Fig. 9. The reference antenna has one element in the short edge of the ground plane with feed offset from the middle toward the corner by 18.5 mm. In the two-element antenna cluster, the second element is placed on the long edge of the ground plane. The length of the element (L_2), offset of the element (X_2), and the offset of the feed (FX_2) are parameters whose effects on the Q -factor and the antenna performance are studied.

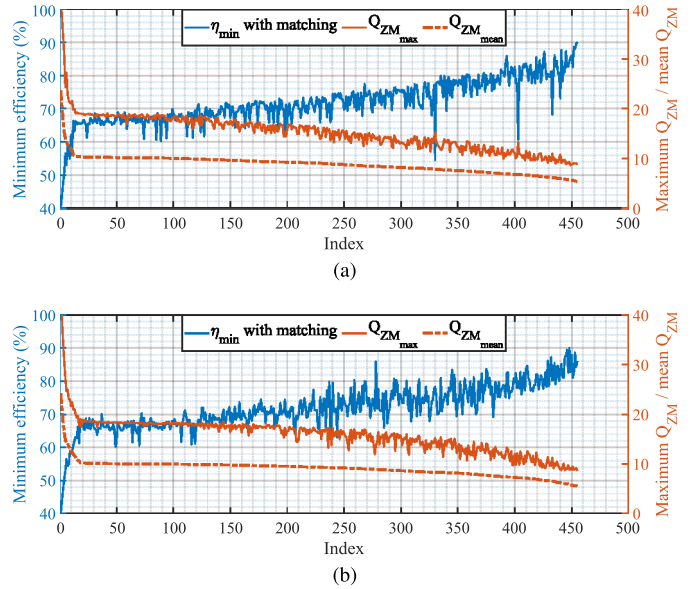


Fig. 10. Maximum and mean Q -factors from (14) and the corresponding minimum efficiency with matching circuits on the target band of 0.7–1.0 GHz for (a) numerically optimized feeding weights and (b) TARC-optimal feeding weights.

The procedure is given as follows. First, EM simulations are performed for a total of 455 different combinations of the three parameters of element 2. Parameter L_2 is varied from 20 to 80 mm, while X_2 and FX_2 are varied from the left maximum to the right one along the longer dimension of the ground plane. Next, Q -factors for these structures are evaluated using the following four approaches:

- 1) from (14) with Q -optimal feeding weights that are numerically optimized as explained in Section III-C;
- 2) from (14) with TARC-optimal feeding weights;
- 3) using approximation $Q_{ZM}^{(1)}$ (16) and feeding weights obtained with (18);
- 4) using approximation $Q_{ZM}^{(2)}$ (19) and feeding weights obtained with (20).

Also, matching networks for all these cases are optimized using the procedure of Section III-B and illustrated in Fig. 5 for the frequency band 0.7–1.0 GHz

The results for the Q -factor of (14) with numerically optimized Q -optimal feeding weights and TARC-optimal feeding weights are shown in Fig. 10 and for approximation $Q_{ZM}^{(1)}$ (16) and approximation $Q_{ZM}^{(2)}$ (19) in Fig. 11. The results show the maximum and mean Q -factors on the target band and the corresponding minimum efficiency with the matching networks applied. All the results have been sorted based on decreasing mean Q -factor for convenience, and the index on the x -axis refers to the 455 different antenna structures. The efficiency results are, therefore, the same in all the cases, but they correspond to different Q -values and can be sorted differently. All the results show a very strong correlation between the Q -factors and the achievable performance except for the case of TARC-optimal

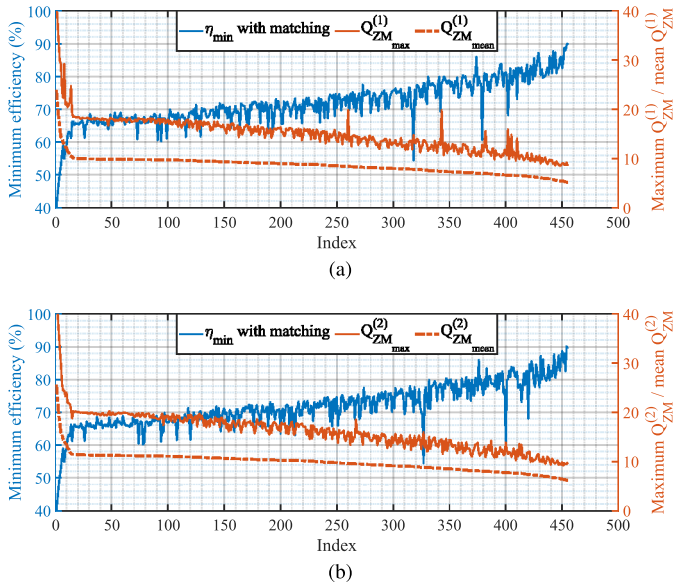


Fig. 11. Maximum and mean Q -factors and the corresponding minimum efficiency with matching circuits on the target band of 0.7–1.0 GHz for (a) approximation $Q_{ZM}^{(1)}$ (16) and (b) approximation $Q_{ZM}^{(2)}$ (19).

TABLE I

COMPUTATION TIMES REQUIRED FOR THE Q -FACTOR CALCULATIONS AND MATCHING NETWORK OPTIMIZATION FOR 455 DIFFERENT ANTENNA STRUCTURES

Operation	Time	Hardware
Matching network optimization	>24 h	2x Intel Xeon Gold 6146 24 cores
Q by optimization	30 min	Intel Core i5-8365U 4 cores
Q by approximation	20 s	Intel Core i5-8365U 4 cores

feeding weights, which shows more variations in the results than the other ones. Although there are few individual data points where the correspondence between these parameters is not so good, i.e., small Q does not agree with good matched efficiency, the general trend of decreasing Q seems to indicate higher efficiency extremely well.

To highlight the benefits of the Q -factor method, Table I presents the required computation time for the matching network optimization, Q -factor evaluation using numerical optimization for the feeding weights, and Q -factor evaluation using the proposed approximations. Also, the used CPU for each operation is presented. The time required for the EM simulations of the antennas is not included in these results. These results show that, even with a powerful workstation with 24 CPU cores, the matching network optimization takes more than 24 h, whereas the Q -factor approximations can be calculated extremely fast even with low-powered mobile-CPU. The numerical optimization of the Q -factors takes longer than the Q -approximations but is still significantly faster than the matching network optimization.

B. Further Analysis

To better understand the behavior of the Q -factors and efficiencies shown in Figs. 10 and 11, we take a closer

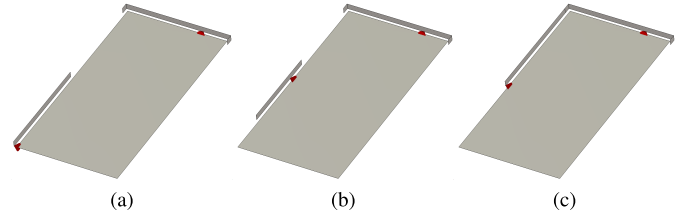


Fig. 12. Two-element antenna clusters A, B, and C.

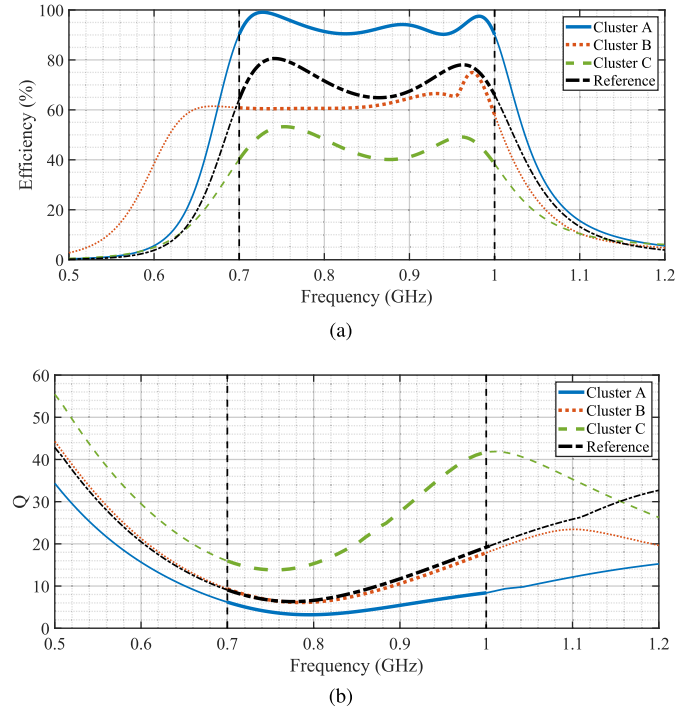


Fig. 13. (a) Efficiencies with matching networks and (b) Q -factors for the antenna structures of Fig. 12 and the reference antenna of Fig. 9(a).

look at a few of the studied structures and the reference antenna. Three different two-element antenna cluster structures are shown in Fig. 12. Efficiencies with matching networks and the corresponding Q -factors for these structures and the reference antenna of Fig. 9(a) are presented in Fig. 13. First, the results confirm the earlier conclusions that Q -factor gives a reliable estimate on the achievable performance level with matching networks. Second, they also show that the proposed Q -factor for multiport antennas directly corresponds to the traditional Q of single-port antennas. This can be seen from the minimum efficiencies of the reference antenna and cluster B that are very close to each other and also the Q -factors for both cases are practically identical. Finally, the results show that Cluster A achieves better than 90% efficiency, while the reference has a minimum efficiency of about 65%. This demonstrates that, when properly designed, multiport antennas can achieve better efficiency than traditional single feed antennas. On the other hand, the results of Clusters B and C show that, when improperly designed, there may be no benefits achieved (Cluster B) or the performance may even be significantly worse (Cluster C) than the traditional single feed antenna.

The proposed Q -factor calculations could be applied to designing multipoint antennas in practice in several different ways. One option is to use a similar approach as the results demonstrated in this work, i.e., simulate a large number of different structures, do the fast evaluation of Q -factors for all of these, and then, finally, do the time-consuming matching networks design only for a limited number of the best candidates based on the Q -factor. Another option is to use the Q -factor directly as an optimization goal for EM simulations [23].

V. CONCLUSION

We have introduced a generalized impedance-based Q -factor formula for antennas with an arbitrary number of feeding ports. The results have been verified with field-based Q -factor with simple pair of dipole antennas and also with a more complex four-port antenna with dimensions matching with modern smartphones. The practical applicability of the Q -factor has been studied using two-port antenna clusters with matching networks in the framework of mobile device antennas operating in the 0.7–1.0 GHz frequency band. We have shown that the proposed approximations for the Q -factor give a reliable figure-of-merit for the performance of multipoint antenna when the feeding weights of the ports have been correctly taken into account. With the proposed formulas, an estimation of the achievable bandwidth can be computed extremely fast even for a very large number of different antenna structures. Furthermore, we have shown that properly designed multipoint antennas can have better performance compared to single-port designs. The tools introduced in this work can help designers to accelerate the design process of multipoint antennas.

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