Generation of Reducts and Threshold Functions Using Discernibility and Indiscerniblity Matrices

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***Abstract*— Dimension reduction of data is an important issue in the data processing and iti is needed for the analysis of higher dimensional data in the application domain. Rough set is fundamental and useful to reduce higher dimensional data to lower one. Reduct in the rough set is a minimal subset of features, which has the same discernible power as the entire features in the higher dimensional scheme. The nearest neighbor relation with minimal distance proposed here has a fundamental information for classification. The nearest neighbor relation plays a fundamental role for generation of reducts and threshold functions using the Boolean reasoning on the discernibility and in discernibility matrices, in which the indiscernibility matrix is proposed here to test the sufficient condition for reduct and threshold function. Then, genetation methods for the reducts and threshold functions based on the nearest neighbor relation are proposed here using Boolean operations on the discernibility and the indiscernibility matrices.**

Keywords*— *reduct; thresholod function; nearest neighbor relation; discernibility matrix; indiscernibility matrix; generation of reduct and threshold function

I. Introduction

Rough sets theory firstly introduced by Pawlak[1,2] provides us a new approach to perform data analysis, practically. Up to now, rough set has been applied successfully and widely in machine learning and data mining. The need to manipulate higher dimensional data in the web and to support or process them gives rise to the question of how to represent the data in a lower-dimensional space to allow more space and time efficient computation. Thus, dimension reduction of data still remains as an important problem. An important task in rough set based data analysis is computation of the attributes or feature reducts for the classification[1,2]. By Pawlak's[1,2] rough set theory, a reduct is a minimal subset of features, which has the discernibility power as using the entire features. Skowlon[3,4] developed the reduct derivation by using the Boolean expression for the discernibility of data. But, generating reducts is a computationally complex task in which the computational complexity may grow non-polynomially with the number of attributes in data set[3,5]. So, a new concept for the efficient generation of reducts is expected. Nearest neighbor relation with minimal distance between different classes proposed here has a basic information for classification. For the classification of data, a nearest neighbor method[6,9,10,11] is simple and effective one. As a classification method, threshold function is well known[14]. Recent studies of threshold functions are of fundamental interest in circuit complexity, game theory and learning theory[13]. We have developed further analysis for the generation of reducts and threshold functions by using the nearest neighbor relations and the Boolean reasoning on the discernibility and indiscernibility matrices. We propose here new generation method for reducts and threshold functions based on the nearest neighbor relation with minimal distance using discernibility and the indiscernibility matrices, in which the indiscernibility matrix tests sufficient conditions for them. Thus, the generation methods based on the nearest neighbor relation are useful for the classified data with groups.

II. Boolean Reasoning of Reducts

Skowron proposed to represent a decision table in the form of the discernibility matrix[3,4]. This representation has many advantages, in particular it enables simple computation of the core, reducts and other concepts[1,2,3]. The discernibility matrix is computed for pairs of instances and stores the different variables(attributes) between all possible pairs of instances that must remain discernible.

Definition 2.1 The discernibility matrix $M(T)$ is defined as follows. Let $T=\{U,A,C,D\}$ be a decision table, with $U=\{x_1,x_2,....x_n\}$, set of instances. A is a subset of C called condition, and D is a set of decision classes. By a discernibility matrix of T, denoted by $M(T)$, which is $n\times n$ matrix defined as

$$c_{ij}=\{a\in C: a(x_i)\neq a(x_j) \wedge (d\in D, d(x_i)\neq d(x_j))\}\ i,j=1,2,...n \quad (1)$$

,where U is the universe of discourse and C is a set of features or attributes.

Definition 2.2 A discernibility function f_A for A is a propositional formula of m Boolean variables, $a_1^*,...,a_m^*$, corresponding to the attributes $a_1,...,a_m$, defined by

$$f_A(a_1^*,...,a_m^*) = \bigwedge_{1\le j<i\le m} \bigvee_{c\in c_{ij}^*, c_{ij}^*\ne\phi} c_{ij}^* \qquad (2)$$

,where $c_{ij}^* = \{a^* : a \in c_{ij}\}$ [3]. In the sequel, a_i is used instead of a_i^*, for simplicity. The symbol $\wedge$ shows Boolean product, while $\vee$ shows Boolean sum.

It can be shown that the set of all prime implicants of f_A determines the set of all reducts , which are derived from the Boolean equation(2).The Boolean equation(2) is the Boolean conjunctive normal form. This equation is simplified to the Boolean disjunctive normal form by using the Boolean absorption law. An example of decision table of the data set is shown in Table 1.The left side data in the column in Table 1 as shown in, $\{x_1, x_2, x_3,..,x_7\}$ is a set of instances, while the data $\{a, b, c, d\}$ on the upper row, shows the set of attributes of the instance. The contents of each row in Table 1 shows numeral values of the corresponding instance. The class shows that the corresponding instance belongs to the numeral class value. In Table 2, the discernibility matrix of the decision table in Table 1 is shown. In case of instance x_1, the value of the attribute a, is a(x_1)=1. That of the attribute b , is b(x_1)=0. Since a(x_1)=1 and a(x_5)=2, a(x_1) $\neq$ a(x_5) holds.

TABLE 1. DECISION TABLE OF DATA EXAMPLE(INSTANCES)

Attribute	a	b	c	d	class
x_1	1	0	2	1	+ 1
x_2	1	0	2	0	+1
x_3	2	2	0	0	-1
x_4	1	2	2	1	-1
x_5	2	1	0	1	-1
x_6	2	1	1	0	+1
x_7	2	1	2	1	-1

The discernibility function is represented by taking the combination of the disjunction expression of the discernibility matrix. In Table 2, the item (b,c,d) in the second row and the first column, implies $b+c+d$ in the Boolean sum expression,

TABLE 2. DISCERNIBILITY MATRIX OF THE DECISION TABLE IN TABLE 1

	x_1	x_2	x_3	x_4	x_5	x_6
x_2	—					
x_3	a,b,c,d	a,b,c				
x_4	b	b,d	—			
x_5	a,b,c	a,b,c,d	—	—		
x_6	—	—	b,c	a,b,c,d	c,d	
x_7	a,b	a,b,d	—	—	—	c,d

which shows the attribute b or c or d appear for the discrimination between instances x_1 and x_3 [3]. By using nearest neighbor relation, generation of reducts is developed in the next session, in which nearest neighbor plays an important role.

A. Nearest Neighbor Relation with Minimal Distance

Nearest neighbor relation with minimal distance is introduced here. The relation with minimal distance plays an important role in the generation of reducts. The nearest neighbor relation with minimal distance implies the items in the discernibility matrix, which have the minimal distance between the different classes. In Table 2, The shaded item $\{a,b\}$ makes the minimal distance , $\sqrt{2}$ between x_1 and x_7 , since the distance x_1 in the class +1 and x_7 in the class -1becomes $\sqrt{2}$. Also, the same minimal distance, $\sqrt{2}$ is computed between x_3 in the class -1 and x_6 in the class +1. Similarly, it is computed between x_5 and x_6. These three items in Table 2 are shown in the shaded items. Boolean characteristics are derived in the discernibility matrix in Table 2 as follows. Instances with different classes are assumed to be measured in the metric distances for the nearest neighbor classification. Then, instances nearest to different classes are picked up in the distance. They are called to be in the nearest neighbor relation with minimum distance. This nearest neighbor relation plays a fundamental role for the generation of reducts from the discernibility matrix.

III. GENERATION OF REDUCTS BASED ON NEAREST NEIGHBOR RELATION AND EXTERNAL SET

We can define a new concept, a nearest neighbor relation with minimal distance, δ . Instances with different classes are assumed to be measured in the metric distances for the nearest neighbor classification.

Definition 3.1 A nearest neighbor relation with minimal distance is a set of pair of instances, which are described in

$$\{(X_i, X_j) : \beta(X_i) \neq \beta(X_j) \wedge |X_i - X_j| \leq \delta\} \quad (3)$$

, where $|X_i - X_j|$ shows the distance between X_i and X_j, and δ is the minimal distance. Then, X_i and X_j in the equation (3) are called to be in the nearest neighbor relation

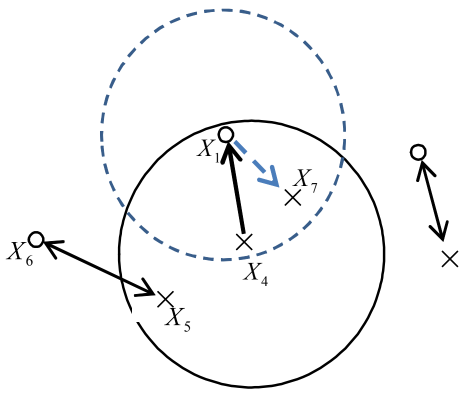

Fig. 1. Search of nearest neighbor relations

with minimal distance δ. To find minimal nearest neighbor relation, the divide and conquer algorithm[12] is applied to the array of data in Table 1with the search of the nearest data so as to be classified to different classes.

In Table1, $(x_6, x_7), (x_5, x_6), (x_1, x_7)$ and (x_3, x_6) are elements of the relation with a distance $\sqrt{2}$. Thus, a nearest neighbor relation with minimal distance $\sqrt{2}$ becomes

$$\{(x_6, x_7), (x_5, x_6), (x_1, x_7), (x_3, x_6)\} \quad (4)$$

Here, we want to introduce the nearest neighbor relation on the discernibility matrix. Assume that the set of elements of the nearest neighbor relation are $\{nn_{ij}\}$. Then, the following characteristics are shown. Respective element of the set $\{nn_{ij}\}$ corrsponds to the term of Boolean sum. As an example, the element $\{a, b, c\}$ of discernibility matrix in the set $\{nn_{ij}\}$ corresponds to a Boolean sum $(a + b + c)$. The following lemmas are derived easily.

Lemma 3.2 Respective Boolean term consisting of the set $\{nn_{ij}\}$ becomes a necessary condition to be reducts in the Boolean expression.

This is trivial, since the product of respective Boolean term becomes reducts in the Boolean expression.

Lemma 3.3 Boolean product of respective terms corresponding to the set $\{nn_{ij}\}$ becomes a necessary condition to be reducts in the Boolean expression.

This is also trivial by the reason of Lemma 3.2. Thus, the relation between Lemma 3.2and Lemma 3.3 is described as

Lemma 3.4 Reducts in the Boolean expression are included in the Boolean term of Lemma 3.2 and the Boolean product in Lemma 3.3.

Fig. 2 shows that nearest neighbor relation with classification is a necessary condition in the Boolean expression for reducts, but not sufficient condition. The distance δ of the nearest neighbor relation in the equation (3) is compared with the distance δ' of the relation in the following theorem.

Theorem 3.5 If the distance δ is greater than the δ', i.e., $\delta > \delta'$ in the equation (3) ,the Boolean expression of the case of δ' includes that of δ.

This is by the reason that the Boolean expression of the nearest neighbor relation is consists of the Boolean product of variables of the relation. The number of variables in the distance δ' are less than that of δ. Thus, the nearest neighbor relation with distance δ' includes the ellipse of δ in Fig.2.

Two sets of attributes(variables) ,[A] and [B] are defined to extract reducts from the nearest neighbor relation $\{nn_{ij}\}$.

[A]: Set of elements in the discernibility matrix includes those of any respective element in $\{nn_{ij}\}$ and those of elements absorbed by $\{nn_{ij}\}$ in the Boolean expression

[B]: Set of elements in the discernibility matrix, which are not absorbed from those of any respective element in $\{nn_{ij}\}$.

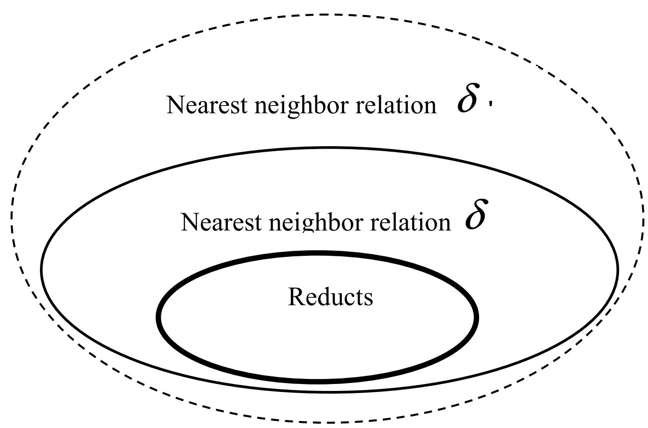

Fig. 2. Boolean condition of nearest neighbor relations and reducts

The Boolean sum of attributes is absorbed in the Boolean sum element with the same fewer attributes in the set $\{nn_{ij}\}$. As an example, the Boolean sum of (a + b + c) in the set [A] is absorbed in the Boolean sum of (a + b) in the set $\{nn_{ij}\}$.

Lemma 3.6 Within set [B] , the element with fewer attributes(variables) plays a role of the absorption for the element with larger attributes(variables).

Then, the absorption operation is carried out within set [B].

Theorem 3.7 Reducts are derived from the nearest neighbor relation by the absorption of variables in set [B] terms in the Boolean expression.

In the relation (x_1, x_7) in Table 1 , the x_1 in the class +1 is nearest to the is x_7 in the class -1. Similarly, (x_5, x_6) and (x_6, x_7) are nearest relations. Then, variables of the set [A] in these relations are shown in shading in Table 2 of the discernible matrix. The Boolean product of these four terms becomes

$$(a+b)\cdot(b+c)\cdot(c+d) = b\cdot c + b\cdot d + a\cdot c \tag{5}$$

,which becomes a candidate of reducts. The third term in the equation (5) is absorbed by the product of variable {b} of the set [B] and the equation (5) . The final reducts equation becomes

$$b\cdot c + b\cdot d \tag{6}$$

Thus, reducts $\{b,c\}$ and $\{b,d\}$ are obtained finally. To search final reducts in the equation (5), the following Boolean reasoning is considered at the set [A] of the nearest neighbor relations. At the step of the Boolean equation (4), some dominant Boolean variables are searched from the bottom up. The candidate dominant Boolean variables are shown in Fig. 3. From the equation (5), three Boolean minters $b\cdot c$, $b\cdot d$ and $a\cdot c$ are derived from the nearest neighbor relations. The common variables are candidates of the dominant variables between three terms. From the set [B], the dominant variables are searched in Table 2. Thus, the variable $\{b\}$ is searched from the set [B] , which is not in the set [A] .

IV.…GENERATION OF REDUCTS BASED ON NEAREST NEIGHBOR RELATION AND INDISCERNIBILITY MATRIX

In this section, we propose another generation method of reducts, which is based on nearest neighbor relation and indiscernibility matrix proposed here. The external set in the previous section 3 is replaced to indiscernibility matrix.

Definition 4.1 Indiscernibility matrix is defined to be $IM(T)$, which is $n \times n$ matrix defined as

$$c_{ij} = \{a \in C : a(x_i) = a(x_j) \wedge (d \in D, d(x_i) \neq d(x_j))\}\ i, j = 1, 2, \ldots n \tag{7}$$

,where U is the universe of discourse, C is a set of features,

The difference between $M(T)$ in the equation (1) and $IM(T)$ in the equation(7) , is shown in the following. Attributes $a(x_i) \neq a(x_j)$ holds in $M(T)$, while $a(x_i) = a(x_j)$ holds in $IM(T)$. The example of the indiscernible matrix $IM(T)$ for Table 1 of dicision table T is shown in Table 3. Indiscernibility shows the instance x_i in the row has the same variables at the instance x_j in the column, in which x_i and x_j belong to the different classes. For example, the instance x_2 in the class +1 and the instance x_4 in the class -1, cannot be discriminated by only variables a and c, since they are the same variables. Since the Boolean operation in the element of the indiscernible matrix is AND, the element value (a,c,d) between x_1 and x_4 shows Boolean product $a\cdot c\cdot d$. The Boolean product $a\cdot b\cdot c$ also implies $a\cdot c$ or $c\cdot d$ or $a\cdot d$.

TABLE 3. INDISCERNIBILITY MATRIX IN TABLE 1

	x_1	x_2	x_3	x_4	x_5	x_6
x_2	—					
x_3	—	d				
x_4	a,c,d	a,c	—			
x_5	d	—	—	—		
x_6	—	—	a,d	—	a,b	
x_7	c,d	c	—	—	—	a,b

In the section 2, the nearest neighbor relation is derived from Table 1. In the relation (x_1, x_7) in Table 1 , the x_1 in the class +1 is nearest to the is x_7 in the class -1. Similarly, (x_5, x_6) and (x_6, x_7) are nearest neighbor relations. Then, variables of the nearest neighbor relations are shown in shading in Table 2 of the discernible matrix. The Boolean product of these four terms becomes

$$(a+b)\cdot(b+c)\cdot(c+d) = b\cdot c + b\cdot d + a\cdot c \tag{8}$$

In the three minterms in the above equation(8), the minterm $a\cdot c$ in equation(8) is checked in the indiscernibility matrix

Table 3. The ($a \cdot c$) is found between x_1 and x_4, also between x_2 and x_4. The minterm $a \cdot c$ cannot discriminate instances between x_1 and x_4, also between x_2 and x_4. Then, the minterm $a \cdot c$ is removed from the equation(8). Thus, the reducts

$$b \cdot c + b \cdot d \quad (9)$$

is obtained, which is the same derived in the equation(6). Thus, reducts $\{b, c\}$ and $\{b, d\}$ are obtained.

A. Relation Between Set [B] and Indiscernibility Matrix

For the one generation method of reducts, set [A] and set [B] are defined in section 3 and the other generation method uses indiscernibility matrix in section 4. The set [B] in Table 2, becomes $\{b, (b, d)\}$, which is remained after Boolean absorption by the set of nearest neighbor relations. In the set [B], the variable (b, d) is expressed in Boolean form as $(b+d)$. The variable b of Boolean product with $(b+d)$ becomes by Boolean absorption law,

$$b \cdot (b+d) = b + b \cdot d = b$$

Thus, the variable b represents the set [B]. The variable b multiplied by the Boolean form of the set [A] derived from the nearest neighbor relation becomes

$$b \cdot (a \cdot c + b \cdot c + b \cdot d) = b \cdot c + b \cdot d$$

The element variable (a, c, d) in indiscernibility table ,which is placed in the same element in discernibility matrix, removes the Boolean implicant $a \cdot c$. Thus, the final Boolean reduct form becomes

$$b \cdot c + b \cdot d$$

Theorem 4.2 The Boolean absorption of the variables in the set [B] multiplied by the Boolean forms derived from the nearest neighbor relations generate reducts, which are also generated by directly removing Boolean variables in the indiscerniblity matrix from the Boolean forms of the nearest neighbor relations.

V. Generation of Threshold Functions using Discernibility and Indiscernibility Matrices

The nearest neighbor relation is also applicable to the generation of threshold functions. The threshold function is a Boolean function based on the n-dimensional cube with 2^n vertices of n components of 1 or 0. The function f is characterized by the hyperplane $WX - \theta$ with the weight vector $W(=(w_1, w_2, ..., w_n))$ and threshold θ. The X is a vertex of the cube 2^n. In the following, the threshold function is assumed to be positive and canonical threshold function, in which the Boolean variables hold the partial order[14].

Definition 5.1 The nearest neighbor relation (X_i, X_j) on the threshold function is defined from Equation (3),

$$\{(X_i, X_j) : \beta(X_i) \neq \beta(X_j) \wedge |X_i - X_j| \leq \delta(=1)\} \quad (10)$$

, where $\delta = 1$ shows one bit difference between X_i and X_j in the Hamming distance (also in the Euclidean distance).

The boundary vector near the hyperplane is defined as follows.

Definition 5.2 The boundary vector X is defined to be the vector which satisfies

$$|WX - \theta| \leq |WY - \theta| \text{ for the } X(\neq Y \in 2^n) \quad (11)$$

Theorem 5.3 The boundary vector X becomes an element of nearest neighbor relation in the threshold function.

This is proved, since the boundary vector X is the nearest data to the hyperplane, which divides the true data and the false data. The nearest neighbor relation is not necessarily boundary vector. Since the boundary vectors determine the hyperplane of the threshold function, the nearest neighbor relation also characterizes the threshold function. The data set is called to be admissible set of f, if the set realizes a threshold function f.

Theorem 5.4 The set of the element of the nearest neighbor relation is admissible set of threshold function.

Theorem 5.5 The vectors X_i and X_j in the nearest neighbor relation (X_i, X_j) are the adjacent vectors, each of which belongs to different class through the hyperplane.

This theorem shows the line between adjacent vectors X_i and X_j with one bit difference is crossed by the hyperplane.

Theorem 5.6 The nearest neighbor relations $\{(X_i, X_j)\}$ in a threshold function f is unique in f.

This is proved by the contradiction. Assume a threshold function has two nearest neighbor relations NNR_A and NNR_B, in which $(X_{iA}, X_{jA}) \neq (X_{iB}, X_{jB})$ holds. First, since X_{iB} is not as true data in the NNR_A, it exists as a false data. Then (X_{iB}, X_{jB}) does not make nearest neighbor relation of f, which contradicts the assertion. Second, when the partial ordering $X_{iA} \prec X_{iB}$ holds[14], X_{iB} does not make nearest neighbor relation of f with X_{jB} in one bit distance. Thus, $\{(X_i, X_j)\}$ is unique in the given threshold function f.

Generation of threshold function is performed as follows. As an example, the three dimensional data is given as follows.
As true valued data, (101), (110) and (111) are given. As false valued data, (000), (010), (100), (001), and (011) are given.

Then, the 3-dimensional cube is shown in Fig.3, in which the black circle belongs to +1 class, while the white circle belongs to o class. The black circle data is (101), (110) and (111), while the white circle data is (000),(001), (010) ,(100) and (011).

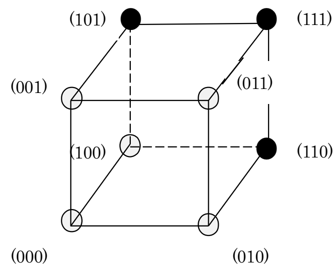

Fig.3. Example of a threshold function in 3-dimensional cube

Boolean reasoning of the discernibility matrix in threshold function is as follows. In Fig.3, a true valued data (101) has nearest neighbor relations as {(101), (001)} and {(101), (100)}.
as shown in shaded cells in Table 4.The Boolean reasoning in these relations becomes a Boolean product $x_1 \cdot x_3$ of the respective relation x_1 and x_3 in Table 4. The Boolean product, called minterm $x_1 \cdot x_3$ satisfies to be 1 for (101), while to be 0 for (001) and (100). Similarly, a true valued valued data (110) has nearest neighbor relations as {(110),(100)} and {(110), (010)}. From these relations, the Boolean miterm $x_2 \cdot x_3$ is generated. Finally, the minterm with one variable x_1 is generated from the relation {(111),(011)}. The discernibility matrix of the nearest neighbor relation shows a necessary condition to be a threshold function. The matrix shows different variables between adjacent vectors with different classes 1 or 0. The discernibility matrix of nearest neighbor data in Fig.3 is shown in Table 4. In Fig.3, a true valued data (101) has nearest neighbor relations as {(101), (001)} and {(101), (100)} as shown in shaded cells in Table 4. In Table 4, the difference of variables between ●(101) and ○(001) is x_1, in which $x_1 = 1$ in (101). Then, x_1 is used, which is based on variables on the true data.

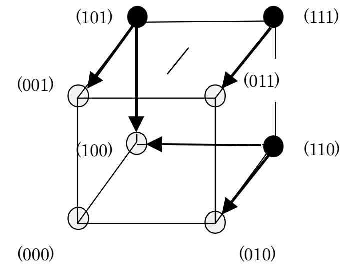

Fig.4. Arrows show nearest neighbor relations in 3-dimensional cube

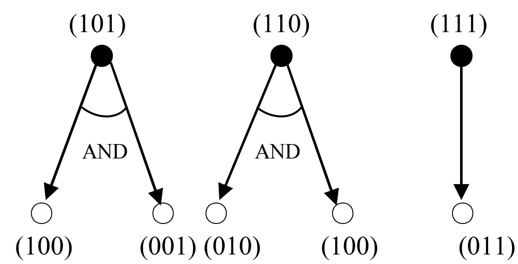

Fig.5. Boolean operation for nearest neighbor relation

TABLE 4 DISCERNIBILITY MATRIX OF NEAREST NEIGHBOR DATA IN FIG.3

	● (101)	● (110)	● (111)
○ (001)	x_1	…	…
○ (100)	x_3	x_2	…
○ (010)	…	x_3	…
○ (011)	…	…	x_1

Similarly, the difference between (101) and (100) is x_3. For the Boolean realization of the difference of these variables is performed by the Boolean product $x_1 \cdot x_3$ also as shown in Fig.5. The Boolean product, called minterm $x_1 \cdot x_3$ satisfies to be 1 for (101), while to be 0 for (001) and (100). Similarly, a true valued valued data (110) has nearest neighbor relations as {(110),(100)} and {(110), (010)}. From these relations, the Boolean miterm $x_2 \cdot x_3$ is generated. Finally, the minterm with one variable x_1 is generated from the relation {(111),(011)}. But, this minterm is removed from the indiscernibility matrix in Table 5, in which the sufficiency condition of these minterms are checked for a threshold function by taking common variables between true and false vectors The common variables x_2 and x_3 are taken as a Boolean product $x_2 \cdot x_3$ in the cell between ●(101) and ○ (001), which is not used for the difference (101) and (001). But , the minterm derived in Table 4, $x_1 \cdot x_3$ discriminates the true vector (101) and the false vector (001). Finally, the minterm x_1 is generated from the relation {(111),(011)}in Table 4 in the shaded cell. The common variable x_1 also exists in Table 5, between true vector (111) and false vector (100). But, to realize the difference between (111) and (100), the minterm $x_1 \cdot x_3$ or $x_2 \cdot x_3$ is applicable to this cell. Thus, the minterm x_1 derived in Table 4, which is only one common variable in Table 5 is removed by checking the discernibility and the indiscernibility matrices.

TABLE 5. INDISCERNIBILITY MATRIX OF DATA IN FIG.3

	● (101)	● (110)	● (111)
○ (001)	$x_2 \cdot x_3$	$\cdots$	x_3
○ (100)	$x_1 \cdot x_2$	$x_1 \cdot x_3$	x_1
○ (010)	$\cdots$	$x_2 \cdot x_3$	x_2
○ (011)	x_3	x_3	$x_2 \cdot x_3$

But, this minterm is removed from the indiscernibility matrix in Table 5. In discernibility matrix for making threshold function for classification, it is necessary to perform AND operation of the terms in the column, while to perform OR operation among different columns. In Table 4, the difference of variables between ● (101) and ○ (001) is x_1 , in which $x_1 = 1$ in (101).

Then, x_1 is used, which is based on variables on the true data.

The minterm x_1 in the relation {(111),(011)} in Table 4, which is shaded in the cell, is exists also in the (111) column in Table 5. Then, the minterm x_1 is removed from the Boolean sum from Table 4. Thus, the Boolean function obtained is

$$f = x_1 \cdot x_2 + x_1 \cdot x_3 \qquad (12)$$

The Boolean function f becomes a threshold function, since a hyperplane exists to satisfy the equation (12).

VI. CONCLUSION

Nearest neighbor relation developed here is the set of pair elements with minimal distance, which classify between different classes. Reduct is introduced as the minimal set for the data classification in the rough set theory. Threshold functions are of fundamental interest in circuit complexity, game theory and learning theory. This paper develops the role of the nearest neighbor relations for the classification of data, which is proposed here. Then, generation of the reducts and threshold functions based on the nearest neighbor relations using the discernibility and the indiscernibility matrices is developed. The necessary condition for the generation of reducts and threshold functions is derived from the discernibility matrix with nearest neighbor relations, while the indiscernibility matrix is proposed to test the sufficient conditions for reducts and threshold functions.

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