Improved Joint Antenna Selection and User Scheduling for Massive MIMO Systems

Yuhan Dong, Yuanyuan Tang, and Kai Zhang
Shenzhen Key Laboratory of Advanced Sensor and Integrated System
Graduate School at Shenzhen, Tsinghua University, China
Email: {dongyuhan@sz, tangyy15@mails, zhangkai@sz}.tsinghua.edu.cn

***Abstract*—Massive multi-input multi-output (MIMO) technology is promising by employing a large number of antennas at the base station to support a large amount of users. However, due to the limitation of analog front-ends at the base station, the antenna selection and user scheduling strategies are essential to achieve spatial diversity and reduce hardware cost at the same time. In this work, we consider the strategy of joint antenna selection and user scheduling (JASUS) for uplink massive MIMO systems and propose a greedy two-step JASUS algorithm referred to as largest minimum singular value based JASUS (LMSV-JASUS). In its first step, a simplified downward branch and bound based JASUS is used to find a near-optimal antenna and user sets whose channel matrix has the near-largest MSV. In its second step, a swapping-based algorithm is proposed to find a better solution by swapping antennas and users between the selected and the discarded. The numerical results suggest that the proposed algorithm outperforms traditional approaches in terms of system sum-rate and computational complexity.**

***Index Terms*—Massive MIMO, antenna selection, user scheduling.**

I. Introduction

Over the past decade, the multi-input multi-output (MIMO) has attracted lots of attention around the world due to its advances in high capacity and frequency efficiency [1]. Without increasing frequency band and transmit power, a MIMO system is capable to increase the channel capacity linearly as the minimum number of antennas employed by the transmitter and receiver [2]. However, The small-scale MIMO system equipped with eight antennas in cellular networks such as 4G systems [3] cannot meet the demand of large amount of mobile data. To break the dilemma, a massive MIMO system employed with tens or hundreds of antennas at base station is considered as one of the most promising and potential technologies for 5G systems [4], [5], [6], [7]. Unfortunately, due to the constraints of device cost and energy consumption, the quantity of analog front-ends (AFEs) applied in base station is limited.

To take the advantage of large antenna array as well as saving AFEs, antenna selection is considered to be a suitable choice [8]. Some researchers have investigated antenna selection in recent years. Due to the monotonicity of minimum singular value (MSV) of a matrix, a bidirectional branch and bound (BAB) algorithm can find the optimal antenna set as the brute-force search (BFS) algorithm as well as reduction of computational complexity [9]. An antenna selection algorithm based on the maximum 2-norm of user channel vector [10] can promote system ability to against bit error rate (BER) when beamforming technique is adapted.

Since the quantity of active users served by base station is no more than the AFEs, user scheduling is necessary when users are more than AFEs. Since the channel state information (CSI) from mobile users to the base station are evaluated distributively, a user scheduling algorithm based on competition to feedback CSI can save time of acquiring CSI and improve system capacity [11]. When zero-forcing beamforming (ZFBF) technique is applied, three joint antenna selection and user scheduling (JASUS) algorithms which can find sub-optimal solution are proposed with limited backhaul capacity [1]. Through swapping antennas and users elements between the sets of waiting for service and the discarded, the algorithm has a higher probability to obtain the global optimal solution [12]. Based on the spatial orthogonality of CSI for different users, authors in [8] has proposed a JASUS algorithm with comparable performance as BFS and reduction of hardware complexity.

Inspired by these prior works [1], [8], [9], [13], [14], we propose a two-step largest MSV based JASUS (LMSV-JASUS) algorithm. The contributions of the paper are:

1) A two-step JASUS algorithm consisting of simplified downward branch and bound based JASUS (SDBAB-JASUS) algorithm and Swapping-based algorithm is proposed, Which can outperform other algorithms in terms of sum-rate.
2) Compared to the primary JASUS algorithm [8], the proposed SDBAB-JASUS can reduce the computational complexity, especially for massive elements, by choosing a appropriate number of candidate antennas.
3) A swapping-based algorithm can further improve sum-rate based on results of fist step.

The rest of this paper is organized as follows. In Section II, we present the system model and problem formulation. In Section III, the processes and weakness of original JASUS are introduced. In Section IV, we propose a two-step JASUS algorithm by searching the largest MSV. We provide the performance comparison of average sum-rate and computational complexity for various algorithms in Section V and draw the conclusion in Section VI.

The corresponding author is K. Zhang.

978-1-5090-5507-4/17/$31.00 ©2017 IEEE
ICIS 2017, May 24-26, 2017, Wuhan, China

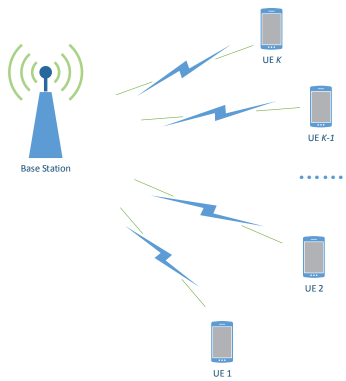

Fig. 1. A Single cell massive MIMO system.

II. System Model and Problem Formulation

A. System model

We consider a single cell massive MIMO system, just like Fig. 1, consisting of one base station equipped with M antennas and N AFEs with $M > N$, and K single transmit antenna users with $K > N$. Because of the limitation of AFEs, each time the base station can only choose N best condition antennas and serve N users at most. At the base station, the signal received from K users is given by

$$y = \sqrt{\rho}\mathbf{H}x + z, \tag{1}$$

where $y \in C^{M\times 1}$ is the received signal vector at base station, ρ is the SNR at the transmitter, $\mathbf{H} \in C^{M\times K}$ is channel matrix from all users to base station. Each single element of channel matrix is assumed to be a quasi-static independently and identically distributed (i.i.d.) Rayleigh fading channel, $x \in C^{K\times 1}$ is transmit signal vector from K users to base station, and $z \in C^{M\times 1}$ is the noise term following an distribution of independent complex Gaussian with zero mean and unit variance, i.e. $z \sim \mathcal{N}(0, 1)$.

At the receiver, the base station can achieve near-best reception when linear zero-forcing (ZF) detection is adopted. Then the received signal vector can be written as

$$\tilde{x} = \mathbf{W}y = \frac{1}{\tau}\left(\sqrt{\rho}x + (\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H z\right), \tag{2}$$

where $\mathbf{W} \in C^{K\times M}$ is linear detection matrix of massive MIMO system. For ZF detector, $\mathbf{W} = (\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H/||(\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H||_2$ with $||\cdot||_2$ as the 2-norm operator, and $\tau = ||(\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H||_2$. Then the receive SNR of i-th user at base station is[9]

$$\begin{aligned}\gamma_i(\mathbf{H}) &= \rho\frac{1}{[(\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H((\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H)^H]_{i,i}}\\ &= \rho\frac{1}{[(\mathbf{H}^H\mathbf{H})^{-1}]_{i,i}}\\ &\geq \rho\lambda^2_{\min}(\mathbf{H})\end{aligned} \tag{3}$$

where $|\lambda|_{\min}(\mathbf{H}) = \lambda_{\min}(\mathbf{H})$ represents the MSV of $\mathbf{H}$.
proof: see Appendix A.

B. Problem Formulation

We denote the transmit power of all users as both P and the quantity of AFEs employed at base station as N. If both P and N are limited, the base station can utilize N antennas to serve at most N users simultaneously, where the selected antenna set and user set are A and U, respectively. Then the useful CSI matrix of system can be expressed as

$$\mathbf{H}_{A,U} = \mathbf{H}(A, U). \tag{4}$$

Since the wireless communication process satisfies stochastic distribution, the expectation of maximum ergodic rate of the sum is

$$R(A, U) = \max_{A,U} E\left\{\sum_{i\in U}\log\left(1 + \gamma_i(\mathbf{H}_{A,U})\right)\right\}, \tag{5}$$

where $E\{\cdot\}$ is mathematical expectation. Besides, the antenna and user sets must satisfy

$$|A| \leq N \quad \text{and} \quad |U| \leq N, \tag{6}$$

where $|\cdot|$ returns the cardinality of a set. According to the analysis of (4), (5), (6), both the quantities of selected antennas and users from A and U are both equal to N, respectively. Furthermore, according to (3) and (5), the maximum rate of system can be obtained when A and U satisfy the following condition

$$(A, U) = \arg\max_{A1,U1}(\lambda_{\min}(\mathbf{H}_{A1,U1})), \tag{7}$$

where $A1$ and $U1$ are sets containing N arbitrary different antennas and N arbitrary different users, respectively. In other words, the problem of obtaining the maximum rate of system can be converted to find the largest MSV of CSI matrix of all the matches of $A1$ and $U1$ [9].

The optimal solution can be obtained by exhaustive search (also known as BFS), which requires a prohibitive computational complexity, especially when the quantities of antennas and users are very large. To solve this problem efficiently, we propose a two-step greedy search method called LMSV-JASUS which outperforms traditional approaches in terms of sum-rate and computational complexity.

III. Original JASUS Algorithm

The original JASUS algorithm[8] consists of DBAB algorithm and semi-orthogonal user selection (SUS) algorithm which can achieve antenna and user selection, respectively.

1) DBAB Scheme: The algorithm can find the global optimal solution if the objective cost function $J(S)$, performance of set S, satisfies downward monotonicity. For the subset $S_m \subseteq S_n$ with $m < n$, if the relationship can be described as

$$\max J(S_n) \geq \max_{S_m\subseteq S_n} J(S_m), \tag{8}$$

then we can say that the set is downward monotonous based on $J(S)$ [9], [13]. When (8) is satisfied, the selection problem can be expressed as

$$J(\tilde{S_n}) = \max_{S_n\subseteq S_N} J(S_n), \tag{9}$$

where $\tilde{S_n}$ denotes the globally optimal search direction or set.

2) SUS: The algorithm depicted in Algorithm 1 can find N best users whose channel have a good level of orthogonality and larger channel gain. The SUS algorithm contains N iterations. In i-th iteration, The absolute channel gains of $(i-1)$ selected users makes up subspace $\{\tilde{g}_1, \dots, \tilde{g}_{i-1}\}$. Based on the channel vector $h(k, A)$ of k-th user to antennas in A, the valuable channel gain $g_{k,A}$ which is orthogonal to the subspace can be obtained by[14]:

$$g_{k,A} = h(k, A) \left(\mathbf{I} - \sum_{j=1}^{i-1} \frac{\tilde{g}_j^H \tilde{g}_j}{\| \tilde{g}_j \|^2} \right) \tag{10}$$

After that, the candidate user whose has maximum norm will be the i-th active user. The N selected users make up the user set U_N.

3) Original JASUS algorithm: The algorithm can obtain a good solution of antenna and user sets by $(M - N)$ iterations, in each iteration a candidate antenna who has worst CSI will be discarded. Besides, three steps are included in each iteration, which is described in detail below:

- First, the candidate antenna set, A, is obtained from last iteration. The quantity of candidate search directions, A_{-i}, is the same as the size of A.
- Next, find *the best* users, U_N, of A_{-i} based on SUS algorithm[14]. The objective cost function of each search direction can be expressed as:

$$J(A_{-i}) = \mathrm{R}(\mathbf{H}(A_{-i}, U_N)) \tag{11}$$

- Finally, the new candidate antenna set, A, is updated by

$$A = \arg\max_{A_{-i}} J(A_{-i}) \tag{12}$$

The algorithm will stop when new candidate antenna set contains only N antennas. Then the solution, A_N and U_N, is obtained. The computational complexity of original JASUS algorithm still increases rapidly when quantity of candidate antennas is large.

IV. Improved JASUS Algorithm

Based on the analysis above, we propose a simple improved LMSV-JASUS algorithm which is achievable in practical scenarios. In the first step, a SDBAB-JASUS algorithm can obtain good solution of antenna and user sets as well as achieve a trade-off between computational complexity and sum-rate. In the second step, based on the first step, a swapping-based algorithm is added to further improve the sum-rate of system. The improved JASUS algorithm mainly consists of two steps, described as algorithm 2. In each step, SUS described in algorithm 1, is to select *the best* users.

A. First Step: SDBAB-JASUS algorithm

The computational complexity of original JASUS algorithm above will increase rapidly when the quantity of antennas at the base station is large. In practice, most of it is worthless, but increases the hardware cost. To reduce computational complexity and hold sum-rate, a SDBAB-JASUS algorithm is proposed by importing trade-off step before original algorithm.

Algorithm 1 Semi-orthogonal user selection function (**SUS**) [14]

Input:
CSI matrix: $\mathbf{H}$
Number of AFEs: N
Available antenna set: A

Initialization
$U \leftarrow \{1, 2, \dots, K\}$
$U_N \leftarrow \emptyset$
$i = 1$
while $i < N$ **do**
  for $k \in U$ **do**
    $g_{k,A} = h(k, A) \left(\mathbf{I} - \sum_{j=1}^{i-1} \frac{\tilde{g}_j^H \tilde{g}_j}{\|\tilde{g}_j\|^2} \right)$
  end
  $i_{\text{opt}} = \arg\max_{k \in U} (\| g_{k,A} \|_2)$
  $U_N \leftarrow U_N \bigcup \{i_{\text{opt}}\}$
  $U = U \setminus U_N$
  $\tilde{g}_i = g_{i_{\text{opt}}, A}$
  $i = i + 1$
end

Output:
The selected users obtained by U_N

Since the selected users and antennas are both N, Gersgorin theorem can be applied to find singular value (the same to eigenvalues here) of CSI matrix $\mathbf{H}_{A,U}$. According to Gersgorin theorem, the singular values of matrix $\mathbf{H} = (h_{i,j}) \in C^{N \times N}$ locate in the discs:

$$D_i = \{\lambda || \lambda - h_{i,i}| \le P_i\} \tag{13}$$

where $P_i = \sum_{j \ne i} |h_{i,j}|$, λ represents the singular values. The (13) can be rewritten as

$$\lambda \le \min_i \sum_j |h_{i,j}| = \min_i |h_i| \tag{14}$$

where h_i represents channel gain of the i-th antenna. Besides, according to the analysis of (3), the cost function is simplified by change the $J(A_{-i})$ from sum-rate to the largest MSV. If the quantity of users is infinite (great orthogonality), the LMSV can be obtained by finding CSI matrix who has largest minimum channel gain based on (3). Although the number of users is limited in practice, the LMSV still can be obtained in a lot of candidate antennas who have larger channel gain.

By importing a parameter t (also called trade-off number, TFN), the algorithm obviously reduces computational complexity (the trade-off step of Algorithm 2). Before finding the largest MSV by original JASUS algorithm, the SDBAB-JASUS will select t $(N < t < M)$ candidate antennas whose channel gains are larger than the rest firstly. Compared to original JASUS, this procedure significantly decreases the computational complexity to obtain a candidate antenna set A which contains t active antennas.

The SDBAB-JASUS algorithm become the original JASUS algorithm when t is equal to M; if t is equal to N, the SDBAB-JASUS is simplified to norm-based JASUS algorithm which contains only one iteration. As a result, by adjusting t, the Simplified DBAB-JASUS algorithm can find a trade-off between the sum-rate and computational complexity [15].

Algorithm 2 Two-step JASUS Algorithm

Input:
CSI matrix: $\mathbf{H}$
Number of AFEs: N
Trade-off Number (TFN): $N < t < M$

First step: SDBAB-JASUS

Initialization
$A \leftarrow \{1, 2, \ldots, M\}$
$i = 1$
Trade-off Step: Reduce complexity
$Ch_t = \text{diag}(\mathbf{H}\mathbf{H}^H)_t$, t candidate antennas with largest Ch_t
$A \leftarrow A_t$, t candidate antennas form new antenna set A
while $i < (t - N)$ **do**
$\max \text{MSV} \leftarrow 0$
for $m \in A$ **do**
$U_N \leftarrow$ **SUS**$(\mathbf{H}(A \backslash \{m\}, :), N)$, **Algorithm 1** to select users
$S_{-m} = \min \text{SVD}(\mathbf{H}(A \backslash \{m\}, U_N))$
if $S_{-m} \geq \max \text{MSV}$ **then**
$\max \text{MSV} = S_{-m}$
$a_{bad} = m$
end
end
$A \leftarrow A \backslash \{a_{bad}\}$
$i = i + 1$
end
$A_N \leftarrow A$
Output:
The selected antennas and users obtained by A_N and U_N, respectively

Second step: Swapping-based JASUS

Initialization
$A_l \leftarrow \{1, 2, \ldots, M\}$
$A \leftarrow A_N$, $U \leftarrow U_N$
$A_t \leftarrow \emptyset$, $U_t \leftarrow \emptyset$
while $A_t \neq A$ **or** $U_t \neq U$ **do**
$A_t \leftarrow A$, $U_t \leftarrow U$
$Ch = \text{diag}(\mathbf{H}_{(:,U)}\mathbf{H}^H_{(:,U)})$
$A_c \leftarrow \arg\max_{A_N \in A_l}(Ch)$
if $R_{\text{ZF}}(A_c, U) > R_{\text{ZF}}(A, U)$ **then**
$A \leftarrow A_c$
end
$U_c \leftarrow$ **SUS**$(\mathbf{H}, A)$, **Algorithm 1** to select users
if $R_{\text{ZF}}(A, U_c) > R_{\text{ZF}}(A, U)$ **then**
$U \leftarrow U_c$
end
end

Output:
The selected antennas and users obtained by A and U, respectively

B. Second Step: Swapping-Based Algorithm

The SDBAB-JASUS algorithm can find near-optimal antenna and user sets, A_N and U_N, respectively. However, the author in [12] has mentioned that the selected users may become redundant users when new users are added. In order to get more outstanding performance, a swapping-based algorithm by swapping the antennas and users between the selected and the discarded is proposed. The processes of the algorithm are depicted in Algorithm 2. In i-th swapping iteration, with the user set fixed, the N antennas who have better channel state (larger channel gain) can be obtained, or vice verse. If the sum-rate of system based on swapped antenna and user sets is larger than the former, the former antenna or user set can be updated by the latest. The algorithm will stop when the swapped antenna and user sets are the same as the former.

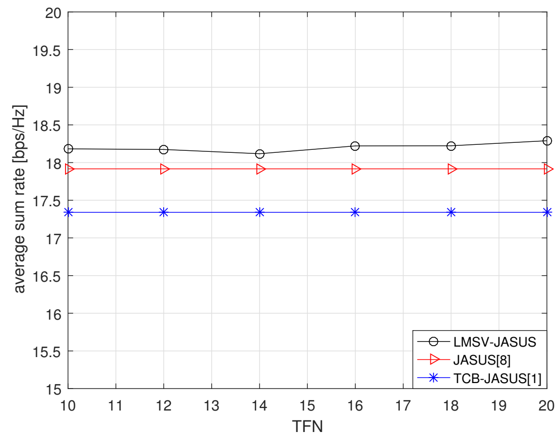

Fig. 2. Average sum-rate versus TFN with SNR=10dB and AEFs=8.

V. Numerical Results

We evaluate the performance of sum-rate and computational complexity of the proposed LMSV-JASUS as well as the comparison with JASUS algorithm [8], TCB-JASUS algorithm [1]. Without loss of generality, the simulation results about parameters with both low and high values will be presented. A massive MIMO system of single cell consisting of $M = 64$ antennas at base station and $K = 25$ single antenna users is considered.

A. Performance Evaluation

Firstly, Fig. 2 shows a performance comparison in term of the sum-rate based on the function of TFN. The SNR of transmitter is 10 dB and the quantity of AFEs is 8. As see from Fig. 2, the sum-rate of LMSV-JASUS is about 0.5bits/Hz higher than original JASUS algorithm and 1bit/Hz higher than TCB-JASUS algorithm with TFN ranges from 10 to 20, respectively. Besides, as the TFN increases, the sum-rate of LMSV-JASUS also improves. However, the increasing of TFN leads to higher computational complexity. To achieve a higher sum-rate as well as reduce computational complexity, the TFN is assumed to 12 when the AFEs employed at base station is 8.

Secondly, Fig. 3 shows a performance comparison of algorithms in terms of the sum-rate which can be achieved based on the function of SNR. Because of the high hardware cost, the quantity of AFEs is limted to 8, $N = 8$. According to the analysis in Section II, the quantity of active antennas and the users is 8 at most. Based on the analysis of Fig. 2, the TFN is assumed to be 12. As seen from the figure, the proposed LMSV-JASUS can achieve higher average sum-rate than original JASUS algorithms and TCB-JASUS algorithm with SNR ranging from 0 dB to 20 dB. What's more, compared to the performance when SNR is small, the gap of sum-rate of proposed algorithm and existed algorithms increases when the

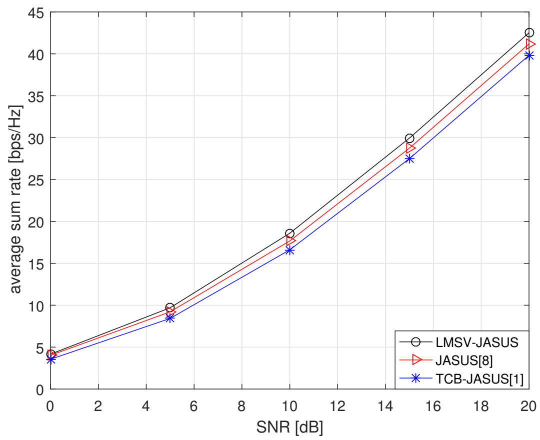

Fig. 3. Average sum-rate versus SNR with AFEs=8 and TFN=12.

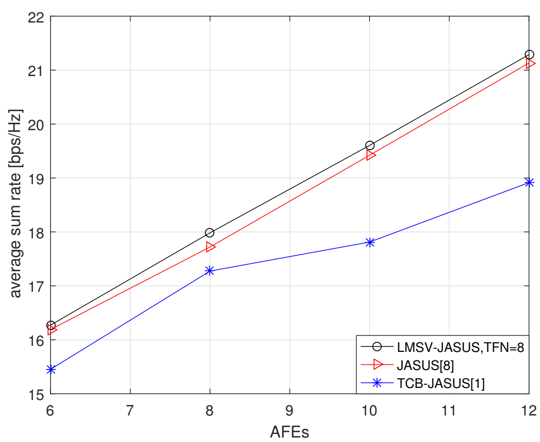

Fig. 4. Average sum-rate versus AFEs whith SNR=10dB and TFN=AFEs+8

SNR at base station is larger. So the proposed LMSV-JASUS algorithm outperform other algorithms when SNR increases.

Finally, Fig. 4 shows a performance comparison in term of the sum-rate based on the function of AFEs. To obtain the influence contributed by AFEs, we vary the quantity of AFEs from 6 to 12. The TFN is 8 larger than the quantity of AFEs, satisfying $TFN = AFEs + 8$, to guarantee that the proposed LMSV-JASUS algorithm can achieve higher performance. As seen from Fig. 4, the achievable average sum-rate of LMSV-JASUS algorithm increases as the number of AFEs increases and is larger than other two algorithms. In other word, the proposed LMSV-JASUS algorithm can outperform other algorithms in term of sum-rate by adjusting TFN when AFEs is different.

B. Complexity Analysis

The computation complexity of JASUS and TCB-JASUS algorithm is $\sum_{i=1}^{M-N}(N+i)o(N^3)$ [8] and $N_{t1}(o(MN^2+o(N^3)))$ [1], respectively, where iteration number N_{t1} is smaller than 6.

The LMSV-JASUS consists of SDBAB-JASUS and Swapping-based algorithms. The computational complexity of MSV of a matrix can reach $o(N^3)$, The computational complexity of the SDBAB-JASUS algorithm is $\sum_{i=1}^{t-N}(N+i)o(N^3)$. The computational complexity of Swapping-based algorithm is mainly caused by sum-rate computation. Because the sum-rate performs a channel inversion at each iteration with a complexity of $o(N^3)$, the total computational complexity of LMSV-JASUS algorithm is about $\sum_{i=1}^{t-N}(N+i)o(N^3)+N_t o(N^3)$, where iteration number N_t is smaller than N_{t1} for all the TFN.

The table. I shows a computational complexity of LMSV-JASUS and original JASUS algorithm who achieve higher sum-rate. The quantity of AFEs employed at base station is $N = 8$ and TFN equals to 12. Based on the general case, the computational complexity of proposed LMSV-JASUS algorithm mainly depends on the TFN while the original JASUS algorithms lies on the numbers of antennas. The computational complexity of LMSV-JASUS algorithm decreases obviously compared to JASUS in the example since the quantity of TFN is obviously smaller than the number of antennas, i.e., $t \ll M$.

Note that the proposed algorithm can still take advantage over traditional algorithms when the quantity of AFEs increases and is larger than 8.

VI. Conclusion

In this paper, we considered the problem of joint antenna selection and user scheduling in uplink of massive MIMO systems where the users transmit signals with equal power. The BFS can find the optimal antenna and user sets to obtain the maximum average sum-rate, but the computational complexity is prohibitive. We proposed a two-step greedy JASUS algorithm which consists of SDBAB-JASUS and swapping-based algorithm. In the first step, the SDBAB-JASUS can find a near-optimal antenna and user sets by finding largest MSV. In the second step, the swapping-based algorithm can find a better solution by swapping antennas and users between the selected and the discarded. A suitable TFN can achieve high sum-rate as well as reduce computational complexity at the same time. The numerical results suggest that the proposed two-step LMSV-JASUS algorithm outperforms traditional algorithms in term of sum-rate and computational complexity.

Appendix A

If all the users are randomly located in the ring of disc, the rank of CSI matrix is equal to the number of users. According to Singular value decomposition of matrix, the channel matrix $\mathbf{H} \in C^{M\times K}$ can be expressed as

$$\mathbf{H} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H \tag{15}$$

where the diagonal matrix $\mathbf{\Sigma} = diag(\lambda_1, \ldots, \lambda_K, \underbrace{0, \ldots, 0}_{M-K})$ contains all the singular values of $\mathbf{H}$. $\mathbf{U} \in C^{M\times M}$ and $\mathbf{V} \in C^{K\times K}$ are both orthogonal matrix, also called unitary matrix, and satisfy the condition:

$$\mathbf{U}\mathbf{U}^H = \mathbf{U}\mathbf{U}^{-1} = \mathbf{I} \tag{16}$$

TABLE I
COMPUTATIONAL COMPLEXITY COMPARISON OF JASUS AND LMSV-JASUS.

Algorithm	general case	$M = 64, N = 8, K = 25, t = 12$
JASUS	$\sum_{i=1}^{M-N}(i+N)o(N^3)$	1.05×10^6
LMSV-JASUS	$\sum_{i=1}^{t-N}(i+N)o(N^3) + N_t o(N^3)$	3.00×10^4

$$\mathbf{V}\mathbf{V}^H = \mathbf{V}\mathbf{V}^{-1} = \mathbf{I} \tag{17}$$

then $\mathbf{V}^{-1} = \mathbf{V}^H$ and $\mathbf{U}^{-1} = \mathbf{U}^H$ can be obtained.

Since the conjugate transpose of $\mathbf{H}$ preserves its singular values, $\mathbf{H}\mathbf{H}^H$ can be expressed as

$$\begin{aligned}[\mathbf{H}\mathbf{H}^H]^{-1} &= [\mathbf{U}\mathbf{\Sigma}\mathbf{V}^H\mathbf{V}\mathbf{\Sigma}\mathbf{U}^H]^{-1} \\ &= \mathbf{U}\mathbf{S}\mathbf{U}^H\end{aligned} \tag{18}$$

where diagonal matrix $\mathbf{S} = (\mathbf{\Sigma}\mathbf{\Sigma})^{-1} = diag(\frac{1}{\lambda_1^2}, \ldots, \frac{1}{\lambda_K^2}, \underbrace{0, \ldots, 0}_{M-K})$.

According to (16) and (17), the unitary matrix $\mathbf{U} = [\mu_1, \ldots, \mu_M]$ satisfies

$$\mu_i^H \mu_i = 1, i = 1, \ldots, M \tag{19}$$

where $\mu_i \in C^{M\times 1}$ is the i-th column of matrix $\mathbf{U}$. So

$$\begin{aligned}[\mathbf{H}\mathbf{H}^H]_{i,i}^{-1} &= [[\mu_1, \ldots, \mu_M]\mathbf{S}[\mu_1, \ldots, \mu_M]^H]_{i,i} \\ &= [\sum_{j=1}^{k} \frac{1}{\lambda_j^2}\mu_j\mu_j^H]_{i,i} \\ &= \sum_{j=1}^{k} \frac{1}{\lambda_j^2}\mu_{j,i}^2\end{aligned} \tag{20}$$

Besides, the singular value of channel matrix $\mathbf{H}$ satisfies

$$\lambda_{\max} \geq \lambda \geq \lambda_{\min} > 0 \tag{21}$$

Based on (19), (20) and (21), $[\mathbf{H}\mathbf{H}^H]_{i,i}^{-1}$ can be limited by

$$[\mathbf{H}\mathbf{H}^H]_{i,i}^{-1} \leq \frac{1}{\lambda_{\min}^2}\sum_{j=1}^{K}\mu_{j,i}^2 \leq \frac{1}{\lambda_{\min}^2} \tag{22}$$

Combining (3) and (22) together, we reach the conclusion:

$$\begin{aligned}\gamma_i(\mathbf{H}) &= \rho\frac{1}{[(\mathbf{H}^H\mathbf{H})^{-1}]_{i,i}} \\ &\geq \rho\lambda_{\min}^2(\mathbf{H})\end{aligned} \tag{23}$$

REFERENCES

[1] G. Xu, A. Liu, W. Jiang, H. Xiang, and W. Luo, “Joint user scheduling and antenna selection in distributed massive MIMO systems with limited backhaul capacity,” *China Commun.*, vol. 11, no. 5, pp. 17–30, May 2014.

[2] F. Rusek *et al.*, “Scaling pp MIMO: Opportunities and challenges with very large arrays,” *IEEE Signal Processing Mag.*, vol. 30, no. 1, pp. 40–60, Jan. 2013.

[3] Z. Lu, J. Ning, Y. Zhang, T. Xie, and W. Shen, “Richardson method based linear precoding with low complexity for massive MIMO systems,” in *Proc. IEEE 81st VTC Spring*, Glasgow, 2015, pp. 1–4.

[4] H. Q. Ngo, E. G. Larsson, and T. L. Marzetta, “Energy and spectral efficiency of very large multiuser MIMO systems,” *IEEE Trans. Commun.*, vol. 61, no. 4, pp. 1436–1449, April 2013.

[5] L. Dai, Z. Wang, and Z. Yang, “Spectrally efficient time-frequency training OFDM for mobile large-scale MIMO systems,” *IEEE J. Sel. Areas Commun.*, vol. 31, no. 2, pp. 251–263, Feb. 2013.

[6] E. G. Larsson, O. Edfors, F. Tufvesson, and T. L. Marzetta, “Massive MIMO for next generation wireless systems,” *IEEE Commun. Mag.*, vol. 52, no. 2, pp. 186–195, Feb. 2014.

[7] Y. Wang and Y. Dong, “A genetic antenna selection algorithm for massive MIMO systems with channel estimation error,” in *Proc. Advances in Wireless and Optical Communications (RTUWO)*, Riga, 2015, pp. 1–4.

[8] M. Benmimoune, E. Driouch, W. Ajib, and D. Massicotte, “Joint transmit antenna selection and user scheduling for massive MIMO systems,” in *Proc. IEEE WCNC*, New Orleans, LA, 2015, pp. 381–386.

[9] Y. Gao, W. Jiang, and T. Kaiser, “Bidirectional branch and bound based antenna seletion in massive MIMO systems,” in *Proc. IEEE 26th PIMRC*, Hong Kong, 2015, pp. 563–568.

[10] S. E. El-Khamy, K. H. Moussa, and A. A. El-Sherif, “On the performance of massive multiuser MIMO with different transmit beamforming techniques and antenna selection,” in *Proc. 1st URSI AT-RASC*, Las Palmas, 2015, pp. 1–10.

[11] X. Xie and X. Zhang, “Scalable user selection for MU-MIMO networks,” in *Proc. IEEE INFOCOM*, Toronto, ON, 2014, pp. 808–816.

[12] S. Huang, H. Yin, J. Wu, and V. C. M. Leung, “User selection for multiuser MIMO downlink with zero-forcing beamforming,” *IEEE Trans. Veh. Technol.*, vol. 62, no. 7, pp. 3084–3097, Sept. 2013.

[13] Y. Cao and V. Kariwala. “Bidirectional branch and bound for controlled variable selection: Part I. principles and minimum singular value criterion.” *Computers & Chemical Engineering*, vol. 32, no. 10, pp. 2306–2319, Oct. 2008.

[14] T. Yoo and A. Goldsmith, “On the optimality of multiantenna broadcast scheduling using zero-forcing beamforming,” *IEEE J. Sel. Areas Commun.*, vol. 24, no. 3, pp. 528–541, March 2006.

[15] J. Joung and S. Sun, “Two-step transmit antenna selection algorithms for massive MIMO,” in *Proc. IEEE ICC*, Kuala Lumpur, 2016, pp. 1–6.