

Research on Torque Ripple Under Healthy and Open-Circuit Fault-Tolerant Conditions in a PM Multiphase Machine

Ali Akay, and Paul Lefley

Abstract— In this paper, research into torque ripple production has been undertaken for both the healthy and open-circuit fault-tolerant conditions of a five-phase permanent magnet (PM) machine by using the instantaneous power (I-Power) approach. When only the fundamental component of the phase currents is applied to the phase windings, it has been shown that the 9th and 11th harmonics of the back-electromotive force (back-EMF) causes torque ripples in a five-phase PM machine and its frequency is ten times the frequency of the fundamental phase currents. When the combined fundamental and third harmonic components of the phase currents are applied to the phase windings, it has been shown that the 7th and 13th harmonic of the back-electromotive force (back-EMF) causes additional torque ripples in a five-phase PM machine. These torque ripples under fault-tolerant conditions have been analyzed analytically, as well. It has been proven that there are interactions between the fundamental component of current and the third harmonic component of the back-EMF and vice versa. These interactions cause torque ripples. A finite element analysis (FEA) model of the five-phase PM machine has been done to validate the analytical results.

Index Terms— Five-phase, harmonic current injection, multiphase machine, permanent magnet machine, torque ripple.

I. INTRODUCTION

IN recent years, multi-phase machines have garnered interest from researchers due to their various beneficial features[1]. The main reason is that the level of current that flows in a semiconductor power device in an inverter is reduced by current sharing when increasing the number of phases[2]–[4]. This has significant implications in the use of high power and fault tolerant drives in critical automotive and aerospace applications. The second reason is to reduce the amplitude of the torque pulsations by increasing their frequency[5]. For better performance, the desired torque response should be flat. Increasing the number of phases creates a net torque production close to being constant.

Additionally, losses in the stator and rotor, along with the vibration and noise of the machine, are reduced in a multi-phase machine compared to a three-phase machine. Multi-phase machines have a better fault tolerance [6], [7]. This means,

multi-phase machines can produce a rotating field even if they have lost one or more phases [7]. Thus, a multi-phase machine can continue to operate under open-circuit fault conditions.

In this paper, research on torque ripples has been undertaken for the healthy condition and open-circuit fault-tolerant conditions of a multiphase PM machine whose back-EMF is non-sinusoidal. Research on torque ripple in a multiphase machine for the open-phase condition was studied by Yin et al. in [8]. They investigated torque ripple in terms of the winding function approach. In [3] and [9], developed an open-circuit fault tolerant control for the five-phase machines by using the I-Power approach theory. They obtained open-circuit fault-tolerant control currents by eliminating the components that cause ripple in the I-Power.

It is important to stress out that the torque of the multiphase machine can be increased by injecting the higher order harmonics of the phase currents, and a multi-phase machine still can be run smoothly even if one or more than one of its coils are open-circuited. I-Power is the product of the phase current and the back-EMF including their harmonics, and torque equation of the PM machine consists of I-Power. Therefore, it is better to research torque ripple in a PM multi-phase machine by using the I-Power approach theory. This approach helps to see the reason of the torque ripple in a PM multi-phase machines easily.

In this study, research on torque ripple has been conducted in terms of the instantaneous power (I-Power) approach. Detailed research on torque ripple has been done for the healthy and the open-circuit fault-tolerant control conditions. In section II, a model of prototype five-phase PM machine is introduced. In section III, firstly the I-Power approach is presented, and then the torque ripple under healthy condition is analyzed by using I-Power approach. An analysis of the torque ripple of the machine under fault-tolerant condition is discussed, in section IV. In section V, the results of a finite element analysis (FEA) of the machine model is presented to observe torque ripple.

II. THE FEA MODEL OF THE FIVE-PHASE PM MACHINE

The FEA model of the five-phase fractional slot PM machine can be seen in Fig.1. The machine is a double layer fractional slot PM machine. The parameters of the machine model are shown in Table I, the back-EMF waveform and the winding distribution of the machine can be seen in Fig.1. The harmonic content of the back-EMF includes 100% fundamental, 3rd harmonic: 9.6%, 5th: 0%, 7th: 3.32%, 9th:3.01%, 11th:0.52%.

Manuscript received April 19, 2020; Revised August 26, 2020; Accepted November 06, 2020. Date of publication December 25, 2020; Date of current version December 18, 2020.

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Digital Object Identifier 10.30941/CESTEMS.2020.00042

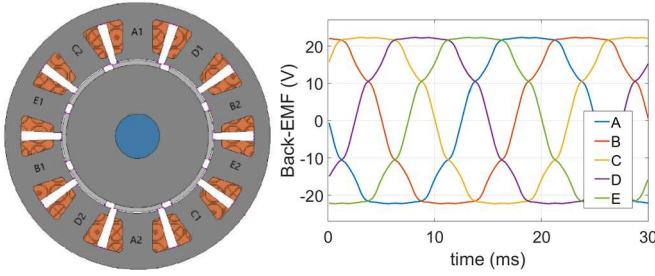


Fig. 1: FEA model and back-EMF of the five-phase PM machine.

III. TORQUE RIPPLE IN A NON-SINUSOIDAL BACK-EMF MACHINE UNDER THE HEALTHY CONDITION

One way of expressing torque in a permanent magnet (PM) five-phase machine can be seen below in (1) [3], [10], where ω_r is the rotor angular speed and $\theta = \omega t$ is the electrical angle. Equation (1) holds for transient and steady state conditions, but it is important to note that the equation only governs the part of the energy conversion process that involves the flux that links both the field and the armature. It describes only the energy that crosses the airgap[11]. Therefore, cogging torque of the machine, magnetic saturation of the stator core, and the losses in the rotor due to spatial harmonics are ignored [12].

TABLE 1.
PARAMETERS OF THE FIVE-PHASE PM MACHINE

Rated Power (kW)	1
Rated Speed (rpm)	2000
Rated Current (Amps)(Peak)	3.39
Rated Torque (Nm)	6.6966
Number of Poles	8
Number of slots	10

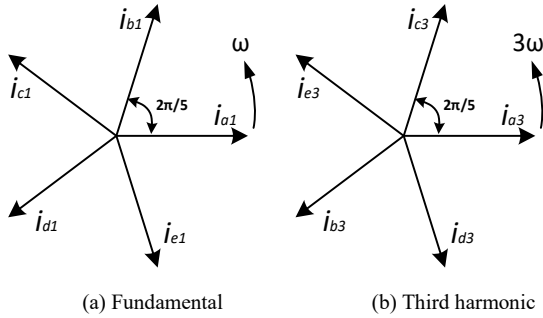


Fig. 2. Phasor diagram of the phase current components.

$$T = \frac{1}{\omega_r} \left\{ \begin{array}{l} i_a(\theta)e_a(\theta) + i_b(\theta)e_b(\theta) + i_c(\theta)e_c(\theta) \\ + i_d(\theta)e_d(\theta) + i_e(\theta)e_e(\theta) \end{array} \right\} \quad (1)$$

Currents waveforms for each phase can be expressed as below in (2).

$$\begin{aligned} i_a(\theta) &= i_{a1}(\theta) + i_{a3}(3\theta) + \dots \\ i_b(\theta) &= i_{b1}(\theta) + i_{b3}(3\theta) + \dots \\ i_c(\theta) &= i_{c1}(\theta) + i_{c3}(3\theta) + \dots \\ i_d(\theta) &= i_{d1}(\theta) + i_{d3}(3\theta) + \dots \\ i_e(\theta) &= i_{e1}(\theta) + i_{e3}(3\theta) + \dots \end{aligned} \quad (2)$$

In a five-phase PM machine, only the fundamental, or both the fundamental plus the third harmonic current components, can be used to supply the windings of the machine. The third

harmonic current component is applied to improve the torque in a five-phase machine. The fundamental and third harmonic current components are given in (3) and (4), respectively, where I_{m1} and I_{m3} are the amplitudes of the fundamental and third harmonic current components, respectively. The phasor diagram of the fundamental and third harmonic current components can be seen in Fig. 2.

$$\begin{aligned} i_{a1}(\theta) &= I_{m1} \sin(\theta) \\ i_{b1}(\theta) &= I_{m1} \sin\left(\theta - \frac{2\pi}{5}\right) \\ i_{c1}(\theta) &= I_{m1} \sin\left(\theta - \frac{4\pi}{5}\right) \end{aligned} \quad (3)$$

$$\begin{aligned} i_{d1}(\theta) &= I_{m1} \sin\left(\theta - \frac{6\pi}{5}\right) \\ i_{e1}(\theta) &= I_{m1} \sin\left(\theta - \frac{8\pi}{5}\right) \\ i_{a3}(3\theta) &= I_{m3} \sin(3\theta) \\ i_{b3}(3\theta) &= I_{m3} \sin\left(3\theta - \frac{6\pi}{5}\right) \\ i_{c3}(3\theta) &= I_{m3} \sin\left(3\theta - \frac{2\pi}{5}\right) \\ i_{d3}(3\theta) &= I_{m3} \sin\left(3\theta - \frac{8\pi}{5}\right) \\ i_{e3}(3\theta) &= I_{m3} \sin\left(3\theta - \frac{4\pi}{5}\right) \end{aligned} \quad (4)$$

The I-Power in each phase is the product of the back-EMF and the current in phase with it. The back-EMF of each phase can be generalized as in (5), where E_n is the amplitude of the related back-EMF harmonic. k in (5) represents the phases in a five-phase machine, where $k = 1, 2, 3, 4, 5$ to represent phases A, B, C, D, and E, respectively.

$$e_k(\theta) = \sum_{n=1,3,5,\dots}^{\infty} E_n \sin\left[n\left(\theta - \frac{(k-1)2\pi}{5}\right)\right] \quad (5)$$

The I-Power for the five-phase machine is expressed in (6) [3], [6], [12]. According to the torque equation, pulsating components in the I-Power will produce ripples in the torque. Amplitudes of the back-EMF harmonics in the FEA model machine have been used as a reference for the analytical (graphical) results. It is assumed that $E_1 = 1$, $E_3 = 0.096$, $E_5 = 0$, $E_7 = 0.0332$, $E_9 = 0.0301$, and $E_{11} = 0.0052$ and $I_{m1} = 1$ and $I_{m3} = 0.2I_{m1}$ to ease the calculations.

$$P = \left\{ \begin{array}{l} i_a(\theta)e_a(\theta) + i_b(\theta)e_b(\theta) + i_c(\theta)e_c(\theta) \\ + i_d(\theta)e_d(\theta) + i_e(\theta)e_e(\theta) \end{array} \right\} \quad (6)$$

A. Torque Ripple due to Fundamental Current Component

In this section, the I-Power for only the fundamental component of the phase currents is analysed according to (6). Applying equation (3) into the I-Power equation in (6) the I-Power is obtained as in (7).

$$\left\{ \begin{array}{l} P = \frac{5}{2} I_{m1} E_1 - \frac{1}{9} I_{m1} E_9 \cos(10\theta) + \frac{1}{11} I_{m1} E_{11} \cos(10\theta) + \dots \\ P = \frac{5}{2} I_{m1} E_1 + I_{m1} \left[\frac{E_{11}}{11} - \frac{E_9}{9} \right] \cos(10\theta) + \dots \end{array} \right. \quad (7)$$

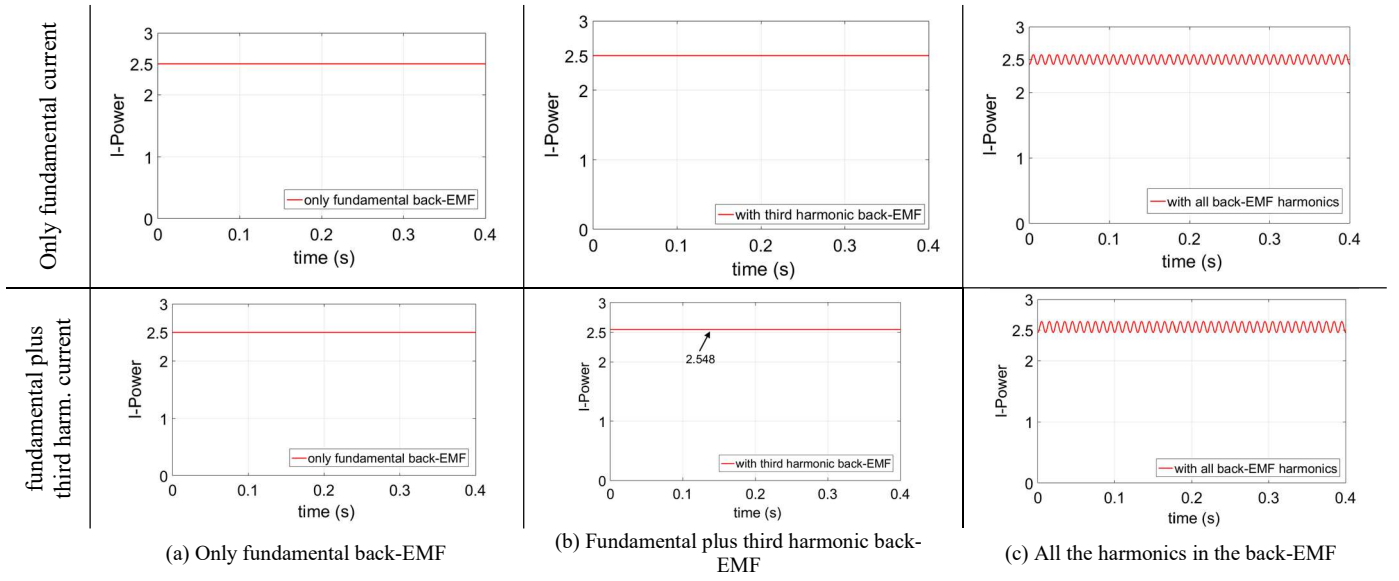


Fig. 3: I-Power regarding to the back-EMF harmonic contents – Healthy condition.

According to equation(7), the first term produces a smooth torque output, as it is a constant value. However, the remaining terms are responsible for the ripples in the torque, as it is a cosine function. The ripple frequency is ten times the electrical frequency and the amplitude of the ripple depends on the 9th and 11th harmonic of the back-EMF. The ripples of the torque will change regarding the harmonic content in the back-EMF. The I-Power is obtained by applying only fundamental current when the back-EMF consists of only the fundamental, combined with the fundamental and third harmonic, and up to the 11th harmonic in Fig. 3.

B. Torque Ripple due to Fundamental Plus Third Harmonic Current Component

The fundamental and third harmonic current components are applied to the I-Power equation in (6). Then the I-Power for the combined fundamental and third harmonic currents is obtained as in (8). Power can be separated into two parts. The first power component is due to the fundamental current component and the third harmonic current component produces the second one.

$$\begin{cases} P = P_1 + P_3 \\ P = \frac{5}{2} I_{m1} E_1 + \frac{5}{2} I_{m3} E_3 + I_{m1} \left[\frac{E_{11}}{11} - \frac{E_9}{9} \right] \cos(10\theta) \\ + I_{m3} \left[\frac{E_{13}}{13} - \frac{E_7}{7} \right] \cos(10\theta) + \dots \end{cases} \quad (8)$$

The resultant power can be expressed below in (9). This equation consists of two parts. The first one produces the smooth torque. This part also proves torque improvement due to the third harmonic current component when the back-EMF of the machine includes the third harmonic inside. The second part of the equation causes ripples in torque output, as it is a cosine function. Ripples produced by the fundamental and the third harmonic current components have the same frequency being ten times the electrical frequency of the fundamental current. Similar to the previous case, the amplitudes of the

ripples depend on the amplitude of the back-EMF harmonics that are in the second part of the equation.

$$\begin{cases} P = \frac{5}{2} [I_{m1} E_1 + I_{m3} E_3] \\ + \left[I_{m1} \left(\frac{E_{11}}{11} - \frac{E_9}{9} \right) + I_{m3} \left[\frac{E_{13}}{13} - \frac{E_7}{7} \right] \right] \cos(10\theta) + \dots \end{cases} \quad (9)$$

The I-Power has been plotted by applying the fundamental plus third harmonic current components when the back-EMF consists of only the fundamental, the fundamental and third harmonic combined, and up to the 11th harmonic in Fig.3. Equation(9), Fig.3(a) and Fig.3(b) prove that, the fundamental current component only interacts with the fundamental component of the fundamental back-EMF, and the same goes for the third harmonic component of the current and back-EMF. For the healthy condition, they can be considered separate machines.

IV. TORQUE RIPPLES IN A NON-SINUSOIDAL BACK-EMF MACHINE UNDER THE FAULT-TOLERANT CONDITIONS

In this section, torque ripple in open-circuit fault-tolerant conditions will be presented. The fault-tolerant method developed by Yi Sui et al. [7] has been considered for the fault-tolerant conditions to examine the ripple components in the machine torque.

The generalized I-Power for the fault-tolerant condition can be written as in (10). Where, P_{mn}^* is the I-Power which is produced by the m^{th} harmonic of current and n^{th} harmonic of the back-EMF.

$$\begin{cases} P^* = P_1^* + P_3^* \\ P_1^* = P_{11}^* + P_{13}^* + P_{15}^* + \dots \\ P_3^* = P_{31}^* + P_{33}^* + P_{35}^* + \dots \end{cases} \quad (10)$$

Each I-Power produces two cosine components. Their frequencies are $(n \mp m)\omega$. For example, the frequencies of the cosine components belonging to the I-Power produced by the

fundamental current component and the fundamental component of the back-EMF are 0ω and 2ω . That is, the I-Power will have a constant value and a cosine component. However, the sum of the cosine components whose frequency is 2ω is equal to zero for the fundamental current component and the fundamental back-EMF under fault-tolerant condition. Another example, the I-Power produced by the fundamental current component and the third harmonic component of the back-EMF produces cosine components whose frequencies are 2ω and 4ω . The sum of these cosine components is zero for the healthy condition. However, these cosine components produce ripples in the torque for the fault-tolerant conditions since the sum of them is not zero. One more example: the I-Power due to the fundamental current component and the 5th harmonic of the back-EMF produces cosine components whose frequencies are 4ω and 6ω .

It is important to note again, the sum of the I-Powers due to the interaction between the fundamental current component and the 3rd, 5th, 7th, 13th, ... is zero for the healthy condition. This means that there is no torque contribution or ripples due to these back-EMF harmonics in the healthy condition. However, the sum of the I-Powers produced by the fundamental current component and these back-EMF harmonics are not zero under fault-tolerant, and they consist of cosine components. That means that there will be ripples in the torque due to these interactions.

A. Fault-Tolerant Control Currents Without Harmonics

1) Single Phase Open-Circuit Condition

Fault-tolerant currents for the single-phase open-circuit (SPOC) condition can be seen in (11) [7], where $\theta = \omega t$ is the electrical angle. It is assumed that Phase A is open-circuited. The phasor diagram of the fundamental component is illustrated in Fig.4.

$$\begin{cases} i_{b1}^*(\theta) = 1.314I_{m1} \sin\left(\theta - \frac{3\pi}{10}\right) \\ i_{c1}^*(\theta) = 1.314I_{m1} \sin\left(\theta - \frac{9\pi}{10}\right) \\ i_{d1}^*(\theta) = 1.314I_{m1} \sin\left(\theta - \frac{11\pi}{10}\right) \\ i_{e1}^*(\theta) = 1.314I_{m1} \sin\left(\theta - \frac{17\pi}{10}\right) \end{cases} \quad (11)$$

The I-Power of the single-phase open condition is expressed in (12) [9].

$$P_1^* = \left\{ \begin{aligned} & i_{b1}^*(\theta)e_b(\theta) + i_{c1}^*(\theta)e_c(\theta) + i_{d1}^*(\theta)e_d(\theta) \\ & + i_{e1}^*(\theta)e_e(\theta) \end{aligned} \right\} \quad (12)$$

Equation (12) can be split into parts as shown in (13), and I-Power produced by the fundamental current component and the fundamental back-EMF is in (14). The resultant I-Power is a constant value.

$$P_1^* = P_{11}^*(0\omega, 2\omega) + P_{13}^*(2\omega, 4\omega) + P_{15}^*(4\omega, 6\omega) + \dots \quad (13)$$

$$P_{11}^*(0\omega, 2\omega) = 2.628I_{m1}E_1 \cos\left(\frac{\pi}{10}\right) \quad (14)$$

The I-Power produced by the fundamental current and the third harmonic component of the back-EMF is expressed in (15).

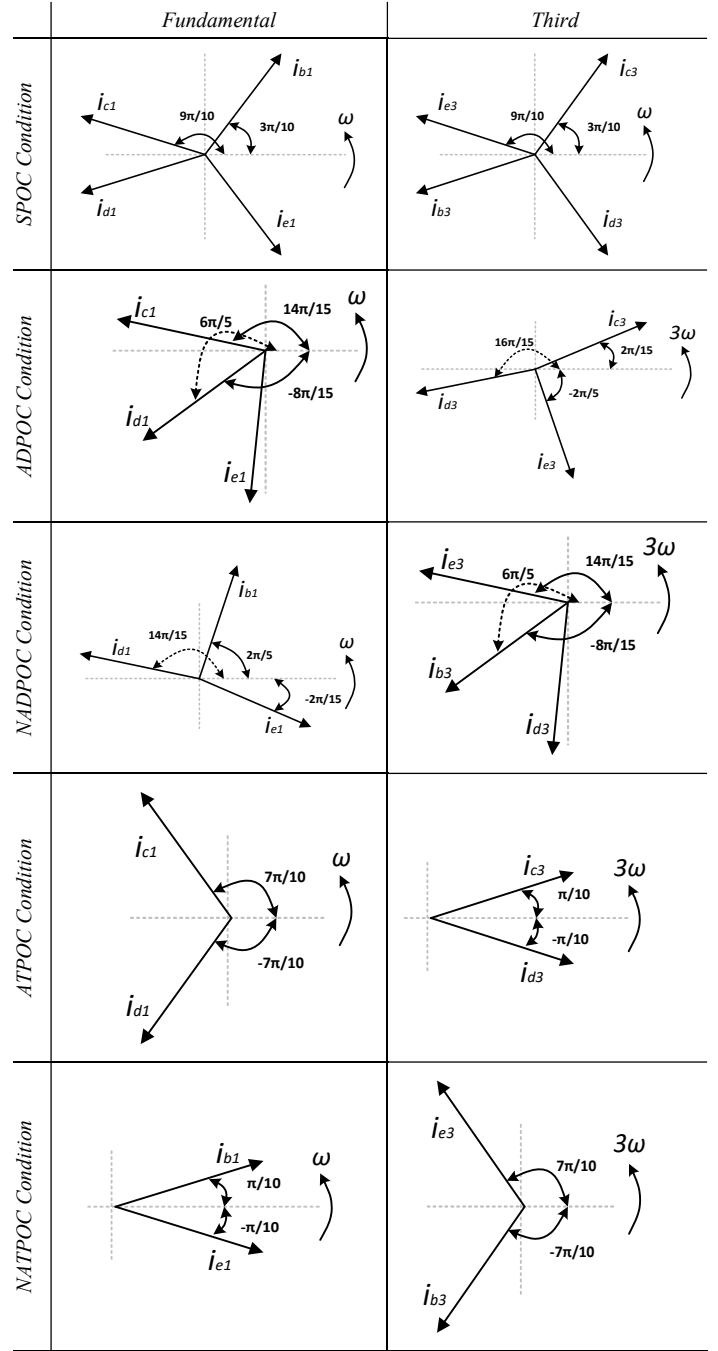


Fig. 4. Phasor diagram of the fault-tolerant currents.

It consists of cosine components, and the sum of these components is not zero. Therefore, these cosine components cause ripples in the torque.

$$P_{13}^*(2\omega, 4\omega) = \frac{2.628I_{m1}E_3}{2} \left\{ \begin{aligned} & \cos\left(2\theta - \frac{5\pi}{10}\right) + \cos\left(2\theta - \frac{9\pi}{10}\right) \\ & + \cos\left(2\theta - \frac{11\pi}{10}\right) + \cos\left(2\theta - \frac{15\pi}{10}\right) \\ & - \cos\left(4\theta - \frac{5\pi}{10}\right) - \cos\left(4\theta - \frac{7\pi}{10}\right) \\ & - \cos\left(4\theta - \frac{13\pi}{10}\right) - \cos\left(4\theta - \frac{15\pi}{10}\right) \end{aligned} \right\} \quad (15)$$

The I-Power produced by the fundamental current and the

remaining back-EMF components also produces ripples in the torque as they consist of cosine components.

The I-Power is obtained for the SPOC condition by applying the fundamental only current component when the back-EMF consists of; only the fundamental, the fundamental and third harmonic combined, and up to the 11th harmonic in Fig.5. The torque ripple due to the back-EMF harmonics can be seen clearly in Fig.5. Notably, the third harmonic component causes higher ripples compared to the other harmonics in back-EMF as the amplitude is higher than the others. The expected torque waveform will be as shown in Fig.5(c). Another result of Fig.5 is that the fundamental current component interacts with the third harmonic component of the back-EMF for the fault-tolerant control. That means that the fundamental and third harmonic components of the back-EMF can no longer be considered as separate machines for the faulty conditions.

2) Double Phase Open-Circuit Condition

The fault-tolerant control currents of the adjacent and nonadjacent double phase open-circuit (ADPOC, NADPOC) condition has been considered. Fault-tolerant currents for the adjacent DPOC condition are in (16) [7], and the I-Power is expressed as in (17) [9]. The phasor diagram of the fault-tolerant currents is illustrated in Fig.4.

$$\begin{cases} i_{c1}^*(\theta) = 1.769I_{m1} \sin\left(\theta - 14\pi/5\right) \\ i_{d1}^*(\theta) = 1.769I_{m1} \sin\left(\theta - 6\pi/5\right) \\ i_{e1}^*(\theta) = 1.769I_{m1} \sin\left(\theta - 22\pi/5\right) \end{cases} \quad (16)$$

$$P_1^* = \left\{ i_{c1}^*(\theta)e_c(\theta) + i_{d1}^*(\theta)e_d(\theta) + i_{e1}^*(\theta)e_e(\theta) \right\} \quad (17)$$

The I-Powers in (17) due to fundamental and third harmonic back-EMFs are given in (18) and (19), respectively. The I-Power due to the fundamental current and fundamental back-EMF is constant value. However, the I-Power produced by the fundamental current and the third harmonic back-EMF consists of cosine components, and the sum of these components is not zero so ripples will be there in the torque.

$$P_{11}^*(0\omega, 2\omega) = 1.769I_{m1}E_1 \left[\cos\left(\frac{2\pi}{15}\right) + 0.5 \right] \quad (18)$$

$$P_{13}^*(2\omega, 4\omega) = \frac{1.769I_{m1}E_3}{2} \left\{ \begin{array}{l} \cos\left(2\theta - \frac{6\pi}{15}\right) + \cos\left(2\theta - \frac{20\pi}{15}\right) \\ + \cos\left(2\theta - \frac{22\pi}{15}\right) - \cos\left(4\theta - \frac{4\pi}{15}\right) \\ - \cos\left(4\theta - \frac{12\pi}{15}\right) - \cos\left(4\theta - \frac{20\pi}{15}\right) \end{array} \right\} \quad (19)$$

Fault-tolerant currents for the NADPOC condition are in (20) [7], and the I-Power is expressed as in (21) [9]. The phasor diagram of the fault-tolerant currents of NADPOC condition is illustrated in Fig.4.

$$\begin{cases} i_{b1}^*(\theta) = 2.139I_{m1} \sin\left(\theta - 2\pi/15\right) \\ i_{d1}^*(\theta) = 2.139I_{m1} \sin\left(\theta - 14\pi/5\right) \\ i_{e1}^*(\theta) = 2.139I_{m1} \sin\left(\theta - 28\pi/15\right) \end{cases} \quad (20)$$

$$P_1^* = \left\{ i_{b1}^*(\theta)e_b(\theta) + i_{d1}^*(\theta)e_d(\theta) + i_{e1}^*(\theta)e_e(\theta) \right\} \quad (21)$$

The I-Powers in (21) due to fundamental and third harmonic back-EMFs are given in (22) and (23), respectively. As in ADPOC condition, the I-Power due to the fundamental current and fundamental back-EMF is a constant value, and the I-Power produced by the fundamental current and the third harmonic back-EMF produces ripples because of including cosine components whose sum is not zero.

$$P_{11}^*(0\omega, 2\omega) = 2.139I_{m1}E_1 \left[\cos\left(\frac{4\pi}{15}\right) + 0.5 \right] \quad (22)$$

$$P_{13}^*(2\omega, 4\omega) = \frac{2.139I_{m1}E_3}{2} \left\{ \begin{array}{l} \cos\left(2\theta - \frac{10\pi}{15}\right) + \cos\left(2\theta - \frac{12\pi}{15}\right) \\ + \cos\left(2\theta - \frac{14\pi}{15}\right) - \cos\left(4\theta - \frac{8\pi}{15}\right) \\ - \cos\left(4\theta - \frac{10\pi}{15}\right) - \cos\left(4\theta - \frac{22\pi}{15}\right) \end{array} \right\} \quad (23)$$

The I-Power is plotted for each adjacent and nonadjacent DPOC condition by applying the only fundamental current component when the back-EMF consists of only the fundamental, the fundamental plus third harmonic, and combined harmonics up to the 11th harmonic in Fig.5. Ripples caused by the higher-order harmonics of the back-EMF can be seen in Fig.5. As in the previous case, the third harmonic component causes higher ripples than the other harmonics in back-EMF. Expected torque waveforms are shown in Fig.5(c).

3) Three Phase Open-Circuit Condition

The fault-tolerant control currents of the adjacent and nonadjacent three-phase open-circuit (ATPOC, NATPOC) condition has been discussed. Fault-tolerant currents for the adjacent and nonadjacent TPOC are in (24) [7] and (28)[7], and I-Powers are in (25) and (29), respectively. The phasor diagram of the fault-tolerant currents can be seen in Fig.4.

$$\begin{cases} i_{c1}^*(\theta) = 2.628I_{m1} \sin\left(\theta - 7\pi/10\right) \\ i_{d1}^*(\theta) = 2.628I_{m1} \sin\left(\theta - 13\pi/10\right) \end{cases} \quad (24)$$

$$P_1^* = \left\{ i_{c1}^*(\theta)e_c(\theta) + i_{d1}^*(\theta)e_d(\theta) \right\} \quad (25)$$

I-Powers in (25) due to fundamental and third harmonic back-EMFs are expressed as in (26) and (27), respectively. As in previous cases, the I-Power due to the fundamental current and fundamental back-EMF is a constant value, and the I-Power produced by the fundamental current and the third harmonic back-EMF consists of cosine components. Since the sum of these components is not zero, torque will have ripples.

$$P_{11}^*(0\omega, 2\omega) = 2.628I_{m1}E_1 \left[\cos\left(\frac{\pi}{10}\right) \right] \quad (26)$$

$$P_{13}^*(2\omega, 4\omega) = \frac{2.628I_{m1}E_3}{2} \left\{ \begin{array}{l} \cos\left(2\theta - \frac{3\pi}{10}\right) + \cos\left(2\theta - \frac{7\pi}{10}\right) \\ - \cos\left(4\theta - \frac{9\pi}{10}\right) - \cos\left(4\theta - \frac{11\pi}{10}\right) \end{array} \right\} \quad (27)$$

$$\begin{cases} i_{b1}^*(\theta) = 4.253I_{m1} \sin\left(\theta - \pi/10\right) \\ i_{e1}^*(\theta) = 4.253I_{m1} \sin\left(\theta - 19\pi/10\right) \end{cases} \quad (28)$$

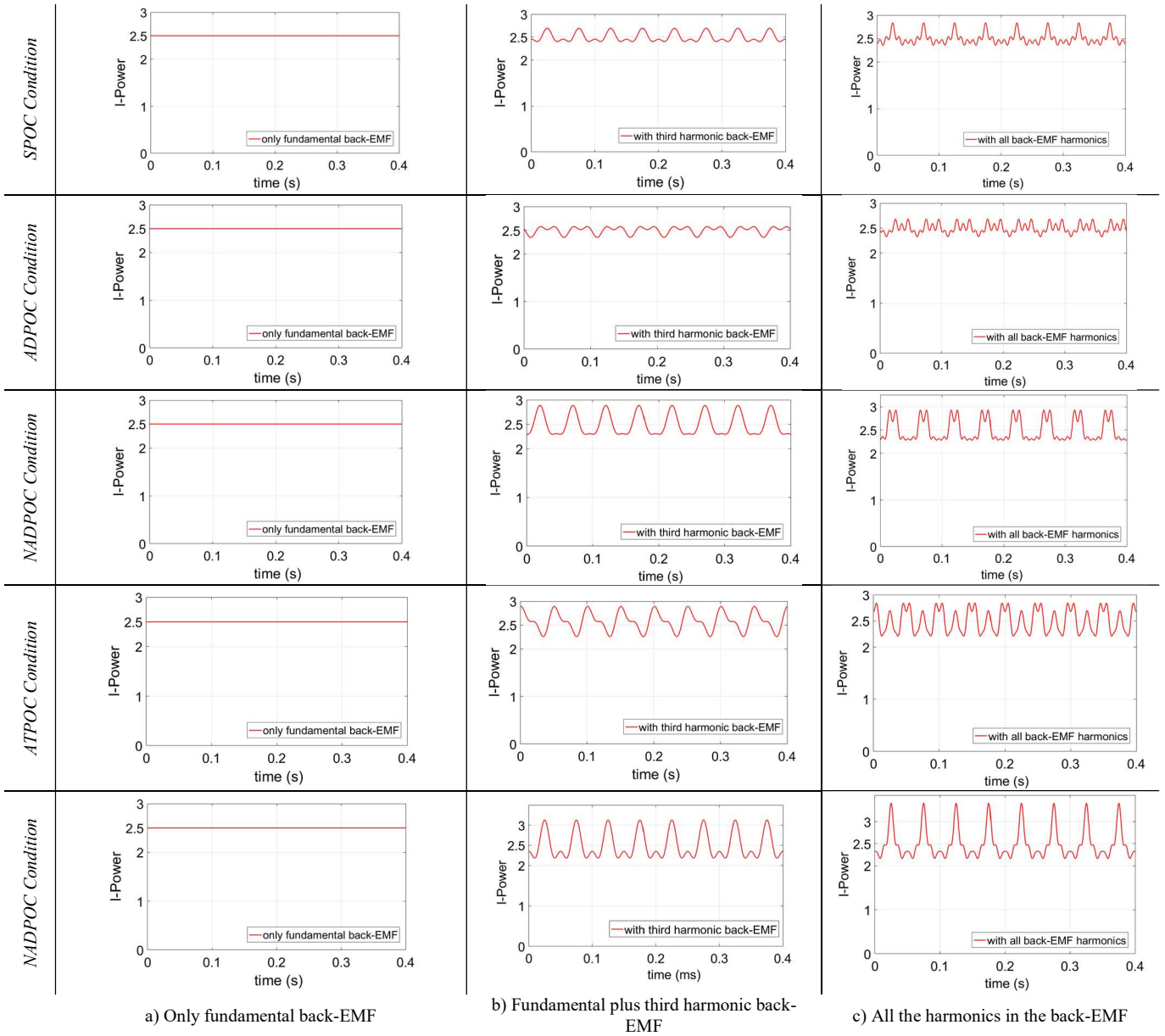


Fig. 5. I-Power of fault-tolerant condition when the only fundamental current is applied.

$$P_1^* = \{i_{b1}^*(\theta)e_b(\theta) + i_{e1}^*(\theta)e_e(\theta)\} \quad (29)$$

Same process goes for the equation (29), and the I-Power produced by the fundamental current and the fundamental back-EMF is in (30) and it is a constant value. The I-Power due to the fundamental current and the third harmonic back-EMF has been analyzed in (31), and it will cause ripples in the torque as the sum of the cosine components is not zero.

$$P_{11}^*(0\omega, 2\omega) = 4.253I_{m1}E_1 \left[\cos\left(\frac{3\pi}{10}\right) \right] \quad (30)$$

$$P_{13}^*(2\omega, 4\omega) = \frac{4.253I_{m1}E_3}{2} \left\{ \begin{array}{l} \cos\left(2\theta - \frac{9\pi}{10}\right) + \cos\left(2\theta - \frac{11\pi}{10}\right) \\ -\cos\left(4\theta - \frac{7\pi}{10}\right) - \cos\left(4\theta - \frac{13\pi}{10}\right) \end{array} \right\} \quad (31)$$

I-Power graphs of these conditions can be seen in Fig.5.

B. Fault-Tolerant Control Currents for the Fundamental Plus Third Harmonic Current Component

In this section, the third harmonic currents for the open-circuit fault-tolerant control strategies are derived. I-Power is obtained by applying the fundamental plus third harmonic fault-tolerant control currents to the five-phase machine when the back-EMF consists of only the fundamental, the fundamental plus third harmonic, and combined harmonics up to the 11th harmonic. Also, the I-Power is plotted by applying the only third harmonic component current to the third harmonic back-EMF to check the third harmonic current components of the fault-tolerant control. In the analysis, it is assumed that, $I_{m3} = 0.2I_{m1}$ and the $E_3 = 0.096E_1$.

1) Single Phase Open-Circuit Condition

Fault-tolerant currents with the third harmonic component

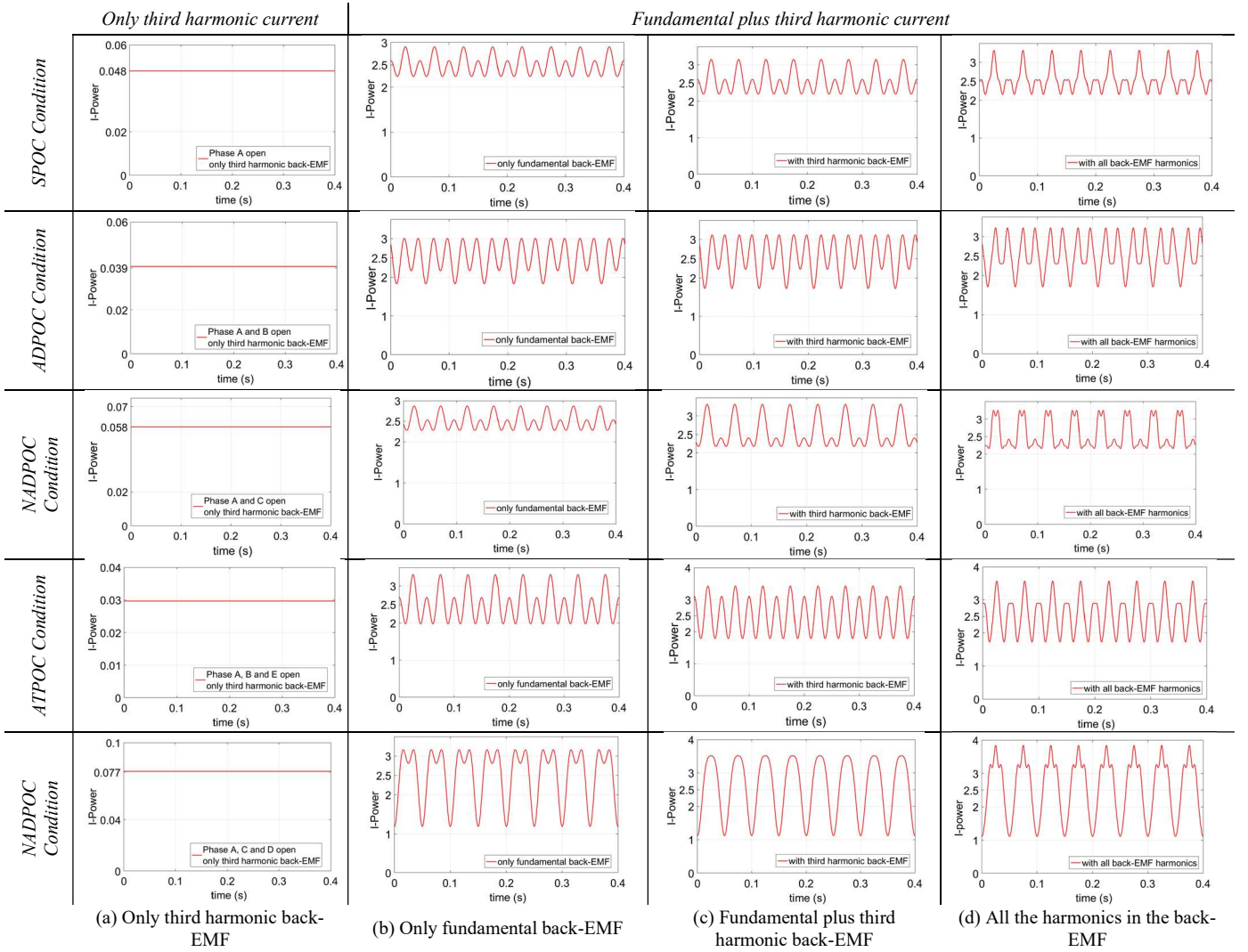


Fig. 6. I-Power of fault-tolerant condition when the fundamental plus third harmonic current is applied.

for the SPOC condition are given in (32). It is assumed that Phase A is open-circuited. The phasor diagram of the fundamental and the third harmonic currents are in Fig. 4. The I-Power for the SPOC and the other open-circuit fault-tolerant conditions can be generalized in (33) when the fundamental plus third harmonic current is applied.

$$\begin{cases} i_b^*(\theta) = i_{b1}^*(\theta) + 1.314I_{m3} \sin\left(3\theta - 11\pi/10\right) \\ i_c^*(\theta) = i_{c1}^*(\theta) + 1.314I_{m3} \sin\left(3\theta - 3\pi/10\right) \\ i_d^*(\theta) = i_{d1}^*(\theta) + 1.314I_{m3} \sin\left(3\theta - 17\pi/10\right) \\ i_e^*(\theta) = i_{e1}^*(\theta) + 1.314I_{m3} \sin\left(3\theta - 9\pi/10\right) \end{cases} \quad (32)$$

$$P^* = P_1^* + P_3^* \quad (33)$$

P_1^* has already been derived in previous section for the five fault-tolerant conditions. P_3^* can also be split into parts. The I-Power due to the third harmonic current and fundamental back-EMF in (34), and the I-Power due to the third harmonic current and the third harmonic back-EMF in (35) for the SPOC condition. Equation (34) causes ripples as it consists of cosine

components, and the sum of them is not zero.

$$P_{31}^*(2\omega, 4\omega) = \frac{2.628I_{m3}E_3}{2} \begin{cases} \cos\left(2\theta - \frac{5\pi}{10}\right) + \cos\left(2\theta - \frac{7\pi}{10}\right) \\ + \cos\left(2\theta - \frac{13\pi}{10}\right) + \cos\left(2\theta - \frac{15\pi}{10}\right) \\ - \cos\left(4\theta - \frac{5\pi}{10}\right) - \cos\left(4\theta - \frac{9\pi}{10}\right) \\ - \cos\left(4\theta - \frac{11\pi}{10}\right) - \cos\left(4\theta - \frac{15\pi}{10}\right) \end{cases} \quad (34)$$

$$P_{33}^*(0\omega, 6\omega) = 2.628I_{m3}E_3 \left[\cos\left(\frac{\pi}{10}\right) \right] \quad (35)$$

The I-Power of SPOC condition for the four cases can be seen in Fig.6. I-Power is obtained by combining the fundamental and the third harmonic current components when the back-EMF consists of only the fundamental, the fundamental plus third harmonic, and combined harmonics up to the 11th harmonic in Fig.6. I-Power is also plotted for the only third harmonic current component and the only third harmonic of the back-EMF, as well.

Fig.6 proves that the third harmonic current component interacts with the fundamental component of the back-EMF and vice versa. This interaction will produce ripples in the torque waveform. Also, the interaction between the other harmonics will cause extra ripples, as seen in Fig.6.

2) Double Phase Open-Circuit Condition

Fault-tolerant currents with the third harmonic component for the ADPOC and the NADPOC conditions are given in (36) and (39), respectively. The phasor diagram of the fundamental and the third harmonic currents are illustrated in Fig.4

$$\begin{cases} i_c^*(\theta) = i_{c1}^*(\theta) + 1.769I_{m3} \sin\left(3\theta - \frac{2\pi}{15}\right) \\ i_d^*(\theta) = i_{d1}^*(\theta) + 1.769I_{m3} \sin\left(3\theta - \frac{8\pi}{15}\right) \\ i_e^*(\theta) = i_{e1}^*(\theta) + 1.769I_{m3} \sin\left(3\theta - \frac{16\pi}{15}\right) \end{cases} \quad (36)$$

The I-Power produced by the third harmonic current and the fundamental back-EMF is in (37), and the I-Power due to the third harmonic current and the third harmonic back-EMF is in (38) for the ADPOC condition.

$$P_{31}^*(2\omega, 4\omega) = \frac{1.769I_{m3}E_1}{2} \left\{ \begin{array}{l} \cos\left(2\theta - \frac{6\pi}{15}\right) + \cos\left(2\theta - \frac{20\pi}{15}\right) \\ + \cos\left(2\theta - \frac{22\pi}{15}\right) - \cos\left(4\theta - \frac{10\pi}{15}\right) \\ - \cos\left(4\theta - \frac{12\pi}{15}\right) - \cos\left(4\theta - \frac{14\pi}{15}\right) \end{array} \right\} \quad (37)$$

$$P_{33}^*(0\omega, 6\omega) = 1.769I_{m3}E_3 \left[\cos\left(\frac{4\pi}{15}\right) + 0.5 \right] \quad (38)$$

$$\begin{cases} i_b^*(\theta) = i_{b1}^*(\theta) + 2.139I_{m3} \sin\left(3\theta - \frac{6\pi}{15}\right) \\ i_d^*(\theta) = i_{d1}^*(\theta) + 2.139I_{m3} \sin\left(3\theta - \frac{22\pi}{15}\right) \\ i_e^*(\theta) = i_{e1}^*(\theta) + 2.139I_{m3} \sin\left(3\theta - \frac{14\pi}{15}\right) \end{cases} \quad (39)$$

For the NADPOC condition, the I-Power produced by the third harmonic current and the fundamental back-EMF is in (40), and the I-Power due to the third harmonic current and the third harmonic back-EMF is in (41).

$$P_{31}^*(2\omega, 4\omega) = \frac{2.139I_{m3}E_1}{2} \left\{ \begin{array}{l} \cos\left(2\theta - \frac{4\pi}{15}\right) + \cos\left(2\theta - \frac{12\pi}{15}\right) \\ + \cos\left(2\theta - \frac{20\pi}{15}\right) - \cos\left(4\theta - \frac{8\pi}{15}\right) \\ - \cos\left(4\theta - \frac{10\pi}{15}\right) - \cos\left(4\theta - \frac{24\pi}{15}\right) \end{array} \right\} \quad (40)$$

$$P_{33}^*(0\omega, 6\omega) = 2.139I_{m3}E_3 \left[\cos\left(\frac{2\pi}{15}\right) + 0.5 \right] \quad (41)$$

3) Three Phase Open-Circuit Condition

Fault-tolerant currents with the third harmonic component for the ATPOC and the NATPOC conditions can be seen in (42) and (45), respectively. The phasor diagram of the fundamental and the third harmonic currents are illustrated in Fig.4.

$$\begin{cases} i_b^*(\theta) = i_{b1}^*(\theta) + 2.628I_{m3} \sin\left(3\theta - \frac{\pi}{10}\right) \\ i_d^*(\theta) = i_{d1}^*(\theta) + 2.628I_{m3} \sin\left(3\theta - \frac{19\pi}{10}\right) \end{cases} \quad (42)$$

The I-Power produced by the third harmonic current and fundamental back-EMF is in (43), and the I-Power due to the third harmonic current and the third harmonic back-EMF is in (44) for the ATPOC condition.

$$P_{31}^*(2\omega, 4\omega) = \frac{2.628I_{m3}E_1}{2} \left\{ \begin{array}{l} \cos\left(2\theta - \frac{3\pi}{10}\right) + \cos\left(2\theta - \frac{7\pi}{10}\right) \\ - \cos\left(4\theta - \frac{9\pi}{10}\right) - \cos\left(4\theta - \frac{11\pi}{10}\right) \end{array} \right\} \quad (43)$$

$$P_{33}^*(0\omega, 6\omega) = 2.628I_{m3}E_3 \cos\left(\frac{3\pi}{10}\right) \quad (44)$$

$$\begin{cases} i_b^*(\theta) = i_{b1}^*(\theta) + 4.253I_{m3} \sin\left(3\theta - \frac{13\pi}{10}\right) \\ i_e^*(\theta) = i_{e1}^*(\theta) + 4.253I_{m3} \sin\left(3\theta - \frac{7\pi}{10}\right) \end{cases} \quad (45)$$

For the NATPOC condition, the I-Power produced by the third harmonic current and the fundamental back-EMF is in (46), and the I-Power due to the third harmonic current and the third harmonic back-EMF is in (47).

$$P_{31}^*(2\omega, 4\omega) = \frac{4.253I_{m3}E_1}{2} \left\{ \begin{array}{l} \cos\left(2\theta - \frac{9\pi}{10}\right) + \cos\left(2\theta - \frac{11\pi}{10}\right) \\ - \cos\left(4\theta - \frac{3\pi}{10}\right) - \cos\left(4\theta - \frac{17\pi}{10}\right) \end{array} \right\} \quad (46)$$

$$P_{33}^*(0\omega, 6\omega) = 4.253I_{m3}E_3 \cos\left(\frac{\pi}{10}\right) \quad (47)$$

V. SIMULATION ON AN FEA CIRCUIT BASED MODEL

In this section of the study, a model five-phase fractional slot PM machine has been undertaken in an FEA software. Torque ripples have been observed for the healthy condition and the fault-tolerant control. Simulations have only been done for the fundamental current of the healthy and the faulty conditions. The FEA model of the five-phase PM machine was introduced in Section II. In all simulations, the rated current of the five-phase machine has been used as a reference current. The back-EMF waveform of the machine is shown in Fig.1. All simulations have been performed at a rotor speed of 600rpm with an electrical excitation frequency of 40Hz.

The FEA torque results are obtained for one mechanical revolution of the PM five-phase machine. As the machine has 4 pole pairs, frequency of the fundamental currents should be 4 times of the mechanical frequency per mechanical revolution. Therefore, period of the fundamental current is 25ms in the simulations. That is, obtained FEA torque results belong to the four-cycle period of the fundamental current.

Six FEA simulations have been undertaken to observe the

TABLE II
TORQUE RIPPLES FOR THE HEALTHY AND OPEN-CIRCUIT FAULT-TOLERANT CONDITION

	No Fault	SPOC	ADPOC	NADPOC	ATPOC	NATPOC
Average Torque	6.697	6.672	6.689	6.503	6.504	6.167
Torque ripple (%)	7.193	19.725	15.370	23.474	22.415	38.291

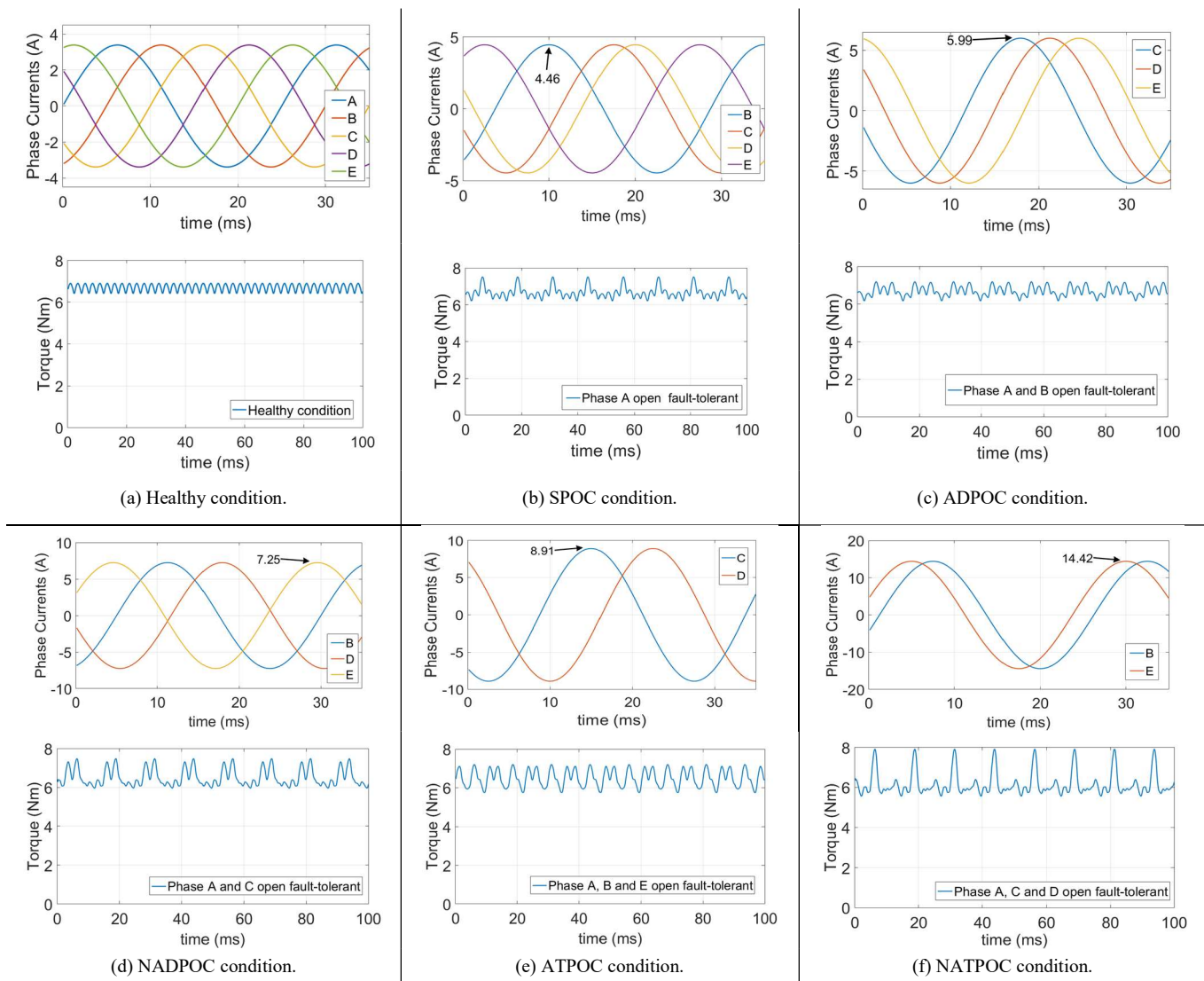


Fig. 7. FEA torque results.

torque results. All simulations have only been carried out for the fundamental component of the phase currents. Firstly, the healthy condition of the five-phase PM machine has been undertaken. A balanced set of five-phase currents, as seen in Fig.7(a), has been applied to the machine's windings. The torque result of the healthy condition has ripples, as expected according to the analytical results in (7). These ripples are caused by the 9th and 11th harmonic of the back-EMF and the cogging torque of the machine.

Secondly, an FEA simulation of five open-circuit fault-tolerant conditions for the five-phase machine has been done. The applied fault-tolerant currents and the FEA torque results can be seen in Fig.7 for all open-circuit fault conditions. The analytical results in section III and the FEA torque results have similar waveforms. The FEA simulation results validate the analytical study. The FEA torque results have ripples, and the third harmonic of the back-EMF causes higher ripples than the other higher-order harmonics as the amplitude of the third harmonic back-EMF is higher than the others. Torque ripples

that belong to the healthy and fault-tolerant conditions can be seen in Table II.

The FEA torque results are obtained for one mechanical revolution of the PM five-phase machine. As the machine has 4 pole pairs, the frequency of the fundamental currents should be 4 times of the mechanical frequency per mechanical revolution. Therefore, the period of the fundamental current is 25ms in the simulations. That is, obtained FEA torque results belong to the four-cycle period of the fundamental current.

It is clear that all fault-tolerant conditions have eight higher peak ripples. These ripples are caused by the interaction of the fundamental current component and the third harmonic back-EMF. The I-Power produced by the fundamental current and the third harmonic back-EMF has been derived in section III. This I-Power consists of cosine components for the all fault-tolerant conditions, and the sum of them is not zero. There are two different cosine components whose frequencies are 2ω and 4ω in this I-Power. The sum of these cosine components produces waveform which looks like in Fig. 5(b) at zero level.

There will be additional ripples due to other harmonics apart from fundamental and the third harmonic back-EMF. However, amplitude of this ripples will be lower than those caused by the third harmonic back-EMF. Therefore, it is clear from the Fig.7, the FEA torque results validate the analytical results.

VI. CONCLUSION

This paper presents research on torque ripples in a multiphase PM machine for both the healthy and the open-circuit fault-tolerant conditions in terms of the I-Power approach. Pulsations in I-Power causes ripples in torque according to the torque equation in (1).

In the healthy condition, if the fundamental component of the phase currents is applied to the phase windings, then the 9th and 11th harmonic of the back-EMF produces power pulsations due to the interaction between the fundamental current and these back-EMF harmonics. Both ripples have the same frequency; it is ten times the electrical frequency. The amplitudes of these pulsations depend on the amplitudes of the 9th and 11th back-EMF harmonics.

If the combined fundamental and third harmonic components of the phase currents is inputted into the phase windings, the I-Power is split into two parts for the healthy condition. In addition to the I-Power due to fundamental component, the third harmonic current component produces another I-Power that has two power components. As in the I-Power produced by the fundamental current component, one of the power components contributes to the torque, while the other produces ripples in torque. The back-EMF harmonics that cause ripples for the third harmonic current are the 7th and 13th ones. Both pulsations have the same frequency and are ten times the electrical frequency. That is, ripples produced by the third harmonic current can positively or negatively contribute to the ripples produced by the fundamental current. This contribution depends on the amplitude of the back-EMF harmonic contents that cause ripples. Additional results for the healthy condition, the fundamental component and the third harmonic component of the back-EMF can be considered as separate machines.

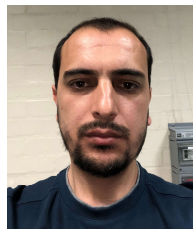
Torque ripples have been discussed analytically in section IV for the fault-tolerant conditions. Analytical solutions have been done for the I-Powers for the fundamental and the third harmonic back-EMF when the only fundamental current or fundamental plus third harmonic current have been applied. Analytical solutions show that the sum of the I-Power produced by the fundamental current and third harmonic back-EMF (or vice versa) is no longer zero. That is the proof of that there is an interaction between fundamental current and the third harmonic back-EMF, and vice versa. That means that the fundamental component and the third harmonic component of the back-EMF can no longer be considered as separate machines for the faulty conditions.

The I-Power has been plotted by applying the only fundamental current component or combining the fundamental plus the third harmonic current components. It is clear that from the graphical results, the fundamental current component

interacts with the third harmonic component of the back-EMF and vice versa. According to the graphical results, these interactions cause higher ripples compared to interactions with the other higher-order harmonics. The amplitude of the ripples depends on the amplitude of the back-EMF harmonics, and the amplitude of the harmonic components of the currents. However, the frequency of the produced ripples due to interaction of the existing harmonics and the applied current components will be same for all PM machine types. FEA torque results in section IV validate the analytical results. For future work, a fault-tolerant control strategy can be developed for non-sinusoidal back-EMF machines.

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