

Cogging Torque Sensitivity considering Imperfect Magnet Positioning for Permanent Magnet Machines of Different Slot and Pole Count

Gerd Bramerdorfer, *Senior Member, IEEE*, Edmund Marth, and Gereon Goldbeck

Abstract—This work is about analyzing surface mounted permanent magnet machines regarding their sensitiveness related to erroneous magnet positioning. A finite element analysis based approach is presented and different topologies in terms of slot and pole count are compared. The study further includes the analysis of multiple magnet widths and stator teeth widths. By contrast to most of previous studies, the work is based on evaluating the cumulative distribution function of the cogging torque in case of non-idealities. A Monte Carlo importance sampling based strategy is focused. This approach facilitates studying arbitrary tolerance distributions. Results reveal that topologies with particularly promising rated cogging torque behaviour exhibit the most significant performance degradation in presence of tolerances. A linear relationship is identified for cogging torque performance as function of the accuracy in magnet positioning. Results emphasize the necessity of tolerance analyses for electric machine design to not overrate their performance in the presence of manufacturing uncertainties.

Index Terms—cumulative distribution function, electric machine, Monte Carlo, PMSM, quantile, sensitivity, tolerance analysis

I. INTRODUCTION

PERMANENT magnet machines are widely applied due to their outstanding performance in terms of efficiency, power density, and power factor [1]. However, the field excitation through permanent magnets also causes undesired cogging torque, which can cause vibrations and noise and, at worst, affects the overall machine operation, e.g., in terms of more frequent bearing failures. Consequently, cogging torque reduction is generally important in electric machine design and in particular for mass-produced arrangements [2], [3]. Besides constructive measures like skewing, auxiliary slots, or special rotor contour shaping [4], [5], general suggestions were provided in [6] in terms of which slot and pole number facilitate low cogging torque.

While the effect of aforementioned measures is usually studied for ideal arrangements, cogging torque characteristics

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might significantly differ in the presence of inevitable tolerances. Thus, researchers try to analyze (i) which tolerance-affected parameters impact the cogging torque performance most and (ii) if there are outstanding designs considering the selection of the topology, materials, and geometric dimensions.

In terms of internal permanent magnet (IPM) machines, magnetization faults [7], changes of the contour of the rotor lamination [8] as well as the impact of different slot- and pole counts [9] were investigated.

Surface mounted permanent magnet (SPM) machines were for instance studied in [10]–[12]. The major work is focused on uncertainties related to the permanent magnets, while some authors consider non-conventional tolerances, e.g., an axial displacement of the rotor relative to the stator position [13].

Tolerance analyses performed by utilizing finite element analysis (FEA) software follows high computational cost. Thus, authors try to find appropriate analytical models for representing the effect of tolerances, e.g., in [14]–[18]. Besides, research is carried out to derive general conclusions regarding the cogging torque sensitiveness of designs with regard to particular magnetomotive force (mmf)- or permeance-harmonics in the air gap field [11], [19].

Most of the work related to tolerance analysis is focused on deriving quantitative measures based on computations for a single machine design and verifying the results by measurements. However, there is further interesting activity, e.g., where authors try to find a systematic approach for identifying the non-idealities of a manufactured design based on measurement results [20]. As measuring (cogging) torque in the process of mass-production of electric machines is costly and difficult to realize, some authors try to find a (simplified) relationship of the easier-to-measure Back-EMF with the cogging [21], [22].

Tolerances can be categorized in terms of the required effort for modeling and evaluation [23]. While, e.g., an error in the rotor angle measurement usually only requires additional evaluations of already existing models, two more groups can be found: (i) ‘symmetric’ tolerances and (ii) ‘asymmetric’ tolerances. The first group for instance includes the analysis of the same change of the magnet height of all utilized permanent magnets. This can be relevant when magnets vary from batch to batch. Such parameters are often considered in optimization scenarios. So, some data is available, but it requires an accurate model and an efficient analysis to study the effect of tolerances during optimization [24]. However, additional simulations might also be required, as a change

of, e.g., the permeability of the magnets, typically is not considered during the design process.

The second group consists of tolerance considerations that usually cause the design to lose its initially present circumferential symmetry. For instance, individual errors in magnet positioning are likely present when manufacturing electric machines. This group constitutes the most challenging one in terms of computational and modeling effort. Most likely, no data is available at all during the regular machine design process for this group of tolerances.

While several authors considered asymmetric tolerances for single machine design analyses, e.g., in [17], [18], [25], the biggest share of the research carried out so far focused on symmetric tolerances, as it was for instance done in [10]. This becomes even more apparent in the case that tolerance analysis shall be incorporated to optimization scenarios [26]–[31].

Most tolerance analyses incorporated to optimization problems are either about identifying the worst case [32] or follow a six sigma based approach [26], [27]. In case of non safety-critical applications, the worst case based robust optimization usually follows too conservative designs. The six sigma based approach is excellent for increasing production quality. In terms of electric machine performance measures, a cumulative density function (cdf) based approach might feature some benefits, as it allows to specify a threshold value for minimum requirements on the machine performance. Consequently, the share of tolerance-affected combinations that fulfills this constraint is evaluated. While previously applied in other engineering domains [33], recently a successful implementation was presented for electric machine design optimization in [31]. There, the work, again, dealt with symmetric tolerances.

As a consequence, here the authors apply the cdf-based evaluation for asymmetric tolerances. The cogging torque of different brushless permanent magnet machines in terms of slot- and pole-count and with regard to geometric dimensions shall be evaluated for uncertainties in the magnet positioning of SPM machines. In contrast to previous studies, general conclusions can be derived thanks to the thorough analysis of different geometric configurations and the innovative sampling and evaluation concept. This gives valuable insights for engineers dealing with electric machine design. It further allows drawing conclusions about the sensitivity of particular slot-/pole-count combinations considering imperfect magnet positioning. This extends basic knowledge, as conventional analyses usually cover only the comparison of rated performance for perfect (=flawless) magnet positioning or only go for single or few design analyses.

The remainder of this paper is organized as follows: Section II introduces the basic model used for this analysis and therewith associated definitions. This will be followed by the analysis of the rated cogging torque for all defined arrangements in case of no tolerances present in Section III. The sampling process and cdf-based evaluation in case of erroneous magnet positioning will be explained in Section IV. The results and a thorough comparison of the tolerance sensitiveness are illustrated in Section V. A conclusion and outlook about future work given in Section VI complete this research activity.

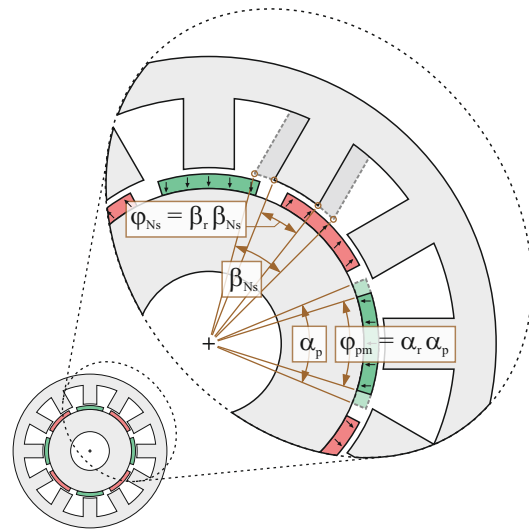


Fig. 1. Basic model of the inner rotor SPM machine applied throughout this study.

II. THE BASIC MODEL

The basic model used for this analysis is illustrated in Fig. 1. The work is focused on the cogging torque for the rated ('ideal') situation and the scenario where tolerance-induced errors in magnet positioning are present. The obtained results should allow deriving general conclusions about which slot-/pole-combination is more or less prone to significantly change its cogging torque performance through the considered non-idealities. The following assumptions and constraints were defined:

- only interior-rotor machines are analyzed,
- no special tooth-shapes are considered,
- magnets are of ring-segment shape and feature parallel magnetization.

Even though results will differ for, e.g., other tooth shapes, radial magnetization, different magnet shapes, or exterior-rotor machines, the overall general characteristics and tendencies for a particular slot-/pole-count combination are expected to not vary significantly.

The magnet pole pitch α_p is defined by

$$\alpha_p = \frac{2\pi}{2p}, \quad (1)$$

where $2p$ gives the number of magnet poles. Consequently, by introducing the relative pole coverage α_r with $0 \leq \alpha_r \leq 1$, the magnet angle φ_{pm} is defined as

$$\varphi_{pm} = \alpha_r \cdot \alpha_p. \quad (2)$$

With constant φ_{pm} along the radial extent of the permanent magnets, they are of ring-segment shape.

In case of the stator teeth, the teeth flanks are defined to be parallel. Here, φ_{Ns} is defined to be the stator tooth angle at the stator inner diameter. Similarly to the magnets, the relative width of the stator teeth can be defined. The (theoretical) maximum stator tooth width β_{Ns} is given by

$$\beta_{Ns} = \frac{2\pi}{N_s}, \quad (3)$$

TABLE I
ANALYZED TOPOLOGIES

Name	Symbol	Topologies		
stator slot- / rotor pole- combinations	$N_s /$ $(2p)$	9/8	12/8	12/10
		$\hat{=}$ 9S8P	$\hat{=}$ 12S8P	$\hat{=}$ 12S10P
goodness factor	C_T	1	4	2
fundamental winding factor	ξ_f	0.945	0.866	0.933

where N_s gives the number of stator slots or number of stator teeth. Again, a relative measure for the stator tooth width compared to its maximum is introduced as β_r with $0 \leq \beta_r \leq 1$. Finally, the circumferential stator tooth width φ_{N_s} can be computed by using:

$$\varphi_{N_s} = \beta_r \cdot \beta_{N_s} . \quad (4)$$

While for the relative magnet pole coverage α_r values close to 1 are favored for high specific power densities, the relative stator tooth width β_r is defined by evaluating the tradeoff maximizing the space for the stator coil while maintaining a reasonable flux density in the stator teeth for efficient electric machine operation. Besides, both ratios α_r and β_r are varied for minimizing the cogging torque.

Besides the above mentioned ratios, the cogging torque is strongly dependent on the combination of number of stator slots N_s and rotor magnets $2p$. Considering a mechanical rotor revolution, the fundamental frequency of the cogging torque n_c is

$$n_c = \text{lcm}(N_s, 2p) . \quad (5)$$

With increase of n_c , the cogging torque is said to be lower. In [6], a factor C_T is empirically defined as

$$C_T = \frac{2p N_s}{n_c} , \quad (6)$$

which defines the ‘goodness’ of the design regarding cogging torque performance. The higher C_T , the larger cogging torque can be expected.

For this analysis, we want to investigate different typical slot-/pole-combinations for fractional horsepower machines regarding C_T and, consequently, compare their rated cogging torque as well as the change of the cogging torque due to errors in magnet positioning. In Table I, all selected combinations and their C_T -values are provided. The 12 slots / 8 poles arrangement features the highest C_T . Care was taken such that it can be directly compared with the design featuring 9 slots / 8 poles in terms of number of poles, while the 12 slots / 10 poles setup facilitates a comparison for same number of stator slots. In the following, they are called 9S8P, 12S8P, and 12S10P-configuration.

The 9S8P-variant exhibits an inherent drawback. The design features no symmetry regarding the circumferential direction when considering the machine cross section. Thus, a considerable net force is likely to be generated into the radial direction. This can cause increased noise and bearing wear.

TABLE II
ANALYZED RELATIVE ROTOR MAGNET POLE AND
STATOR TOOTH WIDTH RATIOS

Name	Symbol	Values
relative magnet pole coverage	α_r	0.75, 0.80, 0.85, 0.90, 0.95
relative stator tooth width		β_r

By contrast, the 12S8P design features a fourfold symmetry, and the 12S10P design a negative twofold symmetry, both featuring zero net force. Nevertheless, the 9S8P constitutes a good configuration for a comparison. From winding factor point of view, both the 9S8P- and the 12S10P-configuration feature a higher winding factor than the 12S8P-topology. Their values are given in the last row in Table I.

The considered values for the relative magnet pole coverage α_r and the relative stator tooth width β_r are given in Table II.

In total, (3 topologies) · (25 configurations) = 75 combinations = $n_{designs}$ are evaluated. The parameter to be studied is the cogging torque peak-to-peak value $t_{cogg,pp}$, which for the i -th design is defined as

$$t_{cogg,pp}^i = \max(t_{cogg}^i(\varphi)) - \min(t_{cogg}^i(\varphi)) , \quad (7)$$

where φ gives the rotor angle. All the nonlinear magnetostatic finite element analyses are done by using the software FEMM [34]. The flux density plot for an exemplary design can be found in Fig. 2. As the main interest is a relative comparison of cogging torque levels, no absolute values are given in the following. Instead, relative cogging torque peak-to-peak values normalized to the overall maximum value over all combinations are compared. The normalized cogging torque of the i -th design is defined by a capital letter $T_{cogg,pp}^i$ and is calculated by

$$T_{cogg,pp}^i = \frac{t_{cogg,pp}^i}{t_{cogg,pp,max}} \quad (8)$$

with

$$t_{cogg,pp,max} = \max_{i=1 \dots n_{designs}} (t_{cogg,pp}^i) . \quad (9)$$

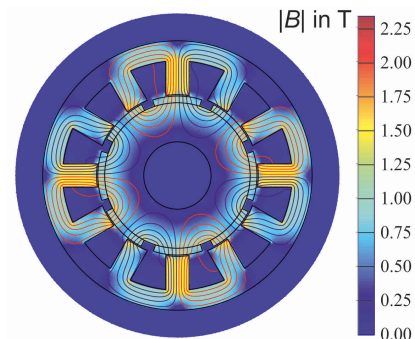


Fig. 2. Flux density plot including field lines for an exemplary machine topology featuring 12 slots and 8 poles.

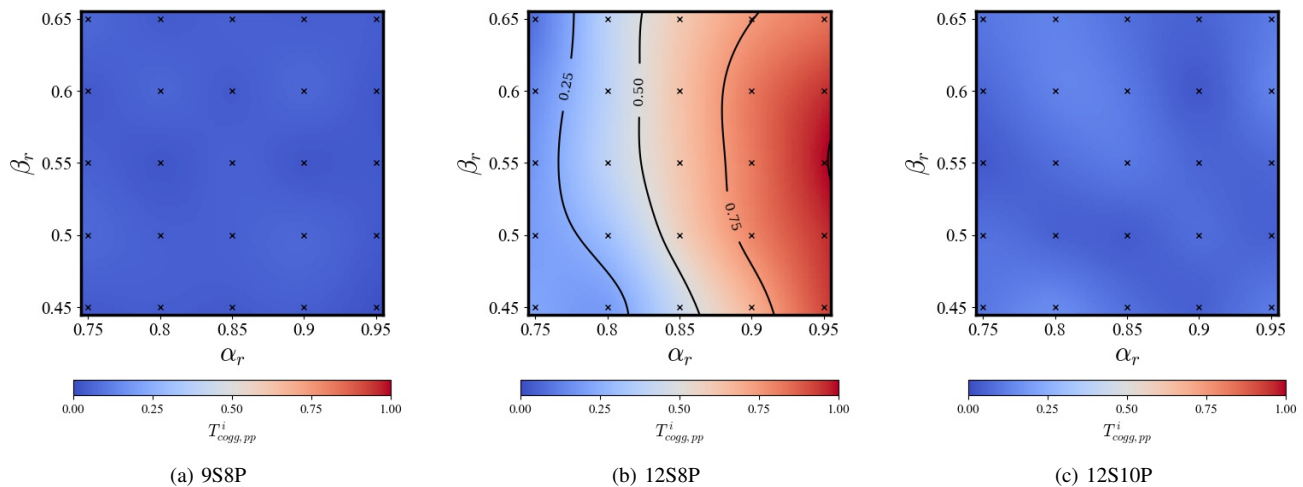


Fig. 3. Rated cogging torque as function of the relative magnet pole coverage α_r and the relative stator tooth width β_r .

The full analysis is done using the software SyMSpace [35]. The main reason is that it allows for fully automatizing the design process, which is defined by the main steps:

- geometry regeneration based on the selected design parameters,
- finite element (FE)-based analysis, and
- post-processing of the data from FE.

Besides, it facilitates utilizing a computer cluster managed through the HTCondor software [36]. While this is not an indispensable requirement in case 75 design configurations are evaluated, it will turn out to be crucial for the subsequent tolerance analysis explained in Sections IV and V.

III. RATED RESULTS

At first, the rated cogging torque for all 25 design variants for all three slot-/pole configurations are analyzed. All the combinations were evaluated by FEA. Consequently, a radial basis function (rbf) based modeling approach was considered to model $T_{cog,pp}$ as function of α_r and β_r . This was done in order to facilitate evaluating a finer grid as basis for the contour plots given in Fig. 3. The results are illustrated over the α_r/β_r -domain. The 25 initial FE-based α_r/β_r -combinations are given by the black crosses.

As can be observed, the 9S8P- and 12S10P-designs, as expected, feature a far lower cogging torque than the 12S8P counterparts. The maximum overall cogging torque is obtained for $\alpha_r = 0.95$ and $\beta_r = 0.55$ for the 12S8P topology, and thus defines $t_{cog,pp,max}$ for the normalization.

The trend of the 12S8P-topology in the considered α_r/β_r -domain is almost convex. By contrast, the 9S8P- and the 12S10P-topology feature multi-modal characteristics.

Even though also significant smaller cogging torque values than $t_{cog,pp,max}$ can be achieved for the 12S8P-topology for smaller relative magnet pole coverage α_r , the normalized individual cogging torque $T_{cog,pp}^i$ still is above the maximum level of the other two slot-/pole-combinations. Besides, a decrease of α_r follows an inevitable decrease of the flux

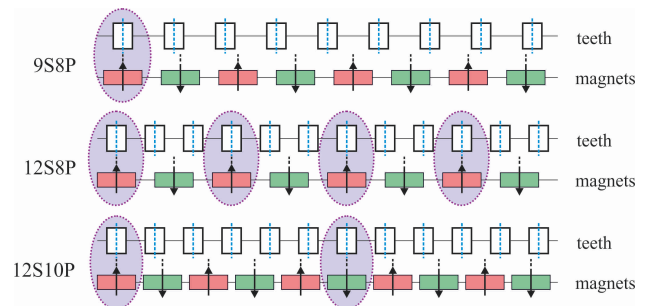


Fig. 4. Cogging torque explanation - dashed ellipses represent similar tooth/magnet-interactions along the circumference.

linkage in the stator coils and thus decreases the utilization of the machine design. Consequently, especially selecting the 12S10P-topology is promising from rated cogging torque point of view and having in mind no net radial force to be generated for the symmetric case.

Besides the goodness factor C_T , another explanation for the differences in cogging torque performance presented in the past [37], [38] is illustrated in the following (cf. Fig. 4): In case of the 12S8P-configuration, 4 stator teeth are aligned with permanent magnets at the same time. Thus, by simplifying the total cogging torque as a sum of individual cogging torque components caused by single stator teeth, four components are aligned. By contrast, the 9S8P topology features different phase shifts for the cogging torque components of all teeth and the 12S10P features only 2 stator teeth aligned with permanent magnets at the same time.

IV. APPLIED SAMPLING AND EVALUATION TECHNIQUE FOR ERRONEOUS MAGNET POSITIONING

As the analysis covers the slot-/pole-combinations 9S8P, 12S8P, 12S10P, the modeling is about 10 magnets and their positioning at maximum. Considering 10 magnets and modeling a displacement range $\Delta\gamma$ in terms of, e.g., 5 distinct

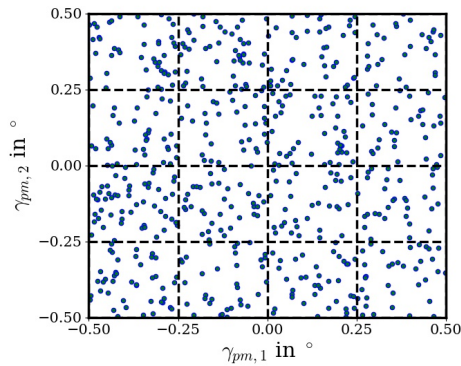


Fig. 5. Scatter plot of sampled points for the displacement of magnets 1 and 2.

positions, such that possible shifts of the position of the x -th magnet are defined to

$$\gamma_{pm,x} \in \left\{ -\frac{\Delta\gamma}{2}, -\frac{\Delta\gamma}{4}, 0, \frac{\Delta\gamma}{4}, \frac{\Delta\gamma}{2} \right\}, \quad (10)$$

the overall number of combinations $n_{pm,pos}$ to be analyzed would follow

$$n_{pm,pos} = 5^{10} = 9\,765\,625 \quad (11)$$

variants. As can be observed, this can hardly be evaluated by means of finite element (FE-) analyses. Even in case of 3 steps, $n_{pm,pos}$ is still ≈ 60000 , which is considered as minimum number of steps for facilitating deriving a surrogate model and, consequently, evaluating the design on the basis of such a model. This is still a non-reasonable number of required FE-analyses. Particularly, due to the asymmetry of the design caused by the magnet displacements, any FE-analysis requires modeling the full machine cross section and analyzing (at least) a rotor angle φ of

$$\varphi = 0 \dots \frac{2\pi}{N_s} \quad (12)$$

with appropriate discretization. In case of the 12S10P-design, the order of the fundamental cogging torque harmonic for symmetric ('ideal') condition n_c can be defined by

$$n_c = \text{lcm}(N_s, (2p)) = 60. \quad (13)$$

Thus, within the range of φ , 5 periods of the fundamental harmonic are present. Consequently, very small rotor angle steps, e.g., 0.25° or 0.50° , are required, which further increases the overall computational burden. Besides, typically tolerance-affected parameters feature a continuous distribution and thus an infinite number of combinations would be required to be evaluated.

When evaluating the effect of some tolerance-affected parameter and its corresponding level, often the worst case scenario is intended to be found. However, for conventional mass-produced electric machines, the worst case measure might not be the best quantity to apply for considering the robustness of designs. Considering a probability density function (pdf) for the parameter(s) featuring tolerance(s), the worst case is very unlikely to occur. Thus, designing a machine by considering the worst case might follow a costly conservative arrangement.

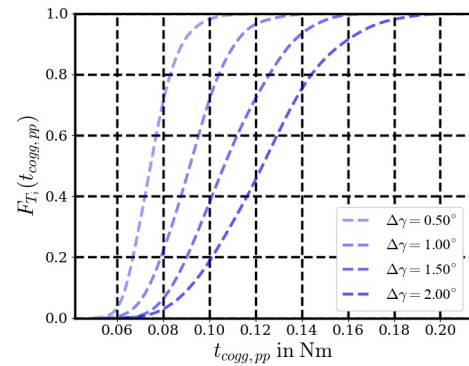


Fig. 6. Exemplary cdf as function of the error in magnet positioning $\Delta\gamma$.

By contrast, the focus of the here presented analysis is on evaluating the performance that 95% of the designs will at least achieve. In terms of the cogging torque evaluation, this follows that a limit can be derived that will not be exceeded by more than 5% of all parameter combinations. The limit defining a certain percentage threshold is typically called 'quantile' q , while here the superscript shall define the targeted percentage, i.e. $q^{0.95}$ gives the cogging torque threshold that 95% of the parameter combinations will not exceed. The cumulative distribution function (cdf) can be applied for determining the quantile level for the quantity of interest, such as the cogging torque.

As was pointed out by authors for other domains of engineering [33], the 'true' cdf can be approximated reasonably accurate by a reduced number of samples of all possible combinations for the tolerance-affected parameters. In order to derive a representative cdf, the sampling shall be based on the probability density functions of the tolerance-affected parameters. This is also called importance sampling, as parameter values featuring higher probability are more likely selected.

Fig. 5 gives exemplary results for randomly selected 600 samples of two permanent magnet angular displacements $\gamma_{pm,1}$, $\gamma_{pm,2}$. In the present case, a symmetric uniform distribution of the displacements in the range of $\pm 0.5^\circ$ is considered. In the following, such symmetric uniform ranges are defined by specifying the tolerance level $\Delta\gamma$, and $\Delta\gamma = 1^\circ$ for this particular case. As can be observed, the overall parameter space is fairly well evaluated. The sampling is not limited to uniform distributions. In case of, e.g., normally distributed parameters, more samples would lie closer to the center of 0° . If more than two permanent magnets are considered, Fig. 5 does not change in general. However, the design space is increased by one dimension for each additional magnet. A tradeoff for the number of samples to evaluate for reasonable computational effort versus an appropriate sampling of the design space has to be found.

In order to find the best compromise, it is crucial to define the robustness measures first. As mentioned above, the focus is on the 95%-quantile $q^{0.95}$ of the cogging torque. Fig. 6 gives examples for the cdfs $F_{T_i}(t_{cogg,pp})$ for different tolerance levels of the magnet displacement $\Delta\gamma$. As can be observed,

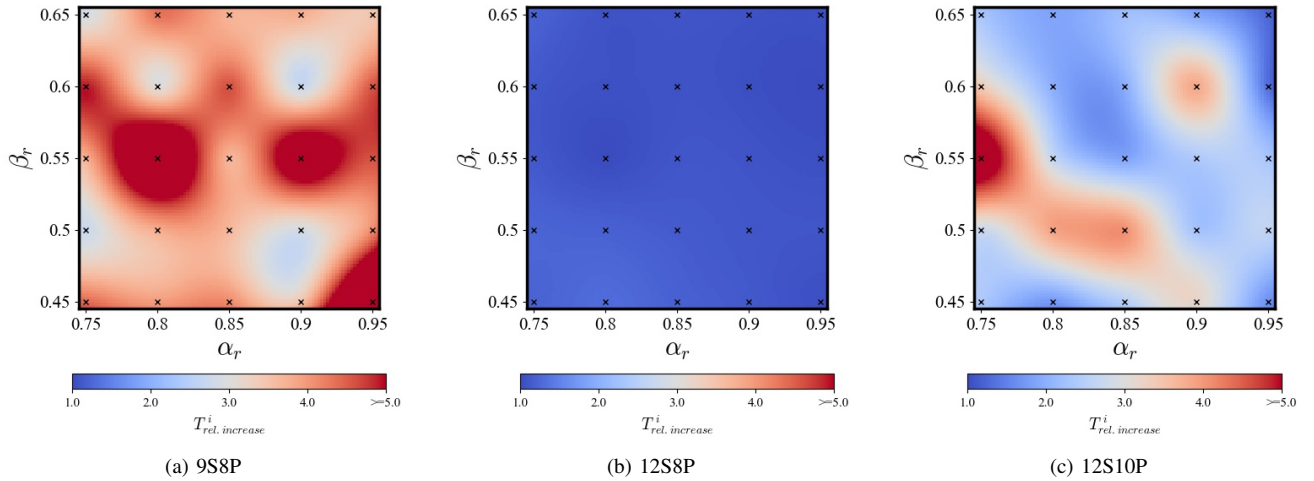


Fig. 7. Relative cogging torque increase $T^i_{rel.increase}$: The evaluation is performed for a uniformly distributed magnet positioning uncertainty $\Delta\gamma=2.00^\circ$, which corresponds to a magnet positioning accuracy of $\pm 1.00^\circ$.

the cdfs typically get more stretched when increasing the tolerance level. From this figure it becomes apparent that a cdf-based evaluation likely is most accurate for the median of the distribution, while close to q^0 and q^1 , which is equivalent to the minimum and maximum of the distribution, the accuracy is decreased. This is due to the fact that a (random) importance sampling likely will not identify the particular parameter settings for best and worst cogging torque performance. Thus, when interested in those quantities, other evaluation techniques with more promising chances for determining these extrema should be applied. In the following, the change of the cogging torque with regard to the magnets' displacements will be analyzed in detail.

V. TOLERANCE-INDUCED COGGING TORQUE CHANGE

As outlined through the last section, a detailed investigation of the change of the cogging torque due to magnet positioning errors was performed. In particular, all the 3 topologies (n_{top}) different in slot and pole count were analyzed for all $5 \times 5 = 25$ α_r, β_r -combinations $n_{\alpha/\beta}$. To allow for a thorough comparison, multiple different tolerance levels for the magnet positioning errors $\Delta\gamma$ were investigated, which are defined by the set \mathcal{T} :

$$\mathcal{T} = \{0.50^\circ, 1.00^\circ, 1.50^\circ, 2.00^\circ\}. \quad (14)$$

Thus, in total 4 tolerance levels $n_{\mathcal{T}} = 4$ were considered. For any $\Delta\gamma \in \mathcal{T}$, all magnets feature a random positioning error that is uniformly distributed within the boundaries $\left[-\frac{\Delta\gamma}{2}, \frac{\Delta\gamma}{2}\right]$. So, the maximum $\Delta\gamma = 2^\circ$ corresponds to a positioning error of $\pm 1^\circ$. The authors consider these values as reasonable for typical manufacturing qualities, even though providing general statements is difficult. The reason is that the positioning depends on the machine's overall dimensions, the manufacturing process, and many more aspects.

For each setting, $n_{samples}=600$ samples were created through Monte Carlo (MC) based importance sampling. This facilitates an accurate determination of the $q^{0.95}$ quantile for

the cogging torque based on the obtained cumulative distribution function for all combinations. Therefore, in total

$$n_{FE} = \underbrace{3}_{n_{top}} \cdot \underbrace{25}_{n_{\alpha/\beta}} \cdot \underbrace{4}_{n_{\mathcal{T}}} \cdot \underbrace{600}_{n_{samples}} = 180\,000 \quad (15)$$

cross sections were evaluated.

Fig. 7 gives the results of this study for $\Delta\gamma=2.00^\circ$. Thereby, for each design variant i , the 95%-quantile $t^{i,0.95}_{cogg,pp}$ is set in proportion to the rated cogging torque of this parameter setting $t^i_{cogg,pp}$. The so defined relative cogging torque increase is called $T^i_{rel.increase}$:

$$T^i_{rel.increase} = \frac{t^{i,0.95}_{cogg,pp}}{t^i_{cogg,pp}}. \quad (16)$$

As can be observed, for $\Delta\gamma = 2.00^\circ$, which corresponds to a maximum error in positioning of the permanent magnets of $\pm 1.00^\circ$, the relative cogging torque increase $T^i_{rel.increase}$ of the 9S8P- and the 12S10P-topology is significant and reaches maximum values of more than 5. By contrast, the cogging torque of the 12S8P-topology stays almost constant. Roughly speaking, the tolerance sensitiveness of the designs is somewhat inverse to the factor C_T introduced in [6] which refers to the rated cogging torque behavior.

To analyze the results in more detail, $T^i_{rel.increase}$ is now evaluated for all considered angular displacement tolerance levels in \mathcal{T} . Fig. 8 gives the relative increase as function of the positioning error $\Delta\gamma$ for all topologies and all α_r/β_r -combinations. Overall, the characteristics can be reasonably approximated as linear relationship for any particular setting, which is in agreement with a single design analysis presented in [39]. As can be observed, the 12S8P-topology shows no noticeable cogging increase, while the 12S10P- and the 9S8P-variant feature up to a 6- and 8-fold increase, respectively. These maximum values are obtained for $\Delta\gamma = 2^\circ$, which corresponds to maximum positioning errors of $\pm 1^\circ$.

Even though these results identify the 9S8P- and 12S10P-topology as very sensitive in terms of cogging torque change

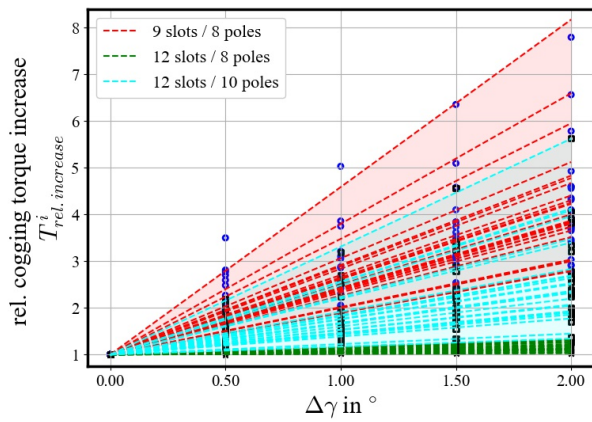


Fig. 8. Relative cogging torque increase $T^i_{rel.increase}$ as function of the magnet positioning tolerance level $\Delta\gamma$ for all topologies and α_r/β_r -combinations.

due to an erroneous magnet positioning, the evaluation was done for the individual cogging torque increase of a particular design variant compared to its performance in case of ideal symmetric magnet placement. A fair analysis should thus further include the direct comparison of the globally and not individually normalized 95%-quantile values $T^i_{cogg,pp}{}^{0.95}$ of all topologies and α_r/β_r -combinations. Fig. 9 provides a plot dedicated to this objective. Again, all topologies and combinations are illustrated against the magnet positioning error $\Delta\gamma$. As can be observed, even in case of the maximum value $\Delta\gamma = 2^\circ$, most α_r/β_r -variants of the 12S8P-topology exceed their counterparts of the other topologies. Thus, even though the relative cogging torque increase for the 9S8P- and 12S10P-topology is significant, the overall cogging torque performance is still at least competitive.

Moreover, an interesting fact is observed here: While for rated cogging torque minimization we are interested in a high least common multiple (lcm) regarding slot and and pole count,

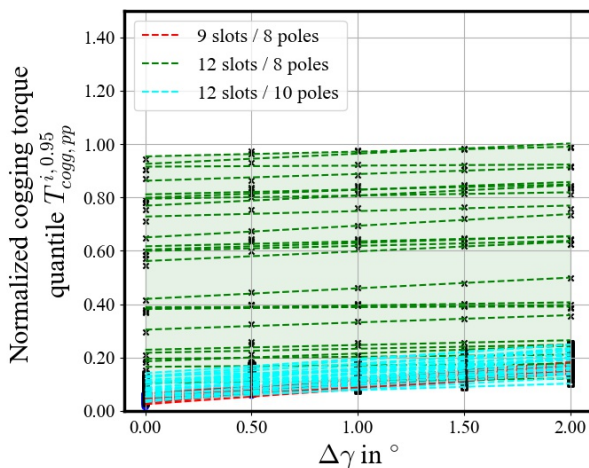


Fig. 9. Normalized cogging torque quantile $T^i_{cogg,pp}{}^{0.95}$ as function of the magnet positioning tolerance level $\Delta\gamma$ for all topologies and α_r/β_r -combinations.

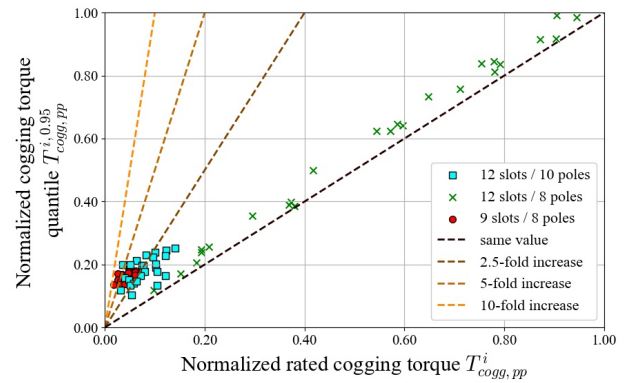


Fig. 10. Rated normalized cogging versus 95%-quantile $T^i_{cogg,pp}{}^{0.95}$.

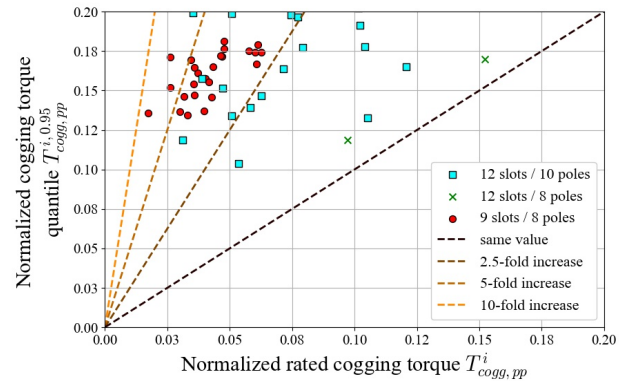


Fig. 11. Zoom of promising region of Fig. 10

which follows a low order of symmetry regarding the machine's cross section. Consequently, the 9S8P design features outstanding characteristics and thus minimum overall rated cogging with some modulation based on α_r and β_r , cf. Figs. 3 and 4 and (5), (6). However, when it comes to cogging torque sensitivity, a shift of the magnets compared to the expected position follows a significant increase of the cogging torque, as magnets interact with stator teeth more simultaneously than for the ideal rated case. Similar characteristics can be observed for the 12S10P configuration. By contrast, the 12S8P design features a fourfold symmetry. Thus, imperfect magnet positioning likely follows that, compared to the ideal case with 4 magnets are interacting equally with the stator teeth, less similar interaction is followed. This qualitative analysis allows understanding the different tolerance sensitiveness of the slot/pole-configurations.

In order to further analyze the fact of different ratios of 95%-quantile cogging torque to the rated cogging, another illustration is presented in Fig. 10, while a corresponding zoom of a certain range is given in Fig. 11. As can be observed, the 12S8P designs feature a solid robustness regarding magnet imperfections. There is no significant increase in the cogging torque. By contrast, the 9S8P features very high sensitiveness. Having a closer look at Fig. 11, one can see that 12S10P designs present the best compromise regarding rated cogging and 95%-quantile.

The here presented analysis revealed the necessity of tolerance analyses. Otherwise, taking the rated cogging torque for ideal symmetric conditions as reference might follow disappointment when measuring a manufactured prototype.

VI. CONCLUSION

This paper was about analyzing the cogging torque change due to tolerances in the magnet positioning of surface mounted permanent magnet machines. Three different slot-/pole-configurations, i.e. the 9S8P-, 12S8P-, and the 12S10P-topology were analyzed.

The analysis is done for different maximum magnet positioning errors. The focus is on levels relevant for mass-produced fractional horsepower machines. As evaluating the overall worst case is time-consuming and typically follows very conservative machine designs, here an alternative approach is applied. It is based on importance sampling and deriving the cumulative distribution function. 600 tolerance combinations are evaluated per investigated machine design, which is defined by the slot-/pole-configuration and the relative magnet pole coverage and the relative stator tooth width. Consequently, the cogging torque for a certain quantile level is evaluated. Particularly, the 95%-quantile was selected here.

The rated cogging torque of the 12S8P-configurations is far higher than the one of the 9S8P and the 12S10P counterparts. This is in agreement with the state of the art, which predicts the cogging torque characteristics based on the order of its fundamental harmonic regarding the angular motion.

The analysis reveals that the cogging torque increase through the magnet positioning error level can be accurately approximated by a linear function. Besides, it can be observed that the promising slot-/pole-configurations 9S8P and 12S10P exhibit a far higher cogging torque increase than the 12S8P-arrangement with highest rated cogging torque. Nevertheless, the net cogging torque levels of the 9S8P and 12S10P designs usually are still lower for reasonable tolerance levels. From this it can be concluded that the promising configurations still shall be favored when minimization of cogging torque is crucial. Nevertheless, either a dedicated tolerance analysis or some safety factor shall be incorporated in the machine design process in order to avoid negative surprises at the test bench when measuring a constructed prototype.

Future work will be about analyzing and comparing the impact of further typical manufacturing tolerances. As with the magnet positioning errors, the focus will be on parameters that cause the machine to lose its symmetry that is applicable for the ideal case, as those tolerances constitute the computationally most challenging analyses. Besides, additional performance measures, as the linked flux, load torque, and efficiency, shall be investigated in upcoming studies.

REFERENCES

- [1] G. Bramerdorfer, S. Silber, G. Weidenholzer, and W. Amrhein, "Comprehensive cost optimization study of high-efficiency brushless synchronous machines," *Proceedings of the 2013 IEEE International Electric Machines and Drives Conference, IEMDC 2013*, vol. 2017, no. January, pp. 1126–1131, 2013.
- [2] M. Islam, S. Mir, and T. Sebastian, "Issues in Reducing the Cogging Torque of Mass-Produced Permanent-Magnet Brushless DC Motor," *IEEE Transactions on Industry Applications*, vol. 40, no. 3, pp. 813–820, may 2004.
- [3] M. S. Islam, R. Islam, T. Sebastian, A. Chandy, and S. A. Ozsoylu, "Cogging Torque Minimization in PM Motors Using Robust Design Approach," *IEEE Transactions on Industry Applications*, vol. 47, no. 4, pp. 1661–1669, jul 2011.
- [4] D. C. Hanselman, "Effect of skew, pole count and slot count on brushless motor radial force, cogging torque and back EMF," *IEE Proceedings: Electric Power Applications*, vol. 144, no. 5, pp. 325–330, 1997.
- [5] G. Bramerdorfer, W. Amrhein, S. Silber, and R. Hagen, "Spectral-field design with respect to minimum cogging torque and maximum output power," *IECON Proceedings (Industrial Electronics Conference)*, pp. 2264–2269, 2010.
- [6] Z. Q. Zhu and D. Howe, "Influence of design parameters on cogging torque in permanent magnet machines," *IEEE Transactions on Energy Conversion*, vol. 15, no. 4, pp. 407–412, 2000.
- [7] I. Coenen, M. van der Giet, and K. Hameyer, "Manufacturing Tolerances: Estimation and Prediction of Cogging Torque Influenced by Magnetization Faults," *IEEE Transactions on Magnetics*, vol. 48, no. 5, pp. 1932–1936, may 2012.
- [8] X. Ge and Z. Q. Zhu, "Influence of Manufacturing Tolerances on Cogging Torque in Interior Permanent Magnet Machines with Eccentric and Sinusoidal Rotor Contours," *IEEE Transactions on Industry Applications*, vol. 53, no. 4, pp. 3568–3578, jul 2017.
- [9] X. Ge and Z. Q. Zhu, "Sensitivity of Manufacturing Tolerances on Cogging Torque in Interior Permanent Magnet Machines With Different Slot/Pole Number Combinations," *IEEE Transactions on Industry Applications*, vol. 53, no. 4, pp. 3557–3567, jul 2017.
- [10] B. H. Lee and J. W. Jung, "Robust design for reducing cogging torque in surface-mounted permanent magnet synchronous motor considering tolerances of sub-components," *IET Electric Power Applications*, vol. 14, no. 3, pp. 448–456, 2020.
- [11] G. Bramerdorfer, "A General Investigation of the Sensitiveness of Brushless Permanent Magnet Synchronous Machines Considering Magnet Tolerances," *IEEE Transactions on Magnetics*, vol. 56, no. 1, pp. 1–9, 2020.
- [12] M. A. Khan, I. Husain, M. R. Islam, and J. T. Klass, "Design of Experiments to Address Manufacturing Tolerances and Process Variations Influencing Cogging Torque and Back EMF in the Mass Production of the Permanent-Magnet Synchronous Motors," *IEEE Transactions on Ind. Appl.*, vol. 50, no. 1, pp. 346–355, jan 2014.
- [13] Y.-K. Kim, J.-J. Lee, J.-P. Hong, and Y. Hur, "Analysis of Cogging Torque Considering Tolerance of Axial Displacement on BLDC Motor by Using a Stochastic Simulation Coupled With 3-D EMCN," *IEEE Transactions on Magnetics*, vol. 40, no. 2, pp. 1244–1247, mar 2004.
- [14] H. Qian, H. Guo, Z. Wu, and X. Ding, "Analytical Solution for Cogging Torque in Surface-Mounted Permanent-Magnet Motors With Magnet Imperfections and Rotor Eccentricity," *IEEE Transactions on Magnetics*, vol. 50, no. 8, pp. 1–15, aug 2014.
- [15] A. J. Pina Ortega, S. Paul, R. Islam, and L. Xu, "Analytical Model for Predicting Effects of Manufacturing Variations on Cogging Torque in Surface-Mounted Permanent Magnet Motors," *IEEE Transactions on Industry Applications*, vol. 52, no. 4, pp. 3050–3061, jul 2016.
- [16] A. J. Piña Ortega and L. Xu, "Investigation of Effects of Asymmetries on the Performance of Permanent Magnet Synchronous Machines," *IEEE Transactions on Energy Conversion*, vol. 32, no. 3, pp. 1002–1011, 2017.
- [17] M. Klausnitzer and A. Mockel, "Quick cogging torque calculation for electronically commutated motors considering combinations of deviant and flawless magnets," in *Intern. Symposium on Power Electronics, Electrical Drives, Automation and Motion*, jun 2012, pp. 66–69.
- [18] S. Gerber and R.-J. Wang, "Statistical analysis of cogging torque considering various manufacturing imperfections," in *2016 XXII International Conference on Electrical Machines (ICEM)*. IEEE, sep 2016, pp. 2066–2072.
- [19] J. Ou, Y. Liu, R. Qu, and M. Doppelbauer, "Experimental and Theoretical Research on Cogging Torque of PM Synchronous Motors Considering Manufacturing Tolerances," *IEEE Transactions on Industrial Electronics*, vol. 65, no. 5, pp. 3772–3783, may 2018.
- [20] N. Taran, V. Rallabandi, D. M. Ionel, P. Zhou, M. Thiele, and G. Heins, "A Systematic Study on the Effects of Dimensional and Materials Tolerances on Permanent Magnet Synchronous Machines Based on the IEEE Std 1812," *IEEE Transactions on Industry Applications*, vol. 55, no. 2, pp. 1360–1371, mar 2019.

- [21] V. Simon-Sempere, M. Burgos-Payan, and J.-R. Cerquides-Bueno, "Influence of Manufacturing Tolerances on the Electromotive Force in Permanent-Magnet Motors," *IEEE Transactions on Magnetics*, vol. 49, no. 11, pp. 5522–5532, nov 2013.
- [22] V. Simon-Sempere, M. Burgos-Payan, and J.-R. Cerquides-Bueno, "Cogging Torque Measurement Using the Electromotive Force in Surface-Mounted Permanent-Magnet Motors," *IEEE Transactions on Magnetics*, vol. 51, no. 7, pp. 1–10, jul 2015.
- [23] G. Bramerdorfer, "Tolerance Analysis for Electric Machine Design Optimization: Classification, Modeling and Evaluation, and Example," *IEEE Transactions on Magnetics*, vol. 55, no. 8, pp. 1–9, aug 2019.
- [24] G. Bramerdorfer, "Computationally Efficient Tolerance Analysis of the Cogging Torque of Brushless PMSMs," *IEEE Transactions on Industry Applications*, vol. 53, no. 4, pp. 3387–3393, jul 2017.
- [25] L. Wu, R. Qu, B. Song, H. Bi, O. Jing, G. Yang, and C. Du, "Analysis of cogging torque in surface permanent magnet machine with manufacturing tolerances," *IECON 2015 - 41st Annual Conference of the IEEE Industrial Electronics Society*, pp. 4732–4737, 2015.
- [26] G. Lei, J. G. Zhu, Y. G. Guo, J. F. Hu, W. Xu, and K. R. Shao, "Robust design optimization of PM-SMC motors for six sigma quality manufacturing," *IEEE Transactions on Magnetics*, 2013.
- [27] G. Lei, C. Liu, Y. Guo, and J. Zhu, "Robust Multidisciplinary Design Optimization of PM Machines With Soft Magnetic Composite Cores for Batch Production," *IEEE Transactions on Magnetics*, vol. 52, no. 3, pp. 1–4, mar 2016.
- [28] B. Ma, J. Zheng, G. Lei, J. Zhu, Y. Guo, and J. Wu, "A Robust Design Optimization Method for Electromagnetic Devices with Interval Uncertainties," *IEEE Transactions on Magnetics*, vol. 54, no. 11, 2018.
- [29] A. C. Zavoianu, E. Lughofer, G. Bramerdorfer, W. Amrhein, and S. Saminger-Platz, "A Surrogate-Based Strategy for Multi-objective Tolerance Analysis in Electrical Machine Design," *Proceedings - 17th International Symposium on Symbolic and Numeric Algorithms for Scientific Computing, SYNASC 2015*, pp. 195–203, 2016.
- [30] G. Bramerdorfer and A.-C. Zavoianu, "Surrogate-Based Multi-Objective Optimization of Electrical Machine Designs Facilitating Tolerance Analysis," *IEEE Transactions on Magnetics*, vol. 53, no. 8, pp. 1–11, aug 2017.
- [31] G. Bramerdorfer, "Robustness Criteria for Concurrent Evaluation of the Impact of Tolerances in Multiobjective Electric Machine Design Optimization," *China Electrotechnical Society Transactions on Electrical Machines and Systems*, vol. 4, no. 1, pp. 3–12, mar 2020.
- [32] Ziyang Ren, D. Zhang, and C.-S. Koh, "Multi-objective worst-case scenario robust optimal design of switched reluctance motor incorporated with FEM and Kriging," in *2013 International Conference on Electrical Machines and Systems (ICEMS)*, Oct. 2013, pp. 716–719.
- [33] B. Rhein, T. Clees, and M. Ruschitzka, "Robustness Measures and Numerical Approximation of the Cumulative Density Function of Response Surfaces," *Communications in Statistics - Simulation and Computation*, vol. 43, no. 1, pp. 1–17, jan 2014.
- [34] David Meeker, "FEMM," 2020 (accessed May 23, 2020). [Online]. Available: <http://www.femm.info/wiki/HomePage/>
- [35] S. Silber, W. Koppelstätter, G. Weidenholzer, G. Segon, and G. Bramerdorfer, "Reducing Development Time of Electric Machines with SyMSpace," in *2018 8th International Electric Drives Production Conference (EDPC)*. IEEE, dec 2018, pp. 1–5.
- [36] T. Tannenbaum and M. Litzkow, "The Condor Distributed Processing System," *Dr. Dobb's Journal of Software Tools*, 1995.
- [37] Z. Zhu, S. Ruangsinchaiwanich, and D. Howe, "Synthesis of cogging torque waveform from analysis of a single stator slot," in *IEEE International Conference on Electric Machines and Drives, 2005.*, no. 4. IEEE, 2005, pp. 125–130.
- [38] G. Heins, T. Brown, and M. Thiele, "Statistical Analysis of the Effect of Magnet Placement on Cogging Torque in Fractional Pitch Permanent Magnet Motors," *IEEE Transactions on Magnetics*, vol. 47, no. 8, pp. 2142–2148, aug 2011.
- [39] G. Heins and M. Thiele, "Superposition techniques for analyzing cogging torque sensitivity to manufacturing errors in fractional pitch PMSM," in *IECON 2011 - 37th Annual Conference of the IEEE Industrial Electronics Society*. IEEE, nov 2011, pp. 2042–2047.



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