

Robustness Criteria for Concurrent Evaluation of the Impact of Tolerances in Multiobjective Electric Machine Design Optimization

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Abstract—This article is about a comparison of different measures for determining the robustness or reliability of electric machine designs in the presence of inevitable tolerances. The selected criteria shall be suitable for concurrent evaluation in the course of solving state-of-the-art large scale multi-objective optimization problems. In the past, besides particularly customized criteria, mainly gradient based measures, worst case information, or standard deviation based quantities were considered. In this work, the quantile measure is introduced for electric machine design optimization and compared with the existing solutions.

The evaluation of a design's robustness is typically examined based on finite element simulations. As for most measures a significant number of parameter combinations and thus computations are required, a surrogate model assisted approach is presented to minimize computational effort and runtime. A test problem is defined and analyzed to illustrate the differences of selected robustness measures. Results reveal the importance of considering robustness in the optimization process. Moreover, a careful choice of appropriate measures has to be taken. Selected designs are compared and conclusions and an outlook on future activities are presented.

Index Terms—electric machine, optimization, robustness, sensitivity, six sigma, tolerance analysis, quantile

I. INTRODUCTION

THE development of advanced mathematical optimization techniques has facilitated solving large scale electric machine optimization problems featuring several design parameters, multiple objectives, and additional constraints. For instance, stochastic evolutionary algorithms allow to efficiently search through a multi-dimensional design space and to change the search direction based on results for many different target values. Prominent examples to apply are the Non-dominated Sorting Genetic Algorithm II (NSGA-II) [1] and the Strength Pareto Evolutionary Algorithm (SPEA2) [2]. Besides, the advancement in computational power and tools for utilizing a computer cluster, e.g., in [3], to gain advantage of multiple computing resources are further key aspects such that (large scale) electric machine optimization got evermore popular.

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Dedicated special sections [4]–[6] and state-of-the-art review papers [7], [8] prove the immense number of research activities conducted through the last years and the interest of the scientific community on the topic.

Nowadays, evermore authors focus on system-level based optimization rather than only considering the machine design's characteristics [9]. Consequently, due to the large number of design parameters and objectives, as well as the significant computational cost of finite element (FE-) based evaluations, multiple research activities on speed improvements, e.g., achieved by surrogate modeling, were recently presented [10]–[12].

Usually, optimization focuses on rated conditions. However, at the presence of tolerances, optimized machine designs might feature very different performance. This is why tolerance analysis and robustness or reliability based evaluations are of significant importance. The major work is done with regard to changes of the cogging torque characteristics. For instance, Ge et al. investigate the impact of rotor outer contour changes for IPM machines [13], [14], while Coenen et al. [15] consider tolerance-affected permanent magnets.

Such post-processing analyses can help avoiding negative surprises when manufacturing a prototype of a particular design variant or in case of considering a series production of electric machines. Nevertheless, it would be beneficial to incorporate the robustness or reliability evaluation directly into the optimization process. Thus, during optimization, a tradeoff regarding rated performances versus robustness can be studied. Consequently, the optimization algorithm can steer towards best regions in the design space for both criteria. If only rated performance is considered, some promising domains might be undiscovered and sensitive designs are likely to be obtained.

Obviously, incorporating robustness evaluations to an optimization problem usually significantly increases the computational cost. In [16], a classification of tolerances related to electric machine design is presented for guidance. Robustness measures must be carefully selected depending on the considered application of the machine design. Consequently, different approaches were followed.

Besides deriving a local sensitivity (gradient) measure, a typical approach is to do a worst case analysis [17]. The worst case can, e.g., be derived by an ‘optimization’ within the local domain defined by all tolerance-affected parameters and their

respective ranges for any design. As an alternative, a grid search considering only extreme values of the tolerance levels can be focused. Worst case analyses are, e.g., important for safety-critical applications.

For standard industrial motors, it is more important to determine the rated performance and its fluctuation due to tolerances. This is due to significantly higher cost for the design and respective manufacturing if a worst case based approach with zero failure strategy is followed. By contrast, typically the expected value and variance, and so called design for six sigma approaches in particular, are focused [18]–[20]. As deriving the variance and, more generally, obtaining distribution based measures, involves a high number of design evaluations, metamodeling techniques are often applied to construct a surrogate instead of computing each considered tolerance-affected design variant by using FE-simulations [21].

Further customized measures exist, e.g., a torque ripple evaluation over full driving cycles and a consequent comparison of the results for different cycles [22]. Another idea is to consider the local topological derivative [23]. As usually the numerical determination of gradients is erroneous, a gradient-free sensitivity index is proposed in [24]. Other authors consider the local hypervolume in the objective space due to tolerances for any design investigated [25]–[29]. Consequently, the volume is considered to be minimized. The hypervolume itself can be, e.g., defined by the worst case values or any other reasonable measure. A general discussion of the role of robustness and a particular example are presented in [30], while in [31] approaches featuring a gradient-based evaluation, a worst case analysis, and a six sigma based study are compared.

While overall many different measures were introduced and compared, no work in the field of electric machine design regarding a quantile-based measure was found. The quantile q_x is defined as the level of a certain quantity y that a certain percentage of $x \cdot 100\%$ of the design variations do not exceed. Thus, it constitutes an interesting measure for observing the performance of a design under given tolerance distributions. Consequently, a certain number of defective designs is tolerated depending on the considered quantile level.

The upcoming sections of this article are organized as follows: Section II gives a qualitative comparison of different robustness or reliability based measures including the quantile-based evaluation. As the latter and also other measures require a lot of design variants to be analyzed, a surrogate-assisted tolerance evaluation is presented in Section III. A test problem is defined in Section IV and results for different measures are presented in the consequent Section V. Finally, a conclusion and outlook complete this activity.

II. CRITERIA FOR ROBUSTNESS CHARACTERIZATION

The range of considered criteria for characterizing the robustness of tolerance-affected designs is manifold. Here, the typically applied quantities are briefly explained, e.g., the sensitivity measure, the worst case analysis, and the standard deviation. Besides, quantiles are introduced as alternative. For visualizing the differences of the presented concepts, simplified examples are illustrated by figures. Only single input / single output problems are focused for reasons of visibility.

A. Local Sensitivity / Gradient

Often, the robustness of a design featuring a parameter x and an output quantity y is evaluated by taking the derivative

$$S' = \left. \frac{dy(x)}{dx} \right|_{x_s}, \quad (1)$$

where x_s gives the x -value of the considered design at which the sensitivity is evaluated. If multiple design and target quantities are focused, the partial derivative(s) must be determined. All designs are defined by introducing a general parameter vector \mathbf{x} , and \mathbf{x}_s gives respective values for a particularly investigated variant. Finally, evaluating the gradient of the i -th target with regard to the j -th parameter is defined as follows:

$$S'_{i,j} = \left. \frac{\partial y_i(\mathbf{x})}{\partial x_j} \right|_{\mathbf{x}_s}. \quad (2)$$

While this approach sounds reasonable at first sight, evaluation must be handled with care for the following reasons:

- Even taking the derivative of given mathematical functions often is not trivial. For optimization problems in the field of electric machines, the target quantities are usually not given in terms of equations. In case of standard optimization problems from the field, the difference quotient must be considered:

$$S_{i,j}^{\Delta} = \left. \frac{\Delta y_i(\mathbf{x})}{\Delta x_j} \right|_{\mathbf{x}_s}. \quad (3)$$

The step width for this evaluation must be carefully defined. On the one hand, it should be large enough to minimize numerical errors, e.g., when dividing by Δx_j . On the other hand, it must not be very large in order to guarantee deriving a local gradient information for evaluating the sensitiveness.

- When comparing the impact of different tolerance-affected parameters, typically the (absolute) local sensitivity is not of major interest. By contrast, one is mainly interested in consequent tolerance-related changes of the output parameter y . Thus, besides the gradient information, it is essential to additionally consider the likeliness of x to change and the respective tolerance level. One way for evaluating the net effect is to multiply the sensitivity by the variance of the design parameter:

$$S_{i,j}^{\sigma'} = S'_{i,j} \sigma_{x_j} \quad \text{and} \quad S_{i,j}^{\sigma\Delta} = S_{i,j}^{\Delta} \sigma_{x_j}. \quad (4)$$

- If x_j is a design parameter used for optimization, then the optimal solution for x_j might be where $S'_{i,j} = 0$ (or the equivalent difference quotient equals zero). This further complicates the analysis, and often the second order derivatives are finally evaluated:

$$S''_{i,jk} = \left. \frac{\partial^2 y_i(\mathbf{x})}{\partial x_j \partial x_k} \right|_{\mathbf{x}_s}. \quad (5)$$

All second-order derivatives represented in terms of a matrix are usually called the ‘Hessian matrix’.

As can be seen, evaluating the local sensitivity for electric machine optimization problems follows several difficulties. This includes the comparison of different designs. For instance,

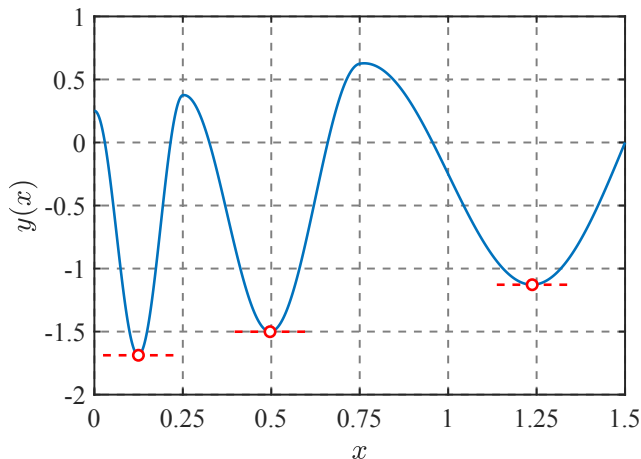


Fig. 1: A simple example for a multi-modal function with several minima and consequent gradients of zero value. However, the steepness of the function around the three minima is different.

Fig. 1 is considered. Three designs featuring same gradient information are highlighted. However, it is obvious which design is most or least sensitive regarding small changes in x . If Δx is defined too high when taking the difference quotient, the sensitivity information might be evaluated wrong due to the multimodality of the function.

B. Worst Case Analysis

The worst case analysis (WCA) is a common method for quantifying the robustness of designs considered through an optimization process. A certain range for the variation of the tolerance-affected parameters is defined. This can be done by introducing some absolute or relative maximum tolerance levels regarding the change of x and defining a respective domain \mathbf{X} , e.g., by

$$\mathbf{X} = \{ \mathbf{x} \mid \mathbf{x}_s - \Delta \mathbf{x}_s \leq \mathbf{x} \leq \mathbf{x}_s + \Delta \mathbf{x}_s \}. \quad (6)$$

Obviously, also asymmetric domains regarding the rated design parameter values can be defined. Considering the domain \mathbf{X} , the extremum of y in \mathbf{X} can be discovered. For convex problems, extreme values are situated at domain boundaries of \mathbf{X} , which minimizes the computational cost for determining. For more complex problems, finding the extremum takes more effort. Usually, either a grid search or a local search based on an optimization algorithm are applied. Depending if the target y is an objective to be minimized or maximized, either the highest or the lowest value of y is of interest.

WCA is comparably easy to implement. Nevertheless, for high dimensional problems, the curse of dimensionality takes effect. Then, the computational effort for the evaluation can be significant. This type of robustness evaluation is essential for safety-critical applications, e.g., in the field of aerospace, that must not fail. For optimization problems with less severe requirements regarding safety, worst case measures might not be the right choice. This is for the following reason: Often, particular distributions are used for defining the probability

of a certain tolerance level to occur. If the worst case scenario applies for a single combination of n parameters with probabilities p_1, p_2, \dots, p_n , the probability for the worst case situation p_{wc} to occur is

$$p_{wc} = p_1 \cdot p_2 \cdot \dots \cdot p_n. \quad (7)$$

Thus, the probability of the worst case to take place might be very unlikely. Considering the manufacturing of electric machines, it is often more important how the overall performance change with regard to tolerances develops, rather than what the overall worst performance would be. Thus, further measures for robustness evaluation were developed, and they are explained in the following.

C. Standard deviation

The standard deviation σ is a very useful measure for quantifying the range of variation for some quantity y featuring a probability density function (pdf). Known from the normal distribution, the standard deviation can be calculated for any type of distribution. In case of a finite number of N samples, it is defined as

$$\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^N (y_i - \mu)^2}, \quad (8)$$

where y_i gives the value of the target quantity y for the i -th parameter combination and μ the mean value of all available samples

$$\mu = \frac{1}{N} \sum_{i=1}^N y_i. \quad (9)$$

In robust design, often the ‘design for six sigma’ (DFSS) approach is focused [32]. Thus, starting from the expected value μ , the range from $\mu - 6\sigma$ up to $\mu + 6\sigma$ is focused. In Fig. 2, an example for the normal distribution in the range of $\mu - 3\sigma$ up to $\mu + 3\sigma$ with domains of equal probability defined by different colors is presented. Besides, the domains themselves are halved in probability by vertical lines. Besides

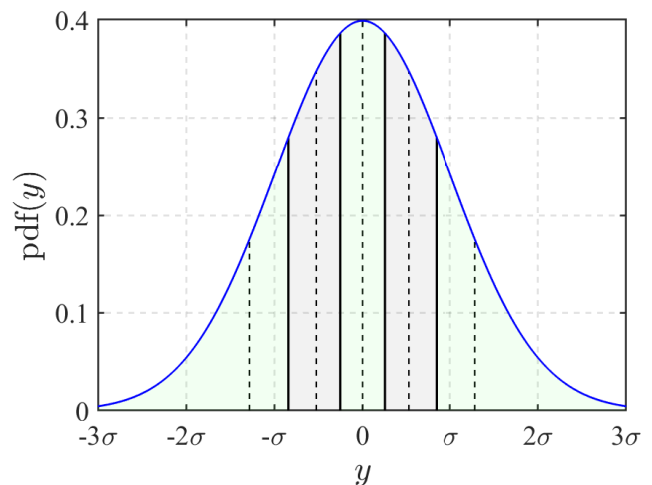


Fig. 2: An example for a probability density function - the normal distribution.

the pdf given in that figure, it is also interesting to evaluate the robustness by considering the cumulative distribution function (cdf), illustrated in Fig. 3.

If this approach is considered during production, it follows that all designs inside the respective range must fulfill some quality level. Consequently, in case of DFSS, only 0.002 defects per million (for the short term analysis) are accepted [32]. For machine design optimization, e.g., in case of cogging torque minimization, besides minimizing the mean value of all samples μ , it can be of interest to minimize the standard deviation σ . Thus, if σ is small, even in case of tolerances, a cogging torque close to the rated cogging will be achieved for almost all samples.

However, both an increase as well as a decrease of some quantity compared to its regular value give a certain standard deviation σ . This might be appropriate for requirements on exact values, e.g., for a shrink fit and the respective two components featuring some diameters that need to show a certain preciseness. In case of cogging also tolerance level combinations that feature a decrease of cogging would follow a non-zero standard deviation. This is usually unwanted, as a decrease of the cogging torque corresponds to an improvement of the performance. Alternatives can be considered if one only is interested in tolerance related effects that either increase or decrease the quality measure. One possibility is to make use of the quantile measure, which will be introduced in the next subsection.

D. Quantile

A quantile q defines a certain percentage out of all evaluated samples in the course of the robustness evaluation. Usually, it is evaluated by making use of the cumulative density function $F(x)$, introduced in Fig. 3. Here, it shall be defined such that q_m gives the threshold value of the quantity y that a relative share m of all samples shall not exceed. By for instance considering the example given in Fig. 3, 50% of all designs feature $y \leq 0$. Thus, $q_{0.5} = 0$. A thorough explanation of quantile-based robustness evaluations can be found in [33].

The quantile measure (QM) is considered more flexible than DFSS. For an electric machine manufacturer, for most applications it is too expensive to have zero failure rate. Depending on the overall situation, sometimes it might be enough that, e.g., 99% designs fulfill the specifications, while in another case it is 99.9%, etc. Thus, applying the QM, two ways for machine design optimization could be focused:

- Given some maximum or minimum value \hat{y} a sample should not exceed or deceed, respectively, the ratio of samples (not) fulfilling this constraint can be determined. The corresponding measure can be added as objective to the optimization. Thus, besides optimizing the rated performance, also the ratio of design variations (not) fulfilling the given constraint(s) shall be evaluated.
- An alternative approach is to fix some certain ratio of designs, e.g., 99%, and compute $q_{0.99}$, which gives the corresponding y -level for the design under investigation. This value can be added as objective in order to analyze

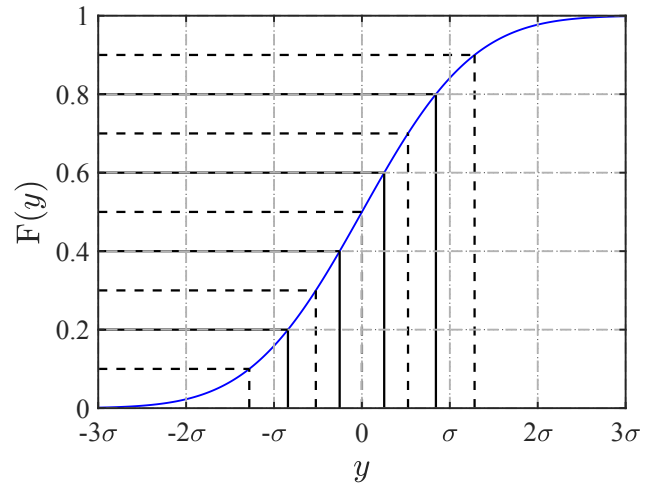


Fig. 3: The cumulative distribution function of the normal distribution.

and compare it for any design investigated, e.g., which cogging torque level 99% of the designs do not exceed.

A beneficial aspect about applying quantile measures is that either too high or too low y -values can be treated as not ok, depending on the definition. This is the usual case required for solving engineering problems. The machine manufacturer usually does not care if the cogging torque due to tolerances is lower than the rated performance. However, the interest is on how many designs feature an increased cogging. The opposite holds for the efficiency. While corresponding measures can be easily evaluated by defining appropriate quantile measures, it is more difficult for DFSS. In DFSS, the approach is more focused on any deviation from the rated performance rather than if the deviation can be considered as ok or not.

E. General

Four different measures for quantifying the robustness were introduced. Determining the local sensitivity and the worst case measure usually require less number of designs to be evaluated. However, the design for six sigma and quantile-based approach can be more useful for many practical applications. In order to allow for considering the latter two for a standard electric machine design optimization problem, it is worth applying surrogate modeling techniques. Thus, based on several particularly selected finite element (FE-) simulations based computations, a surrogate model is created. Afterwards, the evaluation of numerous combinations of tolerance levels of affected parameters can be handled much faster. The here considered approach is introduced in the next section.

III. SURROGATE MODEL ASSISTED DESIGN EVALUATION

A surrogate-based evaluation of any design investigated through the optimization process shall be focused in the following. The here considered approach is explained in detail. Surrogate modeling is a common technique in electric machine design. It is, e.g., applied for modeling the machine's torque or fluxes. The characteristics are generally nonlinear with regard

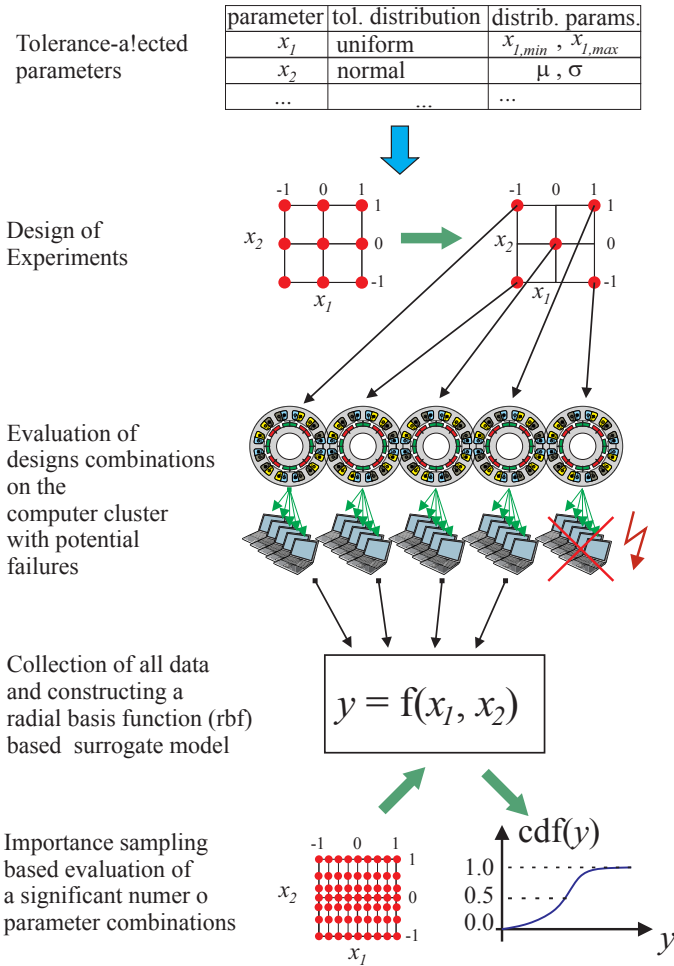


Fig. 4: Approach for evaluating the robustness measures considered for each investigated design variant.

to the applied currents and the rotor position. In the course of the design evaluation, usually several current combinations are evaluated for selected rotor positions by means of finite element simulations. Consequently, a surrogate model (or often called metamodel) is created, as in [10], [34], [35]. Finally, any further current combination for any rotor position is computed using this created model. Thus, the computational cost can be minimized and the required time for analyses is significantly reduced. A further approach is to directly use surrogate models for optimization problems. After conventionally evaluating a certain number of designs, e.g., by applying finite element simulations, a surrogate model can be defined to model all objectives (targets) as functions of the design parameters [11]. If successful, these models again allow minimizing the runtime and saving computational cost.

Here, any design investigated is evaluated based on the approach illustrated in Fig. 4. All tolerance-affected parameters of an optimization problem are defined based on distributions. For any distribution, a selected number of parameter levels is defined for an initial conventional evaluation. Overall, this would follow a grid analysis in the case of multiple tolerances. The number of required analyses thus could be very high. To minimize the number of analyses, a PYTHON-package called

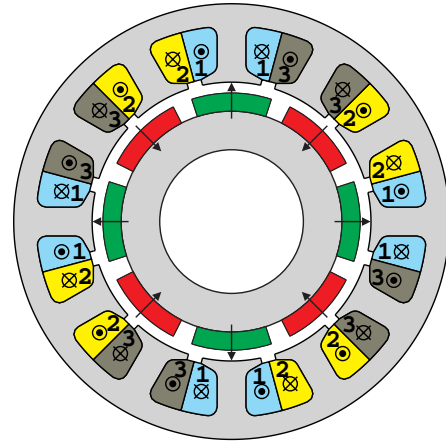


Fig. 5: Considered machine topology for the test problem.

pyDOE2 is applied. It allows for selecting the best tolerance level combinations to gain as much information as possible with a reduced computational effort. The selected combinations are evaluated using a computer cluster by means of finite element simulations. As due to unreliable computers or any unexpected events some results might not be available at the end of this process, a certain threshold for a maximum number of erroneous evaluations is defined. A lower number of not retrieved results still would allow to continue with the surrogate modeling. When the radial basis function (rbf) based model was successfully created using another PYTHON-package called *SciPy*, it can be evaluated for many different tolerance level combinations in short time. The particularly randomly selected levels are based on their likelihood to appear considering the defined distribution functions. This approach is called importance sampling. Thus, a cumulative distribution function and any distribution-related measures can be properly approximated for the target quantities.

IV. TEST PROBLEM

The optimization of an inner-rotor surface permanent magnet (SPM) machine is considered as test problem. Figure 5 gives an illustration of the focused three-phase topology.

As objectives, the material cost, the efficiency at rated load η , and the cogging torque are considered. The specific prices, the rated load point, and further constant parameters are given in Table I.

TABLE I
CONSTANT PARAMETERS

Name	Symbol [Unit]	Value
number of stator slots	N_s / -	12
number of rotor poles	p_z / -	8
air gap width	δ / mm	0.7
rotor inner diameter	d_{ri} / mm	16
coils' temperature	ϑ_{coil} / °C	120
permanent magnets' temperature	ϑ_{pm} / °C	90
rated torque	T_{rate} / Nm	5
rated speed	n_{rate} / rpm	3000
specific permanent magnet cost	c_{pm} / (Euro/kg)	100.0
specific laminated steel cost	c_{lam} / (Euro/kg)	2.0
specific Copper cost	c_{copp} / (Euro/kg)	8.0

TABLE II
CONVECTIONAL DESIGN PARAMETERS

Name	Symbol / Unit	Min.	Step	Max.
stator inner diameter	d_{si} / mm	40.0	1.0	90.0
stator outer diameter	d_{so} / mm	75.0	1.0	140.0
axial length	l_{ax} / mm	35.0	1.0	80.0
stator tooth width	w_{st} / mm	4.0	0.2	10.0

TABLE III
TOLERANCE-AFFECTED DESIGN PARAMETERS

Name	Symbol / Unit	Min.	Step	Max.
magnet radial dimension	h_m / mm	3	0.25	8
- uniform tolerance distrib.	Δh_m / mm	-0.1		0.0
magnet pole pitch	α_m / -	0.5	0.025	1
- uniform tolerance distrib.	$\Delta \alpha_m$ / -	-0.01		0.01
stator slot width	b_{ss} / mm	2.0	0.5	7.0
- uniform tolerance distrib.	Δb_{ss} / mm	-0.01		0.01

TABLE IV
TOLERANCE-AFFECTED MATERIAL PARAMETERS

Name	Symbol / Unit	Min.	Nom.	Max.
magnet residual induction	B_r / T		1.28	
- uniform tolerance distrib.	ΔB_r / T	-0.08		0.02

Four parameters for optimization, three tolerance-affected geometry parameters and one tolerance-affected material parameter are studied, which are defined in detail through the information provided in Tables II, III, IV.

While the efficiency and the material cost are not constrained, the cogging torque peak-to-peak value shall feature a maximum limit, such that

$$T_{cogging,pp} = \frac{\max_{\alpha} T_{cogging}(\alpha) - \min_{\alpha} T_{cogging}(\alpha)}{T_{rate}} \cdot 100\% \leq 10\%, \quad (10)$$

where α gives the rotor angle.

Only worst case measure and quantile-based evaluation are considered for the test problem for the following reasons. Gradient-based information does not give a direct measure for allowing to apply some constraint on cogging torque. Similarly for the six sigma based approach, based on some initially derived expected cogging torque value and its standard deviation, it is not directly possible to apply some threshold value for limiting the cogging torque to some extent. Obviously, it is possible to analyze the standard deviation and to (try to) minimize it. Nevertheless, this approach has some drawback for the present situation: Consider a design A that features a certain rated cogging torque $T_A = T_{cogging,pp,rated}$ and a cogging torque decrease, i.e. an improvement, due to tolerances. Consequently, the standard deviation for this design is non-zero $\sigma_A > 0$. A design B featuring the same rated cogging $T_B = T_A = T_{cogging,pp,rated}$, but a cogging torque increase due to tolerances, might feature the same standard deviation $\sigma_B = \sigma_A$. In case of the six sigma approach, both designs would be treated equally regarding the cogging torque performance. However, obviously design A should be considered as better than design B due to better characteristics in the presence of tolerances.

Using either worst case measure or the quantile-based approach allows for a direct evaluation of these circumstances and a consequent ranking. By contrast, for this situation

considering the standard deviation σ does not allow for a technically reasonable ranking. Consequently, also the six sigma based approach does not allow a ranking based on the respective six sigma domains for design A and B , i.e. $[T_A - 6\sigma_A, T_A + 6\sigma_A]$ versus $[T_B - 6\sigma_B, T_B + 6\sigma_B]$, as the two considered designs would also follow same domains for this particular case.

The six sigma approach, however, is definitely useful for many other applications, also in the field of electric machine design. For instance, consider a system-level based optimization including the Back-EMF of a machine design. If, due to the also considered power electronics and control, some certain number of winding turns is determined for optimal performance, both an increase and a decrease of the Back-EMF due to tolerances can deteriorate the system's overall performance. Here, applying six sigma based robustness evaluations and, consequently, minimizing the standard deviation definitely is a useful approach.

The test problem is solved by utilizing an NSGA-II algorithm featuring a population of 100 individuals. A single design evaluation evolves 6 different i_d/i_q -combinations evaluated over a full electric period by 2-D magnetostatic FE-simulations each lasting about one minute. Afterwards, a nonlinear rbf-based machine model is created to find the best d/q -current combination for maximum efficiency.

The tolerance-related analysis based on the approach illustrated in Fig. 4 involves 32x6 further FE-based evaluations over the full electric period. A share of 14% (= 4 out of 32 variations) is defined as maximum that is allowed to be erroneous to still continue with the surrogate model construction for the objectives as function of the tolerance-affected parameters. Finally, after having successfully created a surrogate model for each objective, they are each run 10^6 times for a distribution-based tolerance evaluation.

The overall process is approached by making use of a computer cluster managed through the software HTCondor [3]. Ten designs are at maximum analyzed in parallel, and a constraint is set to have at most 50 FE-simulations concurrently running. Overall, 6000 designs are evaluated. While the first 1000 variants are defined by applying a Latin-Hypercube-based sampling of the design space, the remaining 5000 machine designs are obtained through the offspring creation by the optimization algorithm.

As could be noticed, even though concurrently evaluated, no robustness criterion was initially selected as objective for the test problem. The intention was to first generally analyze the results for the different criteria. Afterwards, the change of the Pareto fronts depending on which criterion would have been set as constraint for the cogging torque is illustrated.

V. RESULTS

Figure 6 gives the results of the test problem in terms of the Pareto front(s). As three objectives were considered, two 2-D plots are defined. The green squares denote Pareto optimal results. While for two objectives the Pareto optimal designs would define a boundary of the cloud of design points, for three objectives the surface is more complex and thus this

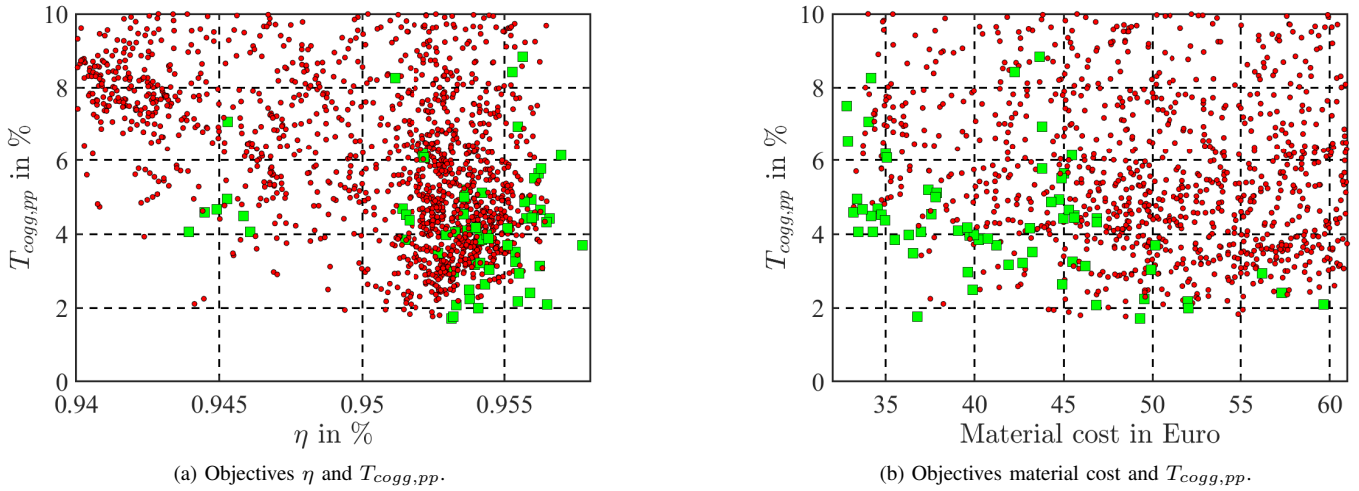


Fig. 6: Optimization results of the test problem.

is no longer a necessary condition. Regarding the relative cogging torque values $T_{cogg,pp}$, only designs satisfying the 10%-constraint are illustrated. It can be noticed that a lot of non-optimal designs feature similar performance than Pareto-optimal counterparts.

In Fig. 7, the comparison of different robustness criteria evaluated for all designs are presented. On the y -axis, in both figures the maximum (=worst case, WC) cogging torque over the 10^6 tolerance related evaluations per design is given.

In Fig. 7a, it is compared with the cogging torque level that 95% of all designs out of the 10^6 evaluations not exceed, i.e. the quantile $q_{0.95}$ is presented along the x -axis. The blue line is identifying same absolute value for both measures. It is obvious that the WC cogging is generally at least as big as the quantile value. Usually, WC is to some extent larger. The gray box denotes designs that would violate the maximum cogging torque level of 10% in case the WC measure would be included as constraint in the optimization, but they would satisfy a $q_{0.95}$ -related constraint. Thus, the optimization results would significantly differ depending on which criterion is applied.

Many Pareto optimal points, given by the green squares, are within this gray box exceeding the cogging torque limit for the WC measure.

By contrast, in Fig. 7b, the WC cogging is compared with the quantile $q_{0.99}$. Consequently, for the latter measure the cogging that 99% of the designs not exceed is presented. As can be seen, by increasing the quantile level, obviously the WC- and quantile-based measure tend to be more equal. It can be followed that using quantiles allows for a more flexible definition of the constraints for particular requirements of the test problem.

In Fig. 8, robustness measures are evaluate for the efficiency. By contrast to the cogging in Fig. 7, here the WC is equal to a minimum efficiency level. Besides, the efficiency level that 95% and 99% of the designs do not deceed are given along the x -axes of Figs. 8a and 8b. For nearly all designs, a constant offset is observed among the measures. While in case of Fig. 8a, i.e. $q_{0.05}$, the offset is about 0.4%, in case of $q_{0.01}$ in 8b it is only 0.25%. Thus, the overall change is small, while again the WC measure obviously gives at least as low values as the

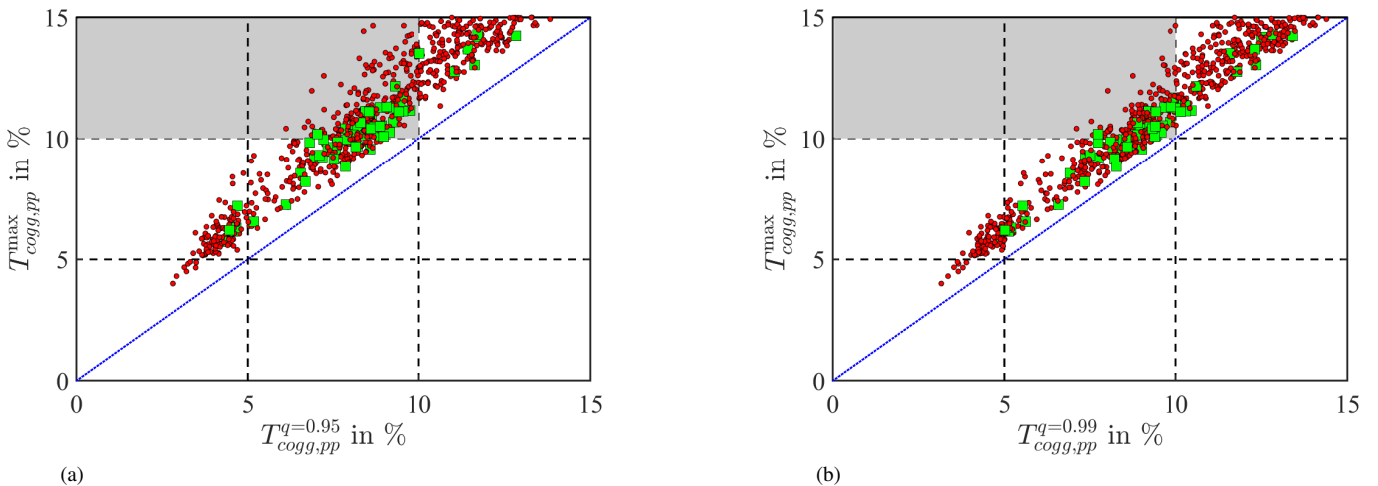


Fig. 7: Comparison of different robustness criteria for the cogging torque measure.

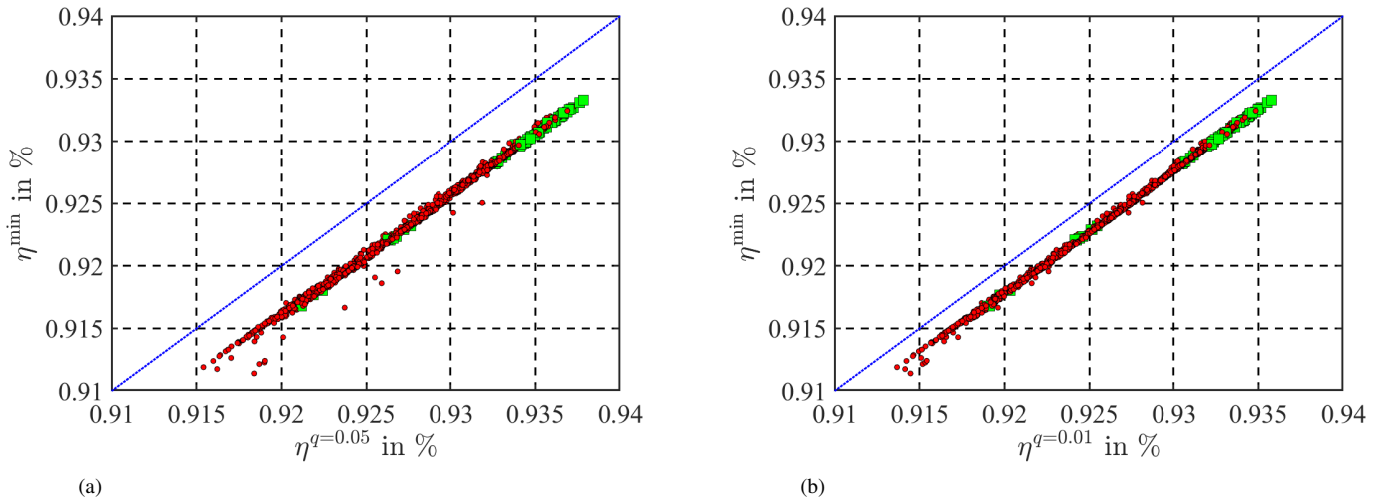
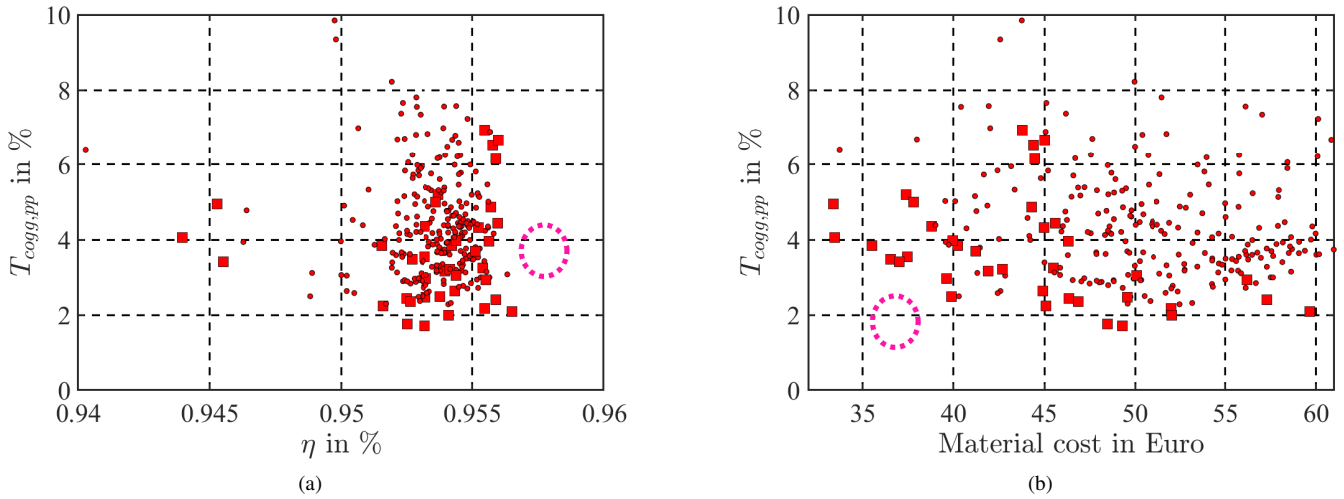
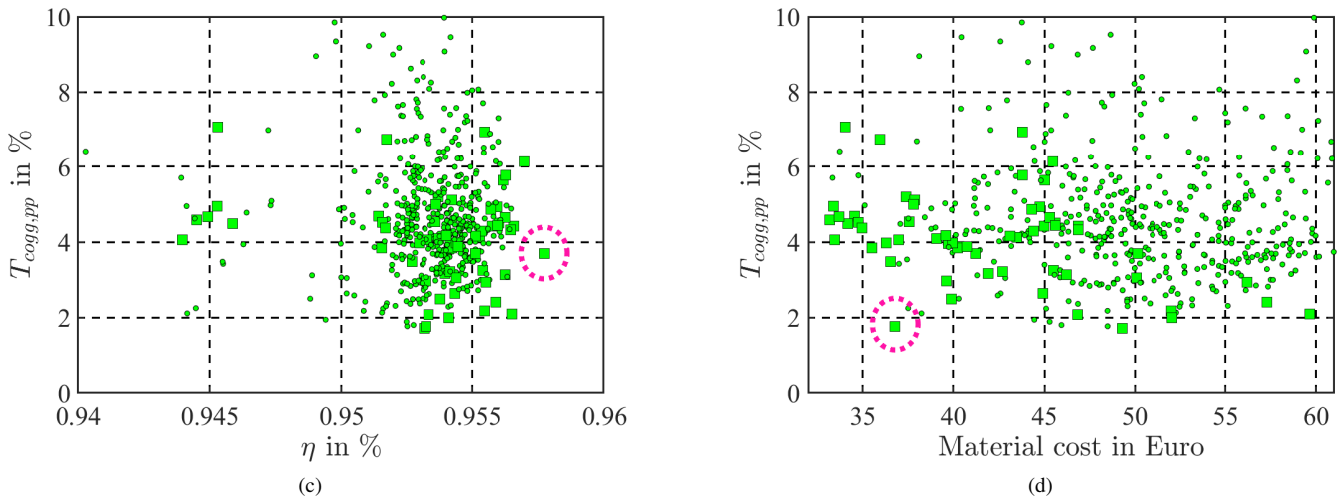


Fig. 8: Comparison of different robustness criteria for the efficiency evaluation.



Figures 9a and 9b define the Pareto optimal results if the worst case measure is used for analyzing the designs' feasibility regarding the fulfillment of the 10% cogging torque constraint.



Figures 9c and 9d define the Pareto optimal results if the $q_{0.95}$ -quantile is used for analyzing the designs' feasibility regarding the fulfillment of the 10% cogging torque constraint.

Fig. 9: Pareto fronts as function of the selected cogging torque robustness criterion for evaluating the 10% cogging constraint.

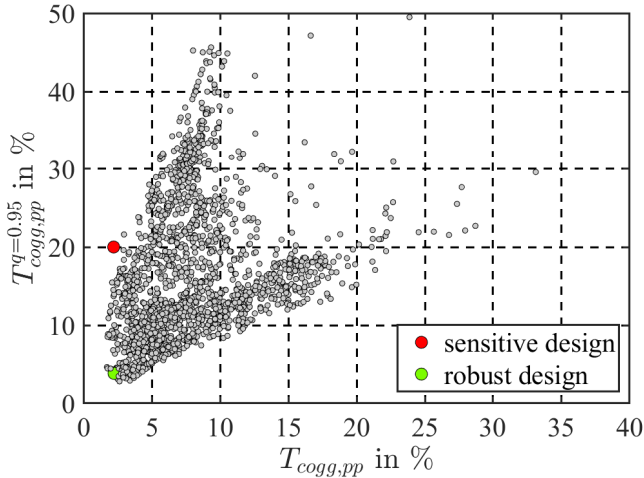


Fig. 10: Comparison of rated cogging versus cogging for the $q_{0.95}$ quantile. A selected sensitive and robust design with similar rated performance are highlighted.

quantile-based measures.

Consequently, depending on the selected robustness measures for evaluating the cogging torque constraint, the optimization results will differ due to more or less designs not fulfilling the respective requirement. In Fig. 9, the upper two figures represent the optimization results that would have been obtained when considering the worst case measure, while the lower two figures give the counterpart if the quantile $q_{0.95}$ would have been used for the cogging torque constraint. Using the worst case measure follows that some designs featuring promising rated performance would have been discarded. Two examples are illustrated by dashed ellipses in the figures. As this might be ok for safety-critical applications, for standard industrial motors the WC measure might be too restrictive. Thus, depending on the application, a proper measure shall be selected.

The general necessity for incorporating robustness measures to electric machine design optimization problems is emphasized in Figs. 10 and 11. Figure 10 gives a comparison of the rated cogging performance versus the $q_{0.95}$ quantile for cogging for the conducted tolerance-based evaluation. As can be seen, the designs feature very different characteristics for these two measures. A very sensitive and a more robust design with similar rated cogging are selected for further investigation. Figure 11 gives the corresponding cumulative distribution function (cdf) for the two designs. As mentioned, both variants feature similar rated cogging performance of about 2.2%. However, when analyzing 10^6 combinations of tolerance-related design modifications, the robust design features only about twice this cogging torque level as maximum over all variations. Moreover, a certain share of tolerance-affected designs features a cogging torque lower than the rated value. By contrast, the cdf of the sensitive design features much worse characteristics, as illustrated in Fig. 11. If no robustness measure is considered during optimization, both designs are treated equally. Consequently, this highly likely follows a lot of sensitive designs in the Pareto front.

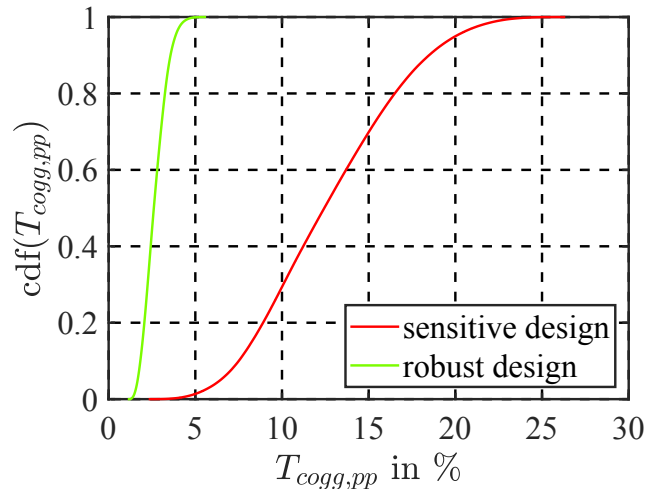


Fig. 11: Comparison of the cumulative distribution functions for the two designs highlighted in Fig. 10.

VI. CONCLUSION

This paper was about measures for evaluating the robustness of electric machine designs. The focus is on a concurrent determination of the designs' sensitiveness while solving dedicated optimization problems. By contrast to post optimization analyses, this allows to include robustness measures as objectives or constraints to the optimization problem. Hence, the optimization algorithm can steer to both promising regions regarding rated performance as well as robustness. This facilitates selecting the best tradeoff after the optimization was completed.

Four different robustness measures were compared from a qualitative perspective: gradient-based sensitivity evaluation, worst case analysis, design for six sigma, and a quantile-based measure. To the author's best knowledge, the latter one has not been considered in articles dealing with electric machine design optimization so far. Both the worst case and the quantile-based measure are selected for investigating a consequently defined optimization problem. They both allow for directly applying a minimum or maximum value as constraint for performance measures of the analyzed design candidates to be maximized or minimized, respectively.

Solving problems with robustness measures included requires a significant number of design evaluations. This particularly holds for six sigma based approaches as well as quantile-based measures. Consequently, a dedicated strategy was presented here to considerably reduce the overall computational effort. The approach is based on a design of experiments technique to minimize the number of initially required finite element based evaluations, and a consequent radial basis function based surrogate modeling. Finally, the obtained models are applied to derive an importance sampling based evaluation of 10^6 different design modifications based on the four considered tolerance-affected parameters and their respective ranges. Such a high number of evaluations is particularly beneficial for six sigma based approaches or quantile-based performance indices.

Results reveal that, depending on the selected robustness

measure, different Pareto optimal results are observed. Thus, depending on the application, the proper quantity has to be selected. Guidance is given by recommendations throughout the article. Besides, the importance of incorporating robustness measures to optimization problems is highlighted by comparing two designs with similar rated performance, but totally different robustness regarding tolerances. The final selection of a particular design can be made by considering the trade off rated performance versus robustness and, consequently, is also application-dependent. Future work will be about analyzing further tolerance-affected parameters, e.g., the eccentricity, and additional objectives, like aspects related to the temperature distribution within electric machines. Moreover, a reliability index based on multiple quantile-measures will be focused.

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