Sub-region Alopex Optimization Method with RSM for Design of Permanent Magnet Machines

Xiaoyu Liu and W. N. Fu

***Abstract*—Based on an Alopex optimization algorithm and a response surface model (RSM), a hybrid sub-region methodology is presented to solve the optimal design problems of permanent magnet (PM) machines. The Alopex optimization method is processed both in subspace and in global solution space. In order to decrease the computing time, a multi quadric radial basis function (MQRBF) is embedded in the optimization. The proposed method speeds up the convergence rate while keeps the accuracy of the solution. A numerical experiment is given to validate the efficiency and effectiveness of the method.**

***Index Terms*—Alopex algorithm, finite element method, multi quadric radial basis function, optimization method, radial basis function.**

I. INTRODUCTION

RECENTLY, high efficient permanent magnet (PM) motors have been applied in wide range of industry, and this has facilitated the attention to novel optimization algorithms for searching the best design of the motors.

The design of PM motors usually involves optimizing several design variables, so the direct optimization can be non-vertex. Since most optimization problems involve minimizing a function subject to certain constraints. Creating the objective functions which relate the performance to the design variables such that an optimization process can be implemented, is not a simple task [1]-[2].A response surface model (RSM) is an efficient approach for re-establishing the mathematical models of PM motors [3]. The RSM is normally introduced to reduce the number of FEM computation for objective function evaluations, which can greatly reduce the cost of computation. There are several types of RSM which have commonly been used for approximation. The radial basis function interpolation method is an important branch of the RSM. Its feature can approximate many functions. Therefore, it has been applied to many fields [4]-[5].Another type of RSM is the moving least squares approximation (MLS)[6],which can also be used to fit both the objective and constraint functions of the electromagnetic design problems [7].

The RSM usually combines in the optimization process to obtain the optimal design. This paper introduces Alopex as the optimization algorithm. It was originally used to solve the problem of determining the shapes of visual receptive fields [8]-[9]. It had been proved to be an effective tool for engineering optimization [10]. In Alopex, the parameters are related with the otherness of the individual variables introduced for the computation of probability, which makes the Alopex can accept a worse solution with a probability and thus improves the climbing capacity [8]. The Alopex process is a stochastic method of finding global extrema of objective function. It has in common with similar approaches the use of "noise" to extricate the process from local extrema [11].

The optimization process involves enormous computational cost, so an analyst is typically willing to decrease the number of function evaluations when carrying out the optimization of the costly objective functions. Our goal in this paper is to develop a global optimization algorithm that produces reasonably good solutions with less computing cost than exist algorithms.

Since the sample points that RSM needs, this paper introduces the idea of subspace. The Alopex will find the extrema point in each subspace and then the RSM uses all these points as its sample point to build a model for the objective. In the end we will randomly choose a point in the solution space and then compare the FEM value with the RSM value. The rate of convergence and the convergence of error between two steps of this algorithm will be shown in this research.

II. ALGORITHM OF ALOPEX-RSM METHOD

A. Response Surface Method

The basic theory of RSM is to randomly choose a set of input variables, and get the value of the objective function on these points by numerical calculation, then build a model for the objective function based on a certain type of RSM. The model that we build is an approximation of the objective function. Building a good model needs first a large scale of sample points, and then a proper basis function.

The RSM employed in this paper is based on the multi quadric radial basis function (MQRBF) [12]. It has widely being used as a global interpolation. It can simulate the objective function smoothly and fit it with limited sampling points [13].

The radial basis function $H(\| x-x_i \|)$ is a monotonic

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function of the distance between a random point x and a certain point x_i. The RSM based on the radial basis function is

$$f(x) = \sum_{i=1}^{N} c_i H_i(r) \tag{1}$$

in which c_i is the coefficient to the ith term associated with the sample point x_i.

The radial basis functions that are popular in electromagnetic computation maintain:

$$H_i(r)=r \tag{2}$$

$$H_i(r)=e^{-h^2 r^2} \tag{3}$$

$$H_i(r)=(r^2+h)^{\beta} \tag{4}$$

$$H_i(r)=(r^2+h)^{-\alpha} \tag{5}$$

here $r=\|x\text{-}x_i\|$. Where h is called shape parameter. It may influence the shape of the function that is built by the MQRBF. When β=0.5, the equation (4) is called the MQRBF.

RSM is applied on many design sample points. The first stage of it is to collect the function values on these sample points and build RSM based on equation (1). After solving the equations, the coefficients of the equation (1) are obtained. In the second stage, with the help of the RSM, the objective function can be simulated and the task of design optimization can be released without computationally expensive finite element electromagnetic analysis at each step.

B. Alopex Algorithm

Alopex is a correlation-based algorithm which possesses the characteristics of both gradient descent and simulated annealing [10]. The algorithm broadcasts a measurement of the global performance of an objective function to all the points in the solution space. The weights are updated by the calculation of the explicit derivative of the objective function [15]. Correlate measure between the change in weight and the value of the function changes is estimated. Then the probability index of the variable going in right direction will make change to the individual weights. As the solution goes to the right direction, the objective function is minimized.

Considering the minimize problem of objective function $F(x_1, x_2,\ldots, x_n)$, x_i (i=1,2,…n) are the variables of the function. The details of the algorithm are given below.

During the tth iteration, the value of the ith variable $x_i(t)$ is updated as,

$$x_i(t)=x_i(t-1)+\delta_i(t) \tag{6}$$

where $\delta_i(t)$ will have a small step of size δ to the positive or negative direction with the following probabilities:

$$\delta_i(t)=\begin{cases} +\delta \ withprobability & p_i(t) \\ \text{-}\delta \ withprobability & (1\text{-}\ p_i(t)) \end{cases} \tag{7}$$

in which the δ is a parameter that relates to the error of last step. So in this paper, we define it as

$$\begin{aligned}\delta=&\lambda_1\cdot\omega\cdot(y(t)-y(t-1))\\ &+\lambda_2\cdot\omega\cdot(y(t-1)-y(t-2))\end{aligned} \tag{8}$$

Here λ_1 and λ_2 are two learning factors, and ω is a pseudo random number between (0,1).

The probability $p_i(t)$ is given by the expression:

$$p_i(t)=1/(1+e^{\Delta_i(t)}/T) \tag{9}$$

where $\Delta_i(t)$ is given by the correlation:

$$\Delta_i(t)=(x_i(t-1)-x_i(t-2))\cdot(F(t-1)-F(t-2)) \tag{10}$$

For the first two iterations, $p_i(t)$ usually is taken as 0.5. In the expression of $p_i(t)$, T is the annealing temperature that determines the effective randomness in the system.

This method reduces the temperature T with an attenuation coefficient λ.

$$T(t)=\lambda\cdot T(t-1), (0<\lambda<1) \tag{11}$$

The attenuation coefficient λ controls the speed of the convergence. If λ is close to 1, then the spend will be slow down and the computing cost will be raised. On the other hand, if λ is close to 0, then the accuracy will be reduced and the possibility confined to the local optimal solution will be increased.

The Alopex is different from other methodologies from the following aspects [14]: (1) the procedure is iterative; (2) the change in the variable x_i depends stochastically upon the change in x_i over the preceding two iterations; (3) all increments in parameter values are retained and carried over into the next iteration; (4) the process is guided by two parameters: the step size δ and the annealing temperature; (5) in order to implement the algorithm, the parameters as the attenuation coefficient λ and the step size δ need to be determined.

C. Sub-region Alopex-RSM

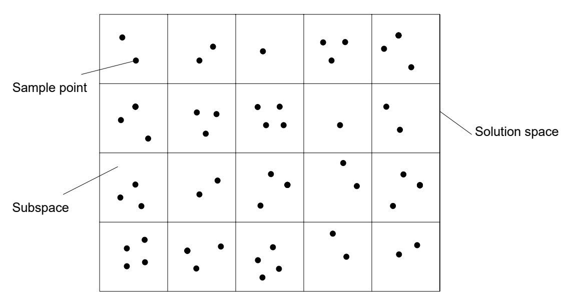


Fig. 1. Distribution of sample points in the solution space.

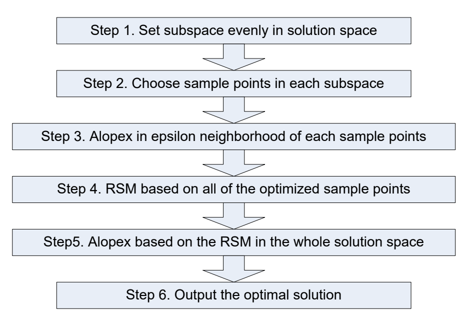


Fig. 2. Block diagram of the optimization process.

RSM based on sub-region Alopex algorithm in this paper is a kind of algorithm that overcomes the insufficient and slow convergence to the local optimal point. The optimization will stop if the number of steps meets the maximum or the optimal solution of the designing is found.

Pre-stage of this algorithm is to divide the solution space into several sub-regions as shown in Fig.1. Then choose several sample points in each subspace and start the Alopex algorithm. These sample points are optimized in each of the subspaces. All the optimal points of every subspace are recorded. To control the computing time of this stage, parallel calculation is employed.

In order to find the global optimal solution, the Alopex assisted with the RSM which is built on all of the recorded optimal points is applied on the whole solution space. Last step is using this model as the approximation of the objective function and start Alopex again.

The whole procedures are described in the block diagram as shown in Fig. 2.

III. Program Description

In order to show how the sub-region Alopex-RSM works, a PM motor as shown in Fig. 3 is taken as an example. The objective function is the power efficiency of the motor. Our purpose of this research is to maximize the power efficiency.

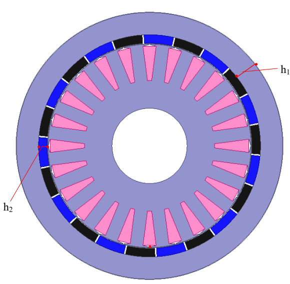


Fig. 3. The permanent magnet motor for numerical experiment.

In this research, only two variables in Table I are involved in the optimization, other parameters are settled. Multi variables optimization can be approached similarly.

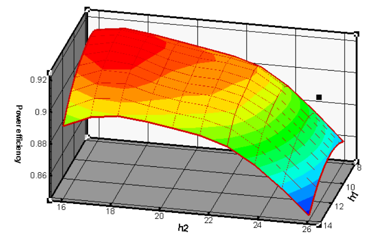
TABLE I
DESIGN PARAMETERS IN THE OPTIMAL DESIGN

Design Parameters	Value
h_1 (thickness of the iron shell of the rotor)	16-24mm
h_2(thickness of the permanent magnets)	8-12mm
Outer radius of the rotor	160mm
Number of the PM	22
Angle of PM with positive radial magnetization	12deg
Number of wires in the slot	58
Wide of the teeth	16mm
Length of the teeth	41mm
Radius of the shaft	44mm

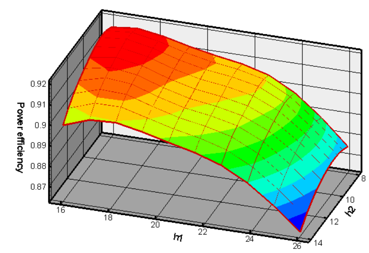
Following the steps of the algorithm, the solution space is firstly divided into 16 subspaces. By means of reducing the cost computing, only 5 sample points in each domain will be chosen. The initial annealing temperature is "T=20", and the attenuation coefficient "$\lambda = 0.9$" . Then start to optimize on each subspace, and get 16 optimal points. Based on these points and their function values, the coefficient of RSM can be confirmed.

IV. Numerical Results

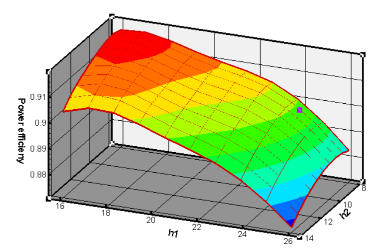
For comparison purpose, the values of points on the whole solution space with different shape parameters are shown in Fig. 4. In Fig.4, the optimized power efficiency of this motor can reaches to91%.It shows that the model fits well toward the objective function. When h=0.8, the results in Fig. 4(b) are simulate to those in Fig. 4(a) and better than other two. So in our case, the power efficiency can reach the highest when h=5 while building the RSM.



(a) h=0.8.



(b) h=5.



(c) h=10.

Fig. 4. The RSM results with different shape parameters.

In order to show the effectiveness of this method, the error $\eta\%=\sqrt{|\tilde{f}-f|/f}$ is introduced in this paper. The Fig. 5 shows the η on the solution space. The $\tilde{f}$ is the FEM calculation result. As shown in Fig. 5, the relative errors are lower than 0.01%.This means that the simulation of the RSM is successful.

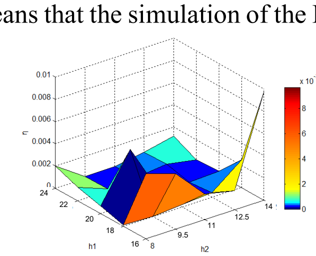


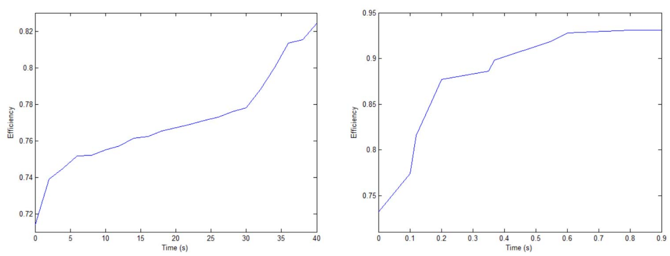
Fig. 5. η in the solution space.

Table II shows the comparison of the proposed algorithm with traditional Alopex method and PSO method. It shows that the proposed method is feasible and capable of reducing computing time of optimization.

TABLE II
COMPARISON OF THE PROPOSED ALGORITHM TO TRADITIONAL METHODS

Methodology	FEM times	Total time (s)	Computer cores used
Proposed Algorithm	192	58	16
Alopex	316	951	1
PSO	308	927	1

Furthermore, to examine the convergence rate of the algorithm, both convergences of the two stages are shown in the Fig. 6. In Fig. 6(a), the first sub-region is chosen to show its convergence as a representation. The initial efficiency of the PM motor was 72% and after optimization by the proposed algorithm, the efficiency reaches 93%. The magnetic flux line of the gear with the optimized parameters is shown in Fig. 7.



(a) (b)

Fig. 6. The convergence of the algorithm. (a). The convergence of the Alopex in sub-region optimization. (b). The convergence of the Alopex after sub-region optimization.

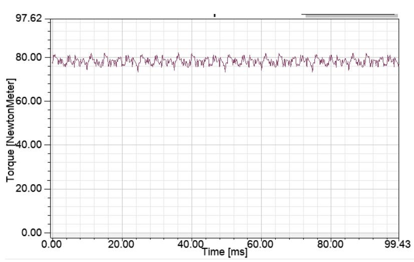


Fig. 7. Torque of optimized structure.

V. CONCLUSION

The proposed algorithm employs RSM methodology and the Alopex optimization to dynamically reestablish the objection function. The rebuilding model is based on the optimal results of Alopex optimization in sub-regions. The numerical example of the optimal design of a PM motor shows that the developed computer program based on the proposed algorithm is capable of building a proper response surface model for the objective function and the accuracy of the this model is quite high.

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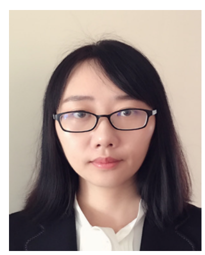

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