Harmonics in the Squirrel Cage Induction Motor Analytic Calculation Part III: Influence on the Torque-speed Characteristic

G. Kovács

***Abstract* —The harmonics that appear in the squirrel cage asynchronous machine have been discussed in great detail in the literature for a long time. However, the systematization of the phenomenon is still pending, so we made an attempt to fill this gap in the previous parts of our study by elaborating formulas for calculation of parasitic torques. It was a general demand among those who work in this field towards the author to verify his formulas with measurements. In the literature, it seems, only one detailed, purposeful series of measurements has been published so far, the purpose of which was to investigate the effect of the number of rotor slots on the torque-speed characteristic curve of the machine. The main goal of this study is to verify the correctness of the formulas by comparing them with the referred series of measurements. Relying on this, the expected synchronous parasitic torques were developed for the frequently used rotor slot numbers - as a design guide for the engineer. Thus, together with our complete table for radial magnetic pull published in our previous work, the designer has all the principles, data and formulas available for the right number of rotor slots for his given machine and for the drive system. This brings this series of papers to an end.**

***Index Terms*—Asynchronous parasitic torques, Induction motor, Squirrel cage rotor, Space harmonics, Synchronous parasitic torques.**

I. Introduction

At the beginning of electric machine design, it was already clear that the harmonics occurring in the machine, and within that the harmonics created by the number of rotor slots, are of decisive importance regarding the torque-speed characteristic curve of the machine. Therefore, in 1930, Möller [5] carried out a series of measurements with 3 stators and 19 rotors, based on a very carefully developed plan, and then attached an evaluation to the results. Some of his evaluations no longer correspond to later realizations, but the work as a whole is still of pioneering importance. One of the results of the evaluation is that Möller was the first to prove that the run-up characteristic curve clearly indicates some behavior like a synchronous machine. The fact that the contemporary Richter [6], in his work considered to be the very basic book of the universal electric machine science, referred to this work and verified his calculations regarding synchronous parasitic torque based just on Möller's measurements, proves the importance of this series of measurements.

In addition, from the fact that Heller and Hamata [7] in 1977, in their widely referenced book specifically dedicated to the effect of harmonics, adopted Möller's entire series of measurements and evaluation unchanged, and Boldea and Nasar [8] in 2010, in their latest edition, did the same, it must be concluded that there is not even a similar work in the literature.

Further, all the books regarding design of induction motors deal with synchronous torques and allowable slot numbers, of which only [9] will be referenced here.

Therefore, looking for suitable measurements, since we want to compare our formulas with a series of measurements recognized by everyone, we found this series of measurements to be the most suitable to "try out" our new formulas developed for synchronous and asynchronous torques. This "trial" is the main purpose of this study.

Of course, nothing else is known about the machines, other than the pole number and the stator and rotor slot number, and the fact given by Möller's text that the rotor slots are not skewed. However, since the formulas do not provide an absolute value, but - as suggested by us first - a ratio, namely the ratio to the breakdown torque in the most expedient way, therefore "following" the measurements by calculation is not a hopeless undertaking. Moreover, it will be seen, this application of a ratio is just the strength of the method. And the result is that one can experience a very surprising match for just the slot numbers that provide the highest synchronous torque.

In the following, first the synchronous, then the asynchronous parasitic torque will be examined in the light of the measurements, and then briefly mentioned the effect of phenomena not taken into account by the formulas.

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G. Kovács is with the Ganz Electric Works, Budapest, Hungary (e-mail: gabor.kovacs.ganz@gmail.com).

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Note that Möller's entire article can even today be downloaded from the AfE website.

The study relies entirely on [3] and [4], it is their direct continuation.

II. List of Symbols

X_m	Magnetizing reactance
X_s	Total leakage reactance
ξ_1, ξ_ν	Winding factor of fundamental wave, harmonic wave
$\eta_{2\nu}^2$	Jordan's coupling factor
s, s_ν	Slip of rotor to fundamental harmonic of stator, to harmonic ν of stator
ν, μ, ν', μ'	Designation of stator harmonics, rotor harmonics
a, b	Designation of harmonics in interaction
g, g_1, g_2, e_1	Different integers
p	Number of pole pairs
Z_1 , Z_2	Stator / rotor slot number
I_1, $I_{2\nu}$	Stator / rotor (harmonic) current
m	Number of phases
q_1, q_2'	Stator / rotor slot number per pole per phase

III. Synchronous Parasitic Torques

A. Calculation of Synchronous Parasitic Torques

Order of harmonics per definition:

$$\nu_a = 6g_1 + 1 \quad \mu_a = eZ_2 / p + \nu_a = e \cdot 2mq_2' + \nu_a \qquad \nu_b = 6g_2 + 1 \tag{1}$$

The formula is see [1] (13):

$$\frac{M_{synchr}}{M_{breakdown}} = \frac{X_m}{X_s} \cdot 2 \sum \frac{\xi_{1\nu_a} \xi_{1\nu_b}}{\mu_a} \eta_{2\nu_a}^2 \frac{1}{\xi_1^2} \tag{2}$$

where $\eta_{2\nu} = \dfrac{\sin \nu \dfrac{p\pi}{Z_2}}{\nu \dfrac{p\pi}{Z_2}} = \dfrac{\sin \nu \dfrac{\pi}{2mq_2'}}{\nu \dfrac{\pi}{2mq_2'}}$.

It is worth repeating our diagram developed in [2], which is the core element of our entire work and which shows the change of $\eta_{2\nu}^2$ in a very visual way.

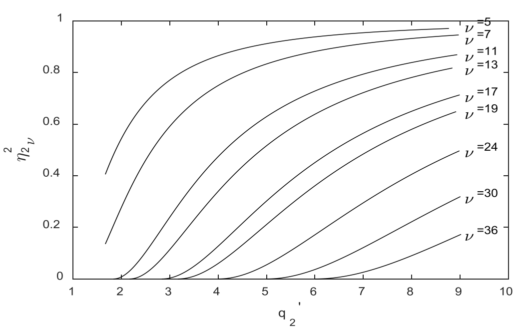


Fig. 1. Representation of the value of $\eta_{2\nu}^2$ as a function of q_2' with ν as a parameter. Higher odd harmonics are replaced mathematically by adjacent (actually with 3-phase not existing) even harmonics only for better transparency.

Based on the formula, the synchronous torques were calculated for the rotor slot numbers measured by Möller, see Attachment Table VI.

The calculation was performed with the value X_m/X_s=15, this is the only data where the estimation of the machine's properties was essential. Möller makes no reference to the chording, so winding factors are calculated without chording.

Odd rotor slot numbers were not dealt with here, even if Möller measured quite a few such numbers. The reason for this decision is that, on the one hand, there is practically no synchronous torque here due to p=2, so the formulas cannot be "checked" based on these measurements. On the other hand, during design, one can not take advantage of this favorable fact anyway, because then there is a risk of radial magnetic force waves of order r=1.

The values contained in Table VI are calculated by the stator fundamental harmonic ν_a=1 only, the rest of stator harmonics ν_a=-5, 7, -11, 13, etc. were not involved. That means they were not added/summated. On the one hand, because their effect is negligible in almost all cases, on the other hand, we specifically wanted to help/encourage the reader to check the results what is much more facilitated in this way. Where summation had still to be applied, this is indicated in remarks attached to the Table VI.

For the complete picture and for further analysis, the value of e and the value of ν_b were also provided. The former is necessary for the calculation of the speed, the latter because of its sign, for the separation of the torques occurring in rotation or standstill, including the determination of motoric or brake operation. After the column of the calculated torques, the sum of the torques occurring in standstill, and just below that, the resultant torque is given. The next column gives the speed where the torque occurs, more precisely the value of 1-s.

Where in Möller's work the synchronous torque is given in mkg, these are definitely the largest values, this value was compared to the breakdown torque measured by the author with a ruler. Where there is no directly measured value, the synchronous torque itself was also determined simply by reading it with a ruler. Where nothing could be read from the curves, next to the calculated value, the corresponding field was left free, but highlighted, simple indicating that the corresponding calculated value was not proven by the measurement; of course, this does not necessarily mean that the measurement would have specifically given zero.

Under the rotor slot number was entered if it represents a special relative one such as: 1/2, 1/3 or 2/3, see III. *G*.

B. Evaluation of the Measurements

In cases where the measured value is given in mkg and these of course represent the largest values the agreement between the calculation and the measurement is striking no matter it is in standstill or during rotation. These measured values are enclosed in a thick frame just to take the attention.

It is not our task to evaluate the measurement itself, but it is clear that in this case an external machine was needed to run up the machine and then the accuracy of the measurement can be considered the most reliable. In the other cases, however, this was not necessary, the torques were determined from a start-up curve, the accuracy of which obviously does not reach that of the previous measurements. Jumping into synchronism was inferred from e.g. sound effects, but a stroboscope was also used to study operation as a synchronous machine.

Regarding the difficulty of the measurement, consider for example the 24/20 slot number. Here, significant synchronous parasitic torques should have been measured and distinguished on slips s=0.96, s=1 and s=1.03. For this purpose, a machine with a much smaller rotor resistance should have been used, where the starting torque of the machine would not have been enough to start the machine, but such a machine can only be built in a much larger size.

To evaluate the whole picture, the following must be added:

1) In the case of some additional moderately significant torques occurring either in rotation or standstill, the agreement is quite good.

2) However, there are many slot numbers where the calculation indicates a significant or medium-sized torque, but when measured, these hardly appear, one might say they disappear.

In any case, it is a safety measure that no torque occurs in the measurement that is not indicated by this calculation before.

1) Synchronous Parasitic Torques in Standstill

The synchronous torque occurring in standstill must be dealt with separately. It is striking that during the measurements, no or only a low torque was measured compared to the theory for a large number of cases. From Möller's evaluation of the measurements, it is clear that, according to the knowledge at the time, the torque occurring in standstill was only expected for the case of Z_1=Z_2, partly for the so-called "a whole number of q_2' ". Therefore, in many cases, the course of the characteristic curve near s=1 is only indicated by a dashed line. According to the theory, however, at many rotor slot numbers, especially the so-called "half q" rotor slot numbers where q_2'=integer + ½ a significant standstill torque should (should have) occurred. The problem will be analyzed in detail later.

It is impossible not to mention the case of Z_1=Z_2=24. Richter [6] (see pp. 198) justifies his calculations on this measurement, that is, he reports his result with specific reference to Möller. His calculated value is 2.13 times the measured one. His torque is determined only by the arithmetical summation of the two torques belonging to e=±1. Richter classifies this result as adequate compared to the difficulty of the topic, and attributes the discrepancy to the neglect of saturation and the extreme difficulty of the measurement, and therefore assuming a smaller measured value than the real one. According to our calculation, the ratio to the breakdown torque is 4.92, which is 1.37 times compared to the measured one 3.6, far more accurate than Richter's calculations. This is all the more surprising since the formulas were developed entirely based on Richter's method. In addition, Richter would, as summation, multiply further by π/2. Here it was multiplied by π/4 see *E*.2 below, so the result is 3.86, which is 1.07 times the measured value: extremely good match.

The reason for the difference of the two approach might rely on two things. On the one hand: every calculation involves a certain amount of error regarding the absolute value of the respective quantity. In the case of an appropriately chosen comparison, however, the basis of the comparison is most likely also burdened by the same error; the two errors can pronounce each other through the comparison: in the opinion of the author, this is the case here. On the other hand: the torques were calculated not only for e=±1, but also summation was made including the rest of e as well and the resultant was composed at the end on a different way see again *E*.2 below.

In general, a good or acceptable match was obtained for more than half of the rotor slot numbers representing integer q_2'.

For the proof of present method, Heller / Hamata's method [7] (see pp. 124.), which specifically applies to torque occurring in standstill shall be quoted here. Where K a value coming from MMF, d is the greatest common divisor of Z_1 and Z_2. For example, if Z_1 and Z_2 are relative primes, then d=1, if Z_1 = Z_2, then d=Z_2.

Let's compare this with the calculations by our formulas and then the measurement for Z_1=24. Everything was compared to the maximum torque generated at Z_2=24.

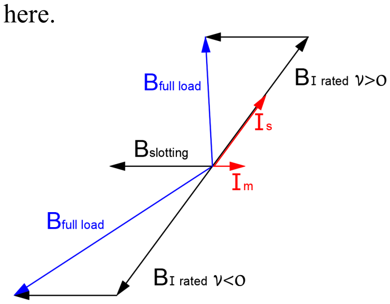
TABLE I
SYNCHRONOUS PARASITIC TORQUES OCCURRING AT STANDSTILL: COMPARISON OF CALCULATIONS AND MEASUREMENT

Z_2	Present formulas	Heller / Hamata method	Measurements
10	0,178	2/24=0,083	
16	0,807	8/24=0,33	
18	0,325	6/24=0,25	0,07
20	0,197	4/24=0,17	
22	0,09	2/24=0,083	
24	1	1	1
26	0,076	2/24=0,083	
28	0,143	4/24=0,17	
30	0,2	6/24=0,25	0,047
32	0,419	8/24=0,33	
36	0,335	12/24=0,5	0,047
44	0,091	4/24=0,17	
48	0,844	24/24=1	0,722

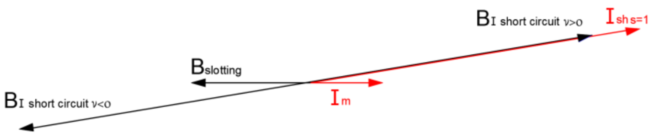
There is a fairly good agreement between the formula and the results of the Heller/Hamata method, but in the cases but one the measurement showed no torque at all, or only a very small one. In such cases, one is forced to handle the measurement with caution.

C. *Effect of the Slot Opening*

The effect of the slot opening will be taken into account based on the same principles as done when calculating the radial magnetic forces [4]. The necessary figure and formula are repeated here.



(a)



(b)

Fig. 2. (a) Vector addition of slotting harmonic due to airgap magnetic conduction fluctuation (being in counter-phase of magnetizing current) with load current winding harmonic (of the same order) [4]. (b) Vector addition of slotting harmonic due to airgap magnetic conduction fluctuation with short circuit current winding harmonic (of the same order).

In Fig. 2(b), the vector position of the short-circuit current is typical for a short-circuit.

It can be seen from the figure that the arithmetical summation of the two vectors is permissible. See also [7].

The slot opening causes a local change in the conductivity of the air gap, which occurs as a result of the open or half-closed slots of the stator and rotor.

The resulting conductivity change causes:

$$B_{slotting} = -(k_c - 1)\frac{\sin g\frac{k_c-1}{k_c}\pi}{g\frac{k_c-1}{k_c}\pi}B_{\nu=1} \quad (3)$$

Induction wave, where k_c Carter factor.

The sign of the effect of slotting is always opposite to the magnetizing current.

B_{slotting} order number $\nu_{slotting}=gZ_1+p$, where now not only $g=\pm1$ but also $g=\pm2$ factor will be included. As one can see, their order number is the same as the order of the winding harmonics $\nu'=\nu p$, called slot harmonics, the two can and should be added together.

For an open slot machine ($k_c\approx1.53$):

$$\text{for } g=\pm1 \qquad \text{for } g=\pm2$$
$$B_{\text{slotting}} \approx 0.45\cdot B_{\nu=1} \quad B_{\text{slotting}} \approx 0.19\cdot B_{\nu=1}$$

Möller's measured motors were certainly not made with open slots. Nevertheless, both stator and rotor part will be counted in this way. In the event of a short-circuit, unlike of radial magnetic forces being interested in no-load and in operation, the leakage paths are heavily saturated, the effect of which is that even the half-closed slot becomes a magnetically open slot.

Since $B_\nu=B_{\nu=1}/\nu$, the synchronous torque calculated with the stator current according to (2) must be multiplied by the following factor for first consideration:

$$g=\pm1 \qquad g=\pm2$$
$$1-\nu\cdot0.45 \quad 1-\nu\cdot0.19$$

However, further considerations must be made here. The first member of the expression is intended to represent the identity with the short-circuit current. The second part of the expression, on the other hand, is proportional not to the short circuit current but to the magnetizing current; therefore, the latter must be divided by the ratio X_m / X_s. As a result, it is obtained[7]:

$$g=\pm1 \qquad g=\pm2$$
$$1-\frac{1}{15}\cdot0.45\cdot\nu_b \quad 1-\frac{1}{15}\cdot0.19\cdot\nu_b \quad (4)$$

It is emphasized that the vector diagram and thus the derived factors are equally valid under the influence of the induction wave created by either the stator or the rotor slot opening. The slotting induction wave is not created by one or the other current, but by the stator and rotor currents together e.g. by the magnetizing current. Therefore, it does not matter whether this slot opening is on the stator or the rotor or both. Let's also examine how to sum up the effects of one slotting harmonic of the stator and one slotting harmonic of the rotor. Consider the effect of the slotting on the magnetic conductivity of the air gap [7]:

$$\lambda(\alpha)\approx\frac{1}{\delta\cdot k_{c1}k_{c2}}\begin{pmatrix}1-a_1k_{c1}\cos Z_1\alpha-b_1k_{c2}\cos Z_2\alpha-\\-a_1k_{c1}\cos 2Z_2\alpha-b_1k_{c2}\cos 2Z_2\alpha-...\end{pmatrix} \quad (5)$$

where λ is the magnetic conductivity of the air gap as a function of the angle, α measured along the circumference. δ, air gap. k_{c1}, k_{c2}, carter factors, stator, rotor. a_1, b_1 constants coming from dimensions.

The formula (5) is used so only that if the above case occurs, the resulting factor will be the following:

$$1-\frac{1}{15}\cdot f_{st}(g)\cdot\nu_{b-stator}-\frac{1}{15}f_{rt}(g)\cdot\mu_{a-rotor} \quad (6)$$

The 2nd part of (6) concerning the harmonics of the stator is easy to calculate in advance, since here in this case it will be dealt with specifying the synchronous torques that appear in rotation, they are only affected by the slot harmonics -11, 13, -17, 19, -23, 25 according to Table VI. The stator slot harmonics: $\nu_b=Z_1/p+1$. For Z_1 = 24 ν= -11, 13, -23, 25. For Z_1 = 36 ν= -17, 19. For Z_1 = 48 no effect by slotting now.

The factors are given in Table II.

TABLE II
EFFECT OF STATOR SLOT OPENING ON SYNCHRONOUS TORQUE

ν_b	Z_1=24	ν_b	Z_1=36
-11	1+0.33=1,33	-17	1+0,51=1,51
13	1-0.39=0,61	19	1-0,57=0,43
-23	1+0.29=1,29	-	-
25	1-0,316=0,684	-	-

If a synchronous torque comes from one of the above harmonics, it means that it is a slot harmonic of the stator winding and then the calculated synchronous torque must be multiplied by the above factor.

In the case of synchronous torque occurring at standstill, the pair-wise sum of the second terms of the two factors e.g. -11 and 13 barely differs from 0, so it is not worth dealing with the torque change caused by stator slotting. In the same way, there is no need to deal with the effect of the rotor as well; otherwise they occur in a pair only when $Z_1=Z_2$.

The rotor harmonics will be $\mu_a=\pm Z_2/p+1$ and $\mu_a=\pm2\cdot Z_2/p+1$. Therefore, if a synchronous torque comes from one of these harmonics, it means that it is a slot harmonic of the rotor cage and then the calculated synchronous torque must be multiplied by a factor calculated also according to (4).

The factors are given in Table III.

TABLE III
EFFECT OF ROTOR SLOTTING ON SYNCHRONOUS TORQUE

Z_2	g	μ_a	Factor	g	μ_a	Factor
10	-1			-2		
	1			2	11	0,86
16	-1	-7	1,21	-2		
	1			2	17	0,78
20	-1			-2	-19	1,24
	1	11	0,67	2		
22	-1			-2		
	1			2	23	0,71
26	-1			-2	-25	1,32
	1			2		
28	-1	-13	1,39	-2		
	1			2	29	0,63
32	-1			-2	-31	1,39
	1	17	0,49	2		
44	-1			-2	-43	1,54
	1	23	0,31	2		

As a numerical example, let's immediately consider the slot number Z_2=10.

Its rotor slotting harmonic is μ_a=11 (g=2), higher g are not dealt with. The factor: 1-0.14=0.86. This applies to all three stator slot numbers.

The $-\mu_a=\nu_b=-11$ harmonic is not a slotting harmonic in the case of stators Z_1=36 and 48, but it is in the case of Z_1=24, this fact can also be seen in the value of ξ_ν in Table VI. This synchronous torque is also affected by the slot opening of both the stator and the rotor. The resulting factor acc. to (6): 1+0.33-0.14=1.19.

At harmonic ν_b=19, in the case of Z_1/Z_2=24/10 and Z_1/Z_2=48/10, neither the slotting of the stator nor the rotor affects the calculated synchronous torque. In the case of Z_1/Z_2=36/10, however, the slot harmonic of the stator reduces the calculated synchronous torque by a factor of 0.43.

The general result: the $\nu'=\nu p=gZ_1-p$ and $\mu'=\mu p=gZ_2$-p harmonics resulting from slotting always increase, the $\nu'=\nu p=gZ_1+p$ and $\mu'=\mu p=gZ_2+p$ harmonics always, sometimes significantly, reduce the respective harmonic value, in the ratio acc. to (6) [7](see chapter 6.7, 6.8 also).

Of course, all these can not be taken into account numerically in the actual Table VI, but only qualitatively, in terms of trends. A statement might be made, in many cases, the disappearance of the synchronous torque occurring during the measurement compared to the calculation in rotation can be explained by this phenomenon. E.g. in the cases of 36/10, 36/20, 24/26 the stator, in the cases of 24/16, 36/16, 24/22, 24/44 the rotor might have a strong reducing effect. In this series of measurements, in case of Z_1=48, neither the slotting of the stator nor the rotor significantly affects the synchronous torques.

D. Effect of Saturation

Here, the effect of saturation will not be examined in general, but specifically in connection with the application of formulas (2) and (25). When deriving the formula for both synchronous and asynchronous torque [1], it was assumed and then noted that the same leakage reactance was substituted which is valid at breakdown torque as well as at the starting current. In reality, however, it is obvious that the starting current saturates the leakage paths compared to the current at breakdown and the leakage reactance decreases. The saturation of X_s can be deduced from the short-circuit characteristic of the machine: $X_s(I\approx 5\cdot I_{\text{rated}})/X_s(I\approx 2.5\cdot I_{\text{rated}})$. Therefore, when measuring - since X_s is in the denominator - a higher torque will be measured compared to the calculated one; not significantly larger, since the leakage paths start to saturate even at breakdown torque with approx. 2.5 times the rated current. Möller made his measurements at 2/3 U_{rated}, i.e. at max. 3.5 I_{rated} where the saturation was certainly not significant yet. Including saturation of X_m in the formula in this way would be a mistake, since the higher harmonics are confined to the region of the tooth heads, they do not reach the entire tooth, and they do not reach the yoke at all, therefore their saturation does not affect the parasitic torques. However, the saturation of X_m must be taken into account in the formula, but it is advisable to calculate the unsaturated value of X_m, because at starting the flux of the main field is approx. half of what occurs in the rated operation, therefore the path of the main field is then certainly unsaturated.

E. Summation of Torques

Consider the torque formula:

If the synchronous torque occurs in the rotation:

$$M = M_{\max}\sin((2+e\frac{Z_2}{p}(1-s))\omega t + e\frac{Z_2}{p}\frac{\pi}{\tau}x_2' + \varsigma_\nu) \qquad (7a)$$

If the synchronous torque occurs in standstill:

$$M = M_{\max}\sin(e\frac{Z_2}{p}(1-s)\omega t + e\frac{Z_2}{p}\frac{\pi}{\tau}x_2' + \varsigma_\nu) \qquad (7b)$$

where ς_ν is the angle acc. to Fig. 3.

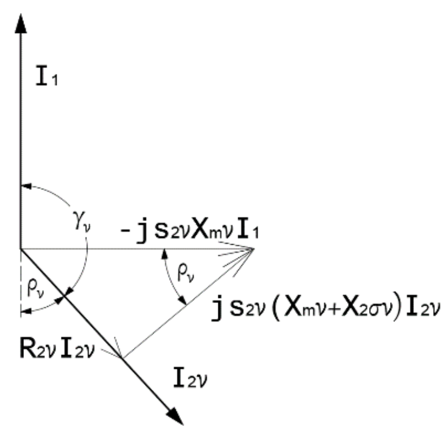


Fig. 3. Vector diagram of stator current and rotor current for the circuit ν [6].

If the slip in (7a) is $s=1+2p/eZ_2$ or in (7b) is s=1 then the term ωt disappears in these formulas. If the torque produced by the stator fundamental harmonic ν_a=1 is used only, then $\varsigma_{\nu=1}\approx 0$, then:

$$M = M_{\max}\sin e\frac{Z_2}{p}\frac{\pi}{\tau}x_2' \qquad (8)$$

In both cases, the torque does not depend on time. The role of the x_2' rotor position can be seen here: x_2'=0 indicates the rotor position when the machine is not currently exerting synchronous torque. Then a rotor tooth is just opposite the center of the magnetomotive wave of the first phase [10].

The formula means that if the rotor is turned by one full rotor slot pitch, the synchronous torque takes on the torques corresponding to one full period; therefore, its maximum, expressed as an angle, is at π/2.

Since these are synchronous torques, let us introduce the δ load angle used for synchronous machines:

$$\delta = \frac{Z_2}{p}\frac{\pi}{\tau}x_2' \quad (9)$$

where τ pole pitch.

Written generally for both cases:

$$M = M_{\max}\sin(e\cdot\delta + \varsigma_v) \quad (10)$$

where ς_v shall be calculated at s_v=1 [2].

If s_{bv}< 1 (and this is usually the case), then ς_v<π/2, usually $\varsigma_v \le \pi/4$, that is, it is not a large value and does not change much. Therefore, a simple arithmetical summation of the torques due to further v_a harmonics at the same e could usually be applied at first.

1) Summation of Torques Occurring in the Rotating State

There is no such summary in the Table VI. Only the effect of the rotor current generated by the fundamental harmonic field of the stator v_a=1 was calculated so that the Reader could easily check the calculated values. The same will be done in the case of torques occurring in standstill as well. This is made possible by the fact that the torques generated by further harmonics of the stator are usually not significant.

If the effect of the further harmonics should be taken into account, then these should only be summarized for the same *e,* and for the same sign; not for different ones, because they occur at different speeds. That is, the further $M_{\max}$ values shall be calculated for v_a=-5, 7, -11, 13, etc. then be added. Where v_b>2mq_1-1, ς_v calculated for the corresponding v_a must be substituted, because v_b is not attenuated, that is, the voltage on X_{mvb} is in phase with I_1; if not, then $\varsigma_v = \varsigma_{va} - \varsigma_{vb}$ must be substituted.

2) Summation of Torques Occurring in Standstill

The way is completely different.

First, the torques produced by the fundamental harmonic stator current v_a=1 are dealt with, where ς_1=0. Torques belonging to coefficients with the same absolute value, e.g. e=-1 and e=1, must be summated arithmetically, with the correct sign. Signed summation means to form their difference, because they are in opposite phase, see the formula. The same goes for e=-2 and e=2 etc. values.

However, the torques belonging to different absolute values of *e* coefficients not only can, but must be added together explicitly. As an example, let's take the rotor slot number Z_1/Z_2=36/36, because here a torque in rotation does not occur at all, the rules leading to the development of the method can be explained simply without other disturbing factors.

The largest torque component naturally occurs at v_b=-17 and v_b=19, with values $\xi_b=\xi_1$ corresponding to the slot harmonics of q_1, where, and this is especially important, ξ_{17} and ξ_{19} have the same sign. Thus, according to $\mu_a = v_b$, the sign of the torques, corresponding to the positive and negative sign of *e*, will always be different. Therefore, their absolute values always add up when forming a difference.

The values calculated with the coefficients e=-2 and e=2 at v_b=-35 and v_b=37 must be added in the same way, since the individual torques, due to the periodicity of the winding factor and to $\xi_{35}=\xi_{37}$, will still have different signs, but their sum will be half of the previous one, consider that: $1/17+1/19=\ldots=2\cdot18/(18^2-1)\approx2/18$; $1/35+1/37=\ldots=2\cdot36/(36^2-1)\approx2/36$.

The rest, with coefficients e=-3 and e=3 etc. the same will be obtained, but their sum will be 1/3, 1/4, etc. of the largest.

Their summation, however, can only be geometrical. The reason for this is that quantities acc. to:

$$M = M_{\max}\sum\frac{1}{e}\sin e\delta \quad (11a)$$

Or space vectors acc. to:

$$M = M_{\max}\sum\frac{1}{e}e^{e\delta} \quad (11b)$$

Shall be summarized, where $M_{\max}$ is the largest torque calculated by the lowest *e*; the lowest *e* is not necessarily $e=\pm1$. It should be noted here that symbol *e* is in 1/*e* and *eδ* an integer coefficient, and in the power expression it is the base of the natural logarithm.

If the rotor is rotated from the initial, equilibrium rotor position by 30° related to the rotor slot pitch, the torque components from *e*=*abs*(2) being otherwise half as large will rotate 60°. Those derived from *e*=*abs*(3) being otherwise third-sized components will rotate 90°; those derived from *e*=*abs*(4) being otherwise quarter-sized components will do 120°, those from *e*=*abs*(5) being otherwise one-fifth the size will do 150°. The geometric sum of these will be the resulting synchronous torque. The maximum of this is being looked for.

The task to be solved is to determine the sum of an infinite series, e.g. the maximum of the sum.

$$M = e^{j\delta} + \frac{1}{2}e^{j2\delta} + \frac{1}{3}e^{j3\delta} + \frac{1}{4}e^{j4\delta} + \frac{1}{5}e^{j5\delta} + \frac{1}{6}e^{j6\delta} + \ldots \quad (12a)$$

where δ is the variable load angle.

Otherwise, since the real part represents the torque.

$$M = \sin\delta + \frac{1}{2}\sin 2\delta + \frac{1}{3}\sin 3\delta + \frac{1}{4}\sin 4\delta + \frac{1}{5}\sin 5\delta + \frac{1}{6}\sin 6\delta \ldots \quad (12b)$$

It will be calculated with (12b), but (12a) will better illustrate the results.

Let us include in the analysis the so-called saw curve, which is defined as follows:

$$f(x) = \frac{\pi - x}{2},\ 0<x<2\pi,\ f(0)=0 \quad (13)$$

This curve is already used in the theory of electric machines, it gives the function of the magnetomotive force created by the current of a single conductor in the air gap [7].

The Fourier series of the curve, now writing δ instead of x:

$$\frac{\pi - \delta}{2} = \sin\delta + \frac{\sin 2\delta}{2} + \frac{\sin 3\delta}{3} + \ldots = \sum_{e=1}^{\infty}\frac{\sin e\delta}{e} \quad (14)$$

It is not difficult to notice that this is precisely the series of synchronous parasitic torques occurring in standstill.

In Fig. 4, both the saw curve itself and the components of the first three harmonics and their sum are shown.

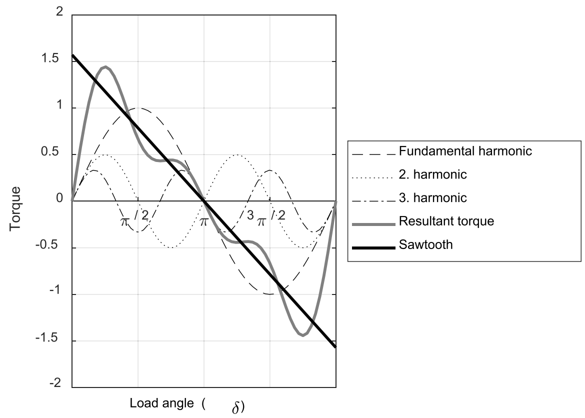


Fig. 4. Summary of torque components occurring in standstill [11].

The maximum occurs at δ=0 and δ=2π, its value is π/2 or -π/2. The fundamental harmonic of the curve is precisely the synchronous torque found with the smallest *e*; the resulting maximum torque will simply be π/2 times this, which occurs at δ=0. According to general opinion[6], [11], this π/2 times maximum torque is the resulting torque and this should be written under the torque calculated with the smallest value of *e* in Table VI and this must be compared with the measured value.

In the present study, this approach will be improved in two aspects.

It is difficult to imagine that with a synchronous machine, and this is what we are talking about here, at the load angle δ=0 the rotor is at rest, more precisely: it does not exert a synchronous torque, but at the same time any smallest $\pm\Delta\delta$ displacement is prevented “immediately” by a ±π/2 times multiple of the torque component calculated with *e*=±1. If, under the influence of an external force, it is still moved with a slight load angle of $\pm\Delta\delta$, the machine immediately enters the unstable range.

The solution is looked for in using of a finite series instead of an infinite one. In any case, this is a legitimate approach based on the explanation according to point F.

Let’s find the location of the maximum in the finite series (12b):

$$\frac{\mathrm{d}M}{\mathrm{d}\delta}=\cos\delta+\frac{1}{2}2\cos2\delta+\frac{1}{3}3\cos3\delta+\frac{1}{4}4\cos4\delta+\frac{1}{5}5\cos5\delta+\frac{1}{6}6\cos6\delta++\frac{1}{7}7\cos7\delta+\frac{1}{8}8\cos8\delta+\frac{1}{9}9\cos9\delta+\frac{1}{10}10\cos10\delta+\frac{1}{11}11\cos11\delta+\frac{1}{12}12\cos12\delta+...=0 \tag{15}$$

Leaving aside the precise solution, by substituting δ=π/6, it can be seen that each group of 6 is the same as the next group of 6, but with the opposite sign, that is, the series gives a zero for every group of 12. This is a local minimum, always with the same load angle. The series also gives a zero after group 6, as well as after every subsequent group of 12, somewhere in the δ-$\Delta\delta$ position, this is a local maximum. It is not worth calculating $\Delta\delta$ with great expense, because it does not advance the matter. Substituting:

$$M=\sin\pi/6+\frac{1}{2}\sin2\pi/6+\frac{1}{3}\sin3\pi/6+\frac{1}{4}\sin4\pi/6+\frac{1}{5}\sin5\pi/6+\frac{1}{6}\sin6\pi/6... \tag{16}$$

The local minima increase monotonically, the local maxima decrease monotonically: both approach to the factor $1.31=5\pi/12$; it is not a big mistake if the first torque found with the smallest value of *e* is multiplied by this factor, regardless of the number of order ν that can be taken into account acc. F.

This result coincides with the fact that if the value of the saw curve is calculated at *x*=δ=π/6, one arrives to the same result

$$\frac{\pi-\pi/6}{2}=\frac{5\pi}{12}=\sin\pi/6+\frac{\sin2\pi/6}{2}+\frac{\sin3\pi/6}{3}+... \tag{17}$$

In any case, the resulting curve means a stable load condition in the range δ= 0 - π/6.

Another part of the previous approach that needs to be clarified is the following: the torque components were all of the same sign; but this is not the case for all slot numbers.

Only positive i.e. identical signs are exclusive at q_1=3, because the periodicity of the slot harmonic winding factors results in always positive sign. However, for q_1=2 and q_1=4, the sign of the slot harmonic winding factors is alternately positive and negative, consequently both exclusively positive e.g. 24/48 slot and alternately positive and negative e.g. 24/24 slot sign occurs. In the case of the latter, the amount, and thus the applicable factor, will obviously be very different.

The series of torques is then:

$$M=e^{j\delta}-\frac{1}{2}e^{j2\delta}+\frac{1}{3}e^{j3\delta}-\frac{1}{4}e^{j4\delta}+\frac{1}{5}e^{j5\delta}-\frac{1}{6}e^{j6\delta}+... \tag{18a}$$

Respectively

$$M=\sin\delta-\frac{1}{2}\sin2\delta+\frac{1}{3}\sin3\delta-\frac{1}{4}\sin4\delta+\frac{1}{5}\sin5\delta-\frac{1}{6}\sin6\delta... \tag{18b}$$

Generally speaking: the sign of the even multiples is opposite to the sign of it and its odd multiples.

$$\frac{\mathrm{d}M}{\mathrm{d}\delta}=\cos\delta-\frac{1}{2}2\cos2\delta+\frac{1}{3}3\cos3\delta-\frac{1}{4}4\cos4\delta+\frac{1}{5}5\cos5\delta-\frac{1}{6}6\cos6\delta...=0 \tag{19}$$

Leaving aside the precise solution, it can be seen that for δ=π/2 the first and third components in each group of four are zero, and the second and fourth terms cancel each other out, that is, the maximum of the sum of the infinite series occurs at the same load angle, where the maximum of the first, of the largest component also occurs; the values from the even coefficients are turned to 180° or 360° = 0°, so they are dropped. And the values from the odd coefficients were turned to either 270° (in opposite phase) or 540°=90°, i.e. in the same phase.

So then it is:

$$1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-...=\frac{\pi}{4}$$

We are dealing with an infinite series whose sum is known.

This result coincides with the fact that if we calculate the value of the saw curve at $x=\delta=\pi/2$, we arrive at the same result:

$$\frac{\pi-\pi/2}{2}=\frac{\pi}{4}=\sin\pi/2+\frac{\sin 2\pi/2}{2}+\frac{\sin 3\pi/2}{3}+... \quad (20)$$

In any case, the resulting curve means a stable load condition in the range δ = 0 - π/2.

In Table VI, the first, largest torque component occurring in standstill was multiplied by the factor π/(12/5)=π/2.4 or π/4, depending on the signs. To determine the signs, by increasing *e*, it is necessary to reach the second slot harmonic. In any case, the result is fundamentally different compared to multiplying by π/2 regardless of everything.

It is noted here that the method used in solving (15) and (19) was of course mathematical in nature. However, according to the analyzing the physics of the phenomenon in point *F* below, definitely finite number of torques are added. Studying Table VI reveals that even in the case of the highest torque-producing slot numbers, a maximum of 10 torque component (usually 3-5 components only) may be added, because the order number of the further torque components is too high to actually form. Therefore, it is not correct to use an infinite series as a basis for summation in [11] and, as a result, to multiply by π/2. Nevertheless, in general, in the first step, it is still recommended using the above two factors, π/(12/5)=π/2.4 or π/4. We will return to this in point *G*, when analyzing Table VI.

It is also noted that torques occurring in standstill but not originating from slot harmonics also occur, see e.g. Z_1=36 / Z_2=24, Z_1=48 / Z_2=24, and many others. However, for these, the winding factors ξ belonging to this harmonic pair are much smaller. But above all, they always have opposite signs, so the torques are subtracted from each other. Thus, there are two reasons why they should be negligible. This fact will be indicated in Table VI by "≈0".

Of course, here, too, the torques created by the harmonics of the stator magnetomotive force v_a can properly be added to the torque from the fundamental harmonic v_a=1 as described above, and only then to proceed to the summation of the torque components belonging to the various *e* coefficients; but thus the rules used and the formula found will not be valid.

F. The Maximum Harmonic Order v to be Taken into Account

The calculation of harmonics, but mainly the effect of harmonics of the stator in the rotor magnetomotive forces and vice versa, is also valid up to very high harmonics, although the effect is obviously decreasing, but it can still be calculated according to the formulas. However, if the half-wavelength of the v^{th} harmonic decreases below twice the air gap, then it is clear, even without an exact space calculation, that the stator MMF does not reach the rotor part, or only a little, and vice versa; the effect calculated so far will not occur, torque will not be produced. With formula:

$$v_b \geq \frac{D\pi}{2p\cdot 2\delta} \quad (21)$$

It would be difficult to express the usual geometric dimensions of different size and manufacturer in even a slightly general way and to derive a formula, but this much can be said that the effect of harmonics above = 150 - 250 should not be taken into account.

It is noted that it can be seen in the literature and in daily practice that the synchronous parasitic torques, which can be considered dangerous and even fatal, sometimes do not prevent the motor from starting up. The subject has a wide literature. However, we do not feel it is our task here to include these phenomena in the calculation and evaluation. Here, the maximum torque generated is specified.

G. Examination of Allowed and not Allowed or Forbidden Slot Numbers

The full examination of a motor currently under design must be carried out by the method used in Table VI. For reasons of limit of length, it is not possible to publish this in advance, for an entire motor series, as a design guide. However, it is possible to deduce from the regularities already found what to expect for certain rotor slot numbers. The method is presented on some special rotor slot numbers. There are regularities, but there are always exceptions, so only a full check of the chosen slot number according to the method in Table VI provides security.

The rotor slot number q_2' can always be written as the sum of an integer and a real fraction:

$$q_2' = q_2 + p/r \quad (22)$$

Here q_2 is a positive integer, unrelated to the stator slot number q_1. *p* and *r* are positive integers: $p \leq r$.

It will be shown as a theorem that the nature of the occurring synchronous torques is determined solely by the real fraction: the denominator *r* is of decisive role, the numerator *p* is of subordinated role.

Harmonics of the rotor magnetomotive force for v_a=1

$$\mu_a = e\cdot 2m\, q_2' + 1 = e\cdot 2mq_2 + e\cdot 2m\cdot p/r + 1 \quad (23)$$

Comparing (23) to v_b in (1) the conditions for generating synchronous torque are easy to see:

1) The value of μ_a must be odd, therefore the sum of the first and second terms in (23) must be even.

2) The first term is not only even, but is divisible by 6 - generally: is divisible by 2*m* - therefore the second term shall also be even: this is satisfied only by suitable values of *e*. In this case, a synchronous torque is generated in rotation, then always $\mu_a=-v_b$

3) If the second term is not only even but also divisible by 6 - by 2*m* -, that means $e\cdot p/r$=integer then synchronous torques are generated in standstill, always in pairs because always $\mu_a=v_b$ for both -*e* and +*e* terms. Since for any value of *r* there is an *e* such that $e\cdot p/r$=integer, therefore it can be stated that any number of rotor slots produces synchronous torque in standstill.

After that, it is simple that if p/r=0 or p/r=1, that is, q_2' is an integer, then very high torques occur and only in standstill. Reason is that then $\xi_b=\xi_1$ sure with low coefficient of e; with $Z_2=Z_1$ sure with $e=\pm1$.

If p/r=1/2 ("of half q"), torques occurring only in standstill are generated, but since m=3 (odd), with $e=\pm2$ and further even values of e. They are still significant.

For further even values of r, e.g. p/r= 1/4, 3/4, furthermore p/r= 1/8, 3/8, 5/8, 7/8 torques occurring only in standstill are generated, but with the smallest value of $e=\pm4$ and $e=\pm8$ respectively. These torques will probably no longer be significant. These values of r will not occur on certain pole numbers.

Let us investigate further rotor slot numbers. Values of r=5 as p/r=1/5, 2/5 etc., r=7 as p/r=1/7, 2/7 etc. produce synchronous torques also *only* in standstill, with the smallest value of $e=\pm5$ and $e=\pm7$ respectively. Such values of r may occur with pole numbers like $2p$=10, 20… and $2p$=14, 28… resp. only.

Let's also examine the often used so-called "of third q" slot numbers.

$$q_2' = q_2 + 1/3 \ or \ q_2 = q_2 + 2/3 \tag{24a}$$

$$e \cdot 2mq_2' = e \cdot 2mq_2 + e \cdot 2m \cdot 1/3 \ or \tag{24b}$$

$$e \cdot 2mq_2' = e \cdot 2mq_2 + e \cdot 2m \cdot 2/3 \tag{24c}$$

Which, by increasing e, will first become divisible by 6 with $e=\pm3$. These will not be small standstill torques either. Despite of this, no any trace of them can be seen during the measurements.

In any case, it can be seen from this example that with a sufficiently high e value one can always detect a synchronous torque occurring in standstill. A theorem can be formulated: there is no rotor slot number where a synchronous parasitic torque in standstill does not occur.

The details of the "of third q" slot numbers are included in Table IV. As can be seen, in both tables the torques calculated with $abs(e)$=1 and $abs(e)$=2 are created alternately in the motoric and the brake range respectively, the torques calculated with the factor e=±3 always occur in standstill. This pattern is repeated for each group of $abs(e)$=4-6, 7-9 etc..

Let's start the investigation with the "of 1/3 q" rotor slot number. All of such rotor slot numbers are generally to be avoided, as they create synchronous torque in the motoric range, namely with the smallest value of e=-1.

It can even be seen further that with stator q_1=2: rotor slot numbers q_2'=2 1/3 and 4 1/3, stator q_1=3: rotor slot number q_2'=3 1/3, stator q_1=4: rotor slot number q_2'=4 1/3, the torque in the motoric range is even created with a winding factor of $\xi_b=\xi_1$; it must therefore be called them explicitly forbidden rotor slot numbers. Also q_2'=2 1/3 with q_1=3 calculated with e=-4 (not shown in the table).

Also forbidden slot numbers are for stator q_1=2: rotor slot number q_2'=1 1/3 and q_2'=3 1/3, stator q_1=4: rotor slot number q_2'=1 1/3, because these, also with a winding factor of $\xi_b=\xi_1$, create a high standstill torque.

TABLE IVa
SYNCHRONOUS PARASITIC TORQUES PRODUCED BY "1/3 Q" ROTOR SLOT NUMBERS

		μ_a=			ν_b			q_1		
q_2	e	e*2m*q_2+	e*2m*1/3	+1	$\nu_b=-\mu_a$	$\nu_b=\mu_a$	operation	q_1=2	q_1=3	q_1=4
1	-1	-6	-2	-7	7		motoric			
	1	6	2	9	n.a.	n.a.				
	-2	-12	-4	-15	n.a.	n.a.				
	2	12	4	17	-17		break		ξ_{slot}	
	-3	-18	-6	-23		-23				
	3	18	6	25		25	standstill	ξ_{slot}		ξ_{slot}
2	-1	-12	-2	-13	13		motoric	ξ_{slot}		
	1	12	2	15	n.a.	n.a.				
	-2	-24	-4	-27	n.a.	n.a.				
	2	24	4	29	-29		break			
	-3	-36	-6	-41		-41				
	3	36	6	43		42	standstill			
3	-1	-18	-2	-19	19		motoric		ξ_{slot}	
	1	18	2	21	n.a.	n.a.				
	-2	-36	-4	-39	n.a.	n.a.				
	2	36	4	41	-41		break			
	-3	-54	-6	-59		-59				
	3	54	6	61		61	standstill	ξ_{slot}		
4	-1	-24	-2	-25	25	-25	motoric	ξ_{slot}		ξ_{slot}
	1	24	2	27	n.a.	n.a.				
	-2	-48	-4	-51	n.a.	n.a.				
	2	48	4	53	-53	53	break		ξ_{slot}	
	-3	-72	-6	-77		-77				
	3	72	6	79		79	standstill			

TABLE IVb
SYNCHRONOUS PARASITIC TORQUES PRODUCED BY "2/3 Q" ROTOR SLOT NUMBERS

		μ_a=			ν_b			q		
q_2	e	e*2m*q_2+	e*2m*2/3	+1=	$\nu_b=-\mu_a$	$\nu_b=\mu_a$	operation	q_1=2	q_1=3	q_1=4
1	-1	-6	-4	-9	n.a.	n.a.				
	1	6	4	11	-11		break	ξ_{slot}		
	-2	-12	-8	-19	19		motoric		ξ_{slot}	
	2	12	8	21	n.a.	n.a.				
	-3	-18	-12	-29		-29				
	3	18	12	31		31	standstill			
2	-1	-12	-4	-15	n.a.	n.a.				
	1	12	4	17	-17		break		ξ_{slot}	
	-2	-24	-8	-31	31		motoric			
	2	24	8	33	n.a.	n.a.				
	-3	-36	-12	-47		-47				
	3	36	12	49		49	standstill	ξ_{slot}		ξ_{slot}
3	-1	-18	-4	-21	n.a.	n.a.				
	1	18	4	23	-23		break	ξ_{slot}		ξ_{slot}
	-2	-36	-8	-43	43		motoric			
	2	36	8	45	n.a.	n.a.				
	-3	-54	-12	-65		-65				
	3	54	12	67		67	standstill			
4	-1	-24	-4	-27	n.a.	n.a.				
	1	24	4	29	-29		break			
	-2	-48	-8	-55	55		motoric		ξ_{slot}	
	2	48	8	57	n.a.	n.a.				
	-3	-72	-12	-83		-83				
	3	72	12	85		85	standstill	ξ_{slot}		

The "of 2/3 q" rotor slot number is generally considered favorable, since it creates the higher torque in the braking range; in the motoric range it is expected that only approx. half as much of the one in breaking due to double absolute value of μ_a i.e. it is considered as less dangerous, even secure.

However, the table shows that stator q_1=3: rotor slot numbers q_2'=1 2/3 and 4 2/3, the torque appearing in the second row, due to $\xi_b=\xi_1$ jumps forward to be the biggest, occurring just in the motoric range. They must therefore be called explicitly forbidden rotor slot numbers. Also forbidden slot numbers for stator q_1=2, rotor slot numbers q_2'=2 2/3 and 4 2/3, stator q_1=4, rotor slot number q_2'=2 2/3, because significant standstill torque is created.

Obviously the synchronous parasitic torques in case of q_2'= 4 1/3, 4 2/3 are less dangerous than those of q_2'= 1 1/3, 1 2/3[2](see Fig. 4).

The method can be continued for further values of p/r, for example:

1) For "of 1/6 q": such rotor slot numbers follow the pattern of those "of 1/3 q" but with e=±2 as lowest value consequently higher μ_a therefore expectedly lower torques will occur.

2) For "of 5/6 q": such rotor slot numbers follow the pattern of those "of 2/3 q" but with e=±2 as lowest value consequently higher μ_a therefore expectedly lower torques will occur except q_2'=3 5/6 at q_1=3 with $\xi_b=\xi_1$ in the motoric range which is therefore forbidden.

3) Above slot numbers may occur on any pole number.

4) For "of 1/12 q", "of 7/12 q": such rotor slot numbers follow the pattern of those "of 1/3 q" but with e=±4 as lowest value consequently even higher μ_a therefore expectedly even lower torques will occur.

5) For "of 5/12 q", "of 11/12 q": such rotor slot numbers follow the pattern of those "of 2/3 q" but with e=±4 as lowest value consequently even higher μ_a therefore expectedly even lower torques will occur.

6) These latter slot numbers "of n/12" cannot occur with pole number 2p=2 and on some further pole numbers.

Based on the above experiences, let's examine what would be the conditions for synchronous torque to occur at all during rotation, again for ν_a=1. Let's write (1) generally for any phase number in a suitable way:

$$e\cdot 2mq_2 + e\cdot 2m\cdot p/r + 1 = -(2mg + 1) \quad (25)$$

After arranging:

$$e\cdot q_2 + g + e\cdot p/r = -1/m \quad (26)$$

Since e, q_2 and g are integers, the equation can be rearranged as follows:

$$e\cdot p/r = -1/m \pm \text{integer} \quad (27)$$

This equation can only be solved only if:

$$r = \text{integer}\cdot m \quad (28)$$

For m=3:

If in (27) e.g. r=3 and p=1 then the solutions are e= -1 - integer ·3 (motor range) and e= 2 + integer ·3 (brake range).

If in (27) e.g. r=3 and p=2 then the solutions are e= 1 + integer 3 (brake range) and e= -2 - integer 3 (motor range) If further e=±3·integer then there is no solution that means no synchronous torque in rotation; but there is synchronous torque in standstill.

All these correspond to Table IVA and IVB and to Table VI.

The method demonstrated in Table IV shows the role of the stator in the generation of synchronous parasitic torques and how this role can be taken into account in the simplest way. As told before, acc. to conception of this series of study the role of the stator is limited to "rearranging" the slot harmonics from one stator slot number to another.

This investigation also shows there is even the most well-proven kind of rotor slot numbers must be checked, because in certain cases - which cannot be determined in advance - the torque of the respective slot number, which is otherwise classified as harmless, might "coincide" with a slot harmonic and becomes dangerous.

When evaluating the synchronous torques occurring in braking range, the oscillating torques created by them also in normal operation must be taken into account. As an example: if a synchronous torque of $M_{\text{synchr}}=M_{\text{breakdown}}$ occurs in the braking range, which is of course declared harmless, then in rated operation this causes an oscillation approx. 8% of rated torque (with a frequency of 600 Hz - 1200 Hz). And considerable radial magnetic forces of r=0 will be created at the same time. Even if the motor is powered via an inverter and only operates in the s=0-s_b range, one must pay attention to the correct rotor slot number.

A detailed analysis according to Table VI is required for the full investigation. In fact, the table must be built by increasing μ_a (by increasing e). One has always to go until the first slot harmonic appears; in case of standstill (for its sign) until the second slot harmonic. It can only be said that a complete picture of the rotor slot number in question is found if such μ_a has been found for the torques that occur both in motoric as well as in brake operation, and in standstill. In practice, this is already found at a relatively low μ_a value for the torques occurring in rotation, but when searching for the largest torques occurring in standstill, in some cases, it is necessary to go to a very high μ_a value; if this value exceeds μ_a = 150 - 250, then that rotor slot number must still be qualified as practically not producing a synchronous torque in standstill.

Möller considered the most important result of the series of his measurements to be that he proved by means of measurements that a synchronous torque-like phenomenon occurs at all in an asynchronous machine. The complete theory was not yet known at that time, only the existence of the synchronous torque in standstill in the case of Z_1=Z_2 was known. Even in the case of "half q" rotor slot numbers, only a small amount of torque caused by the slot openings was thought of. The author of this article intends to explain Möller's approach by this situation namely why the "of half q" and "of third q" rotor slot numbers, which according to today's theory create a considerable torque in standstill, were not searched for during the measurements, the measurement was not carried out in this direction and during these measurements they didn't meet even accidentally either. On the other hand, the highest torques, measured specifically, obviously with a large laboratory apparatus and even given in mkg, agree with the calculations with a high degree of accuracy for the subject.

IV. Asynchronous Parasitic Torques

A. Calculation of Asynchronous Parasitic Torques

The formula is [1], [3]:

$$\frac{M_{\text{breakdown}\,\nu}}{M_{\text{breakdown}}}=\frac{X_m}{X_s}\frac{\xi_\nu^2}{\xi_1^2}\frac{\eta_{2\nu}^2}{\nu}\left(\frac{2}{\frac{s_{b\nu}}{s_\nu}+\frac{s_\nu}{s_{b\nu}}}\right) \tag{29}$$

Here, the calculation of $s_{b\nu}$ harmonic breakdown torque (for calculating not only the maximum but the characteristic of the harmonic asynchronous torque) required an additional estimate, namely the determination of the rotor resistance. This was done on the basis of the $s_b=R_2'/X_s$ relationship: $R_2'=0.04$ was determined by the author based on the $s_b=0.2$ measured from the diagram by a ruler, too, and the $X_s=0.2$ derived from $I_{sh}=5$ estimated. After these, $s_{b\nu}$ will be [3]:

$$s_{b\nu}=R_2'/X_{sm}\cdot\nu^2\cdot\eta_\nu^2 \tag{30}$$

It is advisable substituting into R_2'/X_m in p.u. relative unit.

Significant asynchronous parasitic torque can only be created by the slot harmonics of the stator. These are for $q_1=2$: ν= 11, 13, 23, 25..., for $q_1=3$: ν= 17, 19..., for $q_1=4$: ν= 23, 25..., however, it is clear based on Figure 1, that if $q'_2 < q_1$, the rotor does not respond to them. We also proved in [2], and it is clear just by looking Fig. 1, that by increasing the rotor slot number, more and more harmonics will create asynchronous parasitic torques and they will become larger and larger. Therefore, from Möller's measurements, for reasons of expediency and limit of length, only those with the smallest number of stator slots and the largest numbers of rotor slots will be followed by calculations and reported here. That is why the set of the stator slot number Z_1=24 was chosen for demonstration and the rotor slot numbers $Z_2 > Z_1$ only. In addition to the slot numbers shown, at most the slot numbers Z_1/Z_2 = 36/44 and 36/48 will give significant asynchronous parasitic torque.

B. Evaluation

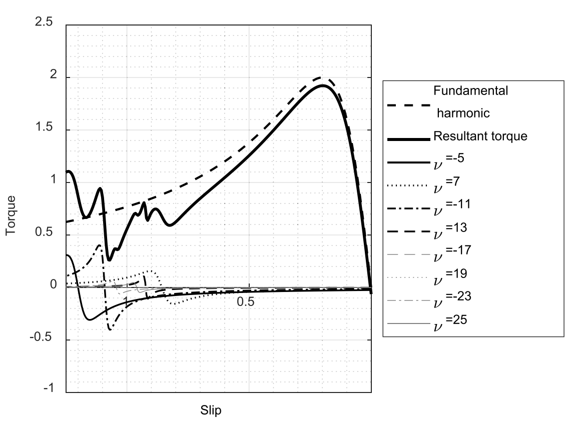


Fig. 5. Asynchronous parasitic torques in case of Z_1= 24, Z_2=32.

Although the measurements show a surprising shape, the figures still follow them quite faithfully. They follow them quantitatively, not only qualitatively. Therefore, it can be seen that the course of the characteristic curves of this torque is

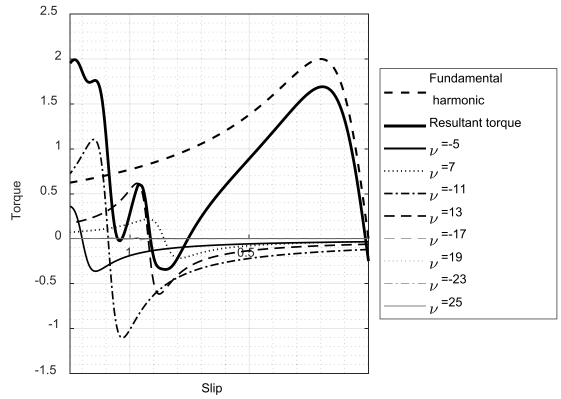


Fig. 6. Asynchronous parasitic torques in case of Z_1= 24, Z_2=44

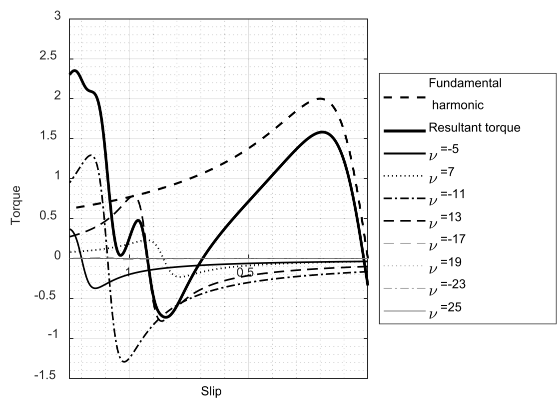


Fig. 7. Asynchronous parasitic torques in case of Z_1= 24, Z_2=48.

really determined by the asynchronous parasitic torques, and that formula (29) gives these torques correctly.

When deriving the formulas, the fundamental breakdown torque was used everywhere as a reference point, that is, all quantities are calculated in such a relative unit. However, Fig. 5 - 7 were calculated based on $M_{\text{breakdown}}$=1 but drawn based on $M_{\text{breakdown}}$=2 so that it is more similar to the characteristic curves usually seen and thus the order of magnitude can be perceived better.

In particular, by studying Fig. 7, it can be observed that such an extremely high rotor slot number reduces the breakdown torque of the machine in an extremely significant way, here precisely to 80% compared to the fundamental harmonic breakdown torque. Additionally, it also reduces the no-load speed, because the machine at s=0 exerts a negative torque i.e. it is in generator mode. Regarding Table VI, this means that the "breakdown torque" that can be read from Fig. 8 with a "ruler" is actually only approx. 80% of it, that is, when the synchronous torque occurring in standstill was read to be as 2.6 times in p.u., the real one is only approx. was 2.08 times in p.u. only.

V. Summation of Asynchronous and Synchronous Torques

Evaluating the measurement of Z_1/Z_2 = 36/28, it can be seen that the synchronous parasitic torque occurring in rotation in the motor range under the influence of the ν=13 harmonic and the asynchronous parasitic torque occurring under the influence of ν=7 exactly coincide. This does not represent a danger, because it exactly coincides with the zero transition of the latter, but it must have caused some difficulties in Möller's measurements.

When measured slot numbers Z_2 = 22, 44 (partially 32) at Z_1=24, the synchronous parasitic torque occurring in rotation caused by the ν=-11 harmonic and the asynchronous parasitic torque also occurring under the effect of ν=-11, however, coincide and reinforce each other. Since this happens during braking, it does not cause any problems in practice, but it apparently made the situation to Möller difficult.

However, it can be a problem if it all happens in the motor range. The condition for this is that the speed of the synchronous parasitic torque coincides with the breakdown slip of the asynchronous parasitic torque in its generator range created by ν=7, 13, 25 etc..

Let us investigate the matter:

$$1-s=-\frac{2p}{e\cdot Z_2}=-\frac{2p}{e\cdot 2pmq_2'}=-\frac{1}{e\cdot mq_2'}=\frac{1}{\nu} \tag{31}$$

Rearranged:

$$q_2'=-\frac{\nu}{e\cdot m} \tag{32}$$

where with $e<0$ provides the synchronous parasitic torque in the motoric range

The resulting rotor slot numbers are given in Table V.

TABLE V
COINCIDENCE OF SYNCHRONOUS AND ASYNCHRONOUS TORQUES

e	q_2'		
ν=	7	13	25
-1	2 1/3	4 1/3	8 1/3
-2	1 1/6	2 1/6	4 1/6
-3	7/9	1 4/9	2 7/9

From this point of view, these rotor slot numbers in themselves are not at all dangerous, because then the synchronous parasitic torque occurring in rotation (if any) coincides with the zero-transition of the asynchronous parasitic torque. On the other hand, if the smaller even number of slots closest to that is chosen, the check must be carried out, because this is where the generatoric "back" of the asynchronous parasitic torque is formed, close to its harmonic breakdown torque $-s_{b\nu}$. The result cannot be determined in advance, because according to [2], the asynchronous parasitic torque increases with the increase in the rotor slot number, and the synchronous parasitic torque decreases, and when even $s_{b\nu}$ should be included in the analysis for a preliminary formula or table the latter would make the entire preliminary analysis extremely voluminous and not transparent.

Anyway, the problem itself only occurs more seriously in machines with such extremely high rotor resistance. In the case of usual machines with a resistance of around 0.01, the asynchronous torque does not appear in a wide range, but only in the form of a peak torque, a dip. That is, the asynchronous and synchronous torque will not add up, and if, as usual, $Z_2<1.25\ Z_1$, then the asynchronous parasitic torque does not significantly affect the torque characteristic of the machine.

Pole-changing Motors

In conventional industrial motors, the rotor slot numbers are not "too far" from the stator slot numbers, so extreme cases do not occur often. However, with pole-changing motors, especially in the case of "wide range" pole number combinations where: $p_2/p_1 = N$, where $N \geq 2$ is a positive integer, very extreme q_2' values, which are sometimes even specifically to be avoided, occur by necessity.

Therefore all the problems involved in this study should be examined and analyzed with the involvement of [2] for both pole numbers. Especially for windings that produce even harmonics.

In light of this, it is really striking that in the literature dealing with pole-changing motors, the stator number of slots, as a consequence of the nature of the matter is examined in as much detail as possible but the rotor number of slots is not touched upon at all.

VI. ATTACHMENTS

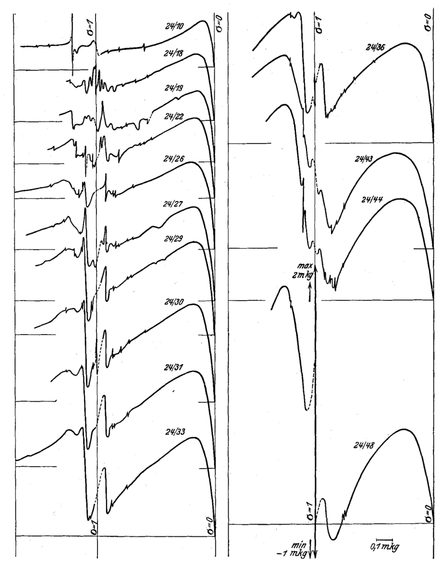


Fig. 8. Measurements with Z_1=24 stator slot number see Möller [5] Fig. 5.

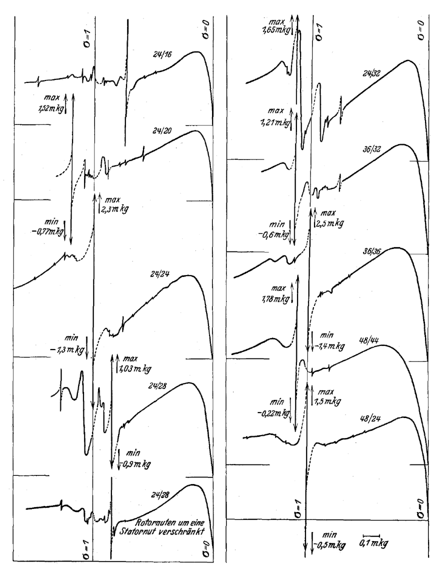


Fig. 9. Highest synchronous torques, the measured values are given in mkg see Möller [5] Fig. 8.

TABLE VI
CALCULATION OF THE MEASURED ARRANGEMENTS (EVEN ROTOR SLOT NUMBERS ONLY)

p=**2**, q_1=**2**, X_m/X_s= 15

Z_2	e	μ_a	ν_b	ξ_v	M/M_b	$\sum M_{s=1}$	1-s	Meas.
10	2	11	-11	**-0,966**	**-2,39**		-0,20	**0,68**
	-4	-19	19	0,259	**-0,37**		0,10	**0**
	-6	-29	-29	0,259	-0,24	≈0		
	-6	31	31	-0,259	-0,23			
	8	41	-41	-0,259	-0,17			
	-10	-49	49	**0,966**	**-0,54**		0,04	
	-12	-59	-59	**-0,966**	0,44	0,88		
	12	61	61	**-0,966**	**-0,43**	**0,69**		
	14	71	-71	**0,966**	**0,37**		-0,03	
	-16	-79	79	-0,259	0,09			
	-18	-89	-89	-0,259	0,08	≈0		
	18	91	91	0,259	0,08			
	-24	-119	-119	0,966	-0,22	-0,44		
	24	121	121	0,966	0,22			
16	-1	-7	7	-0,259	**1,09**		0,25	**0,86**
1/3	2	17	-17	-0,259	**-0,45**		-0,13	**0**
	-3	-23	-23	**0,966**	-1,24	-2,38		
	3	25	25	**0,966**	1,14	**-3,12**		0
	-4	-31	31	-0,259	**0,25**		0,06	
	5	41	-41	-0,259	**-0,19**		-0,05	
	-6	-47	-47	0,966	-,61	-1,19		
	6	49	49	0,966	0,58			
	-7	55	55	-0,259	0,14		0,04	
	8	65	-65	-0,259	-0,12			
18	-2	-17	-17	-0,259	0,45	≈0		
1/2	2	19	19	0,259	0,41			
	-4	-35	-35	**-0,966**	,0,82	1,60		
	4	37	37	**-0,966**	-0,78	**1,26**		**0,25**
	-6	-53	53	0,259	-0,15	≈0		
	6	55	55	-0,259	-0,14			
	-8	-71	-71	0,966	-0,41	-0,80		
	8	73	73	0,966	0,39			
20	1	11	-11	**-0,966**	**-2,64**		-0,20	2.50
2/3	-2	-19	19	0,259	**-0,41**		0,10	
	-3	-29	-29	0,259	-0,27	≈0		
	3	31	31	-0,259	-0,25			
	4	41	-41	-0,259	-0,19		-0,05	
	-5	-49	49	**0,966**	**-0,59**		0,04	
	-6	-59	-59	**-0,966**	0,49	0,97		
	6	61	61	**-0,966**	-0,48	**0,76**		
	7	71	-71	**0,966**	**0,41**		-0,03	
	-9	-89	-89	-0,259	0,09	≈0		
	9	91	-91	0,259	0,09			
	-12	-119	-119					
	12	121	121					
22	2	23	-23	**0,966**	**1,27**		-0,09	**0,2**
	-4	-43	43	0,259	**-0,18**		0,05	**0,06**
	-6	-65	-65	-0,259	0,12	≈0		
	6	67	67	0,259	0,12			
	8	89	-89	-0,259	**-0,09**		-0,02	**0,09**
	-10	-109	109	**-0,966**	**0,27**		0,02	
	-12	-131	-131	**-0,966**	0,22	0,44		
	12	133	133	**-0,966**	-0,22	0,35		
24	-1	-11	-11	**-0,966**	2,67	4,92		
	1	13	13	**-0,966**	-2,26	**3,86**		3,60
	-2	-23	-23	0,966	-1,27	-2,45		
	2	25	25	0,966	1,17			
	-3	-35	-35	-0,966	0,84	1,63		
	3	37	37	-0,966	-0,79			
26	-2	-25	25	**0,966**	**-1,18**		0,08	**0,17**
	4	53	-53	0,259	**0,15**		-0,04	

q_1=**3**

Z_2	e	μ_a	ν_b	ξ_v	M/M_b	$\sum M_{s=1}$	Meas.
10	2	11	-11	-0,177	**-0,44**		**0,12**
	-4	-19	19	0,960	**-1,38**		**0,12**
	-6	-29	-29	-0,177	0,17	≈0	
	-6	31	31	0,218	0,19		
	8	41	-41	0,218	0,15		
	-10	-49	49	0,218	-0,12		
	-12	-59	-59	0,218	-0,10	≈0	
	12	61	61	-0,177	-0,08		
	14	71	-71	**0,960**	**0,37**		
	-16	-79	79	-0,177	0,06		
	-18	-89	-89	**0,960**	-0,29	-0,58	
	18	91	91	**0,960**	0,29	**-0,76**	
	-24	-119		-0,177	0,04	≈0	
	24	121		0,218	0,05		
16	-1	-7	7	-0,177	**0,75**		**0,44**
	2	17	-17	**0,960**	**1,68**		**0,47**
	-3	-23	-23	0,218	-0,28	≈0	
	3	25	25	-0,177	-0,21		
	-4	-31	31	0,218	-0,21		
	5	41	-41	0,218	0,16		
	-6	-47	-47	-0,177	0,11	≈0	
	6	49	49	0,218	0,13		
	-7	-55	55	**0,960**	**-0,52**		
	8	65	-65	-0,177	-0,08		
18	-2	-17	-17	**0,960**	-1,69	-3,21	
	2	19	19	**0,960**	1,52	**-4,21**	**0,30**
	-4	-35	-35	0,960	-0,82	-1,60	
	4	37	37	0,960	0,78		
	-6	-53	53	0,960	-0,54	-1,07	
	6	55	55	0,960	0,52		
	-8	-71	-71	0,960	-0,41	-0,80	
	8	73	73	0,960	0,39		
20	1	11	-11	-0,177	**-0,49**		**0,48**
	-2	-19	19	**0,960**	**-1,53**		**0,05**
	-3	-29	-29	-0,177	0,18	≈0	
	3	31	31	0,218	0,21		
	4	41	-41	0,218	0,16		
	-5	-49	49	0,218	-0,13		
	-6	-59	-59	0,218	-0,11	≈0	
	6	61	61	-0,177	-0,09		
	7	71	-71	**0,960**	**0,41**		
	-9	-89	-89	**0,960**	-0,33	-0,65	
	9	91	-91	**0,960**	0,32	**-0,85**	
	-12	-119	-119				
	12	121	121				
22	2	23	-23	0,218	**0,29**		**0**
	-4	-43	43	-0,177	**0,13**		**0**
	-6	-65	-65	-0,177	0,08	≈0	
	6	67	67	0,218	0,10		
	8	89	-89	**0,960**	**0,33**		
	-10	-109	109	**0,960**	**-0,27**		
	-12	-131	-131	0,218	-0,05	≈0	
	12	133	133	-0,177	-0,04		
24	-1	-11	-11	-0,177	0,49	≈0	
	1	13	13	0,218	0,51		
	-2	-23	-23	0,218	-0,29	≈0	
	2	25	25	-0,177	-0,22		
	-3	-35	-35	**0,960**	-0,84	-1,63	
	3	37	37	**0,960**	0,79	**-2,14**	**0,56**
26	-2	-25	25	-0,177	**0,22**		**0,00**
	4	53	-53	**0,960**	**0,56**		**0,09**

q_1=**4**

Z_2	e	μ_a	ν_b	ξ_v	M/M_b	$\sum M_{s=1}$	Meas.
10	2	11	-11	-0,126	**-0,31**		**0,14**
	-4	-19	19	-0,205	**0,30**		**0,05**
	-6	-29	-29	-0,205	0,19	≈0	
	-6	31	31	0,158	0,14		
	8	41	-41	-0,158	-0,11		
	-10	-49	49	**0,958**	**-0,54**		
	-12	-59	-59	-0,126	0,06	≈0	
	12	61	61	0,126	0,06		
	14	71	-71	**-0,958**	-0,37		
	-16	-79	79	0,158	-0,05		
	-18	-89	-89	-0,158	0,05	≈0	
	18	91	91	0,205	0,06		
	-24	-119		**-0,958**	0,22	0,44	
	24	121		**-0,958**	-0,22		
16	-1	-7	7	-0,158	**0.67**		**0.36**
	2	17	-17	0.158	**0.28**		**0.15**
	-3	-23	-23	**-0.958**	1.24	2.38	
	3	25	25	**-0.958**	-1.14	**1.87**	
	-4	-31	31	0.158	-0.15		
	5	41	-41	-0.158	-0.11		
	-6	-47	-47	0.958	-0.61	-1.19	
	6	49	49	0.958	0.58		
	-7	-55	55	-0.158	0.09		
	8	65	-65	0.158	0.07		
18	-2	-17	-17	0,158	-0,28	≈0	
	2	19	19	-0,205	-0,33		
	-4	-35	-35	0,126	-0,11	≈0	
	4	37	37	-0,126	-0,10		
	-6	-53	53	0,205	-0,12	≈0	
	6	55	55	-0,158	-0,09		
	-8	-71	-71	**-0,958**	0,41	0,80	
	8	73	73	**-0,958**	-0,39	**0,63**	**0,14**
	0		0				
20	1	11	-11	-0,126	**-0,35**		**0,36**
	-2	-19	19	-0,205	**0,33**		
	-3	-29	-29	-0,205	0,21	≈0	
	3	31	31	0,158	0,15		
	4	41	-41	-0,158	-0,12		
	-5	-49	49	**0,958**	**-0,59**		
	-6	-59	-59	-0,126	0,06	≈0	
	6	61	61	0,126	0,06		
	7	71	-71	**-0,958**	**-0,41**		
	-9	-89	-89	-0,158	0,05	≈0	
	9	91	-91	0,205	0,07		
	-12	-119	-119	**-0,958**	0,24	0,48	
	12	121	121	**-0,958**	-0,24		
22	2	23	-23	**-0,958**	**-1,27**		**0,49**
	-4	-43	43	0,205	**-0,15**		
	-6	-65	-65	0,158	-0,07	≈0	
	6	67	67	-0,205	-0,09		
	8	89	-89	-0,158	-0,05		
	-10	-109	109	0,126	-0,04		
	-12	-131	-131	0,126	-0,03	≈0	
	12	133	133	-0,126	-0,03		
24	-1	-11	-11	-0,126	0,35	≈0	
	1	13	13	0,126	0,30		
	-2	-23	-23	**-0,958**	1,27	2,45	
	2	25	25	**-0,958**	-1,17	**1,92**	1,60
	-3	-35	-35	0,126	-0,11	≈0	
	3	37	37	-0,126	-0,10		
26	-2	-25	25	**-0,958**	**1,18**		**>,77**
	4	53	-53	0,205	**0,12**		

	-6	-77	-77	0,259	-0,10	≈0		
	6	79	79	-0,259	-0,10			
	-8	-103	103	-0,259	0,08			
	10	131	-131	**-0,966**	**-0,22**		-0,02	
	-12	-155	-155	**-0,966**	0,19	0,38		
	12	157	157	**-0,966**	-0,19	**0,30**		
	-18	-233	-233					
	18	235	235					
28	-1	-13	13	**-0,966**	**2,27**		0,14	**1,68**
1/3	2	29	-29	0,259	**0,27**		-0,07	**0,21**
	-3	-41	-41	-0,259	0,19	≈0		
	3	43	43	0,259	0,18			
	-4	-55	55	-0,259	0,14		0,04	
	5	71	-71	**0,966**	**0,42**		-0,03	
	-6	-83	-83	**-0,966**	0,36	0,70		
	6	85	85	**-0,966**	-0,35	**0,55**		
	-7	-97	97	**0,966**	**-0,30**		0,02	
	8	113	-113					
	-9	-125	-125					
	9	127	127					
	-12	-167	-167					
	12	169	169					
30	-2	-29	-29	0,259	-0,27	≈0		
1/2	2	31	31	-0,259	-0,26			
	-4	-59	-59	**-0,966**	0,50	0,99		
	4	61	61	**-0,966**	-0,48	**0,77**		**0,17**
	-6	-89	-89	-0,259	0,09	≈0		
	6	91	91	0,259	0,09			
	-8	-119	-119	**0,966**	-0,25	-0,49		
	8	121	121	**0,966**	0,24			
32	1	17	-17	-0,259	**-0,47**	**-1,36**	-0,13	**<1,35**
2-3	-2	-31	31	-0,259	**0,26**		0,06	**0,15**
	-3	-47	-47	**0,966**	-0,63	-1,23		
	3	49	49	**0,966**	0,60	**-1,62**		
	4	65	-65	-0,259	-0,12			
	-5	-79	79	-0,259	0,10			
	-6	-95	-95	**0,966**	-0,31	-0,62		
	6	97	97	**0,966**	0,31			
	7	113	-113	-0,259	-0,07			
	-8	-127	127	-0,259	0,06			
	-9	-143	-143	**0,966**	-0,21	-0,41		
	9	145	145	**0,966**	0,20			
36	-1	-17	-17	-0,259	0,47	≈0		
1/1	1	19	19	0,259	0,42			
	-2	-35	-35	**-0,966**	0,85	1,65		
	2	37	37	**-0,966**	-0,80	**1,30**		**0,17**
	-3	-53	-53	0,259	-0,15	≈0		
	3	55	55	-0,259	-0,14			
	-4	-71	-71	**0,966**	-0,42	-0,83		
	4	73	73	**0,966**	0,41			
44	1	23	-23	**0,966**	**1,30**		-0,09	**0,27**
2/3	-2	-43	43	0,259	**-0,19**		0,05	
	-3	-65	-65	-0,259	0,12	≈0		
	3	67	67	0,259	0,12			
	4	89	-89	-0,259	-0,09			
	-5	-109	109	**-0,966**	**0,27**		0,02	
	-6	-131	-131	**-0,966**	0,23	0,45		
	6	133	133	**-0,966**	-0,22	**0,35**		
	-9	-197	-197	0,259	-0,04	≈0		
	9	199	199	-0,259	-0,04			
48	-1	-23	-23	**0,966**	-1,30	-2,49		
	1	25	25	**0,966**	1,19	**-3,26**		**2,60**
	-2	-47	-47	**0,966**	-0,63	-1,24		
	2	49	49	**0,966**	0,61			
	-3	-71	-71	**0,966**	-0,42	-0,83		
	3	73	73	**0,966**	0,41			

	-6	-77	-77	0,218	-0,09	≈0	
	6	79	79	-0,177	-0,07		
	-8	-103	103	0,218	-0,06		
	10	131	-131	0,218	0,05		
	-12	-155	-155	-0,177	0,04	≈0	
	12	157	157	0,218	0,04		
	-18	-233	-233	**0,960**	-0,13	**-0,25**	
	18	235	235	**0,960**	0,13		
28	-1	-13	13	0,218	**-0,51**		**0,51**
	2	29	-29	-0,177	**-0,19**		**0,08**
	-3	-41	-41	0,218	-0,16	≈0	
	3	43	43	-0,177	-0,13		
	-4	-55	55	**0,960**	**-0,54**		
	5	71	-71	**0,960**	**0,42**		**0,07**
	-6	-83	-83	-0,177	0,07	≈0	
	6	85	85	0,218	0,08		
	-7	-97	97	-0,177	0,06		
	8	113	-113	0,218	0,06		
	-9	-125	-125	**0,960**	-0,24	-0,47	
	9	127	127	**0,960**	0,23		
	-12	-167	-167				
	12	169	169				
30	-2	-29	-29	-0,177	0,19	≈0	
	2	31	31	0,218	0,22		
	-4	-59	-59	0,218	-0,11	≈0	
	4	61	61	-0,177	-0,09		
	-6	-89	-89	**0,960**	-0,33	-0,66	
	6	91	91	**0,960**	0,32	**-0,86**	**0,16**
	-8	-119	-119	-0,177	0,05	≈0	
	8	121	121	0,218	0,06		
32	1	17	-17	**0,960**	**1,74**		**1,36**
	-2	-31	31	0,218	**-0,22**		
	-3	-47	-47	-0,177	0,12	≈0	
	3	49	49	0,218	0,14		
	4	65	-65	-0,177	-0,08		
	-5	-79	79	-0,177	0,07		
	-6	-95	-95	0,218	-0,07	≈0	
	6	97	97	-0,177	-0,06		
	7	113	-113	0,218	0,06		
	-8	-127	127	**0,960**	**-0,23**		
	-9	-143	-143	**0,960**	-0,21	-0,41	
	9	145	145	**0,960**	0,20		
36	-1	-17	-17	**0,960**	-1,75	-3,31	
	1	19	19	**0,960**	1,56	**-4,34**	**2,79**
	-2	-35	-35	0,960	-0,85	-1,65	
	2	37	37	0,960	0,80		
	-3	-53	-53	0,960	-0,56	-1,10	
	3	55	55	0,960	0,54		
	-4	-71	-71	0,960	-0,42	-0,83	
	4	73	73	0,960	0,41		
44	1	23	-23	-0,218	**0,29**		**0,26**
	-2	-43	43	-0,177	**0,13**		
	-3	-65	-65	-0,177	0,08	≈0	
	3	67	67	0,218	0,10		
	4	89	-89	**0,960**	**0,33**		
	-5	-109	109	**0,960**	**-0,27**		
	-6	-131	-131	0,218	-0,05	≈0	
	6	133	133	-0,177	-0,04		
	-9	-197	-197	**0,960**	-0,15	-0,30	
	9	199	199	**0,960**	0,15		
48	-1	-23	-23	0,218	-0,29	≈0	
	1	25	25	-0,177	-0,22		
	-2	-47	-47	-0,177	0,12	≈0	
	2	49	49	0,218	0,14		
	-3	-71	-71	**0,960**	-0,42	-0,83	
	3	73	73	**0,960**	0,41	**-1,09**	**0,21**

	-6	-77	-77	-0,205	0,08	≈0	
	6	79	79	0,158	0,06		
	-8	-103	103	-0,158	0,05		
	10	131	-131	0,126	0,03		
	-12	-155	-155	-0,126	0,02	≈0	
	12	157	157	0,126	0,02		
	-18	-233	-233	-0,158	0,02	≈0	
	18	235	235	0,205	0,03		
28	-1	-13	13	0,126	**-0,30**		**0,26**
	2	29	-29	-0,205	**-0,22**		**0,09**
	-3	-41	-41	-0,158	0,12	≈0	
	3	43	43	0,205	0,15		
	-4	-55	55	-0,158	0,09		
	5	71	-71	**-0,958**	**-0,42**		
	-6	-83	-83	0,126	-0,05	≈0	
	6	85	85	-0,126	-0,05		
	-7	-97	97	**0,958**	**-0,30**		
	8	113	-113	0,158	0,04		
	-9	-125	-125	-0,205	0,05	≈0	
	9	127	127	0,158	0,04		
	-12	-167	-167	**-0.958**	0,18	0,35	
	12	169	169	**-0,958**	-0,17		
30	-2	-29	-29	-0,205	0,22	≈0	
	2	31	31	0,158	0,16		
	-4	-59	-59	-0,126	0,07	≈0	
	4	61	61	0,126	0,06		
	-6	-89	-89	-0,158	0,05	≈0	
	6	91	91	0,205	0,07		
	-8	-119	-119	**-0,958**	0,25	0,49	
	8	121	121	**-0,958**	-0,24	**0,39**	**0,24**
32	1	17	-17	0,158	**0,29**		**0,35**
	-2	-31	31	0,158	**-0,16**		**0,09**
	-3	-47	-47	**0,958**	-0,63	-1,23	
	3	49	49	**0,958**	0,60	**-1,62**	
	4	65	-65	0,158	0,07		
	-5	-79	79	0,158	-0,06		
	-6	-95	-95	**0,958**	-0,31	-0,62	
	6	97	97	**0,958**	0,31		
	7	113	-113	0,158	0,04		
	-8	-127	127	0,158	-0,04		
	-9	-143	-143	**0,958**	-0,21	-0,41	
	9	145	145	**0,958**	0,20		
36	-1	-17	-17	0,158	-0,29	≈0	
	1	19	19	-0,205	-0,34		
	-2	-35	-35	0,126	-0,11	≈0	
	2	37	37	-0,126	-0,11		
	-3	-53	-53	0,205	-0,12	≈0	
	3	55	55	-0,158	-0,09		
	-4	-71	-71	**-0,958**	0,42	0,83	
	4	73	73	**-0,958**	-0,41	**0,65**	**0,33**
44	1	23	-23	**-0,958**	**-1,30**		**1,36**
	-2	-43	43	0,205	**-0,15**		
	-3	-65	-65	0,158	-0,08	≈0	
	3	67	67	-0,205	-0,10		
	4	89	-89	-0,158	-0,06		
	-5	-109	109	0,126	-0,04		
	-6	-131	-131	0,126	-0,03	≈0	
	6	133	133	-0,126	-0,03		
	-9	-197	-197	0,205	-0,03	≈0	
	9	199	199	-0,158	-0,02		
48	-1	-23	-23	**-0,958**	1,30	2,49	
	1	25	25	**-0,958**	-1,19	**1,95**	**2,43**
	-2	-47	-47	**0,958**	-0,63	-1,24	
	2	49	49	**0,958**	0,61		
	-3	-71	-71	**-0,958**	0,42	0,83	
	3	73	73	**-0,958**	-0,41		

Remarks:

The measured values given by Möller in mkg are enclosed in a thick frame.

24/32 slots: torque in rotation: at this slot number, the harmonics generate exceptionally significant torques, they had to be added to the torque generated by the fundamental harmonic v_a=1.

Details: v_a=-5 - μ_a=-v_b=11; v_a=7 - μ_a=-v_b=23; v_a=-11 - μ_a=-v_b=5 pairs are generating additional torques because μ_a=-v_b values in the denominator are very low; there is one slotharmonic, either ξ_a or ξ_b, in each pair; signs are identical to those with v_a=1 so they are added. However, Δ_v shall be taken into the calculation for torques due to v_b=-5 and v_b=-11. Beside the value due to v_a=1, in this case only, the resulting value of 1, 36 was also inserted into the table.

24/48 slots: torques in standstill: at this slot number, the harmonics generate exceptionally significant torques, they increase even further the torque generated by the fundamental harmonic v_a=1 which is given in the table; due to the vectorial position of the torques of the harmonics in phase, however, the increasing effect remains moderate.

VII. Closing Considerations

Consider [4] Table III A and IV. Although Table IIIA only covers 4 poles, the conclusions are general.

The most significant oscillating torques occur at the following rotor slot numbers:

Z_2=2pmq_2, Z_2=2pm(q_2+1/3) and Z_2=2pm(q_2+2/3), as well as Z_2=2pm(q_2+1/2).

Then, of course, significant radial magnetic forces with order number r=0 also arise, primarily on the former 3 slot numbers.

Therefore, the resonance frequency corresponding to r=0 must be checked in every way.

Slot numbers adjacent to the same 3 slot numbers create the most dangerous order number, r=1. These are: Z_2=2pmq_2±1, Z_2=2pm(q_2+1/3)±1, Z_2=2pm(q_2+2/3)±1.

These are odd slot numbers. With a pole number of 2p=4, these completely cover the set of odd numbers, therefore (due to the nature of the phenomenon) these are all forbidden slot numbers, regardless that ξ_b=ξ_1 or ξ_b< ξ_1. If 2p≥6, there are odd slot numbers that do not create r=1 order numbers.

Thinking further about the analysis preceding Table III A of [4], it is clear that the largest oscillating torques in operation corresponding to e=±1 are obtained when Z_2/p = an even integer; a significant torque will be created with e=±2 if Z_2/p = an odd integer. The slot numbers Z_2=2pm(q_2+1/2) belong to the latter.

The slot numbers belonging to the band Z_2=2pm(q_1±1/2) should definitely not be used due to ξ_b=ξ_1 and e=±1.

It is therefore worth looking for the appropriate rotor slot number primarily among the slot numbers not listed above. There is only a small possibility for this in 4 poles, with higher pole numbers the possibilities are wider.

Rotor Slot Numbers for 4 Poles

Therefore, now only 2p=4 will be briefly examined, at the same time presenting the practical application of the approach of this series of articles.

2pm=12 different possibilities are available.

q_2'=q_2+1/12, +2/12, …+12/12.

However, those resulting in odd slot numbers further q_2'= integer and q_2'=q_2+1/2 must be excluded. 4 different possibilities remain.

The table contains the rotor slot numbers remaining in practice at all. For the examination, the "1/6q" table corresponding to Table IV must also be created. Results:

Table VII
Overview of Relative Rotor Slot Numbers for 2P= 4 Poles

Relative rotor slot number q_2'	Synchronous parasitic torque	Oscillating torque in operation	Order number of radial forces	Evaluation	
					Remark exemption
q_1-4/3= q_1-2+**2/3**	brake	high	r=0, r=4	proper	q_1=3,4 forbidden q_1=2 n.a.
q_1-7/6= q_1-2+**5/6**	brake	low	r=2	suggested	q_1=2 n.a.
q_1-5/6= q_1-1+**1/6**	motoric	low	r=2	better than q_1-2/3	except q_1=2
q_1-2/3= q_1-1+**1/3**	motoric	high	r=0, r=4	forbidden	/
q_1+**2/3***	brake	high	r=0, r=4	proper	except q_1=2
q_1+**5/6**	brake	low	r=2	suggested	except q_1=3

*Proposed by Richter, Jordan

"1/6 q" slot numbers generally produce less than half the synchronous/oscillating torque of adjacent "1/3 q" slot numbers, due to more than twice the μ_a values. This also means correspondingly smaller radial magnetic forces see chapter IV.B of [4]. In this context, it is worth considering Fig. 4. and Fig. 5. of [2].

As the conclusion of this series of articles, the author recommends primarily considering the "1/6 q" slot numbers for 2p=4. In inverter operation, when avoiding the synchronous torque occurring in the motor range is not a consideration, the smallest oscillating torque and at the same time the smallest radial magnetic forces are definitely given by the "5/6 q" rotor slot numbers.

We also note that the findings in Table VII are independent of the number of poles that means are valid for all pole numbers. That is the point of the approach of this series of articles. However, there are more and different possibilities for the rest of the pole numbers.

VIII. Conclusion

[1], [2], [3], [4] and this article are built on each other and starting from the derivation of the new formulas to the proof with measurements, they form a complete system. The contribution of this article shall therefore be analyzed and determined in light of this.

The purpose of this article was basically to somehow prove the correctness of our previously developed new formulas. When developing our formulas and then studying [7] and [8], it was found that such a series of measurements exists that are generally accepted in the standard literature; so we searched for the original article and worked only on that basis. It was considered that a comparison with measurements would be stronger evidence than any other, more advanced, more detailed calculation.

In [4], it was proven that the formula for the synchronous parasitic torque and that for the radial magnetic forces can be transferred to each other. If the former can be proven by measurement, then the latter [1] (see equ. (24), (26)) is also proven. Therefore, it was focused now on the synchronous parasitic torque so they were calculated for all measured stator and even rotor slot numbers for comparison.

In the case of the highest synchronous torques, remarkably good agreement was found. The possible reasons for larger differences were pointed out. All the calculations were carried out intentionally in such a way that it can be directly and easily checked by the Reader.

The correct summation of synchronous parasitic torques occurring at the same speed is of fundamental importance. The method for this is well developed in the literature for torques in rotation, but is not complete in the case of torques in standstill. The summation of the latter was made by us mathematically complete. It was briefly pointed out that a finite number of harmonic torque components will be formed only. Nevertheless, if we first calculate with an infinite number of torque components, since it is much simpler, it is also considered a good approximation.

It was proven that the nature of the synchronous parasitic torques, that is, that they will occur in the motor range, in braking mode and/or only in standstill depends solely on the denominator of the number of rotor slots per pole and per phase, expressed in fractional form. It was revealed again that it is better to express the rotor slot number as a relative value rather than by itself, and thus to treat the issue at a completely general level.

A theorem was formulated that if q_2'=integer or if the denominator r is: 2, 4, 5, 7, 8 …, synchronous parasitic torque occurs only in standstill. For the rest, that means r= integer·3, synchronous parasitic torque occurs always in this way: first in the motoric range, then in the brake range or vice versa and finally at standstill as the harmonic order number increases; then this pattern is repeated. For any number of phases: if r= integer·m then synchronous torques occurs in standstill and rotation, for any further r only in standstill.

It was shown that even in the case of an inverter-fed motor, it is worthwhile to avoid the forbidden rotor slot numbers, not only to avoid dangerous radial magnetic forces, but also to avoid high oscillating torques during operation.

When developing the formulas before, the method of calculating the parasitic torques in relation to the breakdown torque was used. That is why the formulas became so conspicuously simple. The expediency of this approach has now been specially proven by the fact that our calculations thus gave a much more accurate result than made by others even if, as in this case, the motor data except pole number and slot numbers are not known for us.

The possible effect of some phenomena not taken into account by the formulas were presented briefly. It was shown that the effect of the slot opening must be taken into account if the synchronous torque occurs in rotation. It was already given a method for incorporating it in [4] before when calculating the radial magnetic forces. Using this as a basis, it was now provided a method and formula for the case of synchronous parasitic torques as well. The slot opening has an influence on those harmonic torques only where the harmonic is a slot harmonic coming either from the stator slot number or rotor slot number or both.

The measurements were also used to verify the formula for asynchronous parasitic torques, with very good results.

The attention was drawn to the possible coincidence of asynchronous and synchronous parasitic torques especially in motoric range, which could cause unpleasant surprises. It was also specified the rotor slot numbers that are dangerous from this point of view.

The analyzes carried out on a very theoretic way in [2] before was also confirmed on one hand by Fig. 5 - Fig. 7 that the asynchronous parasitic torques increase with the increase of the number of rotor slots, and on the other hand by Table VI, that the synchronous parasitic torques increase sharply with the decrease of the number of rotor slots.

From the results, the author concluded that the formulas are accurate enough to be used in daily design practice.

For pole-changing motors, the principles and formulas used in this series of paper must be applied to both pole numbers. Especially for windings that produce even harmonics.

In this series of articles, it was published knowledge and calculation exercises at the level of daily design guide, which enable all designers to determine the machine properties accurately, not only qualitatively, with the help of the new formulas for asynchronous and synchronous parasitic torques as well as for the radial magnetic forces for any rotor slot number and enable him to safely select the rotor slot number applicable for his machine and/or drive system.

This brings our series to an end.

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G. Kovács was born on January 5th 1949, in Budapest, Hungary. He received his Master of Engineering Degree at the Hungarian Technical University Budapest, Hungary in 1972, in the area of Heavy Current Industry, specialized for Electrical Machines.

After graduation, he joined the Ganz Electric Works, Budapest, Hungary. He worked at the Department for Electromagnetic Calculation of Induction Motors during his entire career in different positions, including Head of the Department. Currently he is a Senior Development Manager.

Mr. Kovács was participant of conferences incl. ICEM 82 and 84, IEEE in 90 with his invention "Squirrel cage induction motor with starting disc" produced in the 2-pole MW power range by his company and other topics regarding electromagnetic calculation of high power induction motors. His achievements, nowadays, include the invention of a pole changing winding 3//Y / 3//Y (especially for wide ratio). At present he deals with harmonics of induction machines and - using his formulas - analytic analysis of parasitic torques and radial magnetic forces (ICEM 2020, ICEM 2022).