Harmonics in the Squirrel Cage Induction Motor, Analytic Calculation Part II: Synchronous Parasitic Torques, Radial Magnetic Forces

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Abstract— **The magnetic field generated in the air gap of the cage asynchronous machine and the harmonics of the magnetomotive forces creating that magnetic field, as well as the synchronous parasitic torques, radial magnetic forces have been discussed in great detail in the literature, but always separately, for a long time. However, systematization of the phenomenon still awaits. Therefore, it is worth summarizing the completeness of the phenomena in a single study – with a new approach at the same time - in order to reveal the relationships between them. The role of rotor slot number is emphasized much more than before. New formulas derived for both synchronous torques and radial magnetic forces are used for further investigation. It will be shown that both phenomena in subject must be treated together. Formulas will be provided to take into account attenuation. Design guide will be provided to avoid dangerous rotor slot numbers. It will be shown that the generation of synchronous torques and radial magnetic forces do not depend – in this new approach - on the slot combination, but on the rotor slot number itself.**

Index Terms— **Squirrel cage induction motor, Synchronous parasitic torques, Radial magnetic forces, Winding harmonics.**

I. Introduction

The aim of the article is to summarize and mainly systematize the phenomena given in the title in one single study. Since subject phenomena can be traced back to a single starting point, namely the interaction of stator harmonics and rotor harmonics, it is reasonable to discuss the problem in a unified manner.

However, during the study of the previous works in the literature, it became clear that basic formulas are missing, basic relationships are not explored, and important effects are not taken into account. The goal was therefore to fill the gap, to supplement the missing parts and to include the entire investigation in a unified framework.

The article is a continuation of the Author's previous works [1], [2]. Those articles were developed mainly based on [3] as a very basic work. There are, of course, many valuable works that should also be used, and have been used here, when dealing with such a broad topic [4]-[8], the other countless valuable works cannot be listed here for reasons of length.

Jordan and Oberretl have done fundamental work throughout of life in developing the theme, from which we quote here only examples best fitting the topic of this article [9], [10] and [11]. Möller [12] made measurements in the early days that still serve as a guide today. Ignored phenomena and today's general practical calculations are presented through a few examples [13]-[17]; the method and approach of these latter articles are fundamentally different from the present one.

In the following study, only the winding space harmonics will be dealt with, and only the impact on the subject phenomena will be analyzed, with the usual assumptions. Constant voltage and frequency supply will be assumed. A machine that is symmetrical in all aspects will be taken into consideration, with non-skewed rotor slots; very small slot width, without saturation; phenomena resulting from possible parallel and/or delta connection of the stator, so-called secondary armature reaction, will not be included. The goal is to put the fundamental findings, which are now published, to the point. And also those, which have been known before, but will now be put in a different light. It is intended not to distract attention by what might be important in itself but can only be called a question of detail now. All these can then be taken into account later, in a way already found in the literature.

II. Symbols

X_m	Magnetizing reactance
ξ_1, ξ_ν	Winding factor of fundamental wave, harmonic wave
$\eta_{2\nu}^2$	Jordan's coupling factor
Δ	Attenuation factor
s, s_ν	Slip of rotor to fundamental harmonic of stator, to harmonic ν of stator
ν, μ, ν', μ'	Designation of stator harmonics, rotor harmonics
a, b	Designation of harmonics in interaction
g_1, g_2	Different integers; e_1 integer
p	Number of pole pairs
Z_1, Z_2	Stator/rotor slot number
$I_1, I_{2\nu}$	Stator/rotor current
m	Number of phases
q_1, q_2'	Stator/rotor slot number per pole per phase

III. Synchronous Parasitic Torques

A. Calculation of Synchronous Parasitic Torques

The issue was discussed in detail in [1], but without

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attenuation. The essence and the final result will be repeated, and then the effect of attenuation will be considered.

The ν_a harmonic of the stator produced by the stator current I_1 creates a current layer wave of rotor of order number μ_a, which in turn creates a flux density wave b_{μ_a}. *Independently* from this, there is another ν_b order current layer density wave a_{ν_b} produced by the stator current I_1.

The formulas [3],

$$a_{\nu_b} = \frac{\sqrt{2}m}{p\tau_p} N_1 \xi_{\nu_b} I_1 \cdot \cos(\omega t - \frac{\nu_b \pi}{\tau_p} x_1) \quad (1)$$

where x_1 a point on the periphery relative to the point $x_1 = 0$.

$$b_{\mu_a} = \frac{\sqrt{2}m\mu_0}{\pi\delta'} \frac{1}{p\mu_a} N_1 \xi_{\nu_a} I_1 \eta_{2\nu_a}^2 \cdot \sin((1+(\mu_a - \nu_a)\cdot(1-s))\omega t - \frac{\mu_a \pi}{\tau_p} x_1 + \frac{(\mu_a - \nu_a)\pi}{\tau_p} x_2' - \rho_{\nu_a}) \quad (2)$$

x_2' a point on the periphery of the rotor relative to the point x_2'=0 and its multiplier takes into account that the place x_2'=0 doesn't necessarily coincide with the place $x_1 = 0$.

The torque will be the circumferential integral of product of a_{ν_b} and b_{μ_a}, multiplied by the radius of the rotor expressed (in an expedient manner) in the form $p\tau_p/\pi$ [3]:

$$m_{\mu_a \nu_b} = \frac{p\tau_p l_i}{\pi} \int_0^{2p\tau} a_{\nu_b} b_{\mu_a} dx \quad (3)$$

μ_0 permeability of vacuum
δ' equivalent air-gap
N_1 stator turn number
τ_p circumferential dimension of a pole
l_i equivalent length of stator iron core

Synchronous torque relative to breakdown torque of the machine [1]

$$\frac{M_{synchronous}}{M_{breakdown}} = \frac{X_m}{X_s} \cdot 2\sum \frac{\xi_{1\nu_a}\xi_{1\nu_b}}{\mu_a} \eta_{2\nu_a}^2 \frac{1}{\xi_1^2} \quad (4)$$

Here too, as in the calculation of the asynchronous parasitic torque, a completely new approach compared to the literature was used, by relating the torque not to the starting torque, as is usual, but to the breakdown torque of the machine. A very simple formula was achieved in this way as before for the asynchronous torque. This fact proves that the parasitic torques in accordance with the internal essence of physic of the machine are much closer to each other through the circular diagram and through its diameter, unlike through a slip scale marking the point s=1 somewhere on the circle diagram, even depending on R_2.

In (4), the first quotient represents the machine constants, and the second one provides a proportionality factor independent of the machine constants. For conventional machines, machine constants move in a relatively narrow range: $X_m \approx 3$-4, $X_s \approx 0.2$.

The full formula [3]:

$$M = M_{\max} \sin(e\frac{Z_2}{p}(1-s)\omega t + e\frac{Z_2}{p}\frac{\pi}{\tau} x_2' - \rho_{\nu_a}) \quad (5a)$$

$$M = M_{\max} \sin((2 + e\frac{Z_2}{p}(1-s))\omega t + e\frac{Z_2}{p}\frac{\pi}{\tau} x_2' - \rho_{\nu_a}) \quad (5b)$$

where $\rho_{\nu a}$ is the angle between stator current and rotor harmonic current.

Now it will be turned to consideration of attenuation. Synchronous torque occurs either at a standstill or during running up, on the slip of the motor. If in standstill, then s=1 substituted

$$s_\nu = 1 - \nu_b(1-s) = 1 \quad (6)$$

If during run-up, substituted $s=(\nu_b - 1)/(\nu_b + 1)$

$$s_\nu = 1 - \nu_b\left(1 - \frac{\nu_b - 1}{\nu_b + 1}\right) = ... = 1 - \frac{2\nu_b}{\nu_b + 1} \approx -1 \quad (7)$$

In other words, whether it is in standstill or during start-up, the synchronous torque, based on the symmetry of the circle of $I_{2\nu}$, in both cases the attenuation may be calculated at s_ν=1

Substituted in the formula of $I_{2\nu}$

$$I_{2\nu} = (-\frac{\eta_{2\nu}^2}{s_{b\nu}^2 + 1} - j\frac{s_{b\nu}\cdot\eta_{2\nu}^2}{s_{b\nu}^2 + 1}) I_1 \quad (8)$$

Vector $I_1 + I_{2\nu}$ will be taken relative the vector I_1 which was taken as real

$$\frac{I_1 + I_{2\nu}}{I_1} = 1 + (-\frac{\eta_{2\nu}^2}{s_{b\nu}^2 + 1} - j\frac{s_{b\nu}\cdot\eta_{2\nu}^2}{s_{b\nu}^2 + 1}) = (1 - \frac{\eta_{2\nu}^2}{s_{b\nu}^2 + 1}) - j\frac{s_{b\nu}\cdot\eta_{2\nu}^2}{s_{b\nu}^2 + 1} \quad (9)$$

Absolute value of that expression will provide the Δ_{ν_b} attenuation factor.

The formula containing attenuation is:

$$\frac{M_{szinkron}}{M_{billenő}} = \frac{X_m}{X_s} \cdot 2\sum \Delta_{\nu_b} \frac{\xi_{1\nu_a}\xi_{1\nu_b}}{\mu_a} \eta_{2\nu_a}^2 \frac{1}{\xi_1^2} \quad (10)$$

for those torques where it happens to be $\nu_b < 2mq_1$. This takes place primarily (but not exclusively) in the case of $Z_2 < Z_1$; for higher ν_b : Δ_ν=1.

B. Investigation of Slot Numbers, Not Allowed Slot Numbers

As is known, synchronous torque occurs when the harmonics of the stator field and the harmonics created in the rotor by the stator harmonics have the same order number. This does not include the rotor fields that are generated by the individual stator harmonic fields as "fundamental harmonic fields" in the rotor as "fundamental harmonics". They appear in the formula with e=0, but they produce the asynchronous parasitic torque. In other words, it is customary to say that the order number of a stator field should be equal to the order number of a rotor field created by another stator field.

The usual three-phase stator field always contains the same harmonics. These are: 1, -5, 7, -11, 13, -17, 19, etc. The harmonics created by the rotor, on the other hand, depend on the number of slots and the number of poles of the rotor. In other words, for a certain number of poles, it depends solely on the rotor slot number whether the two sets contain identical in order numbers, that is, whether synchronous torque is produced at all.

When designing asynchronous machines, it is common in public approach to look for the right rotor for the ready stator, that is, to look for number of slot combinations. However, when examining the synchronous torques, just the opposite way shall be definitely recommended: it shall be started from the rotor, that is looking for a rotor slot number, regardless of the stator slot number, which is able to create at least one of the previously defined order numbers that are the same for all stators.

If one is found, the goal is to avoid this, since a synchronous torque is generated by installing such a rotor in any stator. The slot number of the stator affects the synchronous torques only to the extent that it "rearranges" the stator slot harmonics from one order number pair to another, thus rearranging the relationship of the resulting synchronous torques to each other.

A number of measurements were made by Möller in 1930 [12], with different stator and rotor slot number combinations; these were adopted unchanged by [4] pp. 159-162 in 1977 and by [5] Chapter 10.9.7. in 2010. Although the measurements always gave the same nature of torque-speed characteristic curve for the same number of rotor slots, regardless of the stator slots, yet they did not recognize that the result does not depend on the combination, but solely on the rotor slot number.

Consider Annex Table III. In this, the rotor slot numbers for p=2 are given, with which synchronous torque can be generated at all with lowest $\underline{e}$ coefficients. The rotor harmonics created by the fundamental harmonic and v=-5 harmonic of the stator are considered.

The stator order numbers are arranged in two columns: the first one contains the real ν_b order numbers, and the second one contains their pairs with opposite sign. The rotor slot numbers are also arranged in several columns, depending on the value of $\underline{e}$. Here, the negative value of $\underline{e}$ includes the torques that occur in standstill and the motor range, and the positive value of $\underline{e}$ includes those that occur in standstill and in the braking range. The meaning of the row is as follows: slot numbers in the raw of a real order number create synchronous torque at standstill, and those in a row with the opposite sign create a torque in rotation. Therefore, those slot numbers that belong to the positive value of $\underline{e}$ and are in the row with the opposite sign of v do create a synchronous torque in the rotating state, but not in the motor range, but in the brake range: these are put in italics. They can, therefore, be used in machines operating only as motors, but one must be aware that in this case a radial magnetic force with order number r=0 (see later) and an oscillating torque are also generated.

It shall be inserted here a short physical explanation: if ν_b rotates in a positive direction compared to the stator and μ_a in a negative direction that is, "backward" with respect to the direction of rotation of the rotor, there is always a rotor speed where the two fields are at rest relative to each other. If ν_b rotates in the negative direction and μ_a in the positive direction, it is still found the same, but in braking mode. If, on the other hand, both fields rotate in the same direction but obviously at different speeds, such a speed of rotor is not found, they are at rest relative to each other only in the case of a standstill; but if the rotor starts to move in either direction, the two fields move away from each other.

In this way, there is a clear picture of each rotor slot number for each pole pair. Once again, it shall be emphasized that any rotor slot number that appears anywhere in this table will produce synchronous torque no matter what stator it is installed in, the others will not or very low only. Multiple appearance of a certain slot number means more than 1 synchronous torque dip caused by that rotor slot number.

The stator slot numbers are shown for reference purposes only, so that it is easy to see which the slot harmonics of that particular stator are. The rotor slot numbers that appear in the same row with these create a "fatally" high synchronous torque if installed in such stator.

It should also be noted that the rotor slot numbers appearing in the same row create a synchronous torque of the same magnitude, because it only depends on the value of μ. The number of slots appearing in the same column create even if – due to actual stator slot harmonics, not monotonically but decreasing synchronous torque, see [2].

It is also noted that e.g. if q_1=3, then not only waves ν_b= -17 and ν_b=19 are slot harmonics, but due to periodicity of the winding factor, also their multiples: ν_b=-35, 37, -53, 55 etc. Therefore, the slot numbers appearing in the same row as these further stator harmonics should also be avoided. Of course, the synchronous torques created by these further slot numbers are reduced (=are not as "fatal"), namely in inverse proportion to the value of $\nu_b = \pm \mu_a$.

In the case of very low rotor slot numbers, due to very low $\nu_b=\pm\mu_a$ values, a very high synchronous torque is formed with the lowest harmonics of the stator, the 5th and 7th harmonics even if it is installed in a stator with q_1=2. In these cases, chording plays a decisive role in the magnitude of the synchronous torques otherwise not.

If the dangerous slot numbers cannot be avoided, as a rule of thumb $\mu_a/(\xi_a\xi_b\eta_{2va}^2)\geq 120$ value must be adhered to, and then a synchronous torque of around $M\leq 0.5M_{rated}$ is expected.

C. Oscillating Torques

Synchronous parasitic torques must be imagined as synchronous machines, which are mechanically connected to the main machine.

If the torque of the asynchronous machine, fundamental harmonic, is not sufficient to overcome the synchronous torque, they "take control of the machine" and do not allow it to start up.

However, if the machine is able to start up, these synchronous machines continue to operate as out-of-synchronism machines and naturally cause an oscillating torque of the same magnitude as the synchronous torque.

The frequency of the oscillating torques

$$e\cdot Z_2/p\cdot(1-s)\cdot 50\ Hz = e\cdot 2mq_2'\cdot(1-s)\cdot 50\ \text{Hz} \quad (11a)$$

or

$$(2+e\cdot Z_2/p)\cdot(1-s)\cdot 50Hz = (2+ e\cdot 2mq_2')\cdot(1-s)\cdot 50\text{Hz} \quad (11b)$$

The term in parentheses means a frequency of around 600-1200 Hz for rotor slot numbers around $q_2'\approx 3$. The "oscillating synchronous torque" reaches this frequency linearly during start-up.

If the drive system has a torsion resonance frequency in this frequency band, the drive must pass through this resonance. For large machines, the torsion natural frequency is quite small, so the resonance occurs rather at the beginning of the start-up. Whether the resonance is really formed can only be investigated by solving the dynamic flux and motion equations. In the motion equation, (5a) or (5b) must be simply added to the motor torque.

The synchronous torques were calculated with the starting current. If it is taken around 5times, then during operation they are reduced to $1/5^2 = 1/25$. These will be small even in the case of very high synchronous torques. Their frequency typically an order of 600-1200 Hz in operation is too high to create electromagnetic resonance. The latter is typically around 20 Hz, see [18] Fig. 2.21.

IV. Radial Magnetic Forces

A. Calculation of Radial Magnetic Forces

The issue has been discussed in detail in [1], but without attenuation. The essence and the final result will be repeated, and then the effect of attenuation will be considered.

As the relations along the entire periphery of the air-gap shall be surveyed here for this phenomenon rather than a single pole, the orders put down for synchronous torque calculation shall be multiplied by p:

$$\begin{aligned} v_a' &= 6g_1 p + p\cdot\mu_a' = e\,Z_2 + v_a' = e\cdot 2mpq_2' + v_a' \\ v_b' &= 6g_2 p + p \end{aligned} \tag{12}$$

The radial magnetic force wave will be created as a product of the following flux density waves:

The v'_b order flux density wave produced by the stator current I_1

$$\begin{aligned} b_{v'_b} &= \frac{\sqrt{2}m}{\pi}\frac{\mu_0}{\delta'}\frac{1}{pv_b}N_1\xi_{v_b}I_1\cdot\sin(\omega t-\alpha_a) \\ &= B_{v'_b}\cdot\sin(\omega t-\frac{v'_b\pi x_1}{p\tau_p}) \end{aligned} \tag{13}$$

and the same μ'_a flux density wave used to calculate synchronous torque produced by the rotor current

$$b_{\mu_a} = \frac{\sqrt{2}m\mu_0}{\pi\delta'}\frac{1}{p\mu_a}N_1\xi_{v_a}I_1\eta_{2v_a}^2\cdot$$
$$\cdot sin((1+(\mu_a-v_a)\cdot(1-s))\omega t-\frac{\mu_a'\pi}{p\tau_p}x_1+\frac{(\mu_a'-v_a')\pi}{p\tau_p}x_2'-\rho_{v_a}) \tag{14}$$

The tension stress (force acting on a unit surface) is obtained by calculating $B^2/2\mu_0$ from the resulting induction B. It is interesting mainly around $s\approx0$.

After adding and then squaring the inductions, it is obtained:

$$\begin{aligned} B^2 &= B_{v'_b}^2\sin^2(...)+B_{\mu'_a}^2\sin^2(...)+2B_{v'_a}B_{\mu'_a}\sin(...)\sin(...) \\ &=1/2B_{v'_b}^2-1/2B_{v'_b}^2\cos(2\omega t-2v'_b\pi x/p\tau_p)+1/2B_{\mu'_a}^2- \\ &1/2B_{\mu'_a}^2\cos((2+2\mu_a-2v_a)\omega t-2\mu'_a\pi x/p\tau_p)+ \\ &2B_{v'_b}B_{\mu'_a}1/2(\cos((\mu_a-v_a)\omega t-(\mu'_a-v'_b)\pi x_1/p\tau_p)- \\ &\cos((2+\mu_a-v_a)\omega t-(\mu'_a+v'_b)\pi x_1/p\tau_p)) \end{aligned} \tag{15}$$

where $\mu_a - v_a = e\cdot Z_2/p$

For the sake of simplicity, it is permissible here that the location of the rotor $x_2'=0$ at time $t=0$ coincides with the stator location $x_1=0$.

Evaluation is as follows:

1) The squares of the individual induction components are uninteresting in terms of noise and vibration.

2) The third or fourth member must actually be taken into account. One of these results in a low order number of tensile stresses, the other is uninteresting. If $r=\mu'_a-v'_b$ falls in the range $r=0...4$, then the third pulsating force, if $r=\mu'_a+v'_b$ falls in $r=0…4$ range, then the fourth term propagating force will be the determining factor in terms of radial forces.

The formula is valid for all order, but at $r=0$, by definition, only pulsating force occurs.

Here, too, it was derived a short, well-understood formula before [1].

$$f=\frac{1}{(p\tau_p l_i\delta')}(\frac{m}{2}\cdot I_m^2\frac{X_m}{2\pi f})\cdot(\frac{\xi_{v_b}\xi_{v_a}}{v_b\mu_a}\eta_{2v_a}^2\frac{1}{\xi_1^2})=\frac{W_m}{\left(V_{airgap}/2\right)}C$$

$$C=\frac{\xi_{v_b}\xi_{v_a}}{v_b\mu_a}\eta_{2v_a}^2\frac{1}{\xi_1^2} \tag{16}$$

Here f maximum value of tension stress of a cosines radial magnetic force wave (N/m^2)

W_m total magnetic energy of the air gap

V_{airgap} total volume of the air gap

If the magnetizing current is substituted as the current, the maximum value of the tensile stress with cos distribution is obtained by dividing the magnetic energy of the machine by the half-cubic content of the air gap. C is easily determined from the stator winding and from the rotor slot number. It is strikingly similar to (4).

It will be noted that v and μ shall be substituted into the formula of C, not v' and μ'.

The first part in the formulas represents the machine constants. Part C in the formulas can be used generally, since it is independent of the machine constants. Factor C can be used as a proportionality factor when comparing machines with different rotor slot numbers as well as force waves of different origin within one machine.

As already mentioned above, this force is usually of interest during operation, i.e. when $s\approx0$, i.e. $s_v=\infty$. Then in general: $\Delta_v=1-\eta_{2v}^2$.

Attenuation is taken into consideration

$$f_r=(1-\eta_{2v_b}^2)\frac{W_m}{\left(V_{airgap}/2\right)}C \tag{17}$$

for those force waves where it happens to be $v_b<2mq_1$ if any.This takes place primarily but not exclusively in the case of $Z_2<Z_1$. If $v_b\geqslant 2mq_1$ then the attenuation factor for this harmonic v is: $\Delta_v=1$.

This procedure is less important here than with the synchronous parasitic torque. There, the value of the torque alone determines the phenomenon, here, in the case of the force wave, in addition to the absolute value, the mechanical resonance comes into play in an even more decisive way, i.e. whether this force wave coincides one of the motor's primarily yoke natural frequencies.

B. Relationship between Radial Magnetic Forces and Synchronous Torques

1) Frequency of Synchronous Torque Out of Synchronism

and Frequency of Radial Magnetic Forces

Write the complete formula of the synchronous torque, including the time dependence, as a function of $\underline{s}$ [3]. If the synchronous torque occurs at standstill:

$$M = M_{\max} \sin\left(e\frac{Z_2}{p}(1-s)\omega t + e\frac{Z_2}{p}\frac{\pi}{\tau}x_2' + \rho_{va}\right) \quad (18a)$$

If the synchronous torque occurs during run-up:

$$M = M_{\max} \sin\left(\left(2 + e\frac{Z_2}{p}(1-s)\right)\omega t + e\frac{Z_2}{p}\frac{\pi}{\tau}x_2' + \rho_{va}\right) \quad (18b)$$

Formula of the radial magnetic forces where the order of the force wave: $r=\mu'_a \pm v'_b$.

$$\begin{aligned} &2B_{v'_b}B_{\mu'_a}\frac{1}{2}(\cos((\mu_a - v_a)(1-s)\omega t - \\ &(\mu'_a - v'_b)\pi x_1 / p\tau_p) - (\cos((2+\mu_a - v_a)\cdot \\ &(1-s)\omega t - (\mu'_a + v'_b)\pi x_1 / p\tau_p)) \end{aligned} \quad (19)$$

where $\mu_a - v_a = e \cdot Z_2 / p$.

It is clear from the comparison of the formulas that the frequency or more precisely, the set of frequencies, of both phenomena is the same.

This is not a coincidence, because both the harmonic current layer wave and the flux density wave created by the former of the stator and both the harmonic current layer wave and the flux density wave created by the former of the rotor will definitely interact with each other. The two forms of appearance of this phenomenon are the synchronous parasitic torque, which comes from a flux density wave of the rotor and a current layer wave of the stator, i.e. tangential forces, and the radial magnetic forces, which come from the same flux density wave of the rotor and a flux density of the stator. Consequently, the identity of their properties, including their frequency, is a principle. And it can be seen that if two identical order numbers interact, one phenomenon will be a synchronous torque, and the other will be a force wave with the order number r=0: so they always go together. If two waves with different order numbers interact with each other, a radial force wave with serial number $r\geq 1$ is generated, but no synchronous torque. More precisely: circumferential tangential force waves are generated, but they balance each other on the entire circumference the integral is zero, no torque can be felt on the shaft. Circumferential tangential force waves can be felt, however, on the teeth. In some more delicate cases, it is customary to deal with them, e.g. especially in the case of strict noise and vibration requirements. They are usually not dangerous from mechanical strength point of view but might cause tooth bending resonance.

As mentioned an asynchronous machine with harmonics can be modeled by shaft-connected additional asynchronous and synchronous machines. The synchronous machines shall be imagined as having externally excited poles proportional to B'_{v_b} on the stator outer part and externally excited poles proportional to B'_{μ_a} on the rotor. Some of them have the same number of poles: these create the synchronous torques and the r=0 radial magnetic forces. The others have different number of poles: these can not create torque on the shaft but create the tangential force waves that load only the teeth and the radial magnetic forces with order number $r\geq 1$. The rotor part is connected to the main motor via a gear ratio $e\cdot Z_2/p$ corresponding to μ_a and the outer part is rotated at the speed and direction corresponding to v_b.

2) The Absolute Value of Synchronous Torque and of Radial Magnetic Forces

The induction wave of the rotor interacts not only with the induction wave of the stator, but also with the current layer wave that creates the latter. In this case, tangential forces significantly smaller than the radial ones are generated. Ratio of tangential and radial forces [6], [7], [14]

$$f_{tan\,g} / f_{rad} = p\mu_a\delta / R \quad (20)$$

where δ geometric air gap.

R radius of rotor.

In the cited references it was not come aware that just these tangential forces create the synchronous torques, obviously not with magnetizing current but with starting current being $(X_m/X_s)^2 \approx (3\cdot 5)^2$ times higher in this way.

Let's see our formula for the radial magnetic forces again (16). The formula is considered now for the case of r=0, where $\mu_a = \pm\, v_b$. Then f tensile stress is the same in space along the circumference, but only changes in time (pulsates).

In this case, as told, not only radial forces, but also tangential forces occur. Calculate these tangential forces.

Multiply this radial force by the above $f_{\tan g} / f_{rad} = p\mu_a\delta / R$, expression so we get the tangential force occurring at the same time as the radial force.

The circumferential integral of this force turns into a simple multiplication: 2 $p\tau_p l$.

Multiply this force by the radius of the rotor: R. Then the torque acting on the rotor is obtained.

Multiply this torque by the reciprocal of the breakdown torque, $\frac{2\pi f}{p}\frac{2X_s}{mI_m^2X_m^2}$ with this, that torque is related to the breakdown torque. Here, instead of U^2, $I_m^2\cdot X_m^2$ was written.

Finally, it is taken into account that the radial magnetic force was calculated at no-load and the synchronous torque at short-circuit, so let's multiply by X_m^2/X_s^2 ratio. Apparently, our formula of the synchronous torque (4) is received back. With this, not only the relationship itself was proven at the formula level but both of our formulas as well. This cannot be otherwise acc. to the argumentation before.

3) The Basic Equations Regarding Stator and Rotor Harmonic Flux Density Waves and Current Layer Waves of a Rotating Machine

Let us see the physical background of the relationship between tangential forces including synchronous torques and radial forces. Both the flux density waves and the current layer waves of the stator and rotor resp., as a function of time and location (periphery) are written as follows, see in Table I.

$$\begin{aligned} b_s &= B_s \sin(\omega_s t - p_s x), b_r = B_r \sin(\omega_r t - p_r x - \varphi) \\ a_s &= A_s \cos(\omega_s t - p_s x), a_r = A_r \cos(\omega_r t - p_r x - \varphi) \end{aligned} \quad (21)$$

There is the only stipulation that there is a causality relationship between the quantities of both the stator and the rotor: $a=k\cdot db/dx$.

Otherwise, both stator and rotor quantities can be of any origin; between them there is absolutely no forced relationship. Regarding the angular velocity of the fields it is irrelevant whether the angular velocity of the rotor field depends or does not depend on the angular velocity of the rotor.

It is formally stipulated that p_s and p_r, as well as p_s-p_r, can only be positive integers. If the latter is not met, then when writing down the equations, the roles of the stator and rotor are to be exchanged the result will not change. The sign of ω_r is interpreted in relation to positive ω_s.

Then, the general equations of both tangential force/torque and radial force shall be written down in Table I, in parallel columns; then the special conditions will be taken one after the other and will be substituted into the formulas.

The elementary torque is determined as follows [19] see pp. 47.

$$dm = rf = -lr^2 BAdx \tag{22}$$

where r, l radius and length of rotor resp.
x a point on the rotor periphery.

Torque acting on the shaft,

$$M = -lr^2 \int_0^{2\pi} f dx \tag{23}$$

The set of formulas in Table I can be used directly for theoretical considerations and practical calculations related to the harmonics of the stator and rotor of an asynchronous machine and synchronous machine. The results cannot be used to calculate an asynchronous machine and synchronous machine directly, since it was assumed that there is no forced relationship between stator and rotor quantities whatsoever. However, in a fundamental harmonic machine there is always a forced relationship, which is given at the formula level by the fundamental harmonic equivalent circuit diagram.

For the sake of completeness, the following shall be added to the table. General relation between current layer waves and the flux density waves belonging to it as stipulated.

$$\begin{aligned} a &= \frac{\delta}{R\cdot\mu_0}\frac{db}{dx} = \frac{p\delta}{R\cdot\mu_0}B\cos(\omega t - px) \\ &= A\cos(\omega t - px) \end{aligned} \tag{24}$$

where $A = \frac{p\delta}{R\cdot\mu_0}B$

Absolute value of the tangential force,

$$f_{\tan g} = \frac{1}{2}\frac{p\delta}{R\cdot\mu_0}B_r B_s \tag{25}$$

Absolute value of the radial force

$$f_{rad} = \frac{1}{2\mu_0}B^2 = \ldots = \frac{1}{2\mu_0}B_s B_r \tag{26}$$

It follows

$$f_{\tan g} / f_{rad} = \frac{p\delta}{R} \tag{27}$$

The f_{tang} forces corresponding to those f_{rad} forces that are created by the self-taken squares of B_s and B_r are not found among the tangential forces: being consistent with the physics of produce of torque such squares of inductions are not able to create torque.

TABLE I
RELATIONSHIPS BETWEEN RADIAL AND TANGENTIAL FORCES

	Tangential force - Torque	Radial force
Basic formulas	$f_{\tan g} = B_s A_r \sin(\omega_s t - p_s x)\cdot\cos(\omega_r t - p_r x - \varphi)$ $=1/2\cdot B_s A_r[\sin((\omega_s - \omega_r)t - (p_s - p_r)x + \varphi) +$ $\sin((\omega_s + \omega_r)t - (p_s + p_r)x - \varphi)]$	$f_{rad} \approx (B_s + B_r)^2 = B_s^2 + B_r^2 + 2B_s B_r$ $B_s^2/2\cdot(1-\cos(2\omega_s t - 2p_s x)) +$ $B_r^2/2\cdot(1-\cos(2\omega_r t - 2p_r x - 2\varphi)) +$ $2B_s B_r * 1/2\cdot[(\cos((\omega_s - \omega_r)t - (p_s - p_r)x + \varphi)$ $-\cos((\omega_s + \omega_r)t - (p_s + p_r)x - \varphi)]$
$p_s \neq p_r$ $\omega_s \neq \omega_r$	$f_{\tan g} = 1/2\cdot B_s A_r[\sin((\omega_s - \omega_r)t - (p_s - p_r)x + \varphi)$ $+\sin((\omega_s + \omega_r)t - (p_s + p_r)x - \varphi)]$ f_{tang} tangential force waves along the periphery as function of the point on the periphery and the time $M = -lr^2\int_0^{2\pi} f dx = 0$	$f_{rad} \approx \ldots + B_s B_r[\cos((\omega_s - \omega_r)t - (p_s - p_r)x + \varphi) -$ $\cos((\omega_s + \omega_r)t - (p_s + p_r)x - \varphi)]$ Force of order $r \neq 0$
$p_s = p_r$ $\omega_s \neq \omega_r$	$f_{\tan g} = 1/2\cdot B_s A_r[\sin((\omega_s - \omega_r)t + \varphi) +$ $\sin((\omega_s + \omega_r)t - 2px - \varphi)]$ $M = -lr^2\int_0^{2\pi} f dx = -\pi r^2 l B_s A_r \sin((\omega_s - \omega_r)t + \varphi)$ Oscillating torques. Synchronous torques	$f_{rad} \approx \ldots + B_s B_r[\cos((\omega_s - \omega_r)t + \varphi) -$ $\cos((\omega_s + \omega_r)t - 2px - \varphi)]$ Force of order $r = 0$
$p_s = p_r$ $\omega_s = \omega_r$	$f_{\tan g} = 1/2\cdot B_s A_r(\sin\phi + \sin(2\omega t - 2px - \varphi))$ $M = -\pi r^2 l B_s A_r \sin\varphi$ Constant torque in time	$f_{rad} \approx \ldots - B_s B_r(\cos\varphi - \cos(2\omega t - 2px - \varphi))$ Force like a fundamental wave

C. Investigation on Slot Numbers, not Allowed Slot Numbers

As told the statement that small tangential forces are also created together with radial magnetic forces can generally be found in the literature.

Based on the above reasoning, it is clear that synchronous parasitic torques and radial magnetic forces must be treated together, and it is clear that the slot numbers that should be avoided from the point of view of synchronous torque also cause dangerous radial magnetic force waves.

As for radial magnetic forces, not only those slot numbers being excluded from the list of safe slot numbers regarding synchronous torques shall be excluded here, too, but also much more, the ±4 slot bands next to r=0, because not only the r=0 order, but also the slot numbers that cause r=1...4 order must be avoided. These slot numbers must also be included in tables, see Annex Table IV.

The tables also specify which rotor slot number creates which force order wave. The + and – signs show that the force order is created as the sum or the difference of the order of the respective waves: this has a role in calculating the frequency.

This table was created in the same way as Table III, that is, it is emphasized that the rotor slot number that appears anywhere in this table creates a low order radial magnetic force wave with order $r \leq 4$, even if installed in any stator; the stator slot numbers are shown for reference purposes only, so that it is easy to review which the slot harmonics of that particular stator are.

Multiple appearance of a certain rotor slot number in Table IV means multiple number of exciting force waves with different order and/or different frequency. Multiple number of exciting force waves with further order means further natural frequency / frequencies to be avoided.

In case of inverter supplied motors: the motors supplied by frequencies from zero to maximum will go through not only one but more natural frequencies. When calculating exciting frequency, one shall return to the original formula $f=eZ_2/p \cdot f_{supply} \pm 2f_{supply}$ instead of 50 Hz in Table IV.

The rotor slot numbers appearing in the row of the *actual* slot number of the stator due to its slot harmonic will create "fatally" high radial magnetic force waves. This also shows the fundamental physical connection between the two phenomena.

It is also noted that e.g. if q_1=3, then not only waves ν_b= -17 and ν_b=19 are slot harmonics, but due to periodicity of the winding factor, also their multiples: ν_b=-35, 37, -53, 55 etc. Therefore, the slot numbers appearing in the same row as these further stator harmonics should also be avoided. Of course, the radial magnetic forces created by these further slot numbers are reduced (=are not as "fatal"), namely in inverse proportion to the value of μ_a'.

In the case of very low rotor slot numbers, due to very low $\nu_b=\pm\mu_a$ values, very high radial magnetic forces are formed with the lowest harmonics of the stator, with the 5th and 7th harmonics even if it is installed in a stator with q_1=2. In these cases, chording plays a decisive role in the magnitude of the synchronous torques otherwise not.

The magnitude of the tensile stresses f [N/m^2] produced by rotor slots appearing in a group of rows belonging to the same ν_b is almost the same, since ξ_a, ξ_b, $\eta_{2\nu}^2 = \eta_1^2$, and ν_b are the same, only μ_a is different slightly. This group of rows has the same frequency that is why that is given only once.

The number of slots appearing in the same column create decreasing synchronous torque [2]. Stator slot harmonics, however, may create local maximum therefore the synchronous torque does not decrease monotonically. At high slot numbers, therefore, due to the high values of μ_a' and ν_b', the rotor slot numbers declared "fatal" do not produce such fatally high radial magnetic forces, similar to the example of synchronous torques.

Studying the tables, it can be seen that the rotor slot number bands to be avoided "touch" each other at small pole numbers. Therefore, there is no rotor slot number at which a low-order radial force wave does not occur. In fact, one has to be content with avoiding coincidence with the relevant stator slot harmonic. Also, in fact, as a first step, the natural frequencies should be avoided at all costs. In the case of a large number of poles, between two dangerous bands, however, there is rotor slot number that can be safely used.

It can also be seen that the slot numbers that create the most dangerous r=1 order force wave are all odd. But they do not cover the entire odd range, the rest creates r=3 order. But these do not cover the entire range either, the remaining if any odd slot numbers create r=5 order and so on. In any case, if an odd number of rotor slot is not used, one get rid of both r=1 and r=3 force waves. However, with large number of poles, odd numbers of slots can be safely used between the dangerous bands, creating $r \geq 5$ order force waves.

D. *Calculation of Radial Magnetic Forces at No-load and in Operation*

When deriving the formulas, the no-load current was always substituted as current, and *not just because* the formulas could be spectacularly simple and expressive in this way, but also because the values occurring at no-load and full load cannot in all cases be simply converted in proportion to the square of the stator current. That is, the set of constants C according to (16) cannot be used directly.

There are two reasons for this. When deriving the formulas so far, it was assumed that the stator and rotor currents are the same and in opposite phase. This was permissible close to starting when calculating the synchronous torque. Here, however, it was established that this phenomenon is of interest in operation, in any case for slips $0<s<s_b$, but it was not taken this into account so far. With such small slips, the rotor current is not the same as the stator current, even almost disappears at no-load. Therefore, the force waves created by the fundamental harmonic of the stator current are taken into account as a good approximation multiplied by the stator cos φ [7]. On the other hand, the previous assumption is true for the rotor currents and thus the force waves created by the harmonics of the stator current.

The other reason is that the change in magnetic conductivity of the air gap, which occurs as a result of the open stator slots of large machines, cannot be neglected in this phenomenon, and in fact plays a decisive role in the generation of noise. Changing of magnetic conductivity due to the stator open slot [6], [7], [14] creates $B_{slotting}$ induction wave.

$$B_{slotting} = -(k_c - 1)\frac{\sin g \dfrac{k_c - 1}{k_c}\pi}{g \dfrac{k_c - 1}{k_c}\pi} B_{\nu=1} \qquad (28)$$

where k_c Carter factor. The sign of the slotting induction wave is always opposite to the sign of the magnetizing current that creates it.

Order of slotting wave is $\nu_{slotting}=g \cdot Z_1+p$, now only g=±1 factor will be dealt with. As it can be seen, their order is the same as the order of the winding harmonics $\nu'=\nu p$. Therefore, these orders of the winding harmonics are called slot harmonics. Because of the same order the two can be added. In this way, the winding slot harmonics and the conduction slotting harmonics cannot be distinguished in operation. As seen below conduction slotting harmonics are large values, they considerably modify the effect of the winding slot harmonics. Neglecting that means not adding the slotting harmonics to the slot harmonics results in an incorrect calculation.

Rotor conductivity harmonics are out of interest now.

$B_{slotting} \approx 0.45 \cdot B_{\nu=1}$ for high voltage machines is a fairly large value.

Since $B_\nu = B_{\nu=1}/\nu$, the no-load component will be determined:

$C_{no\text{-}load} = C_{basic} \cdot (-1 + abs(\nu) \cdot 0.45)$, if ν positive.

(magnetizing current is such as definition)

$C_{no\text{-}load} = C_{basic} \cdot (1 + abs(\nu) \cdot 0.45)$, if ν negative.

During load, the addition of $C_{basic} \cdot (abs(\nu) \cdot 0.45 \pm 1)$ and $C_{basic} \cdot (I_{rated}/I_m)^2$ must be done geometrically according to Fig. 1.

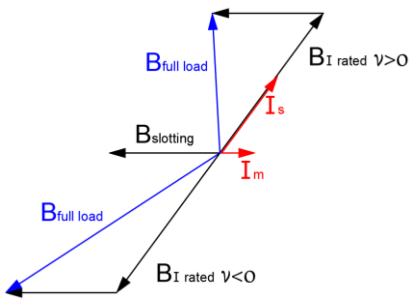

Fig. 1. Vector addition of conduction slotting harmonic (being in counter-phase of magnetizing current) with load current winding harmonic (of the same order) ([14] Fig. 6.10)

Since $0.45 \cdot \nu >> 1$, the slotting harmonics strongly enhance the winding harmonic with the same order, therefore the participation of the slotting harmonic has a decisive effect on the radial magnetic forces.

The table of the calculation of no-load and full-load forces is presented through a numerical example. The values of C proportionality factors have been inserted into the cells of an expedient table. Such a table was proposed by [6], but it can also be found in [7], but such has been filled so far simply by orders and frequencies.

The Annex Table V. shows how to use the proportional factors in practice on a numerical example. It was tried to find an example in which all investigated phenomena occur. Therefore, e.g. a machine without chording was chosen.

The formula was entered into the relevant cells, then entered the necessary data, then calculated and entered the values into the cells, thereby obtaining the basic data required for further calculations.

After that, no-load data will be inserted into the relevant table. This differs from the basic table in two cells only: the cells originated from the fundamental harmonic were multiplied by the no-load $\cos_{\varphi 0}$, and the value created by the slotting was added to the cell affected by the slotting harmonic with the correct sign.

The cell value for the load table is obtained by multiplying the original (base) value of each cell by the square of the quotient of the rated current and the no-load current, which is now taken as a ratio of 10. An exception is the cell affected by the slot harmonic: the field B_{Irated} created by the current I_s is in phase not with the no-load current I_m, but with the stator current I_s; this load data shall be geometrically added to the no-load data.

The result, which is only more or less typical, nevertheless confirms the general experience of the measurements:

- In no-load, the forces resulting from the slot and slotting harmonics mostly dominate.
- When under load, the forces resulting from the winding harmonics dominate, these components typically increase by around 10 dB.
- The resulting forces have increased now by ~4 times, i.e. by 6 dB.

V. Annex

A. Synchronous Torques

TABLE II

THE PROCESS OF CALCULATING SYNCHRONOUS TORQUES

		$\nu_a=1$	$\nu_a=-5$	$\nu_a=7$	$\nu_a=-11$	$\nu_a=13$	$\nu_a=-17$	$\nu_a=19$			
			$(\nu_b=-5)$	$(\nu_b=7)$	$(\nu_b=-11)$	$(\nu_b=13)$	$\nu_b=-17$	$\nu_b=19$	$\nu_b=-23$	$\nu_b=25$	$\nu_b=\ldots$
		ξ_1	ξ_5	ξ_7	ξ_{11}	ξ_{13}	$=\xi_1$	$=\xi_1$	ξ_{23}	ξ_{25}	$\xi_{\nu b}$
	η_a^2	$\eta_{2,1}^2$	$\eta_{2,5}^2$	$\eta_{2,7}^2$	$\eta_{2,11}^2$	$\eta_{2,13}^2$	$\eta_{2,17}^2 \approx 0$	$\eta_{2,19}^2 \approx 0$			
	Δ_b		(Δ_5)	(Δ_7)	(Δ_{11})	(Δ_{13})					
$e=-1$	$\mu_a=$	$-Z_2/p+1$	$-Z_2/p-5$	$-Z_2/p+7$	$-Z_2/p+7$	$-Z_2/p+13$	$-Z_2/p-17$	$-Z_2/p+19$			
	$C\sim$	$\xi_1 \cdot \xi_b \cdot \eta_{2,1}^2/\mu_a$	$\xi_5 \cdot \xi_b \cdot \eta_{2,5}^2/\mu_a$	$\xi_7 \cdot \xi_b \cdot \eta_{2,7}^2/\mu_a$	$\xi_{12} \cdot \xi_b \cdot \eta_{2,11}^2/\mu_a$	$\xi_{13} \cdot \xi_b \cdot \eta_{2,13}^2/\mu_a$	$\xi_{17} \cdot \xi_b \cdot \eta_{2,17}^2/\mu_a$	$\xi_{19} \cdot \xi_b \cdot \eta_{2,19}^2/\mu_a$			
$e=1$	$\mu_a=$	$Z_2/p+1$	$Z_2/p-5$	$Z_2/p+7$	$Z_2/p+7$	$Z_2/p+13$	$Z_2/p-17$	$Z_2/p+19$			
	$C\sim$	$\xi_1 \cdot \xi_b \cdot \eta_{2,1}^2/\mu_a$	$\xi_5 \cdot \xi_b \cdot \eta_{2,5}^2/\mu_a$	$\xi_7 \cdot \xi_b \cdot \eta_{2,7}^2/\mu_a$	$\xi_{11} \cdot \xi_b \cdot \eta_{2,11}^2/\mu_a$	$\xi_{13} \cdot \xi_b \cdot \eta_{2,13}^2/\mu_a$	$\xi_{17} \cdot \xi_b \cdot \eta_{2,17}^2/\mu_a$	$\xi_{19} \cdot \xi_b \cdot \eta_{2,19}^2/\mu_a$			
$e=-2$	$\mu_a=$	$-2 \cdot Z_2/p+1$	$-2 \cdot Z_2/p-5$	$-2 \cdot Z_2/p+7$	$-2 \cdot Z_2/p-11$	$-2 \cdot Z_2/p+13$					
	$C\sim$	$\xi_1 \cdot \xi_b \cdot \eta_{2,1}^2/\mu_a$	$\xi_5 \cdot \xi_b \cdot \eta_{2,5}^2/\mu_a$	$\xi_7 \cdot \xi_b \cdot \eta_{2,7}^2/\mu_a$	$\xi_{11} \cdot \xi_b \cdot \eta_{2,11}^2/\mu_a$	$\xi_{13} \cdot \xi_b \cdot \eta_{2,13}^2/\mu_a$					
$e=2$	$\mu_a=$	$2Z_2/p+1$	$2Z_2/p-5$	$2 \cdot Z_2/p+7$							
	$C\sim$	$\xi_1 \cdot \xi_b \cdot \eta_{2,1}^2/\mu_a$	$\xi_5 \cdot \xi_b \cdot \eta_{2,5}^2/\mu_a$	$\xi_7 \cdot \xi_b \cdot \eta_{2,7}^2/\mu_a$							
$e=-3$	$\mu_a=$	$-3 \cdot Z_2/p+1$									
	$C\sim$	$\xi_1 \cdot \xi_b \cdot \eta_{2,1}^2/\mu_a$									

e= 3	μ_a=	$3Z_2/p+1$
	C~	$\xi_1{\cdot}\xi_b{\cdot}\eta_{2,1}^2/\mu_a$

In general, the synchronous torques generated shall be determined as follows: e.g. q_1=3.

Table II shows how to use the proportionality factor in practice.

If the calculated μ_a values contain those equal to a ν_b value, then a synchronous torque is generated there. The relationship of the signs shows whether they occur in standstill or in starting-up. Under the harmonics of the stator are given the quantities and the formula that should be used in the calculation. The following can be concluded about them:

$\eta_{2\nu}^2$ decreases with ν monotonically; it is already quite low just approaching the slot harmonic. ξ_5 and ξ_7 are low even without chording.

Based on this, it can be stated that the synchronous torque created by the fundamental harmonic stator wave, if any, will be of the decisive importance.

Based on the above, tables have been developed for the most commonly used pole pair numbers, where the fundamental harmonic produces synchronous torque.

The attenuation will be taken into account as follows: the attenuation of stator field ν_b (=± μ_a) will be involved if the rotor slot number produces any of μ_a=±5, ±7, ±11, ±13 order.

Some regularity can be discovered in the tables for example with ν_a=1.

Stator harmonics: ν_b= 6g + 1 = 2mg+1.

Rotor harmonics produced by $\nu_a=1$:$\mu_a=e\,Z_2/p+1= e{\cdot}2mq_2'+1$

From this, already after an initial analysis, some conditions can be seen, the fulfillment of which leads to produce synchronous torque:

- $e{\cdot}2mq_2'$ = even integer.
- q_2' = integer (s_{synchr}=1).
- mq_2' =integer; this is obtained when q_2' is expressed as a fraction by the denominator m=3: these are: q_2' = 2⅓, 3⅓, 4⅓ etc. (s_{synchr}<1); 1⅔, 2⅔, 3⅔, 4⅔ etc. (s_{synchr}>1).
- $\underline{e}$ should be equal to the denominator of q_2' when expressed in fractional form; unless the denominator is 3, because then it can be anything.
- Since q_2' can always be expressed as the quotient of two integers never irrational, there is always an $\underline{e}$ for which $e{\cdot}2mq_2'$ = even integer. That is, after all, there is no any Z_2 that does not produce synchronous torque; of course, these with larger values of $\underline{e}$ are getting smaller.

TABLE III A

ROTOR SLOT NUMBERS THAT PRODUCE SYNCHRONOUS TORQUE DUE TO FUNDAMENTAL HARMONIC FOR p=2

p=2				e=		e=		e=	
				-1	1	-2	2	-3	3
q_1	Z_1	ν_b μ_a=ν_b	-ν_b μ_a= -ν_b						
		-5		12		6		4	
			5		8		4		
		7			12		6		4
			-7	16		8			
2	24	-11		24		12		8	
			11		20		10		
		13			24		12		8
			-13	28		14			
3	36	-17		36		18		12	
			17		32		16		
		19			36		18		12
			-19	40		20			
4	48	-23		48		24		16	
			23		44		22		
		25			48		24		16
			-25	52		26			
5	60	-29		60		30		20	
			29		56		28		
		31			60		30		20
			-31	64		32			
6	72	-35		72		36		24	
			35		68		34		
		37			72		36		24
			-37	76		38			
7	84	-41		84		42		28	
			41		80		40		
		43			84		42		28
			-43	88		44			
8	96	-47		96		48		32	
			47		92		46		
		49			96		48		32
			-49	100		50			
9	108	-53		108		54		36	
			53		104		52		
		55			108		54		36
			-55	112		56			
10	120	-59		120		60		40	
			59		116		58		
		61			120		60		40
			-61	124		62			

TABLE III B

ROTOR SLOT NUMBERS THAT PRODUCE SYNCHRONOUS TORQUE DUE TO HARMONIC -5TH, FOR p=2

p=2				e=		e=		e=	
				-1	1	-2	2	-3	3
q_1	Z_1	ν_b $\mu_a = \nu_b$	-ν_b μ_a =- ν_b						
		-5		0		0		0	
			5		20		10		
		7			24		12		8
			-7	4		2			
2	24	-11		12		6		4	
			11		32		16		
		13			36		18		12
			-13	16		8			
3	36	-17		24		12		8	
			17		44		22		
		19			48		24		16
			-19	28		14			
4	48	-23		36		18		12	
			23		*56*		28		
		25			60		30		20
			-25	40		20			
5	60	-29		48		24		16	
			29		68		34		
		31			72		36		24

			-31	52		26			
6	72	-35		60		30		20	
			35		80		40		
		37			84		42		28
			-37	64		32			
7	84	-41		72		36		24	
			41		92		46		
		43			96		48		32
			-43	76		38			
8	96	-47		84		42		28	
			47		104		52		

		49			108		54		36
			-49	88		44			
9	108	-53		96		48		32	
			53		116		58		
		55			120		60		40
			-55	100		50			
10	120	-59		108		54		36	
			59		128		64		
		61			132		66		44
			-61	112		56			

B. Radial Magnetic Forces

TABLE IV
ROTOR SLOT NUMBERS THAT PRODUCE LOW-ORDER RADIAL MAGNETIC FORCES WITH THE COEFFICIENT Ε=±1 (ADDITIONALLY FOR ε=±2 AND ε=±3), FOR POLE PAIRS p = 2-4, CAUSED BY THE FUNDAMENTAL HARMONIC. THE FREQUENCY OF THE FORCES FOR 50 HZ NETWORK SUPPLY

				p=	2												
				r=	0	r =	1	r =	2	r =	3	r =	4	r =	5	r =	6
				+	-	+	-	+	-	+	-	+	-	+	-	+	-
q_1	Z_1	ν_b	ν'_b														
		-5	-10	8	12	9	13	10	14	11	15	12	16	13	17	14	18
			frequency Hz	300	300	325	325	350	350	375	375	400	400	425	425	450	450
		-5	-10			7	11	6	10	5	9	4	8	3	7	2	6
			frequency Hz			275	275	250	250	225	225	200	200	175	175	150	150
		7	14	16	12	17	13	18	14	19	15	20	16	21	17	22	18
			frequency Hz	300	300	325	325	350	350	375	375	400	400	425	425	450	450
		7	14			15	11	14	10	13	9	12	8	11	7	10	6
			frequency Hz			275	275	250	250	225	225	200	200	175	175	150	150
2	24	-11	-22	20	24	21	25	22	26	23	27	24	28	25	29	26	30
			frequency Hz	600	600	625	625	650	650	675	675	700	700	725	725	750	750
			$e=\pm2$	10	12			11	13			12	14			13	15
			$e=\pm3$		8	7					9	8					10
		-11	-22			19	23	18	22	17	21	16	20	15	19	14	18
			frequency Hz			575	575	550	550	525	525	500	500	475	475	450	450
			$e=\pm2$					9	11			8	10			7	9
			$e=\pm3$					6			7			5			6
		13	26	28	24	29	25	30	26	31	27	32	28	33	29	34	30
			frequency Hz	600	600	625	625	650	650	675	675	700	700	725	725	750	750
			$e=\pm2$	14	12			15	13			16	14			17	15
			$e=\pm3$		8			10			9			11			10
		13	26			27	23	26	22	25	21	24	20	23	19	22	18
			frequency Hz			575	575	550	550	525	525	500	500	475	475	450	450
			$e=\pm2$					13	11			12	10			11	9
			$e=\pm3$			9					7	8					6
3	36	-17	-34	32	36	33	37	34	38	35	39	36	40	37	41	38	42
			frequency Hz	900	900	925	925	950	950	975	975	1000	1000	1025	1025	1050	1050
			$e=\pm2$	16	18			17	19			18	20			19	21
			$e=\pm3$		12	11					13	12					14
		-17	-34			31	35	30	34	29	33	28	32	27	31	26	30
			frequency Hz			875	875	850	850	825	825	800	800	775	775	750	750
			$e=\pm2$					15	17			14	16			13	15
			$e=\pm3$					10			11			9			10
		19	38	40	36	41	37	42	38	43	39	44	40	45	41	46	42
			frequency Hz	900	900	925	925	950	950	975	975	1000	1000	1025	1025	1050	1050
			$e=\pm2$	20	18			21	19			22	20			23	21
			$e=\pm3$		12			14			13	15	13	15			14
		19	38			39	35	38	34	37	33	36	32	35	31	34	30
			frequency Hz			875	875	850	850	825	825	800	800	775	775	750	750
			$e=\pm2$					19	17			18	16			17	15
			$e=\pm3$			13					11	12					10
4	48	-23	-46	44	48	45	49	46	50	47	51	48	52	49	53	50	54
			frequency Hz	1200	1200	1225	1225	1250	1250	1275	1275	1300	1300	1325	1325	1350	1350
			$e=\pm2$	22	24			23	25			24	26			25	27
			$e=\pm3$		16	15					17	16					18
		-23	-46			43	47	42	46	41	45	40	44	39	43	40	42

			frequency Hz			1175	1175	1150	1150	1125	1125	1100	1100	1075	1075	1100	1050
			e=±2					21	23			20	22			20	21
			e=±3					14			15			13			14
		25	50	52	48	53	49	54	50	55	51	56	52	57	53	58	54
			frequency Hz	1200	1200	1225	1225	1250	1250	1275	1275	1300	1300	1325	1325	1350	1350
			e=±2	26	24			27	25			28	26			29	27
			e=±3		16			18			17			19			18
		25	50			51	47	50	46	49	45	48	44	47	43	46	42
			frequency Hz			1175	1175	1150	1150	1125	1125	1100	1100	1075	1075	1050	1050
			e=2					25	23			24	22			23	21
			e=3			17					15	16					14
5	60	-29	-58	56	60	57	61	58	62	59	63	60	64	61	65	62	66
			frequency Hz	1500	1500	1525	1525	1550	1550	1575	1575	1600	1600	1625	1625	1650	1650
			e=±2	28	30			29	31			30	32			31	33
			e=±3		20	19					21	20					22
		-29	-58			55	59	54	58	53	57	52	56	51	55	50	54
			frequency Hz			1475	1475	1450	1450	1425	1425	1400	1400	1375	1375	1350	1350
			e=±2					27	29			26	28			25	27
			e=±3					18			19			17			18
		31	62	64	60	65	61	66	62	67	63	68	64	69	65	70	66
			frequency Hz	1500	1500	1525	1525	1550	1550	1575	1575	1600	1600	1625	1625	1650	1650
			e=±2	32	30			33	31			34	32			35	33
			e=±3		20			22			21			23			22
		31	62			63	59	62	58	61	57	60	56	59	55	58	54
			frequency Hz			1475	1475	1450	1450	1425	1425	1400	1400	1375	1375	1350	1350
			e=±2					31	29			30	28			29	27
			e=±3			21					19	20					18
6	72	-35	-70	68	72	69	73	70	74	71	75	72	76	73	77	74	78
			frequency Hz	1800	1800	1825	1825	1850	1850	1875	1875	1900	1900	1925	1925	1950	1950
			e=±2	34	36			35	37			36	38			37	39
			e=±3		24	23					25	24					26
		-35	-70			67	71	66	70	65	69	64	68	63	67	62	66
			frequency Hz			1775	1775	1750	1750	1725	1725	1700	1700	1675	1675	1650	1650
			e=±2					33	35			32	34			31	33
			e=±3					22			23			21			22
		37	74	76	72	77	73	78	74	79	75	80	76	81	77	82	78
			frequency Hz	1800	1800	1825	1825	1850	1850	1875	1875	1900	1900	1925	1925	1950	1950
			e=±2	38	36			39	37			40	38			41	39
			e=±3		24			26			25			27			26
		37	74			75	71	74	70	73	69	72	68	71	67	70	66
			frequency Hz			1775	1775	1750	1750	1725	1725	1700	1700	1675	1675	1650	1650
			e=±2					37	35			36	34			35	33
			e=±3			25					23	24					22
7	84	-41	-82	80	84	81	85	82	86	83	87	84	88	85	89	86	90
			frequency Hz	2100	2100	2125	2125	2150	2150	2175	2175	2200	2200	2225	2225	2250	2250
			e=±2	40	42			41	43			42	44			43	45
			e=±3		28	27					29	28					30
		-41	-82			79	83	78	82	77	81	76	80	75	79	74	78
			frequency Hz			2075	2075	2050	2050	2025	2025	2000	2000	1975	1975	1950	1950
			e=±2					39	41			38	40			37	39
			e=±3					26			27			25			26
		43	86	88	84	89	85	90	86	91	87	92	88	93	89	94	90
			frequency Hz	2100	2100	2125	2125	2150	2150	2175	2175	2200	2200	2225	2225	2250	2250
			e=±2	44	42			45	43			46	44			47	45
			e=±3		28			30			29			31			30
		43	86			87	83	86	82	85	81	84	80	83	79	82	78
			frequency Hz			2075	2075	2050	2050	2025	2025	2000	2000	1975	1975	1950	1950
			e=±2					43	41			42	40			41	39
			e=±3			29					27	28					26
8	96	-47	-94	92	96	93	97	94	98	95	99	96	100	97	101	98	102
			frequency Hz	2400	2400	2425	2425	2450	2450	2475	2475	2500	2500	2525	2525	2550	2550
			e=±2	46	48			47	49			48	50			49	51
			e=±3		32	31					33	32					34
		-47	-94			91	95	90	94	89	93	88	92	87	91	86	90
			frequency Hz			2375	2375	2350	2350	2325	2325	2300	2300	2275	2275	2250	2250
			e=±2					45	47			44	46			43	45

			e=±3					30			31			29			30
		49	98	100	96	101	97	102	98	103	99	104	100	105	101	106	102
			frequency Hz	2400	2400	2425	2425	2450	2450	2475	2475	2500	2500	2525	2525	2550	2550
			e=±2	50	48			51	49			52	50			53	51
			e=±3		32			34			33			35			34
		49	98			99	95	98	94	97	93	96	92	95	91	94	90
			frequency Hz			2375	2375	2350	2350	2325	2325	2300	2300	2275	2275	2250	2250
			e=±2					49	47			48	46			47	45
			e=±3			33					31	32					30

9	108	-53	-106	104	108	105	109	106	110	107	111	108	112	109	113	110	114
			frequency Hz	2700	2700	2725	2725	2750	2750	2775	2775	2800	2800	2825	2825	2850	2850
			e=±2	52	54			53	55			54	56			55	57
			e=±3		36	35					37	36					38
		-53	-106			103	107	102	106	101	105	100	104	99	103	98	102
			frequency Hz			2675	2675	2650	2650	2625	2625	2600	2600	2575	2575	2550	2550
			e=2					51	53			50	52			49	51
			e=3					34			35			33			34
		55	110	112	108	113	109	114	110	115	111	116	112	117	113	118	114
			frequency Hz	2700	2700	2725	2725	2750	2750	2775	2775	2800	2800	2825	2825	2850	2850
			e=±2	56	54			57	55			58	56			59	57
			e=±3		36			38			37			39			38
		55	110			111	107	110	106	109	105	108	104	107	103	106	102
			frequency Hz			2675	2675	2650	2650	2625	2625	2600	2600	2575	2575	2550	2550
			e=±2					55	53			54	52			53	51
			e=±3			37					35	36					34
10	120	-59	-118	116	120	117	121	118	122	119	123	120	124	121	125	122	126
			frequency Hz	3000	3000	3025	3025	3050	3050	3075	3075	3100	3100	3125	3125	3150	3150
			e=±2	58	60			59	61			60	62			61	63
			e=±3		40	39					41	40					42
		-59	-118			115	119	114	118	113	117	112	116	111	115	110	114
			frequency Hz			2975	2975	2950	2950	2925	2925	2900	2900	2875	2875	2850	2850
			e=±2					57	59			56	58			55	57
			e=±3					38			39			37			38
		61	122	124	120	125	121	126	122	127	123	128	124	129	125	130	126
			frequency Hz	3000	3000	3025	3025	3050	3050	3075	3075	3100	3100	3125	3125	3150	3150
			e=±2	62	60			63	61			64	62			65	63
			e=±3		40			42			41			43			42
		61	122			123	119	122	118	121	117	120	116	119	115	118	114
			frequency Hz			2975	2975	2950	2950	2925	2925	2900	2900	2875	2875	2850	2850
			e=±2					61	59			60	58			59	57
			e=±3			41					39	40					38
11	132	-65	-130	128	132	129	133	130	134	131	135	132	136	133	137	134	138
			frequency Hz	3300	3300	3325	3325	3350	3350	3375	3375	3400	3400	3425	3425	3450	3450
			e=±2	64	66			65	67			66	68			67	69
			e=±3		44	43					45	44					46
		-65	-130			127	131	126	130	125	129	124	128	123	127	122	126
			frequency Hz			3275	3275	3250	3250	3225	3225	3200	3200	3175	3175	3150	3150
			e=±2					63	65			62	64			61	63
			e=±3					42			43			41			42
		67	134	136	132	137	133	138	134	139	135	140	136	141	137	142	138
			frequency Hz	3300	3300	3325	3325	3350	3350	3375	3375	3400	3400	3425	3425	3450	3450
			e=±2	68	66			69	67			70	68			71	69
			e=±3		44			46			45			47			46
		67	134			135	131	134	130	133	129	132	128	131	127	130	126
			frequency Hz			3275	3275	3250	3250	3225	3225	3200	3200	3175	3175	3150	3150
			e=±2					67	65			66	64			65	63
			e=±3			45					43	44					42
12	144	-71	-142	140	144	141	145	142	146	143	147	144	148	145	149	146	150
			frequency Hz	3600	3600	3625	3625	3650	3650	3675	3675	3700	3700	3725	3725	3750	3750
			e=±2	70	72			71	73			72	74			73	75
			e=±3		48	47					49	48					50
		-71	-142			139	143	138	142	137	141	136	140	135	139	134	138

			frequency Hz			3575	3575	3550	3550	3525	3525	3500	3500	3475	3475	3450	3450
			e=±2					69	71			68	70			67	69
			e=±3					46			47			45			46
		73	146	148	144	149	145	150	146	151	147	152	148	153	149	154	150
			frequency Hz	3600	3600	3625	3625	3650	3650	3675	3675	3700	3700	3725	3725	3750	3750
			e=±2	74	72			75	73			76	74			77	75
			e=±3		48			50			49			51			50
		73	146			147	143	146	142	145	141	144	140	143	139	142	138
			frequency Hz			3575	3575	3550	3550	3525	3525	3500	3500	3475	3475	3450	3450
			e=±2					73	71			72	70			71	69
			e=±3			49					47	48					46
13	156	-77	-154	152	156	153	157	154	158	155	159	156	160	157	161	158	162
			frequency Hz	3900	3900	3925	3925	3950	3950	3975	3975	4000	4000	4025	4025	4050	4050
			e=±2	76	78			77	79			78	80			79	81
			e=±3		52	51					53	52					54
		-77	-154			151	155	150	154	149	153	148	152	147	151	146	150
			frequency Hz			3875	3875	3850	3850	3825	3825	3800	3800	3775	3775	3750	3750
			e=±2					75	77			74	76			73	75
			e=±3					50			51			49			50
		79	158	160	156	161	157	162	158	163	159	164	160	165	161	166	162
			frequency Hz	3900	3900	3925	3925	3950	3950	3975	3975	4000	4000	4025	4025	4050	4050
			e=±2	80	78			81	79			82	80			83	81
			e=±3		52			54			53			55			54
		79	158			159	155	158	154	157	153	156	152	155	151	154	150
			frequency Hz			3875	3875	3850	3850	3825	3825	3800	3800	3775	3775	3750	3750
			e=±2					79	77			78	76			77	75
			e=±3			53					51	52					50
14	168	-83	-166	164	168	165	169	166	170	167	171	168	172	169	173	170	174
			frequency Hz	4200	4200	4225	4225	4250	4250	4275	4275	4300	4300	4325	4325	4350	4350
			e=±2	82	84			83	85			84	86			85	87
			e=±3		56	55					57	56					58
		-83	-166			163	167	162	166	161	165	160	164	159	163	158	162
			frequency Hz			4175	4175	4150	4150	4125	4125	4100	4100	4075	4075	4050	4050
			e=±2					81	83			80	82			79	81
			e=±3					54			55			53			54
		85	170	172	168	173	169	174	170	175	171	176	172	177	173	178	174
			frequency Hz	4200	4200	4225	4225	4250	4250	4275	4275	4300	4300	4325	4325	4350	4350
			e=±2	86	84			87	85			88	86			89	87
			e=±3		56			58			57			59			58
		85	170			171	167	170	166	169	165	168	164	167	163	166	162
			frequency Hz			4175	4175	4150	4150	4125	4125	4100	4100	4075	4075	4050	4050
			e=±2					85	83			84	82			83	81
			e=±3			57					55	56					54
15	180	-89	-178	176	180	177	181	178	182	179	183	180	184	181	185	182	186
			frequency Hz	4500	4500	4525	4525	4550	4550	4575	4575	4600	4600	4625	4625	4650	4650
			e=±2	88	90			89	91			90	92			91	93
			e=±3		60	59					61	60					62
		-89	-178			175	179	174	178	173	177	172	176	171	175	170	174
			frequency Hz			4475	4475	4450	4450	4425	4425	4400	4400	4375	4375	4350	4350
			e=±2					87	89			86	88			85	87
			e=±3					58			59			57			58
		91	182	184	180	185	181	186	182	187	183	188	184	189	185	190	186
			frequency Hz	4500	4500	4525	4525	4550	4550	4575	4575	4600	4600	4625	4625	4650	4650
			e=±2	92	90			93	91			94	92			95	93
			e=±3		60			62			61			63			62
		91	182			183	179	182	178	181	177	180	176	179	175	178	174
			frequency Hz			4475	4475	4450	4450	4425	4425	4400	4400	4375	4375	4350	4350
			e=±2					91	89			90	88			89	87
			e=±3			61					59	60					58

				p=	3								
				r=	0	r=	1	r=	2	r=	3	r=	4
				+	-	+	-	+	-	+	-	+	-
q_1	Z_1	ν_b	ν'_b										
2	36	-11	-33	**30**	**36**	**31**	**37**	**32**	**38**	**33**	**39**	**34**	**40**

				frequency Hz	600	600	617	617	633	633	650	650	667	667
				e=±2	15	18			16	19			17	20
				e=±3	10	12					11	13		
		-11	-33				29	35	28	34	27	33	26	32
				frequency Hz			583	583	567	567	550	550	533	533
				e=±2			15	18	14	17	14	17	13	16
				e=±3			10	12	9	11	9	11	9	11
		13	39		42	36	43	37	44	38	45	39	46	40
				frequency Hz	600	600	617	617	633	633	650	650	667	667
				e=±2	21	18			22	19			23	20
				e=±3	14	12					15	13		
		13	39				41	35	40	34	39	33	38	32
				frequency Hz			583	583	567	567	550	550	533	533
				e=±2					20	17			19	16
				e=±3							13	11		
3	54	-17	-51		48	54	49	55	50	56	51	57	52	58
				frequency Hz	900	900	917	917	933	933	950	950	967	967
				e=±2	24	27			25	28			26	29
				e=±3	16	18					17	19		
		-17	-51				47	53	46	52	45	51	44	50
				frequency Hz			883	883	867	867	850	850	833	833
				e=±2					23	26			22	25
				e=±3							15	17		
		19	57		60	54	61	55	62	56	63	57	64	58
				frequency Hz	900	900	917	917	933	933	950	950	967	967
				e=±2	30	27			31	28			32	29
				e=±3	20	18					21	19		
		19	57				59	53	58	52	57	51	56	50
				frequency Hz			883	883	867	867	850	850	833	833
				e=±2					29	26			28	25
				e=±3							19	17		
4	72	-23	-69		66	72	67	73	68	74	69	75	70	76
				frequency Hz	1200	1200	1217	1217	1233	1233	1250	1250	1267	1267
				e=±2	33	36			34	37			35	38
				e=±3	22	24					23	25		
		-23	-69				65	71	64	70	63	69	62	68
				frequency Hz			1183	1183	1167	1167	1150	1150	1133	1133
				e=±2					32	35			31	34
				e=±3							21	23		
		25	75		78	72	79	73	80	74	81	75	82	76
				frequency Hz	1200	1200	1217	1217	1233	1233	1250	1250	1267	1267
				e=±2	39	36			40	37			41	38
				e=±3	26	24					27	25		
		25	75				77	71	76	70	75	69	74	68
				frequency Hz			1183	1183	1167	1167	1150	1150	1133	1133
				e=±2					38	35			37	34
				e=±3							25	23		
5	90	-29	-87		84	90	85	91	86	92	87	93	88	94
				frequency Hz	1500	1500	1517	1517	1533	1533	1550	1550	1567	1567
				e=±2	42	45			43	46			44	47
				e=±3	28	30					29	31		
		-29	-87				83	89	82	88	81	87	80	86
				frequency Hz			1483	1483	1467	1467	1450	1450	1433	1433
				e=±2					41	44			40	43
				e=±3							27	29		
		31	93		96	90	97	91	98	92	99	93	100	94
				frequency Hz	1500	1500	1517	1517	1533	1533	1550	1550	1567	1567
				e=±2	48	45			49	46			50	47
				e=±3	32	30					33	31		
		31	93				95	89	94	88	93	87	92	86
				frequency Hz			1483	1483	1467	1467	1450	1450	1433	1433
				e=±2					47	44			46	43
				e=±3							31	29		
6	108	-35	-105		102	108	103	109	104	110	105	111	106	112
				frequency Hz	1800	1800	1817	1817	1833	1833	1850	1850	1867	1867

			e=±2	**51**	**54**			**52**	**55**			**53**	**56**
			e=±3	**34**	**36**					**35**	**37**		
		-35	-105			**101**	**107**	**100**	**106**	**99**	**105**	**98**	**104**
			frequency Hz			1783	1783	1767	1767	1750	1750	1733	1733
			e=±2					**50**	**53**			**49**	**52**
			e=±3							**33**	**35**		
		37	111	**114**	**108**	**115**	**109**	**116**	**110**	**117**	**111**	**118**	**112**
			frequency Hz	1800	1800	1817	1817	1833	1833	1850	1850	1867	1867
			e=±2	**57**	**54**			**58**	**55**			**59**	**56**
			e=±3	**38**	**36**					**39**	**37**		
		37	111			**113**	**107**	**112**	**106**	**111**	**105**	**110**	**104**
			frequency Hz			1783	1783	1767	1767	1750	1750	1733	1733
			e=±2					**56**	**53**			**55**	**52**
			e=±3							**37**	**35**		
		-41	-123	**120**	**126**	**121**	**127**	**122**	**128**	**123**	**129**	**124**	**130**
			frequency Hz	2100	2100	2117	2117	2133	2133	2150	2150	2167	2167
			e=±2	**60**	**63**			**61**	**64**			**62**	**65**
			e=±3	**40**	**42**					**41**	**43**		
		-41	-123			**119**	**125**	**118**	**124**	**117**	**123**	**116**	**122**
			frequency Hz			2083	2083	2067	2067	2050	2050	2033	2033
			e=±2					**59**	**62**			**58**	**61**
7	126		e=±3							**39**	**41**		
		43	129	**132**	**126**	**133**	**127**	**134**	**128**	**135**	**129**	**136**	**130**
			frequency Hz	2100	2100	2117	2117	2133	2133	2150	2150	2167	2167
			e=±2	**66**	**63**			**67**	**64**			**68**	**65**
			e=±3	**44**	**42**					**45**	**43**		
		43	129			**131**	**125**	**130**	**124**	**129**	**123**	**128**	**122**
			frequency Hz			2083	2083	2067	2067	2050	2050	2033	2033
			e=±2					**65**	**62**			**64**	**61**
			e=±3							**43**	**41**		
		-47	-141	**138**	**144**	**139**	**145**	**140**	**146**	**141**	**147**	**142**	**148**
			frequency Hz	2400	2400	2417	2417	2433	2433	2450	2450	2467	2467
			e=±2	**69**	**72**			**70**	**73**			**71**	**74**
			e=±3	**46**	**48**					**47**	**49**		
		-47	-141			**137**	**143**	**136**	**142**	**135**	**141**	**134**	**140**
			frequency Hz			2383	2383	2367	2367	2350	2350	2333	2333
			e=±2					**68**	**71**			**67**	**70**
8	144		e=±3							**45**	**47**		
		49	147	**150**	**144**	**151**	**145**	**152**	**146**	**153**	**147**	**154**	**148**
			frequency Hz	2400	2400	2417	2417	2433	2433	2450	2450	2467	2467
			e=±2	**75**	**72**			**76**	**73**			**77**	**74**
			e=±3	**50**	**48**					**51**	**49**		
		49	147			**149**	**143**	**148**	**142**	**147**	**141**	**146**	**140**
			frequency Hz			2383	2383	2367	2367	2350	2350	2333	2333
			e=±2					**74**	**71**			**73**	**70**
			e=±3							**49**	**47**		

				***p*=**	**4**								
				r=	**0**	r=	**1**	r=	**2**	r=	**3**	r=	**4**
				+	-	+	-	+	-	+	-	+	-
q_1	Z_1	v_b	v'_b										
		-11	-44	**40**	**48**	**41**	**49**	**42**	**50**	**43**	**51**	**44**	**52**
			frequency Hz	600	600	613	613	625	625	638	638	650	650
			e=±2	**20**	**24**			**21**	**25**			**22**	**26**
			e=±3		**16**			**14**		**14**	**17**		
		-11	-44			**39**	**47**	**38**	**46**	**37**	**45**	**36**	**44**
			frequency Hz			588	588	575	575	563	563	550	550
2	48		e=±2					**19**	**23**			**18**	**22**
			e=±3			**13**					**15**	**12**	
		13	52	**56**	**48**	**57**	**49**	**58**	**50**	**59**	**51**	**60**	**52**
			frequency Hz	600	600	613	613	625	625	638	638	650	650
			e=±2	**28**	**24**			**29**	**25**			**30**	**26**
			e=±3		**16**	**19**					**17**	**20**	
		13	52			**55**	**47**	**54**	**46**	**53**	**45**	**52**	**44**
			frequency Hz			588	588	575	575	563	563	550	550

			e=±2					**27**	**23**			**26**	**22**
			e=±3					**18**			**15**		
3	72	-17	-68	**64**	**72**	**65**	**73**	**66**	**74**	**67**	**75**	**68**	**76**
			frequency Hz	900	900	913	913	925	925	938	938	950	950
			e=±2	**32**	**36**			**33**	**37**			**34**	**38**
			e=±3		**24**			**22**			**25**		
		-17	-68			**63**	**71**	**62**	**70**	**61**	**69**	**60**	**68**
			frequency Hz			888	888	875	875	863	863	850	850
			e=±2					**31**	**35**			**30**	**34**
			e=±3			**21**					**23**	**20**	
		19	76	**80**	**72**	**81**	**73**	**82**	**74**	**83**	**75**	**84**	**76**
			frequency Hz	900	900	913	913	925	925	938	938	950	950
			e=±2	**40**	**36**			**41**	**37**			**42**	**38**
			e=±3		**24**	**27**					**25**	**28**	
		19	76			**79**	**71**	**78**	**70**	**77**	**69**	**76**	**68**
			frequency Hz			888	888	875	875	863	863	850	850
			e=±2					**39**				**38**	**34**
			e=±3					**26**			**23**		
4	96	-23	-92	**88**	**96**	**89**	**97**	**90**	**98**	**91**	**99**	**92**	**100**
			frequency Hz	1200	1200	1213	1213	1225	1225	1238	1238	1250	1250
			e=±2	**44**	**48**			**45**	**49**			**46**	**50**
			e=±3		**32**			**30**			**33**		
		-23	-92			**87**	**95**	**86**	**94**	**85**	**93**	**84**	**92**
			frequency Hz			1188	1188	1175	1175	1163	1163	1150	1150
			e=±2					**43**	**47**			**42**	**46**
			e=±3			**29**					**31**	**28**	
		25	100	**104**	**96**	**105**	**97**	**106**	**98**	**107**	**99**	**108**	**100**
			frequency Hz	1200	1200	1213	1213	1225	1225	1238	1238	1250	1250
			e=±2	**52**	**48**			**53**	**49**			**54**	**50**
			e=±3		**32**	**35**					**33**	**36**	
		25	100			**103**	**95**	**102**	**94**	**101**	**93**	**100**	**92**
			frequency Hz			1188	1188	1175	1175	1163	1163	1150	1150
			e=±2					**51**	**47**			**50**	**46**
			e=±3					**34**			**31**		

A simple scheme can be used to determine the frequencies. Consider (15). Where the resulting number r is created as the difference between the stator and rotor waves, the frequency is simply their difference, i.e. $abs(e)\cdot Z_2/p\cdot 50$ Hz , if the mains frequency is 50 Hz. This is created when the two waves are the same sign.

Where the resulting order $\underline{r}$ is created as the sum of the stator and rotor waves, the frequency differs from the previous one by ±100 Hz: that is, $abs(e)\cdot Z_2/p\cdot 50$ Hz ±100 Hz. This is created when the two waves are different sign.

Arrange the formulas in a small table.

			ν	
			+	-
μ	+	$e>0$	$abs(e)\cdot Z_2/p\cdot 50Hz$	$abs(e)\cdot Z_2/p\cdot 50Hz+100Hz$
	-	$e<0$	$abs(e)\cdot Z_2/p\cdot 50Hz-100Hz$	$abs(e)\cdot Z_2/p\cdot 50Hz$

It must be specifically emphasized that the phenomenon is not simply that the frequency increases with further values of coefficient e=±2 and e=±3. As $\mu_a{}'$ increases, this now creates a low-order force wave with another $\nu_b{}'$. In other words, the respective slot number is likely to "move" under another order r with a different natural frequency.

C. Radial Magnetic Forces – a Numeric Example

TABLE V
NUMERIC EXAMPLE FOR CALCULATING THE PROPORTIONALITY FACTORS OF RADIAL MAGNETIC FORCES WHEN NO-LOAD AND AT LOAD

TABLE VA:
BASIC TABLE OF PROPORTIONALITY FACTORS

p=3	**ν_a=1**	$\mu_a{}'=eZ_2+p$				
q_1=3	q_2= 2 1/3	e=-1	e=1	e=-2	e=2	
Z_1=54	Z_2=42	$\mu_a{}'$	$\mu_a{}'$	$\mu_a{}'$	$\mu_a{}'$	
ν_b	$\nu_b{}'$	-39	45	-81	87	
-11	-33					
13	39	r=+0 f=600 Hz* $C=(1-\eta_{13}^2)\frac{\xi_1\xi_{13}}{\xi_1^2}\frac{\eta_1^2}{\mu_a\nu_b}$ $C\cdot 10^5$=**131**				ξ_1=0.96 ξ_{13}=0.217 $\eta_1{}^2$=0.98 μ_a=-13 ν_b= 13 $\eta_{13}{}^2$=0.006
-17	-51					
-29	-87				r=-0 f=1500 Hz* $C=\frac{\xi_1\xi_{29}}{\xi_1^2}\frac{\eta_1^2}{\mu_a\nu_b}$ $C\cdot 10^5$=**21**	ξ_1=0.96 ξ_{13}=-0.178 $\eta_1{}^2$=0.98 μ_a= 29 ν_b=-29
31	93					

p=3	**ν_a=-5**	$\mu_a{}'=eZ_2+\nu_a p$				
q_1=3	q_2= 2 1/3	e=-1	e=1	e=-2	e=2	
Z_1=54	Z_2=42	$\mu_a{}'$	$\mu_a{}'$	$\mu_a{}'$	$\mu_a{}'$	
ν_b	$\nu_b{}'$	-57	27	-99	69	
-17	-51					
19	57	r=+0 f=600 Hz				ξ_5=0.217 ξ_{19}=0.96 $\eta_5{}^2$=0.64 μ_a=-19

		$C = \frac{\xi_5 \xi_{19}}{\xi_1^2} \frac{\eta_5^2}{\mu_a \nu_b}$ $C \cdot 10^5=$**40**				$\nu_b= 19$
-23	-69				r=-0 f=1500 Hz $C = \frac{\xi_5 \xi_{23}}{\xi_1^2} \frac{\eta_5^2}{\mu_a \nu_b}$ $C \cdot 10^5=$**6**	ξ_5=0.217 ξ_{23}=0.217 η_5^2=0.64 μ_a= 23 ν_b=-23
25	75					

*$f=(2+e \cdot Z_2/p) \cdot 50$ Hz

TABLE VB
TABLE OF PROPORTIONALITY FACTORS AT NO-LOAD

p=3	ν_a=1	$\mu_a'=eZ_2+\nu_a p$				
q_1=3	q_2= 2 1/3	e=-1	e=1	e=-2	e=2	
Z_1=54	Z_2=42	μ_a'	μ_a'	μ_a'	μ_a'	
ν_b	ν_b'	-39	45	-81	87	
13	39	r=+0 f=600 Hz $C \cdot 10^5 \cdot \cos\varphi_0$ **~ 13**				cos φ_0~0.1
25	75					
-29	-87				r=-0 f=1500 Hz $C \cdot 10^5 \cdot \cos\varphi_0$**~2**	
31	93					
p=3	**ν_a=-5**	$\mu_a'=eZ_2+\nu_a p$				
q_1=3	q_2= 2 1/3	e=-1	e=1	e=-2	e=2	
Z_1=54	Z_2=42	μ_a'	μ_a'	μ_a'	μ_a'	
ν_b	ν_b'	-57	27	-99	69	
-17	-51					
19	57	r=+0 f=600 Hz $C \cdot 10^5 \cdot$ $\cdot(-1+\nu_{b*}0.45)$ =**302**				
-23	-69				r=-0 f=1500 Hz $C \cdot 10^5=$**6**	

TABLE VC
TABLE OF PROPORTIONALITY FACTORS AT RATED LOAD

p=3	ν_a=1	$\mu_a'=eZ_2+\nu_a p$				
q_1=3	q_2= 2 1/3	e=-1	e=1	e=-2	e=2	
Z_1=54	Z_2=42	μ_a'	μ_a'	μ_a'	μ_a'	
ν_b	ν_b'	-39	45	-81	87	
13	39	r=+0 f=600 Hz $C \cdot 10^5 \cdot 10=$ **1114**				$(I_n/I_0)^2=10$ cos φ_r=0.85
-17	-51					
-29	-87				r=-0 f=1500 Hz $C \cdot 10^5 \cdot 10=$ **184**	$(I_n/I_0)^2=10$
p=3	ν_a=-5	$\mu_a'=eZ_2+\nu_a p$				
q_1=3	q_2= 2 1/3	e=-1	e=1	e=-2	e=2	
Z_1=54	Z_2=42	μ_a'	μ_a'	μ_a'	μ_a'	
ν_b	ν_b'	-57	27	-99	69	
-17	-51					
19	57	r=+0 f=600 Hz *$C \cdot 10^5=$ **364**				cosφ_n~0.85 φ_n~32°
-23	-69				r=-0 f=1500 Hz $C \cdot 10^5 \cdot 10=$ **62**	$(I_n/I_0)^2=10$

*Calculation of proportionality factor C at load for slot harmonics

VI. SUMMARY

It were derived hitherto non-existent formulas for calculating synchronous torques and radial magnetic forces before. Now those formulas were used for further considerations.

It has been shown that synchronous parasite torque (as well as the asynchronous parasite torque) should be related not to the starting torque, but to the breakdown torque.

It has been shown that it is not the slot combination that the creation of synchronous torque and radial magnetic forces depends on, but in this approach, only the number of rotor slots itself.

Design guides were provided for both phenomena in subject.

The correlation between synchronous torque and radial magnetic forces, resulting from the physics of the phenomena, has been proven.

As for future perspectives, it is intended to perform further analysis with different calculation methods in order to verify subject new formulas.

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