Harmonics in the Squirrel Cage Induction Motor Analytic Calculation - Part I: Differential Leakage, Attenuation, Asynchronous Parasitic Torques

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Abstract—**The magnetic field generated in the air gap of the cage asynchronous machine and the harmonics of the magnetomotive forces creating that magnetic field, as well as the related differential leakage, attenuation, asynchronous parasitic torques have been discussed in great detail in the literature, but always separately, for a long time. However, systematization of the phenomenon still awaits. Therefore, it is worth summarizing the completeness of the phenomena in a single study – with a new approach at the same time - in order to reveal the relationships between them. The role of rotor slot number is emphasized much more than before. An existing, commonly used, but still impractical basic figure has been modified to more clearly demonstrate the response of the rotor for the harmonics of the stator. The need to treat differential leakage, asynchronous parasitic torques and attenuation together will be demonstrated: new formula for asynchronous parasitic torque is derived; the long-used characteristic curves for differential leakage and attenuation used separately so far was merged into one, correct curve in order to provide a correct design guide for the engineers.**

***Index Terms*— Asynchronous parasitic torque, Attenuation, Differential leakage, Squirrel cage induction motor, Winding harmonics.**

I. Introduction

The aim of the article is to summarize and mainly systematize the phenomena given in the title in one single study. Since all subject phenomena can be traced back to a single starting point, namely the interaction of stator harmonics and rotor harmonics, it is reasonable to discuss the problem in a unified manner.

However, during the study of the previous works in the literature, it became clear that basic formulas are missing, basic relationships are not explored, and important effects are not taken into account. The goal was therefore to fill the gap, to supplement the missing parts and to include the entire investigation in a unified framework.

The article is a continuation of the Author's previous works [1], [2]. Those articles were developed mainly based on [3] as a very basic work. There are, of course, many valuable works that should also be used when dealing with such a broad topic [4]-[7], the other countless valuable works cannot be listed here for reasons of length.

In the following study, only the winding harmonics will be dealt with, and only the impact on the subject phenomena will be analyzed, with the usual assumptions. Constant voltage and frequency supply will be assumed. A machine that is symmetrical in all aspects will be taken into consideration, with non-skewed rotor slots; very small slot width, without saturation; phenomena resulting from possible parallel and/or delta connection of the stator (so-called secondary armature reaction) will not be included. To be more precise, a machine in which only winding space harmonics according to (1) and (2) occur will be taken as subject; with no time harmonics. The goal is to put the fundamental findings, which are now published first to the point. And also those, which have been known before, but will now be put in a different light. It is intended not to distract attention by what might be important in itself but can only be called a question of detail now. All these can then be taken into account later, in a way already found in the literature.

II. Symbols

R_1	Stator ohmic resistance
X_{s1} - $X_{1\sigma}$	Stator leakage without differential leakage
X_m	Magnetizing reactance
X_{s2}’	Rotor leakage
$X_{\sigma 2}$’	Rotor differential leakage
R_2’	Rotor resistance reduced to stator
$X_{m\nu}=X_m \cdot 1/\nu^2 \cdot \xi_\nu^2/\xi_1^2$	Magnetizing reactance of harmonic ν
$X_{\sigma 2\nu}$	Rotor differential leakage of harmonic circuit ν not reduced to stator
$R_{2\nu}=R_2 \cdot \xi_\nu^2/\xi_1^2$	Rotor resistance of harmonic circuit ν not reduced to stator
ξ_1, ξ_ν	Winding factor of fundamental wave, harmonic wave
$\eta_{2\nu}^2$	Jordan’s coupling factor
Δ	Attenuation factor
s, s_ν	Slip of rotor to fundamental harmonic of stator, to harmonic ν of stator
ν, μ	Designation of stator harmonics, rotor harmonics
ε	Small positive number $\varepsilon \ll 1$
a, b	Designation of harmonics in interaction
g_1, g_2	Different integers; e_1 integer
δ’	Equivalent air-gap
p	Number of pole pairs
Z_1, Z_2	Stator / rotor slot number
m	Number of phases
q_1, q_2’	Stator / rotor slot number per pole per phase

Manuscript received January 29, 2023; revised February 23, 2023; accepted March 06, 2023. Date of publication September 25, 2023; date of current version June 28, 2023.

The author gratefully acknowledges the contributions of István Laczkó, Chief Engineer of the United Electric Machine Factory, EVIG, Budapest, Hungary, for his valuable comments.

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Digital Object Identifier 10.30941/CESTEMS.2023.00034

III. Equivalent Circuit

If harmonics are to be taken into account, the equivalent circuit derived for the fundamental harmonic shall be extended by such circuits representing the higher harmonics and connected them into series. Consider Fig. 1.

The leakage reactance of the ν^{th} harmonic circuit includes only the differential leakage reactance; the slot leakage and the end-winding leakage may be neglected [3]-[5].

The equivalent circuit was drawn here in a different way from the usual one, both from a didactic point of view and to bring it closer to physical background. The harmonic circuits were placed first and then the fundamental equivalent circuit, to emphasize that they are leakage from the stator's point of view. At the same time, those were placed above the fundamental harmonic magnetization reactance, vertically in line with it, to express that the phenomenon takes place in the air gap, contrary to the classical leakage definition; these fields do reach the rotor, but still do not contribute to the useful fundamental harmonic operation.

Orders of the MMF harmonics of stator in usual cases

$$\nu = 6g + 1 \tag{1}$$

where g= 0, ±1, ±2 etc., ν are the same for all machines.

Orders of the MMF harmonics of rotor in usual cases

$$\mu = e \cdot Z_2/p + \nu = e \cdot 2mq_2' + \nu \tag{2}$$

where e= 0, ±1, ±2 etc. μ are different from rotor to rotor depending from rotor slot number.

From now on the calculations will be based on q_2' (rotor slot per phase per pole), since it is more general and much better suited to the approach of a machine designer.

The fundamental harmonic of the stator with g=0 and that belonging to the rotor with e=0 are included in the fundamental circuit. The harmonics of the stator with g=±1, ±2, etc. and that belonging to the rotor also with e=0 are included in the harmonic circuits. The rest, with e= ±1, ±2, etc. rotor harmonics are not part of the equivalent circuit.

All the harmonics of the stator can be part of the diagram because they induce a mains frequency voltage in the stator in the same way as all other elements of the diagram.

The cage rotor responds to the stator harmonic by generating torque, while according to Lenz's law it also reduces the amplitude of the harmonics. The elements of the rotor reduced to the stator can be the elements of the equivalent circuit because they can always be converted from the frequency occurring in the rotor to the mains frequency, 50/60 Hz. The others, e= ±1, ±2, etc. rotor harmonics are of basic importance for the calculation of synchronous parasitic torques and radial magnetic forces, but since the stator does not react to them (no secondary armature reaction now), they induce a frequency other than mains frequency and therefore they cannot be reduced to mains frequency. They are only mentioned as a reference next to the relevant harmonic circuit; this mentioning will support the evaluations and explanations later.

Any asynchronous motor can be, therefore, represented by a series of mechanically connected induction motors having different pole numbers, whose stator windings are connected in series [5], [6].

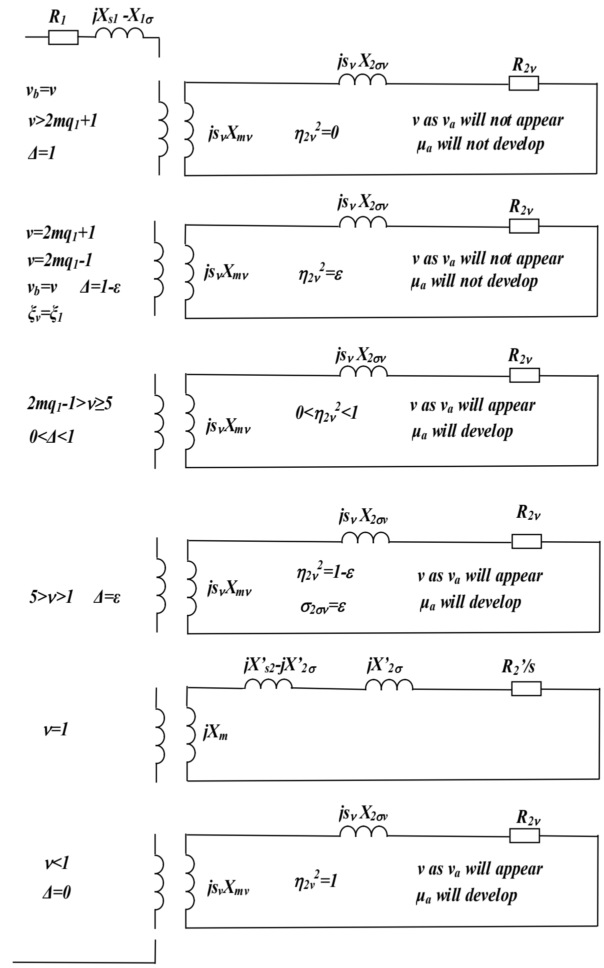


Fig. 1. Equivalent circuit including the circuits for the higher harmonics.

IV. Differential Leakage, Asynchronous Parasitic Torque, Attenuation

A. Differential Leakage

The stator differential leakage may be calculated by the winding factors as follows:

$$X_{\sigma 1} = \sigma_1 \cdot X_m \tag{3}$$

where as definition

$$\sigma_1 = \Sigma \frac{1}{\nu^2} \frac{\xi_\nu^2}{\xi_1^2} \quad \text{but } \nu \neq 1$$

However, the rotor differential leakage, especially for harmonics, cannot be expressed in this way. Therefore, the other method, the Görges diagram, is used and the differential leakage of the rotor is expressed as the quotient of the energy of the harmonics and the fundamental harmonic:

$$X_{\sigma 2\nu} = \sigma_{2\nu} \cdot X_{m\nu} \tag{4}$$

where

$$\sigma_{2\nu} = \frac{1}{\eta_{2\nu}^2} - 1 \quad \text{(see (268) [3], pp. 154)}$$

Of course, $\sigma_{2\nu}$ calculated with ν=1 (being a very small value) is included in the fundamental harmonic circuit.

The $\eta_{2\nu}{}^2$ factor will be needed in all considerations and formulas from now on, so it will be drawn and analyzed separately. The factor is called the Jordan coupling factor. Based on [3].

$$\eta_{2v} = \frac{\sin v \frac{p\pi}{Z_2}}{v \frac{p\pi}{Z_2}} = \frac{\sin v \frac{\pi}{2mq_2'}}{v \frac{\pi}{2mq_2'}} \tag{5}$$

As v increases, the value of η_{2v}^2 gradually becomes negligible as v approaches $Z_2/p=2mq_2'$.

As it can be seen, if $Z_2=2mq_1+1$ or $Z_2=2mq_1-1$, and ν reaches $v=2mq_1+1$ or $v=2mq_1-1$ then $\eta_{2v}^2=0$, the v^{th} harmonic cannot drive current in the cage.

According to the physical background, if $vp/Z_2=1$ (or any further integer), then the electric angle between two adjacent cage bars is exactly 2π, so this harmonic does not drive current in the rotor cage.

The value of η_{2v}^2 can be taken as zero for harmonics with a higher order number than this, i.e. these harmonics - more precisely, the rotor currents generated by these harmonics - do not need to be dealt with.

This appears in (4) in the way that the rotor differential leakage factor σ_{2v}, if $v\geq 2mq_1\pm 1$, becomes very large, even infinite. Compare with [5], see Fig. 17, as well, this means also that these harmonics are not attenuated by the rotor.

However, Fig. 2 is difficult for practice to follow. Therefore, η_{2v}^2 is transformed as a function of q_2', which is much more expressive for the designer, see Fig. 3. In turn, ν is then taken as a parameter. Thus, it is possible to show directly, in a completely universal and illustrative way, which harmonics the rotor responses to and, in particular, to what extent.

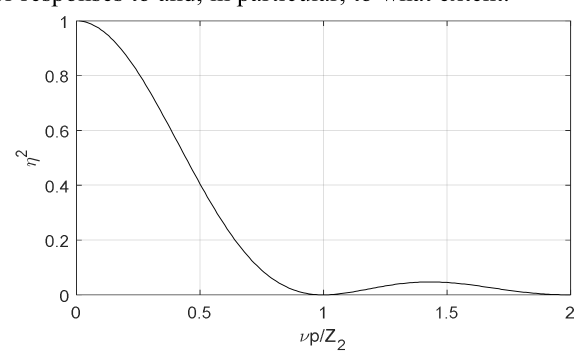


Fig. 2. Plotting the value of η_{2v}^2 as a function of vp/Z_2 ([3] p.154. Fig. 107).

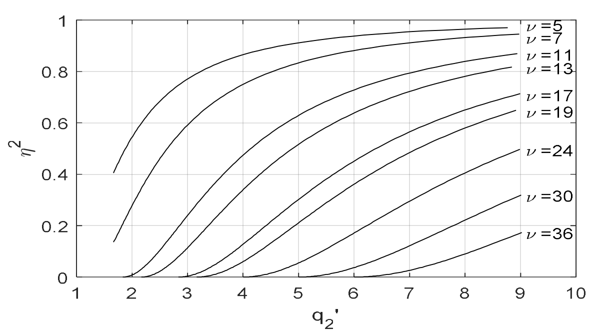


Fig. 3. Representation of the value of η_{2v}^2 as a function of q_2' with ν as a parameter. Higher odd harmonics are replaced mathematically by adjacent (actually with 3-phase not existing) even harmonics only for better transparency.

B. Calculation of Asynchronous Parasitic Torque

Consider the v-th circle. Now all its elements and I_1 also, are reduced to the rotor.

Sum of the reactance in a harmonic circuit:

$$X_{2mv} + X_{\sigma 2v} = \ldots = \frac{X_{mv}}{\eta_{2v}^2} \tag{6}$$

Acc. to voltage equation:

$$I_{2v} = -j\frac{s_v X_{2mv}}{R_{2v} + js_v(X_{2mv} + X_{2\sigma v})} I_1 \tag{7}$$

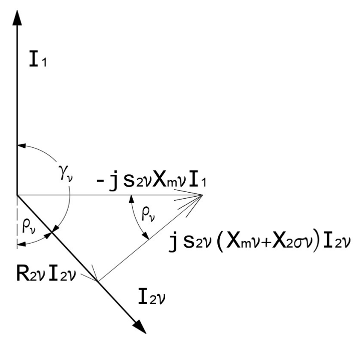


Fig. 4. Vector diagram of stator current and rotor current for the circuit v ([3], Fig. 128).

γ_{va} and ρ_{va} resp. is the angle between stator current and rotor harmonic current.

The end point of I_{2v} describes a circle as a function of s_v, see Fig. 5.

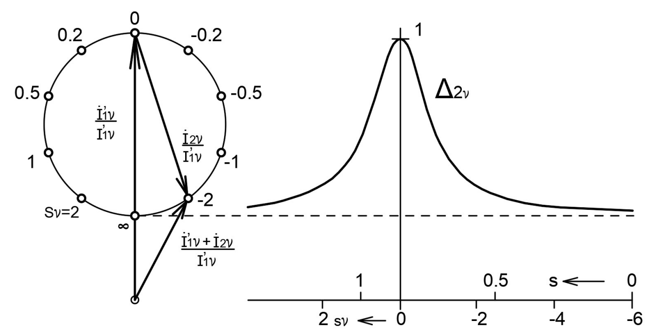


Fig. 5. Vector diagram of the currents of the harmonic circuit v=7 (see [3] Richter Vol. IV p. 150, Fig. 106) and the attenuation factor Δ belonging to this circuit (=to this harmonic).

It is clear that I_{2v} will not be in phase with I_1 (except, of course, the points s_v=0 and $s_v=\infty$). It is easy to see that a torque is thereby generated, namely through the component of I_{2v} perpendicular to I_1. This will obviously be an asynchronous torque, i.e. it occurs at all rotor speeds.

Let us choose I_1 as real. Then the imaginary part of I_{2v} will be proportional to the torque. Divide each term of the expression (7) by the sum of the reactance

$$I_{2v} = -j\frac{s_v X_{2mv} / (X_{2mv} + X_{2\sigma v})}{R_{2v} / (X_{2mv} + X_{2\sigma v}) + js_v(X_{2mv} + X_{2\sigma v}) / (X_{2mv} + X_{2\sigma v})} I_1 \tag{8}$$

The first member of the denominator is obviously the breakdown slip, since the resistance and overall reactance in the circle will be the same, giving out the maximum torque.

This harmonic breakdown slip shall be expressed later in terms of the circuit resistance and reactance of the fundamental harmonic circuit. Using (6)

$$s_{b\nu}=\frac{R_{2\nu}}{(X_{2m\nu}+X_{2\sigma\nu})}=\frac{R_2}{X_m}\nu^2\eta_{2\nu}^2 \tag{9}$$

Returning to (8), introducing $s_{b\nu}$ and arranging

$$I_{2\nu}=-j\frac{s_\nu\cdot\eta_{2\nu}^2}{s_{b\nu}+js_\nu}I_1=...=(-\frac{s_\nu^2\cdot\eta_{2\nu}^2}{s_{b\nu}^2+s_\nu^2}-j\frac{s_\nu s_{b\nu}\cdot\eta_{2\nu}^2}{s_{b\nu}^2+s_\nu^2})I_1 \tag{10}$$

Arranged further

$$I_{2\nu}=(-\frac{s_\nu/s_{b\nu}\cdot\eta_{2\nu}^2}{\frac{s_{b\nu}}{s_\nu}+\frac{s_\nu}{s_{b\nu}}}-j\frac{\eta_{2\nu}^2}{\frac{s_{b\nu}}{s_\nu}+\frac{s_\nu}{s_{b\nu}}})I_1 \tag{11}$$

$$\begin{cases} I_{2\nu}=(-\frac{\eta_{2\nu}^2}{2}-j\frac{\eta_{2\nu}^2}{2})I_1 & \text{At } s_\nu=s_{b\nu} \\ I_{2\nu}=(-\frac{\eta_{2\nu}^2}{2}+j\frac{\eta_{2\nu}^2}{2})I_1 & \text{At } s_\nu=-s_{b\nu} \\ I_{2\nu}=0 & \text{At } s_\nu=s_{b\nu} \\ I_{2\nu}=\eta_{2\nu}{}^2\cdot I_1 & \text{At } s\approx 0, \text{ that means at } s_\nu=\infty \end{cases} \tag{12}$$

Physically, this is when the largest attenuation occurs, since $I_{2\nu}$ is then the largest and at the same time is in opposite phase to the current I_1. However, this category - since it was introduced in the literature on another way - will be clarified later.

It can be seen that the radius of the circle is: $\eta_{2\nu}{}^2/2\cdot I_1$.

Substituting $I_{2\nu}$ at $s_{b\nu}$, the maximum effective power

$$P_{max}=3\cdot U_{1\nu}\cdot I_{2\nu}=3\cdot I_1 jX_{2m\nu}(-j\frac{\eta_{2\nu}^2}{2})I_1 \tag{13}$$

It can be seen from the formula that the imaginary part of $I_{2\nu}$ will indeed be parallel to the vector $I_1{\cdot}jX_{m\nu}$ (since I_1 was assumed to be real). The maximum torque

$$M_{b\nu}=P_{\max}/(\omega_0/(p\nu)) \tag{14}$$

In the formula for P_{max}, the reduction factors for I_1 and $X_{m\nu}$ just cancel each other out, since the reduction, by definition, takes place in such a way that the power (either real or imaginary) remains unchanged. Therefore, the calculation can be performed with reduced values either for the stator or for the rotor.

For the same reason, the introduction of $s_{b\nu}$ also proved to be expedient, since it is also a relative number. At most, it had to be taken into account that in the ν^{th} circuit, R_2 is only recalculated in proportion to the square of the number of turns and winding factors, while X_m is recalculated further in proportion to ν^2.

The slip scale is completely different from the fundamental harmonic, and the two semicircles, $s_\nu>0$ and $s_\nu<0$, are completely symmetrical; that means both breakdown torques, no matter motoric or generatoric, are the same (reason is that R_1 has no role here).

The question arises, regarding $R_{2\nu}$ how to proceed with a deep bar or double-cage machine. In the case of small value of harmonic slip s_ν~0 the low-frequency distribution must be calculated, while of high value of harmonic slip $s_\nu=\infty$ the high-frequency distribution.

No noticeable error will be committed if the current I_1 is taken as the starting current.

$$I_1=U/X_s \tag{15}$$

where X_s is the entire leakage reactance.

Further

$$X_{m\nu}=X_m\frac{1}{\nu^2}\frac{\xi_\nu^2}{\xi_1^2} \tag{16}$$

Substituted

$$M_{\max}=3\cdot\frac{U^2}{X_s^2}X_m\frac{1}{\nu^2}\frac{\xi_\nu^2}{\xi_1^2}\frac{\eta_{2\nu}^2}{2}\frac{p\nu}{\omega_0} \tag{17}$$

This torque will be (completely different to the literature) compared to the fundamental breakdown torque

$$M_b=\frac{3}{2}\frac{U^2}{X_s}\frac{1}{\omega_0/p} \tag{18}$$

It yields finally the harmonic breakdown torque

$$\frac{M_{b\nu}}{M_b}=\frac{X_m}{X_s}\frac{\xi_\nu^2}{\xi_1^2}\frac{\eta_{2\nu}^2}{\nu} \tag{19a}$$

Substituting the imaginary part of (11) into (13) the torque-slip characteristic due to that stator harmonic is obtained. The final formula

$$\frac{M_{b\nu}}{M_b}=\frac{X_m}{X_s}\frac{\xi_\nu^2}{\xi_1^2}\frac{\eta_{2\nu}^2}{\nu}(\frac{2}{\frac{s_{b\nu}}{s_\nu}+\frac{s_\nu}{s_{b\nu}}}) \tag{19b}$$

Using the expression $s_\nu=1-\nu(1-s)$ it yields that (after substituting and arrangement) the slip of this harmonic breakdown torque will be

$$s=\pm\frac{s_{b\nu}}{\nu}+1-\frac{1}{\nu}=\pm\frac{s_{b\nu}}{\nu}+s(s_\nu=0) \tag{20}$$

In an equal distance $\pm s_{b\nu}/\nu$ next to the slip $s_\nu=0$.

Let us also examine the value of $s_{b\nu}$, i.e. the course of the slip scale based on (9). This obviously depends (by $\eta_{2\nu}{}^2$) on q_2', as well as on the ν^{th} harmonic cycle currently under investigation. Taking R_2 as 0.02 – 0.06 and X_m as 3-4 in relative units, it can be seen that $s_{b\nu}$ varies within wide limits, $s_{b\nu}>1$ is also possible for a small machine (R_2~0.06), and generally $s_{b\nu}<1$ for a large machine (R_2~0.02). If $s_{b\nu}$~1 means that the parasitic torque of that harmonic has perceptible effect at the starting torque test in the test room.

The torques generated by the 7[th], 13[th], 19[th], etc. harmonics fall into the motor range. Among these, the torque of the 7[th] harmonic can be eliminated by chording, if necessary, the rest decrease with the increase of the harmonic order.

Let us calculate two values with the data of Fig. 4.

R_2 = 0.066, X_m=3, X_s=0.2 in relative units.

1) For 7[th] harmonic with no chording
for q_2'~ 3:
ξ_7/ξ_1=0.184; $\eta_{2,7}{}^2$=0.6 (see Fig. 3.):
$M_{br.d.7}/M_{br.d.1}$ = 3/0.2·0.184²·0.6/7=0.044
roughly 10% of rated torque.
s_ν=0 falls to s=6/7=0.857, close to standstill.
$s_{b\nu}$=0.066/3·7²·0.6=0.65; $s_{b\nu}$/7=0.093;
positive peak value is at s=0.95, *that* of negative one at s=0.764.

2) For the torque of the slot harmonic (note that it is zero for $Z_2<Z_1$).
also for q_2'~ 3:
ξ_{19}/ξ_1=1; $\eta_{2,19}{}^2\approx$0.05 (see Fig. 3.): if $Z_2>Z_1$
$M_{br.d.19}/M_{br.d.1}$=3/0.2·1²·0.05/19=0.039, (roughly 0.09M_{rated}),

slightly less than the previous one.

s_{bv}=0 falls to s=18/19=0.947 , very close to standstill.

s_{bv}=0.066/3·19²·0.05=0.397, s=0.397/19=0.021.

Positive peak value is at s=0.968, that of negative one at s=0.926.

These two torques add up together (if Z_2>Z_1) only above s=0.947. In this range, the torque of the machine can increase by up to 10-15% of the rated torque. In the test room, at s=1, it can be measured up to 10% more than the fundamental harmonic starting torque; approx. half of it comes from the slot harmonic. For large machines, (with R_2~0.02), both s_{bv} are approx. a third of the above, then the two torque curves do not "slip together", both torques appear as separate torque peaks.

Consider Fig. 6 as an *illustration* of the entire phenomenon discussed in this chapter. In Fig. 6(a), the small vector diagram of Fig. 5 was copied (not inserted) onto the usual circle diagram, with the circle described by I_{2v}, at some characteristic slips. In Fig. 6b, the vector diagrams appearing at s_v, $+s_{bv}$, and $-s_{bv}$ are shown enlarged.

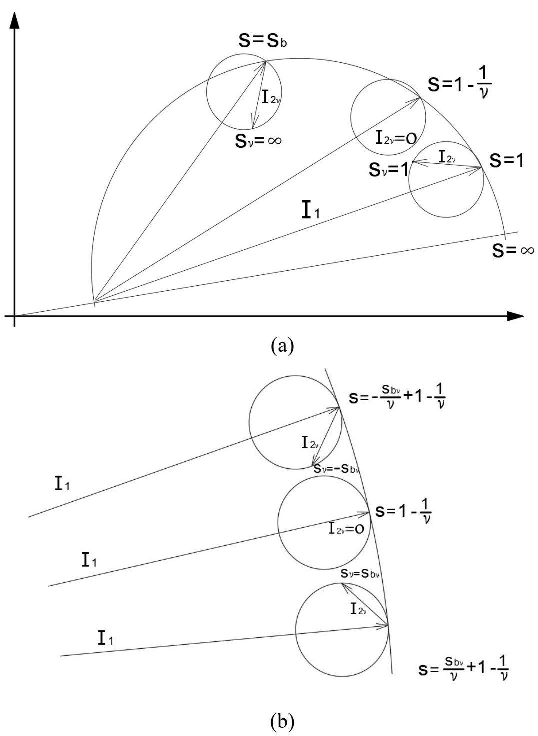


Fig. 6. Copying the v^{th} harmonic circuit (v>0) onto the circuit diagram, merely for illustration.

C. Attenuation

1) Interpretation of Attenuation

In contrast to the fundamental harmonic circuit, which is of a voltage source type and therefore attenuation cannot be interpreted, harmonic circuits are current source type arrangements, and therefore attenuation is one of their properties.

As can be seen from Fig. 5, in the case of s_v=0, no rotor current flows, therefore there is no attenuation, the rotor speed is in synchronism with respect to the v^{th} field. The largest attenuation is at the point s_v=∞, at which point I_{2v} has a completely demagnetizing effect.

When calculating the differential leakage, the attenuation is usually important around $s \approx s_b$, for the accurate calculation of the breakdown torque. Based on the relationship s_v=1-v(1-s), since at s=0 s_v=1-v no noticeable mistake will be made if this point is made as equivalent to the point s_v=∞ . At this point

$$I_{2v} = -\eta_{2v}^2 \cdot I_1 \, , \; \frac{I_1 + I_{2v}}{I_1} = 1 - \eta_{2v}^2 \tag{21}$$

That is, the v^{th} field is attenuated by $-\eta_{2v}^2$, expressed on another way it decreases acc. to a factor of

$$\Delta_v = (1 - \eta_{2v}^2) \tag{22}$$

It must be clarified here that it is not I_1 which would decrease in the above Δ_v ratio, in fact it does not decrease at all, because one element of the differential leakage is negligibly small compared to the entire circle. The (vector) voltage equation should have been correctly written as

$$\frac{I_1 jX_{mv} + I_{2v} jX_{mv}}{I_1 jX_{mv}} = 1 - \eta_{2v}^2 \tag{23}$$

This equation shows that the voltage across the harmonic reactance is the one that decreases by a factor of Δ_v. This phenomenon is treated *as* if the reactance jX_{mv} had decreased by a factor of Δ_v.

According to this, the differential leakage is attenuated by a value of

$$\sum_{v \neq 1}^{2mq_1+1} \eta_{2v}^2 \frac{1}{v^2} \frac{\xi_v^2}{\xi_1^2} X_m \tag{24}$$

that means it will be less by this value. Components of differential leakage shall be calculated acc. to

$$\sum_{v \neq 1}^{2mq_1+1} \frac{1 - \eta_{2v}^2}{v^2} \frac{\xi_v^2}{\xi_1^2} X_m \tag{25}$$

The fields v>2mq_1+1 are practically not attenuated.

After rearrangement, the definition of the attenuation factor according to Richter is obtained

$$\Delta = 1 - \frac{1}{\sigma_1} \sum_{v \neq 1}^{2mq_1+1} \eta_{2v}^2 \frac{1}{v^2} \frac{\xi_v^2}{\xi_1^2} \tag{26}$$

where by definition

$$\sigma_1 = \sum_{v \neq 1}^{\infty} \frac{1}{v^2} \frac{\xi_v^2}{\xi_1^2} \tag{27}$$

According to Richter [3], this value can also be given with a closed formula.

Without chording

$$\sigma_1 = \frac{5q_1 + 1}{12q_1} \tag{28}$$

with chording by $k_2 \leq q_1$ slot

$$\sigma_1 = \frac{(5q_1 + 1) + k_2^3 / 4q_1 - 3k_2^2 / 2 - k_2 / 4q_1}{12q_1} \tag{29}$$

Otherwise, this formula, like (4) and (5), was derived using the Görges diagram, as the ratio of the energy of the harmonics to that of the fundamental harmonic.

It follows from the derivation that the above calculation can be considered accurate at no-load and in operation, but not at low speeds. The synchronous speed of the harmonic fields is very close to the s=1 point and at start-up the motor passes through the non-attenuated s_v=0 points one after the other, see Fig. 5. These are e.g. in the case of q_1=3, the s_v=0 points of the harmonics v=13 and v=7 occurring on the slips s=1-1/13 and s=1-1/7. This means that then the attenuation factor for that harmonic is much closer to 1; on those slips is exactly 1 i.e. the total leakage reactance and the starting current fluctuate slightly at the beginning of the start-up. During short-circuit (i.e. in the test room), this is detected only to the extent as much the attenuated value of the differential leakage at s=1 differs from the operating value at s~0, see also Fig. 5, where the shape of the curve corresponds to the ratios that exist in practice (although for a low-power motor was drawn). This can also be verified by calculation if $s_\nu = s = 1$ is substituted in (10).

2) Diagram of Attenuation Factor Effect of Chording

[3] Richter IV. p. 155. Fig. 108 (calculated acc. to (26)) is generally used to consider attenuation; it is usually adopted, always without changes, in the literature, e.g. [5] p. 48. Fig. 21, [6] p. 154 Fig. 6.5., [7] p. 233. Fig. 4.7. The attenuation factor is given in function of rotor slot number, with q_1 as parameter.

This time, however, there is a need to make a number of changes, both in form and content as follows:

On the horizontal axis, the attenuation factor should not be plotted as a function of Z_2/p, but as a function of q_2', in line with the approach of a machine designer.

Instead of q_1 curves given for a very wide range of rotor slot numbers, the attenuation factor will now be plotted only around the points $q_2'=q_1$, namely for $q_2'=q_1\pm2/3$ and $q_1\pm4/3$, since Z_2 is never too far from Z_1, i.e. q_2' is not far from q_1.

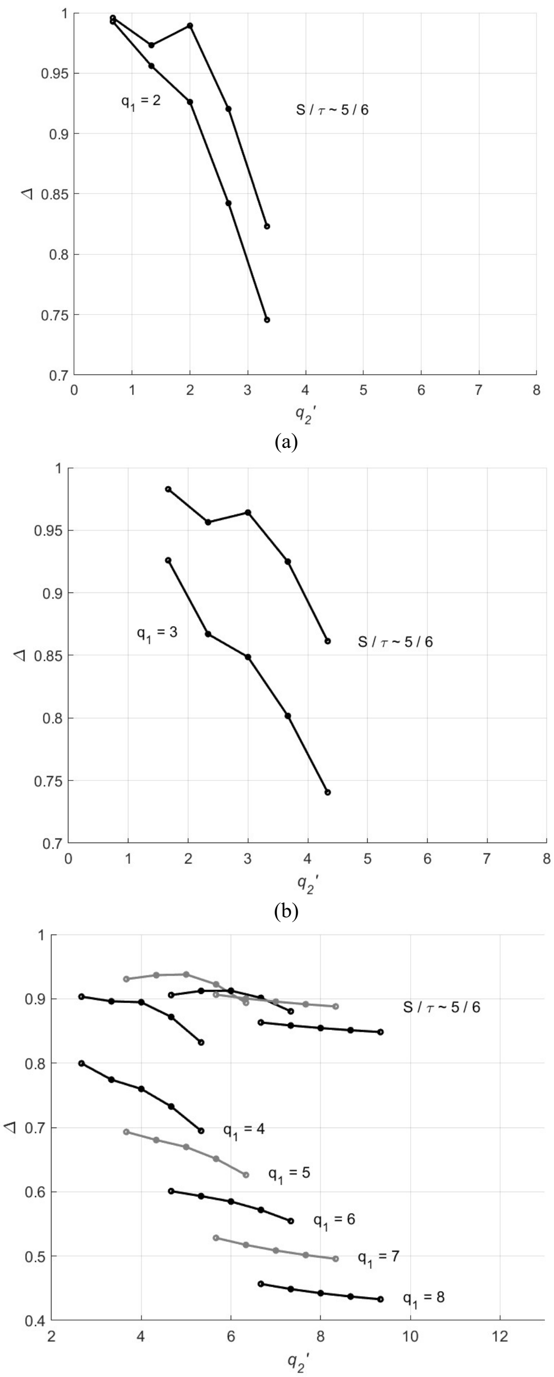
The original points acc. to (26) were calculated without chording; this becomes clear by substituting. This is objectionable in theory. Therefore, for each q_1, the points of the ideal, appr. 5/6 chording were also calculated here acc. to (26) then drawn exactly above the ones calculated with no chording. In the two cases, the rotor cage system is facing to two different harmonic contents of stator field and will attenuate them differently. Without chording, the dominant 5th and 7th harmonics would cause a larger differential leakage of stator, but this is strongly attenuated by the rotor, see Fig. 3. With chording, however, these almost disappear, leaving fewer harmonics, but the remaining ones, on the other hand, are less attenuated by the rotor. With other words: the attenuation does not depend only on stator slot number but also on chording as well. In the end, the effect of chording, despite the difference that seemed significant at first, becomes minor.

From a theoretical point of view, the widespread public conception is strongly objectionable that the resulting differential leakage is formed from an accurately calculated stator differential leakage multiplied by a vaguely estimated attenuation factor being independent of the stator's chording.

Therefore, just as the harmonic content of the stator is accurately calculated, the rotor attenuation effect must be calculated, in line with the previous one, also accurately. Thus, different values and relationships will be obtained.

In the characteristics, the "bottom" curves refer to windings without chording. The curves coincide with Richter's wider rotor slot range quoted above [3].

The "top" curves refer to windings with "ideal" chording. As expected, the attenuation factors are very high, around Δ~0.9, which shows that the fields created by such stator windings (already with a low harmonic content) are only slightly attenuated.



(c)

Fig. 7. (a) Characteristic of attenuation factors acc. to Richter ([3], p. 155. Fig. 108) modified by the Author, for q_1=2, with no chording and with a chording of S/τ~5/6. (b) Characteristic of attenuation factors acc. to Richter ([3], p. 155. Fig. 108) modified by the Author, for q_1=3, with no chording and with a chording of S/τ~5/6. (c) Characteristic of attenuation factors acc. to Richter ([3], p. 155. Fig. 108) modified by the Author, for q_1=4-8, with no chording and with a chording of S/τ~5/6.

3) Combined Characteristics of Stator Differential Leakage Factor and Its Attenuation (by the Rotor)

Characteristic of stator differential leakage is given in each basic book in the literature, Fig. 101(a) of [3], pp. 43 Fig. 16 of [5], pp. 153 Fig. 6.3 of [5], pp. 233 Fig. 4.7 of [7]: in function of chording, with q_1 as parameter. Attenuation is given also see the beginning of chapter 2 above, but always separately.

Now, in possession of the precise attenuation factors, it is high time to modify (supplement) the characteristics of the stator differential leakage σ_1. Now, these characteristics will be combined with the attenuation curves (calculated correctly, not only in function of rotor slot number but also in function of chording). In this way, such curves are obtained that simultaneously show the value of the stator differential leakage factor in function of chording k_2, with stator slot number q_1 and rotor slot number q_2' as parameters, see Fig. 8. The values of $\Delta\sigma_1$ were calculated in dependence from the following rotor slot numbers: $q_2'=q_1-4/3$, $q_2'=q_1-2/3$, $q_2'=q_1$, $q_2'=q_1+2/3$, $q_2'=q_1+4/3$; then inserted just under the existing σ_1 characteristics.

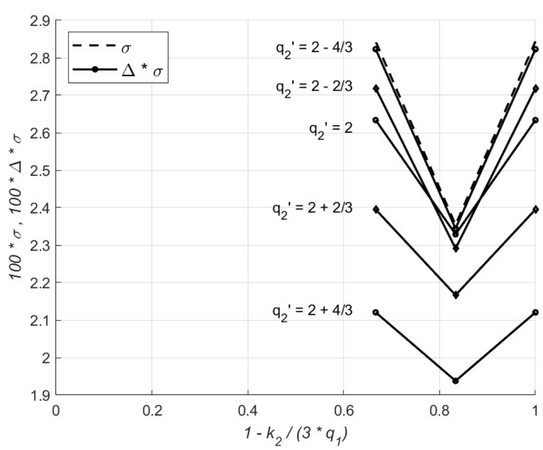
With a dashed line, the differential leakage factors of stator windings, calculated without attenuation, are also drawn, as found in the literature [3], [5], [6], [7]. A continuous line was inserted below the dashed line to provide the precise attenuation curves to be used *instead*; for q_1=2 and q_1=3 also in function of the rotor slot number.

From the curves it is possible to draw clear conclusions:

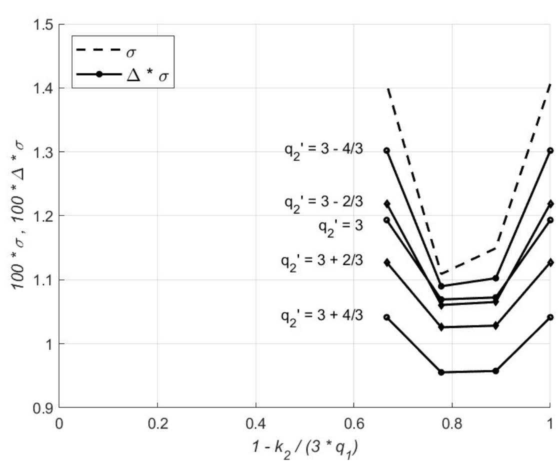
(1) The inclusion of accurate attenuation in the calculation was necessary not only theoretically, but also practically.

(2) For q_1=2, 3, i.e. for small slot numbers, the attenuation effect of the rotor is less, in accordance with Fig. 3, the attenuation factor is close to 1 and strongly dependent on the rotor slot number (less attenuation with $Z_2<Z_1$, higher with $Z_2>Z_1$). Some impact of the ideal chording remains but it will be significantly smaller.

(3) For $q_1\geq4$, the attenuation is, as expected, much stronger; the differential leakage without chording diminishes down to close to the level of the chorded one. The rotor slot number (meaning whether $Z_2<Z_1$ or $Z_2>Z_1$) has only a barely noticeable effect, so only the characteristic calculated with the rotor slot number $q_2'=q_1$ was inserted. In this case, no fatal mistake will be made if the resulting differential leakage is taken always equal to that with the ideal chording with no attenuation, regardless of actual chording and rotor slot number. In other words, at higher slot numbers, the rotor alone will produce what expected from chording.



(a)



(b)

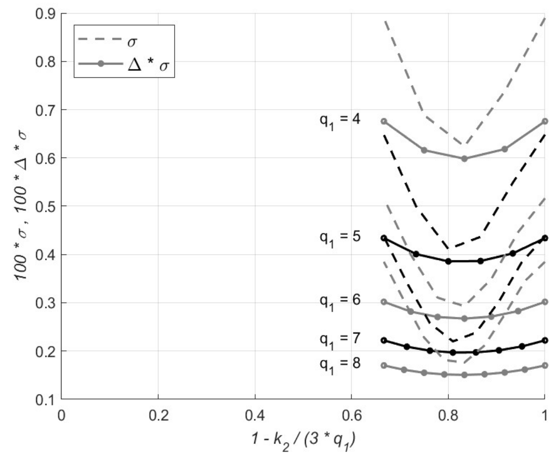


Fig. 8. (a) Differential leakage factors with no attenuation and with attenuation, in the function of the rotor slot number and the chording of k_2 slots, for q_1=2. (b) Differential leakage factors with no attenuation and with attenuation, in the function of the rotor slot number and the chording of k_2 slots, for q_1=3. (c) Differential leakage factors with no attenuation and with attenuation, in the function of the chording of k_2 slots, for $q_2'=q_1$.

D. Detailed Analysis of Fig. 1.

After that, in possession of the clear and complete picture regarding attenuation, it can be turned to the detailed analysis of the individual domains of Fig. 1. The evolution of η_{2v}^2 will be examined, from which Δ will be calculated. The characteristics of the respective domain will be derived from those observations.

1) v<1

These are called sub harmonics, so this part of the circuit is only created with windings with a fractional number of slots per pole and phase. $\eta_{2v}^2=1$ and $\Delta_v=0$ can be assumed here. This means two things:

(1) when calculating the stator differential leakage, this component does not need to be included in σ_1, because it disappears

(2) σ_{2v} disappears and as a result $X_{\sigma 2v}$ disappears, too, the largest possible rotor current occurs, possibly causing synchronous torque and certainly additional radial magnetic forces

In the case of positive v, the asynchronous torque is in generator mode, in the case of negative ν, it is in braking mode, so both cases are uninteresting from the point of view of a motor.

2) ν=1

This is a normal equivalent circuit of the machine; it does not require a separate analysis

3) $1<\nu<5$

This domain is created by winding with a fractional number of slots (per pole and phase) and some pole changing winding.

$\eta_{2\nu}^2$ slightly differs from 1, Δ_ν slightly differs from zero that is why ε was introduced.

Therefore, a significant asynchronous torque develops which falls in the motor range in the case of positive ν, but does not fall in the case of negative ν. According to the author's calculations, ν=4 can still be handled, but ν<4 hardly any more. Therefore, a winding with a fractional number of slots (per pole and phase), which creates positive harmonics in the range $1<\nu<4$, should not or only be used after a very careful calculation. With a rough estimate, it can be said that a value of $\xi_\nu<0.2$ shall be targeted, further $Z_2<Z_1$ shall be used.

The high rotor currents create additional synchronous torques and additional radial magnetic forces.

When calculating the stator differential leakage, these components do not have to be included in σ_1 either, because they also disappear as a result of $\Delta_\nu=\varepsilon$.

4) $2mq_1-1>\nu\geq5$

This is the domain to which all calculations apply. In this domain, all the phenomena discussed in this article occur, i.e. rotor harmonic current, causing attenuation and asynchronous parasitic torque. They are also involved in the creation of the additional synchronous parasitic torque and the radial magnetic forces.

The harmonics discussed so far in points *1), 2)* and *3)* will go into the formulas in the form $\nu_a=\nu$. It cannot be ruled out that they are also included in the form $\nu_b=\nu$, rather - but not exclusively - in the case of $Z_2<Z_1$.

5) $\nu=2mq_1-1$ and $\nu=2mq_1+1$

It is worth dedicating a special point to these two harmonics. These are called first slot harmonics.

Their most special feature is that $\xi_\nu=\xi_1$.

They are located on the border of the previous and subsequent domains. Some attenuation and some asynchronous parasitic torque may develop if $Z_2>Z_1$, but none if $Z_2<Z_1$. It depends on the rotor slot number therefore whether they go into the formulas at all in the form $\nu_a=\nu$. For the sake of generality, they were now included in the attenuation formula.

However, in the form $\nu_b=\nu$, these two harmonics definitely go into the formulas. It is noted that when choosing the number of slots for the rotor, due to $\xi_\nu=\xi_1$, the designer makes still every effort to prevent this situation from occurring (see the extensive literature on a number of slots rules).

6) $\nu>2mq_1+1$

The harmonics created in this range practically cannot create rotor currents, therefore no asynchronous parasitic torque is created, and these harmonics, and their differential leakage) are not attenuated.

It follows from all of this that they are never included in the formulas in the form $\nu_a=\nu$, but are always included in the form $\nu_b=\nu$, that is, non-attenuated.

Synchronous parasitic torque is primarily created by the interaction of rotor fields created by the fundamental harmonic and its low-order attenuated harmonics of the stator with the high-order non-attenuated $\nu_b\geq2mq_1$ fields of the stator.

Radial magnetic force is primarily created by the interaction of the non-attenuated harmonic residual fields of the rotor with the non-attenuated $\nu_b\geq2mq_1$ fields of the stator.

V. Annex

Table I provides the 6-digit values of differential leakage and attenuation. These values are plotted in the form of characteristic curves in Fig. 7 and Fig. 8.

TABLE I (A)
DIFFERENTIAL LEAKAGE $100\cdot\sigma_1$

	q_1=	2	3	4	5	6	7	8	9
chording	0	2.843709	1.406144	0.889584	0.648117	0.516317	0.436631	0.384825	0.349267
	1	2.354159	1.149451	0.737533	0.548538	0.446312	0.384820	0.344969	0.317675
	2	2.843709	1.109003	0.623886	0.436587	0.349012	0.302525	0.275511	0.258710
	3		1.406144	0.688463	0.411377	0.292935	0.238168	0.211656	0.198667
	4			0.889584	0.499542	0.310651	0.219993	0.176318	0.155892
	5				0.648117	0.399841	0.257745	0.181424	0.141705
	6					0.516317	0.341447	0.227963	0.160696
	7						0.436631	0.304655	0.210393
	8							0.384825	0.280187
	9								0.349267

TABLE I (B)
DIFFERENTIAL LEAKAGE WITH ATTENUATION $100\cdot\Delta\sigma_1$ IN FUNCTION OF ROTOR SLOT NUMBER

$q_2 = q_1 - 4/3$								
	q_1	2	3	4	5	6	7	8
chording	0	2.822877	1.30206	0.711245	0.449145	0.310294	0.230657	0.175763
	1	2.344338	1.102513	0.626891	0.406493	0.285826	0.215363	0.165574
	2	2.822877	1.089922	0.594431	0.383986	0.270759	0.205049	0.158267

	3		1.30206	0.621855	0.382818	0.265337	0.199744	0.153825
	4			0.711245	0.403944	0.269629	0.199386	0.152187
	5				0.449145	0.284306	0.204132	0.153382
	6					0.310294	0.214387	0.15754
	7						0.230657	0.164904
	8							0.175763
$q_2 = q_1$ - 2/3								
	q_1	2	3	4	5	6	7	8
chording	0	2.718407	1.21918	0.688739	0.440986	0.306288	0.225888	0.172667
	1	2.290927	1.06534	0.619793	0.404552	0.284762	0.212161	0.163385
	2	2.718407	1.060648	0.595694	0.386188	0.271848	0.203063	0.156811
	3		1.21918	0.616863	0.385321	0.267224	0.198391	0.152819
	4			0.688739	0.402741	0.270937	0.198086	0.151352
	5				0.440986	0.283574	0.202286	0.152435
	6					0.306288	0.211353	0.156184
	7						0.225888	0.162816
	8							0.172667
$q_2 = q_1$								
	q_1	2	3	4	5	6	7	8
chording	0	2.633647	1.193389	0.675881	0.433973	0.301934	0.222103	0.170203
	1	2.329039	1.072405	0.618195	0.402293	0.282774	0.209673	0.161682
	2	2.633647	1.069243	0.59833	0.386496	0.271376	0.201485	0.155661
	3		1.193389	0.61591	0.385763	0.267299	0.197281	0.151998
	4			0.675881	0.400789	0.270587	0.197009	0.150653
	5				0.433973	0.281757	0.200795	0.151654
	6					0.301934	0.208962	0.155098
	7						0.222103	0.161173
	8							0.170203
$q_2 = q_1$ + 2/3								
	$q1$	2	3	4	5	6	7	8
chording	0	2.395184	1.127279	0.651591	0.422063	0.295207	0.219019	0.168209
	1	2.166663	1.028321	0.602101	0.394094	0.277968	0.207671	0.160329
	2	2.395184	1.025714	0.584995	0.380113	0.267714	0.200199	0.154737
	3		1.127279	0.600121	0.379461	0.264042	0.19636	0.15132
	4			0.651591	0.392758	0.267001	0.19611	0.150064
	5				0.422063	0.277051	0.199566	0.151003
	6					0.295207	0.207022	0.154217
	7						0.219019	0.159858
	8							0.168209
$q_2 = q_1$ + 4/3								
	q_1	2	3	4	5	6	7	8
chording	0	2.120566	1.041368	0.617986	0.405795	0.286287	0.216461	0.166573
	1	1.937767	0.957494	0.574646	0.38078	0.270619	0.206024	0.159236
	2	2.120566	0.955288	0.559652	0.368219	0.261267	0.199133	0.153985
	3		1.041368	0.572897	0.367631	0.257912	0.195588	0.150753
	4			0.617986	0.379567	0.260606	0.195355	0.149563
	5				0.405795	0.269767	0.198542	0.150455
	6					0.286287	0.205416	0.153493
	7						0.216461	0.158787
	8							0.166573

VI. SUMMARY

Starting from the basic physical background, a hitherto non-existent, final formula was derived for calculating the asynchronous parasitic torque. It is suitable not only for calculating its "breakdown torque", but also for its torque-slip characteristic curve.

It was shown that the asynchronous parasitic torque should be related not to the starting torque, but to the breakdown torque.

Richter's attenuation characteristic has been redesigned in a way that is more conducive to practical use, and the theoretical objection (ignoring the effect of chording) has been eliminated.

It was shown that chording does not have as much effect on the stator differential leakage as it is usually attributed to.

The previous, widely used characteristics of the stator differential leakage factor (σ_1) have been supplemented by the attenuation in function of chording and rotor slot number i.e. it was provided such a correct, final characteristic curve ($\Delta^*\sigma_1$) instead of the previous one what the design engineer actually needs.

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Mr. Kovács was participant of conferences incl. ICEM'82 and ICEM'84, IEEE in '90 with his invention "Squirrel cage induction motor with starting disc" produced in the 2-pole MW power range by his company and other topics regarding electromagnetic calculation of high power induction motors. His achievements, nowadays, include the invention of a pole changing winding 3//Y / 3//Y (especially for wide ratio). At present he deals with harmonics of induction machines and – using his formulas - analytic analysis of parasitic torques and radial magnetic forces (ICEM'20, ICEM'22).