

# Integrated Sliding Mode Velocity Control of Linear Permanent Magnet Synchronous Motor with Thrust Ripple Compensation

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**Abstract**—In this paper, a compound sliding mode velocity control scheme with a new exponential reaching law (NERL) with thrust ripple observation strategy is proposed to obtain a high performance velocity loop of the linear permanent magnet synchronous motor (LPMSM) control system. A sliding mode velocity controller based on NERL is firstly discussed to restrain chattering of the conventional exponential reaching law (CERL). Furthermore, the unavoidable thrust ripple caused by the special structure of linear motor will bring about velocity fluctuation and reduced control performance. Thus, a thrust ripple compensation strategy on the basis of extend Kalman filter (EKF) theory is proposed. The estimated thrust ripple will be introduced into the sliding mode velocity controller to optimize the control accuracy and robustness. The effectiveness of the proposal is validated with experimental results.

**Index Terms**—Linear permanent magnet synchronous motor (LPMSM), Sliding mode velocity control, New exponential reaching law, Thrust ripple compensation.

## NOMENCLATURE

$u_d, u_q$	$dq$ -axis stator voltages, V.
$i_d, i_q$	$dq$ -axis stator currents, A.
$L_d, L_q$	$dq$ -axis inductances, H.
$R_s$	Stator resistance, $\Omega$ .
$v$	Mover velocity, m/s.
$\tau$	Pole pitch, m.
$\Psi_f$	PM flux linkage, Wb.
$n_p$	Pole pairs.
$F_e$	Electromagnetic thrust, N.
$F_r$	Thrust ripple, N.
$m$	mover mass, Kg.

## I. INTRODUCTION

WITH the rapid progress of high-speed machining and magnetic levitation technology, linear motor has become

a research hotspot [1], among which linear permanent magnet synchronous motor (LPMSM) have received extensive attention owing to its good controllability and high thrust density [2], [3]. Nowadays, LPMSM has extensive use in 3C products manufacturing and completesets of equipment, high-speed logistics and cordless elevators [4]-[6].

To achieve precise control of LPMSM drives, the velocity loop and current loop should track the command values in real time and resist disturbances effectively. Since the current loop tracking performance is only related to motor parameters, the output voltage is often calculated by the precise mathematical model to achieve good current tracking performance. However, due to the problems of strong coupling, strong nonlinearity and susceptibility to interference, the control system should have a velocity control strategy with high decoupling and strong robustness.

In a word, high-performance velocity control is very important to obtain precise control of LPMSM drives. At present, common velocity loop control methods mainly include PI control [7]-[9], internal model control (IMC) [10], [11], active disturbance rejection control (ADRC) [12], [13], intelligent control [14] and sliding mode control (SMC) [15], [16]. However, the proportional integral coefficient does not change once it is set. When the external disturbance occurs suddenly, the system cannot quickly restore the equilibrium and stable state. Since IMC has only one tunable parameter, and the dynamic and robust performance of the system would be compromised. The active disturbance rejection control has many parameters, thus these parameters are difficult to adjust to optimal value. The intelligent control has the shortcomings such as large calculation and high hardware requirements. Among these methods, SMC is famous for its perfect dynamic response, strong robustness in various working conditions.

Because of the simple structure and reliable control performance, SMC has extensive use in various fields [17]-[19], especially motor control [20]. The implementation of the SMC is divided into two stages: reaching stage and sliding mode stage. Reaching law describes the convergence trajectory of system state, which largely determines system's steady and transient performance. However, major weakness of SMC is chattering caused by discontinuous part existing in the input. Therefore, some chattering suppression methods have been proposed by scholars, such as fractional-order sliding-mode [21], nonsingular terminal SMC [22], high-order SMC [23] and reaching law method [24]. The reaching law method acts on the

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$$\vec{B} = B_x n_x + B_y n_y. \quad (3)$$

Thereby, the thrust force  $\vec{F}$  is deduced as,

$$F = \frac{1}{2\mu_0} \int_S [n_x (B_x^2 - B_y^2) + 2n_y B_x B_y] dS. \quad (4)$$

Due to the special structure of the end breaking, the LPMSM has obvious thrust fluctuation, which further leads to velocity fluctuation and control performance degradation. The motor thrust can be expressed as the sum of electromagnetic thrust and thrust ripple, where the thrust fluctuation is denoted by  $F_r$ .

$$F = F_e + F_r. \quad (5)$$

Since the air gap of the LPMSM is relatively large, it can be considered that the inductance in  $dq$  coordinate system is approximately equal, that is,  $L_d = L_q = L_s$ , so the electromagnetic thrust  $F_e$  of the LPMSM is as follows,

$$F_e = \frac{3\pi n_p \psi_f i_q^*}{2\tau} \quad (6)$$

where  $k_t$  is the thrust coefficient,  $k_t = (3\pi n_p \psi_f) / (2\tau)$ .

Thrust ripple is motor position-dependent pulsation mainly caused by cogging force. Write it in Fourier series as,

$$F_r = \sum_{n=1}^{\infty} f_n \cos\left(\frac{n\pi}{\tau} x + \delta_n\right) \quad (7)$$

where  $f_n$  and  $\delta_n$  are the amplitude and initial angle of the  $n$ th-order harmonic, respectively.

Therefore, the dynamic equation of the LPMSM is,

$$m \frac{dv}{dt} = F_e + F_r. \quad (8)$$

It can be seen from (8) that thrust ripple will bring about velocity fluctuation, and it will cause performance deterioration of LPMSM, which restricts the application of the LPMSM in precision manufacturing equipment. Therefore, the suppression and compensation technology of thrust ripple is very important for the LPMSM drive system to obtain superior performance.

### III. VELOCITY REGULATION SCHEME BASED ON NERL-SMC

#### A. CERL Design and Analysis

As mentioned above, the complete SMC process contains of two parts: reaching stage and sliding mode stage. In SMC, only the state variable is required to arrive at sliding mode surface from any initial position within a limited time, while the trajectory of the switching function composed of the system state in the reaching stage is not specified. The reaching law defines the dynamic characteristic of the switching function  $s$ . According to the requirement of different scenarios, different reaching laws are used to design the system state operation trajectories in the reaching stage, which can improve the performance. The CERL is defined as follows,

$$\frac{ds}{dt} = -\gamma \operatorname{sgn}(s) - \varepsilon s, \gamma > 0, \varepsilon > 0 \quad (9)$$

where  $\gamma$  and  $\varepsilon$  are the coefficients of reaching rate,  $s$  is sliding-mode surface function,  $\gamma \operatorname{sgn}(s)$  is the isokinetic reach term, and  $\varepsilon s$  is the index reach term.

For (9), the isokinetic reach term  $\gamma \operatorname{sgn}(s)$  is designed so that

the reaching speed is  $\gamma$  instead of 0 when  $s$  is close to 0. However, even though the accessibility problem is solved by adding  $\gamma \operatorname{sgn}(s)$ , the parameter value will affect the speed of reaching the sliding surface, which makes a contradiction between increasing the reaching speed and reducing the chattering.

In (9), when  $s > 0$ , we can get that,

$$\frac{ds}{dt} = -\gamma - \varepsilon s \quad (10)$$

Integrate (9) from 0 to  $t$ , the reaching time is,

$$t^* = \frac{1}{\varepsilon} \left\{ \ln\left[s(0) + \frac{\gamma}{\varepsilon}\right] - \ln\frac{\gamma}{\varepsilon} \right\}. \quad (11)$$

From (11) we can see that reaching speed increases with  $\varepsilon$ . However, an excessively large  $\varepsilon$  will cause speed exceeds when approaching sliding surface and aggravate chattering. Therefore, the contradiction between reaching speed and the gain size can be solved by setting the coefficient  $\varepsilon$  as a variable.

#### B. The Proposed NERL-Based SMC

A NERL is revealed to diminish chattering in CERL in this section. The NERL establishes a relationship between the controller coefficient and the distance between system state and sliding mode surface, so as to achieve the purpose of reducing chattering of SMC. The NERL is designed as,

$$\frac{ds}{dt} = -\gamma |x|^\alpha \operatorname{sgn}(s) - \varepsilon |s|^{\beta \cdot \operatorname{sgn}(|s|-1)} s \quad (12)$$

where  $x$  is the state variable.  $\lim_{t \rightarrow \infty} |x| = 0, \gamma > 0, \varepsilon > 0, 0 < \alpha < 1, 0 < \beta < 1$ .

According to (12) we can see that when the system is away from sliding surface, there is  $s > 1$ , then  $\operatorname{sgn}(|s|-1) = 1$ , and the system state reaches sliding mode surface according to  $\gamma |x|^\alpha \operatorname{sgn}(s)$  and  $\varepsilon |s|^\beta s$ . Meanwhile, if  $|s|$  diminishes,  $|s|^\beta s$  reaches 1,  $\varepsilon |s|^\beta s$  converges to  $\varepsilon$ . On the other hand, due to the limitation of absolute value sign function in NERL, then  $\operatorname{sgn}(|s|-1) = -1$  and reaching speed of the variable exponential term is  $\varepsilon |s|^{-\beta} s$ , then  $\varepsilon |s|^{-\beta} s > \varepsilon |s|^\beta s$ , which enhances the reaching speed of the variable exponential reaching law. From the above analysis, it can be seen that the reaching speed of NERL is faster than that of CERL during the reaching process, and the chattering phenomenon in CERL is effectively suppressed.

#### C. Chattering Analysis

The actual motor drive digital control system is a discrete system, so this section analyzes the proposed NERL from the perspective of discretization, and compares the NERL with CERL under the condition of discretization.

If the sliding-mode surface  $s$  is close to 0, the NERL in (12) can be simplified as  $ds/dt \approx -\gamma |x|^\alpha \operatorname{sgn}(s)$ . The first order Euler discretization of the NERL is carried out and then the following equation can be obtained,

$$s(k+1) - s(k) = -\gamma \text{sgn}(s(k)) T_s |x|^\alpha \quad (13)$$

where  $T_s$  is the sampling period.

When  $s > 0$ , the system trajectory reaches the sliding surface, then  $s(k)=0^+$ , and in the next period there is,

$$s(k+1) = -\gamma T_s |x|^\alpha. \quad (14)$$

When  $s < 0$ , the system trajectory arrives the sliding surface, then  $s(k)=0^-$ , and in the next period there is,

$$s(k+1) = \gamma T_s |x|^\alpha. \quad (15)$$

Then the discrete switching bandwidth of the NERL can be expressed as,

$$\Delta_{NERL} = 2\gamma T_s |x|^\alpha. \quad (16)$$

When  $s$  is close to 0, the CERL reaching speed is determined by  $\gamma \text{sgn}(s)$ , and its discrete form is,

$$s'(k+1) - s'(k) = -\gamma' \text{sgn}(s'(k)) T_s. \quad (17)$$

Thus, the discrete switching bandwidth of the CERL is,

$$\Delta_{CERL} = 2\gamma' T_s. \quad (18)$$

Fig. 2(a) shows the state trajectory of CERL, from which we can see that the switching bandwidth of CERL is a fixed value, and it is difficult to reach the equilibrium origin (0,0) stably, there will be chattering between  $(-\gamma' T_s, \gamma' T_s)$ . In Fig. 2 (b), the power term of the state variable introduced in (16) can make the system stably converge to the equilibrium origin (0, 0), so as to effectively suppress the chattering phenomenon.

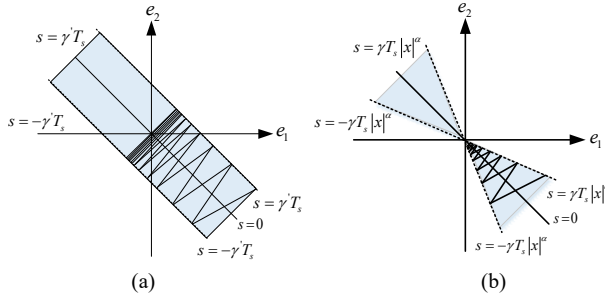


Fig. 2. State trajectory comparison. (a) CERL (b) NERL

#### D. NERL-SMC Design

The LPMSM state variables are,

$$e_1 = v^* - v \quad (19)$$

$$e_2 = \int_{-\infty}^t (v^* - v) dt \quad (20)$$

where  $v^*$  is the mover velocity command value. To enhance system's dynamic response, the integral structure of the state variable is added to the sliding surface design. The sliding surface  $s$  is designed as,

$$s = e_1 + c e_2, \quad c > 0 \quad (21)$$

where  $c$  is a constant.

Selected the velocity tracking error  $x = e_1$  as the state variable, which is the difference between velocity command value and feedback value. Combining the NERL (12) with (19), (20) and (21), the output of the NERL-SMC controller can be expressed as,

$$i_q^* = \frac{m}{k_t} [\dot{v}^* + \frac{1}{m} \hat{F}_r + c e_1 + \gamma |x|^\alpha \text{sgn}(s) + \varepsilon |s|^{\beta \text{sgn}(|s|^{-1})} s] \quad (22)$$

According to the above description, Fig. 3 shows the block diagram of the NERL-SMC.

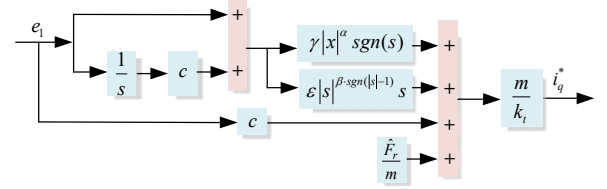


Fig. 3. Block diagram of the NERL-SMC.

#### E. Stability Proof

For proving the stability of NERL-SMC, the Lyapunov function is selected as,

$$V = s^2/2, \quad s \neq 0. \quad (23)$$

Then, we can obtain that,

$$\begin{aligned} \dot{V} &= s\dot{s} = s[-\gamma |x|^\alpha \text{sgn}(s) - \varepsilon |s|^{\beta \text{sgn}(|s|^{-1})} s] \\ &= -s[\gamma |x|^\alpha \text{sgn}(s) + \varepsilon |s|^{\beta \text{sgn}(|s|^{-1})} s] \leq 0. \end{aligned} \quad (24)$$

Since the accessibility condition of SMC is  $s\dot{s} < 0$ , then  $\dot{V}(x) < 0$ . According to the Lyapunov stability theory, it can be concluded that the NERL-SMC velocity controller is asymptotical stable.

When the state variables arrive at the sliding mode surface, there is,

$$s = e_1 + c e_2 = \dot{e}_2 + c e_2 = 0. \quad (25)$$

Solving the homogeneous linear differential equation of one variable, we can obtained that,

$$e_2 = v^* - v = c_0 e^{-\frac{t}{c}} \quad (26)$$

where  $c_0$  is a constant. With  $t$  increases,  $e_2$  exponentially approaches zero, which enables overshoot-free tracking.

Through the above reaching law, the contradiction between the reaching time and chattering in SMC with traditional exponential reaching law is solved. However, there are still some problems in the motion of linear motor, such as the thrust ripple. The thrust ripple directly results in the velocity fluctuation, which will deteriorate the control performance of LPMSM. Therefore, to improve the velocity regulation performance, it is necessary to identify the thrust ripple accurately. Therefore, a thrust ripple observer with extended Kalman filter is designed in the next section, and thrust ripple is observed online and then the feed-forward compensation is carried out.

#### IV. DESIGN OF THE THRUST RIPPLE OBSERVER BAED ON EKF THEORY

In this section, based on the EKF principle and combined with the basic characteristics of the LPMSM, a thrust ripple observer is designed, which enables the system to estimate the thrust ripple, and update it to the controller and compensate the estimation results to the forward channel, so as to restrain the velocity fluctuation and enhance the control precision of the

LPMSM drive system.

### A. Principle of EKF

#### 1) Linear Kalman Filter

For linear systems with the following equation of state,

$$x_k = \Phi_k x_{k-1} + \Gamma_k v_k \quad (27)$$

$$y_k = H_k x_k + w_k \quad (28)$$

where  $x_k$  is the discrete state variables,  $\Phi_k$  is the state matrix of discrete system,  $\Gamma_k$  is the system noise driven array,  $H_k$  is the output matrix,  $v_k$  and  $w_k$  are system noise and measurement noise and they satisfy the following assumptions,

$$\text{cov}(v_k, v_i) = E\{v_k v_i^T\} = \begin{cases} Q_k, i = k \\ 0, i \neq k \end{cases} \quad (29)$$

$$\text{cov}(w_k, w_i) = E\{w_k w_i^T\} = \begin{cases} R_k, i = k \\ 0, i \neq k \end{cases} \quad (30)$$

$$\text{cov}(w_k, v_i) = E\{w_k v_i^T\} = 0, \text{ arbitrary } i \text{ and } k \quad (31)$$

where  $Q_k$  and  $R_k$  are the time varying covariance matrix of system noise and measurement noise, respectively. Based on the above assumptions, the linear Kalman filter can be expressed as the follows:

##### 1) State prediction

$$\bar{x}_k = \Phi_k \hat{x}_{k-1} \quad (32)$$

##### 2) Modified state prediction

$$\hat{x}_k = \bar{x}_k + K_k (y_k - H_k \bar{x}_k) \quad (33)$$

##### 3) Calculate the filter gain

$$K_k = \bar{P}_k H_k^T (H_k \bar{P}_k H_k^T + R_k)^{-1} \quad (34)$$

##### 4) Calculate error covariance of prediction state

$$\bar{P}_k = \Phi_k \hat{P}_{k-1} \Phi_k^T + \Gamma_k Q_k \Gamma_k^T \quad (35)$$

##### 5) Corrected prediction state error covariance

$$\hat{P}_k = (I - K_k H_k) \bar{P}_k \quad (36)$$

Fig. 4 shows the calculation flow of the linear Kalman filter algorithm, which is composed of state calculation loop and gain calculation loop. The online estimation methods based on Kalman filter are all based on this framework, while the calculation methods of the state update and the gain matrix update are different.

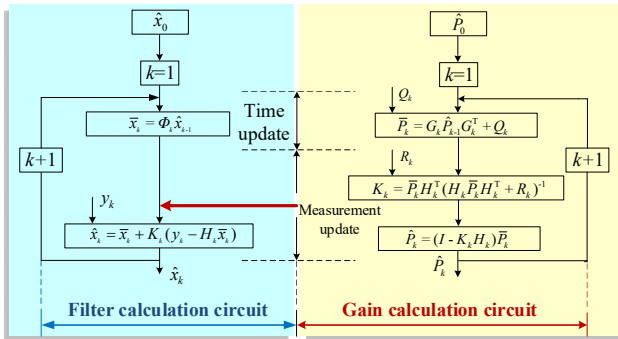


Fig. 4. Flow of Kalman Filter Algorithm.

#### 2) Extended Kalman Filter

Suppose the nonlinear system model is as follows,

$$x_k = f(x_{k-1}, u_k, v_k) \quad (37)$$

$$y_k = h(x_k, w_k) \quad (38)$$

where,  $u_k$  is the control variable;  $y_k$  is the measurement variable,  $f(\bullet)$  is a nonlinear system function,  $h(\bullet)$  is a nonlinear observation function.

At time  $k$ , the nonlinear system function is linearized near the working point, then we can obtain,

$$\begin{aligned} x_k &= f(x_{k-1}, u_k, v_k) = f(\hat{x}_{k-1} + \Delta x, u_k, 0 + \Delta w) \\ &\approx f(\hat{x}_{k-1}, u_k, 0) + \left. \frac{\partial f}{\partial x} (x_{k-1}, u_k, v_k) \right|_{(\hat{x}_{k-1}, u_k, 0)} \Delta x \\ &\quad + \frac{1}{2!} \left. \frac{\partial^2 f}{\partial x^2} (x_{k-1}, u_k, v_k) \right|_{(\hat{x}_{k-1}, u_k, 0)} \Delta x^2 \\ &\quad + \frac{1}{3!} \left. \frac{\partial^3 f}{\partial x^3} (x_{k-1}, u_k, v_k) \right|_{(\hat{x}_{k-1}, u_k, 0)} \Delta x^3 + \dots \\ &\quad + \left. \frac{\partial f}{\partial v} (x_{k-1}, u_k, v_k) \right|_{(\hat{x}_{k-1}, u_k, 0)} \Delta v \\ &\quad + \frac{1}{2!} \left. \frac{\partial^2 f}{\partial v^2} (x_{k-1}, u_k, v_k) \right|_{(\hat{x}_{k-1}, u_k, 0)} \Delta v^2 \\ &\quad + \frac{1}{3!} \left. \frac{\partial^3 f}{\partial v^3} (x_{k-1}, u_k, v_k) \right|_{(\hat{x}_{k-1}, u_k, 0)} \Delta v^3 + \dots \end{aligned} \quad (39)$$

Ignoring the higher-order terms, formula (39) becomes,

$$\begin{aligned} x_k &= f(x_{k-1}, u_k, v_k) = f(\hat{x}_{k-1} + \Delta x, u_k, 0 + \Delta v) \\ &\approx f(\hat{x}_{k-1}, u_k, 0) + \left. \frac{\partial f}{\partial x} (x_{k-1}, u_k, v_k) \right|_{(\hat{x}_{k-1}, u_k, 0)} \Delta x \\ &\quad + \left. \frac{\partial f}{\partial v} (x_{k-1}, u_k, v_k) \right|_{(\hat{x}_{k-1}, u_k, 0)} \Delta v \end{aligned} \quad (40)$$

Define the following expression,

$$f(\hat{x}_{k-1}, u_{k-1}, 0) = \bar{x}_k, G_k = \left. \frac{\partial f}{\partial x} (\hat{x}_{k-1}, u_k, v_k) \right|_{(\hat{x}_{k-1}, u_k, 0)},$$

$$V_k = \left. \frac{\partial f}{\partial v} (\hat{x}_{k-1}, u_k, v_k) \right|_{(\hat{x}_{k-1}, u_k, 0)}$$

(39) can be rewritten as,

$$x_k = f(\hat{x}_{k-1}, u_k, v_k) \approx \bar{x}_k + G_k (x_{k-1} - \hat{x}_{k-1}) + V_k v_k \quad (41)$$

Similarly, if  $(\bar{x}_k, 0)$  is regarded as the current working point of  $h(\cdot)$ , then multidimensional Taylor expansion is carried out.

$$\begin{aligned} y_k &= h(x_k, w_k) = h(\bar{x}_k + \Delta x, 0 + w_k) \\ &\approx h(\bar{x}_k, 0) + \left. \frac{\partial h}{\partial x} (x_k, w_k) \right|_{(\bar{x}_k, 0)} \Delta x + \frac{1}{2!} \left. \frac{\partial^2 h}{\partial x^2} (x_k, w_k) \right|_{(\bar{x}_k, 0)} \Delta x^2 \\ &\quad + \frac{1}{3!} \left. \frac{\partial^3 h}{\partial x^3} (x_k, w_k) \right|_{(\bar{x}_k, 0)} \Delta x^3 + \dots + \left. \frac{\partial h}{\partial w} (x_k, w_k) \right|_{(\bar{x}_k, 0)} \Delta w \\ &\quad + \frac{1}{2!} \left. \frac{\partial^2 h}{\partial w^2} (x_k, w_k) \right|_{(\bar{x}_k, 0)} \Delta w^2 + \frac{1}{3!} \left. \frac{\partial^3 h}{\partial w^3} (x_k, w_k) \right|_{(\bar{x}_k, 0)} \Delta w^3 + \dots \end{aligned} \quad (42)$$

$$\text{Define } W_k = \left. \frac{\partial h}{\partial v} (x_k, w_k) \right|_{(\bar{x}_k, 0)}, \quad L_k = \left. \frac{\partial h}{\partial x} (x_k, w_k) \right|_{(\bar{x}_k, 0)},$$

$h(\bar{x}_k, 0) = \bar{y}_k$ , after omitting the high-order terms, (42) is written as follows,

$$y_k = h(x_k, w_k) \approx \bar{y}_k + L_k (x_k - \hat{x}_k) + W_k w_k \quad (43)$$

Through the above derivation, the state estimation problem

of nonlinear system is transformed into a linear system problem. The EKF algorithm is obtained by using the calculation method of classical linear Kalman filter framework.

### B. Design of the Thrust Ripple Observer of the LPMSM

The dynamic equation of the LPMSM is expressed in (8). The discrete state space model of LPMSM is as follows,

$$\begin{cases} \mathbf{x}(k+1) = \mathbf{f}[\mathbf{x}(k), \mathbf{x}(k)] + \mathbf{w}(k) \\ \mathbf{y}(k) = \mathbf{H}[\mathbf{x}(k)] + \mathbf{v}(k) \end{cases} \quad (44)$$

Based on the mechanical motion equation of the LPMSM, the thrust ripple compensation scheme based on EKF is established. The state variables are selected as  $\mathbf{x} = [v \ \hat{F}_r]^T$ . The algorithm process is as follows:

(1) Initialization of state variables

$$\hat{\mathbf{x}}_0 = \mathbf{E}[\mathbf{x}_0] \quad (45)$$

$$\mathbf{P}_0 = \mathbf{E}[(\mathbf{x}_0 - \hat{\mathbf{x}}_0)(\mathbf{x}_0 - \hat{\mathbf{x}}_0)^T] \quad (46)$$

where  $\mathbf{x}_0$  is the initial value of the state variable and  $\mathbf{P}_0$  is the initial value of the error covariance matrix.

(2) Calculate Jacobian matrix  $\mathbf{G}$  and  $\mathbf{H}$

$$\mathbf{G}_k = \frac{\partial}{\partial \mathbf{x}} (\mathbf{A}_{k-1} \mathbf{x} + \mathbf{B}_{k-1} u) \Big|_{\mathbf{x}=\bar{\mathbf{x}}_k} = \begin{pmatrix} 1 & [u_{k-1} - F_r] T_s \\ 0 & 1 \end{pmatrix} \quad (47)$$

$$\mathbf{H}_k = \frac{\partial}{\partial \mathbf{x}} (\mathbf{C}_{k-1} \mathbf{x}) \Big|_{\mathbf{x}=\bar{\mathbf{x}}_k} = \begin{pmatrix} 1 & 0 \end{pmatrix} \quad (48)$$

(3) One step prediction of the state variables

$$\bar{\mathbf{x}}_{k/k} = \mathbf{f}(\hat{\mathbf{x}}_{k/k-1}, u_{k-1}) \quad (49)$$

where  $\hat{\mathbf{x}}_{k/k-1}$  is the best estimate of the previous time,  $\bar{\mathbf{x}}_{k/k}$  is the predicted value of the state variable at the current moment.

(4) Forecast the error covariance matrix

$$\bar{\mathbf{P}}_{k/k} = \mathbf{G}_k \hat{\mathbf{P}}_{k/k-1} \mathbf{G}_k^T + \mathbf{Q} \quad (50)$$

where  $\hat{\mathbf{P}}_{k/k-1}$  is the error covariance matrix of the upper time,  $\bar{\mathbf{P}}_{k/k}$  is the predicted value of the current time,  $\mathbf{Q}$  is the system noise covariance matrix.

(5) Calculate the Kalman filter gain matrix

$$\mathbf{K}_k = \bar{\mathbf{P}}_{k/k} \mathbf{H}_k^T (\mathbf{H}_k \bar{\mathbf{P}}_{k/k} \mathbf{H}_k^T + \mathbf{R})^{-1} \quad (51)$$

where  $\mathbf{K}_k$  is gain matrix of Kalman filter,  $\mathbf{R}$  is the measurement covariance matrix of control system.

(6) Estimation of state variables

$$\hat{\mathbf{x}}_{k/k} = \bar{\mathbf{x}}_{k/k} + \mathbf{K}_k (y_k - \mathbf{g}(\bar{\mathbf{x}}_{k/k}, u_k)) \quad (52)$$

where  $y_k$  is the actual output,  $\mathbf{g}(\bar{\mathbf{x}}_{k/k}, u_k)$  is the predicted value of the output.

The Kalman filter gain matrix  $\mathbf{K}_k$  is used as the coefficient of the difference between  $y_k$  and  $\mathbf{g}(\bar{\mathbf{x}}_{k/k}, u_k)$ , so as to modify the predicted value of the state variable, and finally obtain the best estimated value  $\hat{\mathbf{x}}_{k/k}$  of the state variable.

(7) Covariance matrix of state variable estimation error

$$\hat{\mathbf{P}}_{k/k} = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \bar{\mathbf{P}}_{k/k} \quad (53)$$

where  $\mathbf{I}$  is the identity matrix. Through  $\mathbf{K}_k$  to modify the current time  $\bar{\mathbf{P}}_{k/k}$ , the best estimate of  $\hat{\mathbf{P}}_{k/k}$  is obtained. Fig. 5 shows the block diagram of the thrust ripple estimation.

## V. EXPERIMENTAL RESULTS

To verify the validity of the proposal, experiments based on one LPMSM system are conducted. Parameters of the studied LPMSM are listed in Table I. Fig. 6 indicates the block diagram of the experimental setup, which is a dual closed-loop system contains a velocity outer loop and two current inner loops. The velocity loop is the key factor for obtaining good velocity control performance. Thus, the proposed scheme to achieve superior velocity control is obtained. The PI controller is adopted as the automatic current regulator (ACR), and a cascade structure combined with a NERL-based SMC velocity controller is formed. An EKF-based thrust ripple estimator is designed to estimate and compensate the thrust ripple.

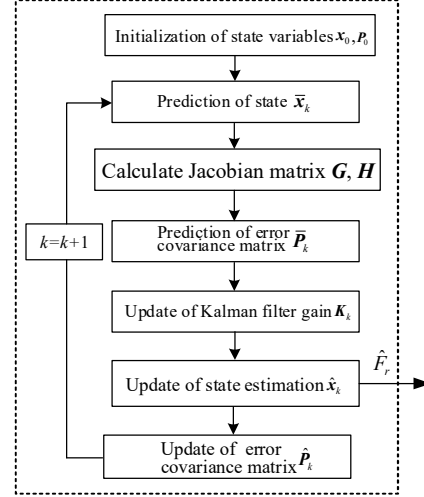


Fig. 5. Block diagram of thrust ripple estimation based on the EKF.

TABLE I  
PARAMETERS OF THE LPMSM.

Parameter	Value	Unit
Pole pitch	15	mm
PM flux linkage	0.2	Wb
Thrust constant	48.6	N/A
Mover mass	0.7	kg
Stator resistance	8.4	$\Omega$
Stator inductance	37.1	mH

The experimental setup of the LPMSM is shown in Fig. 7. The testing setup consists of one main circuit, one DSP control board, one linear encoder, and some other tools, and the core controller is a DSP TMS320F28335 of TI company. The

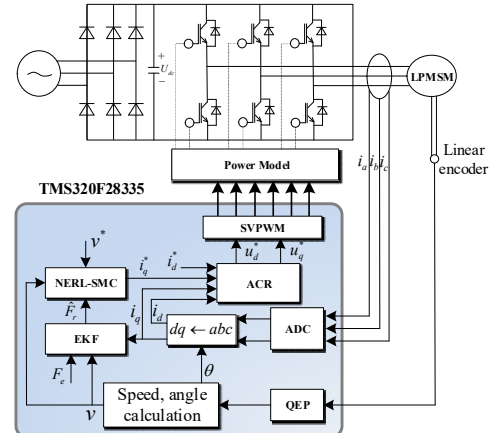


Fig. 6. Block diagram of the experimental bench.

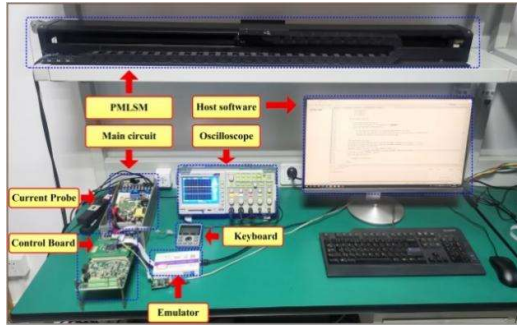


Fig. 7. Photograph of the experimental setup.

sampling frequency of control system is 8 kHz. The proposed algorithm is written by C language in CCS6.0.

The software flow of the LPMSL control system mainly consists of initialization program, main function program, PWM interrupt program, current polarity detection program, etc. The PWM interrupt cycle is 125 $\mu$ s, which mainly completes coordinate transformation, thrust ripple estimation algorithm, SVPWM generation, dead time compensation and other functions. Fig.8 (a) and Fig.8 (b) show the flow charts of the PWM interruption and thrust ripple estimation, respectively.

To fully verify the validity of the proposal, the performance of the proposal has been carried out under these cases: 1) overall performance of the proposed method; 2) validity verification of the NERL-SMC; 3) thrust ripple compensation.

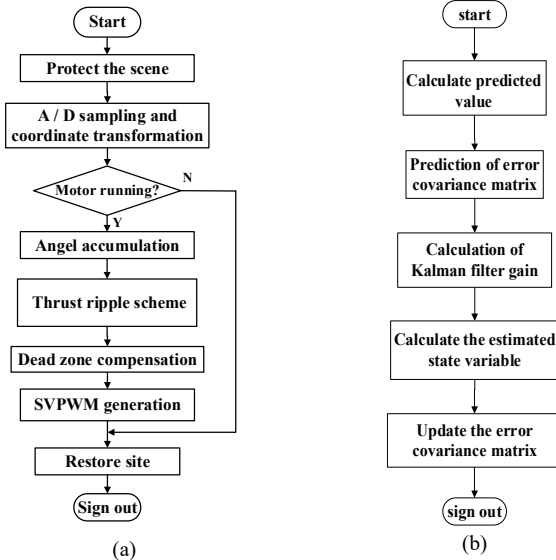


Fig. 8. Flowchart of key software. (a) PWM interrupt program (b) EKF-based thrust ripple estimation method subprogram

### A. Overall Performance Verification

When the tested motor starting from standstill to 0.45m/s, the experimental results based on the proposed compound robust sliding mode velocity control with thrust ripple estimation are shown in Fig. 9, which shows the mover velocity command value, the mover velocity feedback value, and  $u$ -phase current, respectively. The velocity command value increases from standstill to 0.45m/s through a ramp way, and the velocity feedback value follows the velocity command value quickly, which shows that the feedback value of the proposal can quickly track the given value, and has good control

performance.

Fig. 10 presents the control performance of the proposal when the LPMSM runs from start to 0.45m/s. The waveforms are mover velocity feedback value,  $q$ -axis current and  $u$ -phase current. As seen from the experimental results, the speed feedback value has almost no overshoot and fluctuation, and the phase current has good sinusoidal degree. It shows that the proposal has good transient performance and can effectively suppress the velocity ripple caused by the thrust ripple.

Fig. 11 shows the corresponding results when the velocity reversal from -0.375m/s to 0.375m/s by using the proposed method, in which the waveforms are mover velocity command value, mover velocity feedback value, velocity error and  $u$ -phase current. When the LPMSM is in steady state operation, the motor decelerates from 0.375m/s to 0, and then reversely accelerates to the velocity command value quickly. From the experimental results, the velocity response of the LPMSM is fast and the current waveform has little oscillation during velocity reversal. It can be seen from Fig. 11 that the proposed compound method has excellent

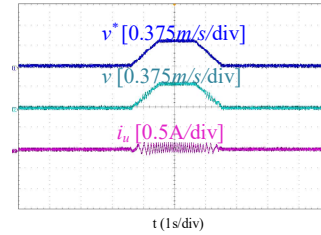


Fig. 9. Velocity and  $u$ -phase current response under the proposed method.

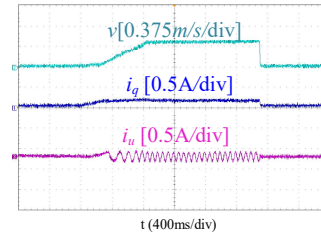


Fig. 10. No-load starting response under the proposed method.

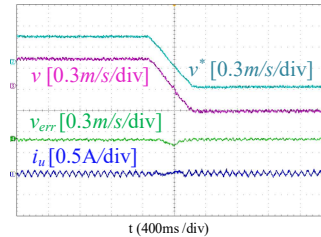


Fig. 11. Reverse response under the proposed method. dynamic performance during the velocity reversal.

### B. Validity Verification of the NERL-SMC

To verify the dynamic performance of the proposed compound method, the comparison experiments are conducted. Fig. 12(a) shows the corresponding results with the CERL-SMC method at 0.45m/s with no-load start-up, and Fig. 12(b) shows the experimental results with the NERL-SMC method at 0.45m/s with no-load start-up. From top to bottom, the mover velocity feedback value, the  $q$ -axis current, and the  $u$ -phase current are shown, respectively. As shown in Fig. 12, the overshoot and settling time of the velocity feedback value

are 4.3% and 150ms, under the CERL-SMC method, respectively. In addition, the settling time of the velocity feedback value are 85ms and the overshoot is nearly 0, under the NERL-SMC method. Besides, the fluctuation of the  $q$ -axis current under the CERL-SMC method is larger than the  $q$ -axis fluctuation of the NERL-SMC method. Therefore, the experimental waveforms show that the proposed NERL-SMC has good transient performance and improves the chattering phenomenon.

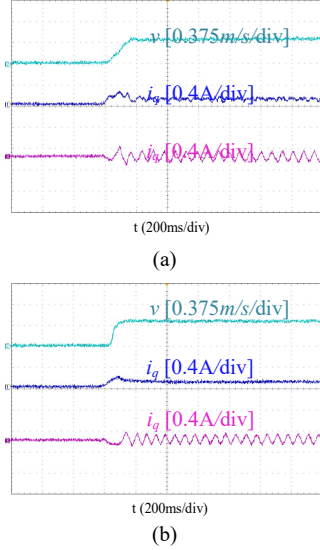


Fig.12. No-load starting response. (a) CERL-SMC (b) NERL-SMC.

In this test, the motor starts from no-load, and then the velocity reference is step changed from standstill to 0.45m/s at 0.8s. The motor operated at 0.45m/s and then back to standstill. Fig. 13 compares the corresponding results with the two sliding mode control methods with different exponential convergence laws. Fig. 13(a) shows the experimental waveforms based on the CERL-SMC method, and Fig. 13(b) shows the experimental waveforms based on the NERL-SMC method. From top to bottom, the mover velocity command value, the mover velocity feedback value, the velocity tracking error, and the u-phase current are given, respectively. The settling times of the CERL-SMC and the NERL-SMC after the sudden velocity changes are 800ms and 120ms, respectively. The corresponding results show that the proposed NERL-SMC has better dynamic performance than the CERL-SMC.

Fig. 14 shows the experimental results with the two different methods at 0.45m/s with a step load. Fig. 14 (a) and Fig. 14 (b) show the results under the CERL-SMC and NERL-SMC, respectively. The settling time of the mover velocity feedback is 220ms with the CERL-SMC method and it is 150ms with the

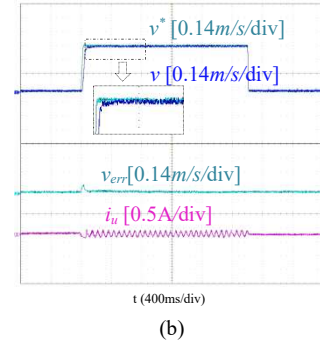
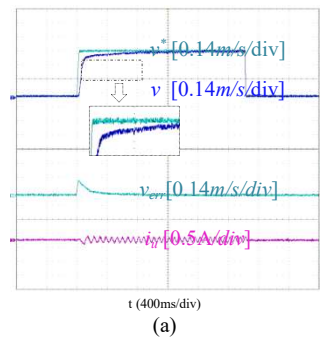


Fig. 13. Experimental results under sudden speed changes. (a) CERL-SMC (b) NERL-SMC

NERL-SMC method, respectively. The velocity drops under the NERL-SMC method is obviously smaller than that of the CERL-SMC method. In a word, compared with CERL-SMC, the proposed NERL-SMC method has better transient performance and anti-load disturbance performance.

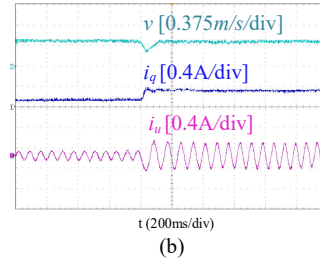
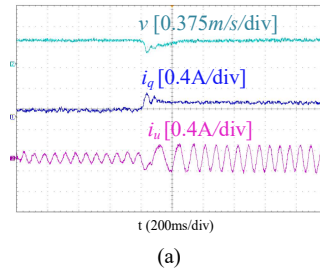


Fig. 14. Comparative results under step load. (a) CERL-SMC (b) NERL-SMC

### C. Thrust Ripple Compensation

To better verify the validity of the thrust ripple estimation algorithm, the performance criterion to evaluate the thrust ripple is the velocity fluctuation factor (VFF),  $VFF = (v_{pk-pk} / v_{rated}) \times 100\%$ . The velocity fluctuation factor is defined as the ratio between the peak value and the rated value of the LPMSM.

The LPMSM will inevitably generate thrust ripple due to the end disconnection. The thrust ripple will cause the motor velocity fluctuation and deteriorate the control performance, thus restricting the application of LPMSM in precision manufacturing equipment. Therefore, thrust ripple suppression is very necessary. In this part, an EKF-based observation technology is designed to estimate and compensate for thrust ripple, thereby realizing thrust ripple suppression. In the LPMSM control system, PI regulators are employed in current loops, and  $k_p$  and  $k_i$  are set to  $k_p = 0.8$  and  $k_i = 0.01$ . The velocity controller employs the NERL-SMC method. The parameters of the SMC are  $c = 300$ ,  $\gamma = 20$ ,  $\varepsilon = 0.01$ .

Fig. 15 shows the velocity and position waveforms of



LPMSM from standstill to 0.18 m/s before and after the adoption of thrust ripple compensation method. After the acceleration stage, the velocity reaches the command value and runs stably at 0.18 m/s. As shown in Fig. 15(a), the velocity fluctuation is obvious, and the velocity feedback value fluctuates near the velocity command value. After adopting the proposed EKF-based thrust ripple compensation method, as shown in Fig. 15(b), the velocity fluctuation is significant reduced, which shows the validity of the thrust ripple compensation method. Compared with the experimental results before and after adopting the thrust fluctuation suppression method, the performance of velocity and position information after compensation is better. Fig. 15

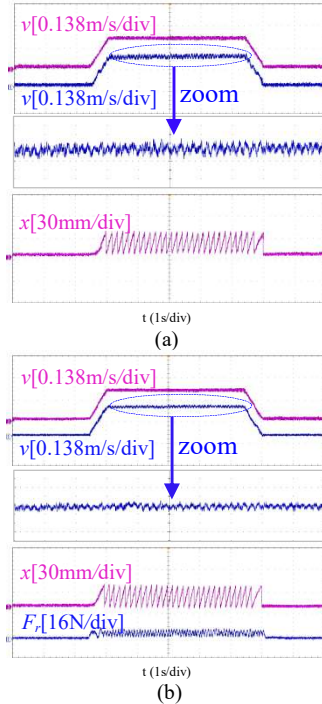


Fig. 15. Velocity tracking performance of 0.18m/s. (a) without thrust ripple estimation (b) with thrust ripple estimation

TABLE II.  
COMPARISON RESULTS WITHOUT AND WITH THRUST RIPPLE COMPENSATION.

Methods	Without compensation	With compensation
VFF	2.55%	1.28%

shows the effectiveness of the proposed thrust fluctuation compensation method.

## VI. CONCLUSION

A compound SMC with EKF-based thrust ripple estimation technology is proposed, which enhances the robustness and transient performance of LPMSM drive system. The NERL-based SMC could effectively reduce chattering of SMC, and enhance convergence speed as well. Then, since the thrust ripple will increase the amplitude of discontinuities and aggravates chattering in SMC, a thrust ripple estimation technology is presented to restrain the thrust ripple based on the extend Kalman filter theory. It feeds back the estimated thrust ripple to SMC with improved reaching law. Finally, the effectiveness of the proposal is validated with experimental

results.

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