Stability Analysis of Sampled-data Control System and Its Application to Electric Power Market*

Tian Zhang[1, 2] *and Shenping Xiao*[1, 2*]

(1. School of Electrical and Information Engineering, Hunan University of Technology, Zhuzhou 412007, China;
2. Key Laboratory for Electric Drive Control and Intelligent Equipment of Hunan Province, Zhuzhou 412007, China)

Abstract: The stability of sampled-data systems is investigated using a new type of Lyapunov functional. The interval from the sampling point t_k to t_{k+1} is assumed to be a sampling interval. By fully utilizing the characteristic information on the whole sampling interval, a new two-sided closed-loop Lyapunov functional is proposed, which utilizes the information on both the intervals from the sampling point t to t_k and from t to t_{k+1}. Based on the two-sided closed-loop Lyapunov functional and modified free-matrix-based inequality, a less conservative stability criterion is derived for a sampled-data control system, and three numerical examples are provided to verify the effectiveness and reduced conservativeness of the proposed method. Furthermore, the proposed method is applied to solve the stability problem of electric power markets, and the practical significance of reducing the conservativeness is discussed.

Keywords: Stability, sampled-data control system, two-sided closed-loop Lyapunov functional, electric power market

1 Introduction

With the rapid development of communication and computer technology, sampled-data control has become a core technology in the field of automatic control. Since sampled-data control only needs the state information of a system at the sampling time, the transmission of information is greatly reduced, and the control efficiency improved. Therefore, sampled-data control has been widely used in digital systems, network systems, and other fields, and has received extensive attention from many researchers [1-4]. It is known that the sampling period is an important indicator for measuring the conservativeness of the stability criterion in a sampled-data control system. The larger the sampling period for a stable system, the lower the conservativeness, and the lower the requirements for communication rate, capacity, and bandwidth of the system. This indicates that the larger the allowed sampling period, the lower the requirements for the system's hardware, and sampling performance, and in turn, lower the cost of sampled-data control equipment. Therefore, it is of great scientific and practical significance to investigate the stability of sampled-data control systems and derive the conditions for low conservativeness, thereby obtaining the largest sampling period.

A sampled-data control system is a hybrid system with both continuous and discrete signals, which cannot be directly analyzed and designed using the theory of continuous systems. Currently, the methods of analyzing and designing sampled-data control systems mainly include the following methods: discretiza- tion, pulse control, and input time-delay. In Refs. [5-6], the discretization method was used to analyze the stability of sampled-data control systems. However, the discretization method had significant limitations in dealing with the systems with uncertainties or variable periods. In Refs. [7-8], the stability problem of the variable-period sampled-data control system was discussed using the pulse control method. This method transformed the sampled-data control system into a pulse system and used pulse control theory to analyze the transformed system. In Ref. [9], the input time-delay method was proposed, which transformed the input signal of the sampled-data controller into a time-delay signal input by the system. This indicates that the state quantity at the time of sampling was regarded as the system state with a time

* Corresponding Author, Email: 244282881@qq.com
* Supported by the National Natural Science Foundation of China (61672225)
Digital Object Identifier: 10.23919/CJEE.2020.000005

delay, so that the sampled-data control system was modeled as a linear time-varying time-delay system. The Lyapunov functional and integral inequality techniques were used to analyze and design the transformed system. This method overcomes the difficulty of dealing with uncertain and variable-period systems faced by the discretization method.

Currently, the input time-delay method is widely used to deal with various types of sampled-data control problems. For example, the stability and stabilization of sampled-data control systems with data lost was discussed in Ref. [10]. In Ref. [11], the sampling synchronization control problem of neural networks was analyzed. In Ref. [12], the H_∞ output tracking control problem of a T-S fuzzy network control system was researched. In early studies, the sawtooth structure features of a sampled-data control system were not fully considered, and therefore the conclusions obtained were still largely conservative. To overcome this disadvantage, some researchers have proposed new types of Lyapunov functional, based on the input time-delay method, such as time-dependent Lyapunov functional[13], time-dependent discontinuous Lyapunov functional[14-15] and closed-loop function methods[16-18]. These methods lower conservativeness as they considered the sawtooth structure features. Recently, a time-dependent discontinuous Lyapunov functional by modified free-matrix integral inequality[19-21] was established in Ref. [19], and the stability problem of sampled-data control systems was discussed, concluding with less conservativeness. However, only the information of the interval between the current sampling point t and a previous sampling point t_k was considered, and the information of the interval between the current sampling point t and a future sampling point t_{k+1} was ignored. Additionally, in Ref. [22], the state information of a future sampling point was integrated into the closed-loop function for the first time, and a new closed-loop function was proposed that effectively reduced the conservativeness of the stable results of the sampled-data control system. However, this study still ignored a large amount of future sampling information, and needs to be further improved. Simultaneously, it also had some conservativeness to the processing of functional derivative. Therefore, this paper aims to construct a new Lyapunov functional, for reducing the conservativeness by effectively mining future sampling information.

The theoretical research on sampled-data control systems has attracted extensive attention [1-6, 10-11, 13-19]. In the above research, the conservativeness of the obtained criterion was usually compared by simple numerical examples, which is difficult to use to explain the practical significance of reducing conservativeness effectively. If an actual system is used to demonstrate the conservativeness of the obtained criterion, it will be more helpful to explain the practical significance of reducing conservativeness. Therefore, this study takes the electric power market as a practical example to demonstrate the significance of reducing conservativeness, with the aim of maintaining the supply and demand balance of electric power in the grid. Considering the market clearing time (i.e., the time of updating power price), the electric power market is modeled as a linear sampled-data control system, and then the conservativeness of the obtained criterion is discussed.

For variable-period sampled-data control systems, this study proposes a new stability analysis method and applies it to the electric power market. Compared with the existing research, the main contributions of this study are summarized as follows:

(1) The information of the whole sampling interval is fully utilized. Further, a two-sided closed-loop Lyapunov functional is proposed, which not only considers the information of the intervals from the current state $x(t)$ to the previous state $x(t_k)$ and from the current state $x(t)$ to the future state $x(t_{k+1})$, but also considers the state information of one and double integral terms and the intervals of $[t_k,t)$ and $[t,t_{k+1})$. Additionally, the cross-information of the states mentioned, is incorporated in the two-sided closed-loop Lyapunov functional. By using the two-sided closed-loop Lyapunov functional and the modified free-matrix inequality, the stability criterion of the sampled-data control system is derived. The advantages and low conservativeness of the proposed method are verified by two examples.

(2) The proposed method is employed to solve the stability problem of the electric power market. This study not only investigates the impact of clearing time on power system stability but also discusses the

practical significance of reducing the conservativeness of the stability criterion.

In this paper, $\mathbf{R}^n$ and $\mathbf{R}^{n\times m}$ represent the n-dimensional vector space and the $n\times m$ matrix space of the real number field, respectively. $\boldsymbol{Q}^{-1}$ and $\boldsymbol{Q}^{\mathrm{T}}$ represent the inverse and transpose operations of a matrix, respectively. "**0**" and "$\boldsymbol{I}$" represent the zero matrix and unit matrix of the appropriate dimension, respectively. $\boldsymbol{R}>0$ represents a positive definite symmetric matrix. $\mathrm{sym}\{\boldsymbol{M}\}=\boldsymbol{M}+\boldsymbol{M}^{\mathrm{T}}$ represent the sum of matrix $\boldsymbol{X}$ and transposed matrix $\boldsymbol{X}$. "*" represents the symmetric terms in a matrix.

2 Problem formulation

The following linear control system is considered

$$\dot{x}(t)=\boldsymbol{A}x(t)+\boldsymbol{B}_I u(t) \tag{1}$$

Here, $x(t)\in\mathbf{R}^n$ is the system state and $\boldsymbol{A}\in\mathbf{R}^n$, $\boldsymbol{B}_I\in\mathbf{R}^n$ are known as real constant matrices. The system control input $u(t)$ satisfies the following form

$$u(t)=\boldsymbol{K}x(t_k)\ \ t\in\left[t_k,t_{k+1}\right) \tag{2}$$

where, $\boldsymbol{K}\in\mathbf{R}^{m\times n}$ is the control gain matrix, $t_k\,(k\in\mathbf{N})$ is the sequence of sampling time, and the control signal between adjacent sampling points is maintained by a zero-order hold (ZOH). The variable sampling time t_k satisfies $0=t_0<t_1<\cdots<t_k<\cdots$ and

$$t_{k+1}-t_k=h_k\ \ h_k\in\left[h_1,h_2\right] \tag{3}$$

where h_k is a certain sampling period and has different values for different k values, which reflects the variable periodicity of sampling, and h_2 is the maximum sampling period.

Bringing formula (2) into formula (1), a closed-loop control system is obtained, which is given by

$$\dot{x}(t)=\boldsymbol{A}x(t)+\boldsymbol{B}_c x(t_k)\ \ t\in\left[t_k,t_{k+1}\right) \tag{4}$$

where $\boldsymbol{B}_c=\boldsymbol{B}_I\boldsymbol{K}$.

For a given control gain matrix $\boldsymbol{K}$, this study aims to derive the condition to guarantee the stability of the system represented by formula (4), thereby obtaining the largest sampling period.

To analyze the stability of the sampling system shown in formula (4), the following lemmas are introduced.

Lemma 1 [16]: For a given scalar $0<h_m\leqslant h_M$, and continuous differentiable Lyapunov function $V:\mathbf{R}^n\to\mathbf{R}^+$, suppose there are positive real numbers of $\mu_1<\mu_2$, and p that satisfy

$$\mu_1\,|\,x\,|^p\leqslant V(x)\leqslant\mu_2\,|\,x\,|^p \tag{5}$$

then, the following two conditions are equivalent

(1) For $\forall k\in\mathbf{N},h_k\in[h_m,h_M]$, the Lyapunov function is strictly a negative definite.

$$\Delta V(k)=V(\chi_k(h_k))-V(\chi_k(0))<0 \tag{6}$$

(2) There is a continuous differentiable function $V_0:\left[0,h_m\right]\times\mathbf{K}\to\mathbf{R}$, $\forall z\in\mathbf{K}$, and it satisfies

$$V_0(h,z(\cdot))=V_0(0,z(\cdot))\ \ \forall h\in\left[h_m,h_M\right] \tag{7}$$

Hence, for $\forall(k,h_k,\tau)\in\mathbf{N}\times[h_m,h_M]\times[0,h_k]$, the following formula is obtained

$$W_0(\tau,\chi_k)=\frac{\mathrm{d}\tau}{\mathrm{d}}\left[V(\chi_k(\tau))+V_0(\tau,\chi_k)\right]<0 \tag{8}$$

Furthermore, the sampled-data control system shown in formula (4) is asymptotically stable if one of the above two conditions is met.

Lemma 2 [21]: For a given positive definite matrix $\boldsymbol{R}\in\mathbf{R}^{n\times n}$, arbitrary matrices $\boldsymbol{G}_1$ and $\boldsymbol{G}_2$, and a vector $\boldsymbol{\xi}_0$, for all the functions x that are successively differentiable on $[a,b]\to\mathbf{R}^n$, the following integral inequality is satisfied

$$-\int_a^b\dot{x}^{\mathrm{T}}(s)\boldsymbol{R}\dot{x}(s)\mathrm{d}s\leqslant$$

$$(b-a)\boldsymbol{\xi}_0^{\mathrm{T}}\left[\boldsymbol{G}_1^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{G}_1+\frac{(b-a)^2}{3}\boldsymbol{G}_2^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{G}_2\right]\boldsymbol{\xi}_0+$$

$$\mathrm{sym}\left\{\boldsymbol{\xi}_0^{\mathrm{T}}\bar{\boldsymbol{G}}_1\boldsymbol{\varpi}(t)\right\}+(b-a)\mathrm{sym}\left\{\boldsymbol{\xi}_0^{\mathrm{T}}\bar{\boldsymbol{G}}_2\boldsymbol{\varpi}(t)\right\} \tag{9}$$

where $\bar{\boldsymbol{G}}_1=[\boldsymbol{G}_1^{\mathrm{T}},-\boldsymbol{G}_1^{\mathrm{T}},-2\boldsymbol{G}_2^{\mathrm{T}}]$ $\quad\bar{\boldsymbol{G}}_2=[\boldsymbol{G}_2^{\mathrm{T}},\boldsymbol{G}_2^{\mathrm{T}},0]$

$$\boldsymbol{\varpi}(t)=\left[x^{\mathrm{T}}(b)\ \ x^{\mathrm{T}}(a)\ \ \int_a^b x^{\mathrm{T}}(s)\mathrm{d}s\right]^{\mathrm{T}}$$

3 Two-sided looped Lyapunov functional

To simplify the process of deriving the stability condition for the system shown in formula (4), the following symbols are defined

$$\alpha_1(t)=\int_{t_k}^{t}x(s)\mathrm{d}s$$

$$\alpha_2(t)=\int_{t}^{t_{k+1}}x(s)\mathrm{d}s$$

$$\alpha_3(t)=\int_{t_k}^{t}(t-s)x(s)\mathrm{d}s$$

$$\alpha_4(t)=\int_{t}^{t_{k+1}}(s-t)x(s)\mathrm{d}s$$

$$\boldsymbol{\eta}_1(t)=[x^{\mathrm{T}}(t)-x^{\mathrm{T}}(t_k)\quad\alpha_1^{\mathrm{T}}(t)\quad\alpha_3^{\mathrm{T}}(t)]^{\mathrm{T}}$$

$$\boldsymbol{\eta}_2(t)=[x^{\mathrm{T}}(t)-x^{\mathrm{T}}(t_{k+1})\quad\alpha_2^{\mathrm{T}}(t)\quad\alpha_4^{\mathrm{T}}(t)]^{\mathrm{T}}$$

$$\boldsymbol{\eta}_3(t_k)=[x^{\mathrm{T}}(t_k) \quad x^{\mathrm{T}}(t_{k+1})]^{\mathrm{T}}$$
$$\boldsymbol{\beta}_1(t)=[x^{\mathrm{T}}(t) \quad \dot{x}^{\mathrm{T}}(t) \quad x^{\mathrm{T}}(t_k) \quad x^{\mathrm{T}}(t_{k+1})]^{\mathrm{T}}$$
$$\boldsymbol{\xi}(t)=[\beta_1^{\mathrm{T}}(t) \quad \alpha_1^{\mathrm{T}}(t) \quad \alpha_2^{\mathrm{T}}(t) \quad \alpha_3^{\mathrm{T}}(t) \quad \alpha_4^{\mathrm{T}}(t)]^{\mathrm{T}}$$
$$e_i=[\mathbf{0}_{n\times(i-1)n}\,\boldsymbol{I}_n \quad \mathbf{0}_{n\times(8-i)n}] \quad i=1,2,\cdots,8$$

For the stability of the sampled-data control system shown in formula (4), the following two-sided closed-loop Lyapunov functional is constructed.

$$V(t)=V_o(t)+\mathcal{V}_C(t) \quad t\in[t_k,t_{k+1}) \tag{10}$$

Here, $V_o(t)$ is a quadratic Lyapnov functional expressed as

$$V_o(t)=x^{\mathrm{T}}(t)\boldsymbol{P}x(t)$$

$\mathcal{V}_C(t)$ is an augmented closed-loop function expressed as

$$\mathcal{V}_C(t)=\sum_{i=1}^{4}\mathcal{V}_{Ci}(t)$$

where

$$\mathcal{V}_{C1}(t)=2\boldsymbol{\eta}_1^{\mathrm{T}}(t)\boldsymbol{Q}\boldsymbol{\eta}_2(t)$$
$$\mathcal{V}_{C2}(t)=(t_{k+1}-t)(t-t_k)\boldsymbol{\eta}_3^{\mathrm{T}}(t_k)\boldsymbol{M}\boldsymbol{\eta}_3(t_k)$$
$$\mathcal{V}_{C3}(t)=(t_{k+1}-t)\boldsymbol{\eta}_1^{\mathrm{T}}(t)\left[\boldsymbol{Z}_1\boldsymbol{\eta}_1(t)+2\boldsymbol{Z}_2\boldsymbol{\eta}_3(t_k)\right]+ (t_{k+1}-t)\int_{t_k}^{t}\dot{x}^{\mathrm{T}}(s)\boldsymbol{R}_1\dot{x}(s)\mathrm{d}s$$
$$\mathcal{V}_{C4}(t)=(t-t_k)\boldsymbol{\eta}_2^{\mathrm{T}}(t)\left[\boldsymbol{Z}_3\boldsymbol{\eta}_2(t)+2\boldsymbol{Z}_4\boldsymbol{\eta}_3(t_k)\right]- (t-t_k)\int_{t}^{t_{k+1}}\dot{x}^{\mathrm{T}}(s)\boldsymbol{R}_2\dot{x}(s)\mathrm{d}s$$

The definition of each matrix in the two-sided closed-loop Lyapunov functional in formula (10) can be found in **Lemma 1**. It is noted that $\mathcal{V}_C(t)$ is the boundary condition that satisfies formula (7) in **Theorem 1**, that is, $\mathcal{V}_{Ci}(t_k)=\mathcal{V}_{Ci}(t_{k+1})=0, i=1,2,3,4$. In Ref. [16], the function that satisfies such conditions is called a closed-loop function.

Note 1: Refs. [15-19] only consider the sampling characteristics between the current sampling point t and the previous sampling point t_k (the one-sided sampling characteristic). By contrast, the two-sided closed-loop Lyapunov functional in formula (10) considers the sampling characteristics of the other side by adding a new term $\mathcal{V}_{C4}(t)$, that is, the sampling characteristics between the current sampling point t and the future sampling point t_{k+1}. Additionally, compared with Refs. [15-19], the functional in formula (10) incorporates more information about the sampled-data control system in formula (4) by the augmenting vectors $\boldsymbol{\eta}_1(t)$ and $\boldsymbol{\eta}_2(t)$, such as the sampling state $x(t_{k+1})$, the sampling interval $[t_k,t)$, and the double integral state $\alpha_3(t)$. The sampling interval $[t,t_{k+1})$ reflects the information of the cross-terms between the one integral state $\alpha_2(t)$ and $\alpha_4(t)$ as well as their respective states. However, this information was not considered in Refs. [15-19]. The stability conditions derived from the functional in formula (10) are, thus, less conservative than those in Refs. [15-19].

Note 2: Since $\mathcal{V}_C(t_k)=\mathcal{V}_C(t_{k+1})=0$, i.e., $\mathcal{V}_C(t)$ satisfies the condition of closed-loop function and considers the two-sided sampling characteristics of the system, and $V_0(t)$ is a standard Lyapunov functional, then $V(t)$ is called a two-sided closed-loop Lyapunov functional. This function can be applied not only to sampled-data control systems, but also to the sampled-data control of other complex dynamic systems such as fuzzy control systems [12, 23], neural network systems [11, 21, 24], and multi-agent systems [25].

4 Stability analysis of sampled-data control system

In this section, the proposed closed-loop Lyapunov functional in formula (10) is used to derive the stability criterion of the sampled-data control system in formula (4), and the effectiveness and low conservativeness of the proposed method are verified by two examples.

4.1 Stability criteria

Theorem 1: For a given scalar $0<h_1\leqslant h_2$, if there is a symmetric matrix of the appropriate dimension, $\boldsymbol{P}>0$, $\boldsymbol{R}_1>0$, $\boldsymbol{R}_2>0$, $\boldsymbol{M},\boldsymbol{Z}_1,\boldsymbol{Z}_3$ and a matrix $\boldsymbol{Q},\boldsymbol{Z}_2,\boldsymbol{Z}_4,\boldsymbol{G}_1,\boldsymbol{G}_2,\boldsymbol{D}_1,\boldsymbol{D}_2,\boldsymbol{N},\boldsymbol{Y}_1,\boldsymbol{Y}_2$, that satisfies the linear matrix inequalities in formula (11) and formula (12), then

$$\begin{bmatrix}\Pi_1+h_k\Pi_2 & h_k\boldsymbol{D}_1 & h_kh_2\boldsymbol{D}_2\\ * & -h_k\boldsymbol{R}_2 & 0\\ * & * & -3h_k\boldsymbol{R}_2\end{bmatrix}<0 \tag{11}$$

and

$$\begin{bmatrix}\Pi_1+h_k\Pi_3 & h_k\boldsymbol{G}_1 & h_kh_2\boldsymbol{G}_2\\ * & -h_k\boldsymbol{R}_1 & 0\\ * & * & -3h_k\boldsymbol{R}_1\end{bmatrix}<0 \tag{12}$$

Here,

$$\Pi_1 = \text{sym}\{e_1^{\mathrm{T}}\boldsymbol{P}e_2 + \Xi_1^{\mathrm{T}}\boldsymbol{Q}\Xi_4 + \Xi_2^{\mathrm{T}}\boldsymbol{Q}\Xi_3 - \Xi_1^{\mathrm{T}}\boldsymbol{Z}_2\Xi_5 + \Xi_3^{\mathrm{T}}\boldsymbol{Z}_4\Xi_5 + \overline{\mathcal{G}}_1\Xi_6 + \overline{\mathcal{D}}_1\Xi_7 - \boldsymbol{Y}_1^{\mathrm{T}}\Xi_9 - \boldsymbol{Y}_2^{\mathrm{T}}\Xi_{10} + \boldsymbol{N}^{\mathrm{T}}\Gamma\} - \Xi_1^{\mathrm{T}}\boldsymbol{Z}_1\Xi_1 + \Xi_3^{\mathrm{T}}\boldsymbol{Z}_3\Xi_3$$

$$\Pi_2 = \text{sym}\{\Xi_2^{\mathrm{T}}[\boldsymbol{Z}_1\Xi_1 + \boldsymbol{Z}_2\Xi_5] + \overline{\mathcal{D}}_2\Xi_7 + \boldsymbol{Y}_2^{\mathrm{T}}\Xi_8\} + \Xi_5^{\mathrm{T}}\boldsymbol{M}\Xi_5 + e_2^{\mathrm{T}}\boldsymbol{R}_1e_2$$

$$\Pi_3 = \text{sym}\{\Xi_4^{\mathrm{T}}[\boldsymbol{Z}_3\Xi_3 + \boldsymbol{Z}_4\Xi_5] + \overline{\mathcal{G}}_2\Xi_6 + \boldsymbol{Y}_1^{\mathrm{T}}\Xi_8\} - \Xi_5^{\mathrm{T}}\boldsymbol{M}\Xi_5 + e_2^{\mathrm{T}}\boldsymbol{R}_2e_2$$

$$\overline{\mathcal{G}}_1 = [\boldsymbol{G}_1^{\mathrm{T}}, -\boldsymbol{G}_1^{\mathrm{T}}, -2\boldsymbol{G}_2^{\mathrm{T}}]$$

$$\overline{\mathcal{D}}_1 = [\boldsymbol{D}_1^{\mathrm{T}}, -\boldsymbol{D}_1^{\mathrm{T}}, -2\boldsymbol{D}_2^{\mathrm{T}}]$$

$$\overline{\mathcal{G}}_2 = [\boldsymbol{G}_2^{\mathrm{T}}, \boldsymbol{G}_2^{\mathrm{T}}, 0] \quad \overline{\mathcal{D}}_2 = [\boldsymbol{D}_2^{\mathrm{T}}, \boldsymbol{D}_2^{\mathrm{T}}, 0]$$

$$\Xi_1 = [e_1^{\mathrm{T}} - e_3^{\mathrm{T}} \;\; e_5^{\mathrm{T}} \;\; e_7^{\mathrm{T}}]^{\mathrm{T}} \quad \Xi_2 = [e_2^{\mathrm{T}} \;\; e_1^{\mathrm{T}} \;\; e_5^{\mathrm{T}}]^{\mathrm{T}}$$

$$\Xi_3 = [e_1^{\mathrm{T}} - e_4^{\mathrm{T}} \;\; e_6^{\mathrm{T}} \;\; e_8^{\mathrm{T}}]^{\mathrm{T}} \quad \Xi_4 = [e_2^{\mathrm{T}} \;\; -e_1^{\mathrm{T}} \;\; -e_6^{\mathrm{T}}]^{\mathrm{T}}$$

$$\Xi_5 = [e_3^{\mathrm{T}} \;\; e_4^{\mathrm{T}}]^{\mathrm{T}} \quad \Xi_6 = [e_1^{\mathrm{T}} \;\; e_3^{\mathrm{T}} \;\; e_5^{\mathrm{T}}]^{\mathrm{T}}$$

$$\Xi_7 = [e_4^{\mathrm{T}} \;\; e_1^{\mathrm{T}} \;\; e_6^{\mathrm{T}}]^{\mathrm{T}} \quad \Xi_8 = \boldsymbol{B}_c e_3$$

$$\Xi_9 = e_1 - e_3 - \boldsymbol{A}e_5 \quad \Xi_{10} = e_4 - e_1 - \boldsymbol{A}e_6$$

$$\Gamma = \boldsymbol{A}e_1 + \boldsymbol{B}_c e_3 - e_2$$

The sampled-data control system in formula (4) is then considered asymptotically stable.

Proof The two-sided closed-loop Lyapunov functional $V(t)$ is derived along with the sampled-data control system in formula (4) with respect to time t.

$$\dot{V}_o(t) = 2x^{\mathrm{T}}(t)\boldsymbol{P}\dot{x}(t) = 2\boldsymbol{\xi}^{\mathrm{T}}(t)e_1^{\mathrm{T}}\boldsymbol{P}e_2\boldsymbol{\xi}(t) \tag{13}$$

$$\dot{\mathcal{V}}_{C1}(t) = 2\boldsymbol{\eta}_1^{\mathrm{T}}(t)\boldsymbol{Q}\dot{\boldsymbol{\eta}}_2(t) + 2\dot{\boldsymbol{\eta}}_1^{\mathrm{T}}(t)\boldsymbol{Q}\boldsymbol{\eta}_2(t) = 2\boldsymbol{\xi}^{\mathrm{T}}(t)\left[\Xi_1^{\mathrm{T}}\boldsymbol{Q}\Xi_4 + \Xi_2^{\mathrm{T}}\boldsymbol{Q}\Xi_3\right]\boldsymbol{\xi}(t) \tag{14}$$

$$\dot{\mathcal{V}}_{C2}(t) = [(t_{k+1} - t) - (t - t_k)]\boldsymbol{\eta}_3^{\mathrm{T}}(t_k)\boldsymbol{M}\boldsymbol{\eta}_3(t_k) = \boldsymbol{\xi}^{\mathrm{T}}(t)\{(t_{k+1} - t)\Xi_5^{\mathrm{T}}\boldsymbol{M}\Xi_5 - (t - t_k)\Xi_5^{\mathrm{T}}\boldsymbol{M}\Xi_5\}\boldsymbol{\xi}(t) \tag{15}$$

$$\begin{aligned}\dot{\mathcal{V}}_{C3}(t) &= -\boldsymbol{\eta}_1^{\mathrm{T}}(t)[\boldsymbol{Z}_1\boldsymbol{\eta}_1(t) + 2\boldsymbol{Z}_2\boldsymbol{\eta}_3(t_k)] + (t_{k+1} - t)\{2\dot{\boldsymbol{\eta}}_1^{\mathrm{T}}(t)[\boldsymbol{Z}_1\boldsymbol{\eta}_1(t) + \boldsymbol{Z}_2\boldsymbol{\eta}_3(t_k)] + \\ &\dot{x}^{\mathrm{T}}(t)\boldsymbol{R}_1\dot{x}(t)\} - \int_{t_k}^{t}\dot{x}^{\mathrm{T}}(s)\boldsymbol{R}_1\dot{x}(s)\mathrm{d}s = \\ &\boldsymbol{\xi}^{\mathrm{T}}(t)\{-\Xi_1^{\mathrm{T}}(\boldsymbol{Z}_1\Xi_1 + 2\boldsymbol{Z}_2\Xi_5) + (t_{k+1} - t)\times \\ &[2\Xi_2^{\mathrm{T}}(\boldsymbol{Z}_1\Xi_1 + \boldsymbol{Z}_2\Xi_5) + e_2^{\mathrm{T}}\boldsymbol{R}_1e_2]\}\boldsymbol{\xi}(t) - \\ &\int_{t_k}^{t}\dot{x}^{\mathrm{T}}(s)\boldsymbol{R}_1\dot{x}(s)\mathrm{d}s\end{aligned} \tag{16}$$

$$\begin{aligned}\dot{\mathcal{V}}_{C4}(t) &= \boldsymbol{\eta}_2^{\mathrm{T}}(t)[\boldsymbol{Z}_3\boldsymbol{\eta}_2(t) + 2\boldsymbol{Z}_4\boldsymbol{\eta}_3(t_k)] + (t - t_k)\{2\dot{\boldsymbol{\eta}}_2^{\mathrm{T}}(t)[\boldsymbol{Z}_3\boldsymbol{\eta}_2(t) + \boldsymbol{Z}_4\boldsymbol{\eta}_3(t_k)] + \\ &\dot{x}^{\mathrm{T}}(t)\boldsymbol{R}_2\dot{x}(t)\} - \int_{t}^{t_{k+1}}\dot{x}^{\mathrm{T}}(s)\boldsymbol{R}_2\dot{x}(s)\mathrm{d}s = \\ &\boldsymbol{\xi}^{\mathrm{T}}(t)\{\Xi_3^{\mathrm{T}}(\boldsymbol{Z}_3\Xi_3 + 2\boldsymbol{Z}_4\Xi_5) + (t - t_k)\times \\ &[2\Xi_4^{\mathrm{T}}(\boldsymbol{Z}_3\Xi_3 + \boldsymbol{Z}_4\Xi_5) + e_2^{\mathrm{T}}\boldsymbol{R}_2e_2]\}\boldsymbol{\xi}(t) - \\ &\int_{t}^{t_{k+1}}\dot{x}^{\mathrm{T}}(s)\boldsymbol{R}_2\dot{x}(s)\mathrm{d}s\end{aligned} \tag{17}$$

The integral term of the functional derivative $\dot{\mathcal{V}}_{C3}(t)$ is processed by the modified free-matrix integral inequality in **Lemma 1.** Then

$$\begin{aligned}&-\int_{t_k}^{t}\dot{x}^{\mathrm{T}}(s)\boldsymbol{R}_1\dot{x}(s)\mathrm{d}s \leqslant \\ &\boldsymbol{\xi}^{\mathrm{T}}(t)\left[(t - t_k)\left(\boldsymbol{G}_1^{\mathrm{T}}\boldsymbol{R}_1^{-1}\boldsymbol{G}_1 + \frac{(t - t_k)^2}{3}\boldsymbol{G}_2^{\mathrm{T}}\boldsymbol{R}_1^{-1}\boldsymbol{G}_2 + \right.\right. \\ &\left.\left.2\overline{\mathcal{G}}_2\Xi_6\right) + 2\overline{\mathcal{G}}_1\Xi_6\right]\boldsymbol{\xi}(t) \leqslant \\ &\boldsymbol{\xi}^{\mathrm{T}}(t)\left[(t - t_k)\left(\boldsymbol{G}_1^{\mathrm{T}}\boldsymbol{R}_1^{-1}\boldsymbol{G}_1 + \frac{h_2^2}{3}\boldsymbol{G}_2^{\mathrm{T}}\boldsymbol{R}_1^{-1}\boldsymbol{G}_2 + \right.\right. \\ &\left.\left.2\overline{\mathcal{G}}_2\Xi_6\right) + 2\overline{\mathcal{G}}_1\Xi_6\right]\boldsymbol{\xi}(t)\end{aligned} \tag{18}$$

Similarly, by processing the integral term of the functional derivative $\dot{\mathcal{V}}_{C4}(t)$,

$$\begin{aligned}&-\int_{t}^{t_{k+1}}\dot{x}^{\mathrm{T}}(s)\boldsymbol{R}_2\dot{x}(s)\mathrm{d}s \leqslant \\ &\boldsymbol{\xi}^{\mathrm{T}}(t)\left[(t_{k+1} - t)\left(\boldsymbol{D}_1^{\mathrm{T}}\boldsymbol{R}_2^{-1}\boldsymbol{D}_1 + \frac{(t_{k+1} - t)^2}{3}\boldsymbol{D}_2^{\mathrm{T}}\boldsymbol{R}_2^{-1}\times\right.\right. \\ &\left.\left.\boldsymbol{D}_2 + 2\overline{\mathcal{D}}_2\Xi_7\right) + 2\overline{\mathcal{D}}_1\Xi_7\right]\boldsymbol{\xi}(t) \leqslant \\ &\boldsymbol{\xi}^{\mathrm{T}}(t)\left[(t_{k+1} - t)\left(\boldsymbol{D}_1^{\mathrm{T}}\boldsymbol{R}_2^{-1}\boldsymbol{D}_1 + \frac{h_2^2}{3}\boldsymbol{D}_2^{\mathrm{T}}\boldsymbol{R}_2^{-1}\boldsymbol{D}_2 + \right.\right. \\ &\left.\left.2\overline{\mathcal{D}}_2\Xi_7\right) + 2\overline{\mathcal{D}}_1\Xi_7\right]\boldsymbol{\xi}(t)\end{aligned} \tag{19}$$

The integral operation is performed on the time interval $[t_k,\ t)$ and $[t,\ t_{k+1})$ on the two sides of the sampled-data control system in formula (4).

$$x(t) - x(t_k) = \boldsymbol{A}\alpha_1(t) + (t - t_k)\boldsymbol{B}_c x(t_k) \tag{20}$$

$$x(t_{k+1}) - x(t) = \boldsymbol{A}\alpha_2(t) + (t_{k+1} - t)\boldsymbol{B}_c x(t_k) \tag{21}$$

Thus, there is a matrix $\boldsymbol{Y}_1, \boldsymbol{Y}_2$ of any suitable dimension that satisfy

$$2\boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{Y}_1^{\mathrm{T}}[(t - t_k)\Xi_8 - \Xi_9]\boldsymbol{\xi}(t) = 0 \tag{22}$$

$$2\boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{Y}_2^{\mathrm{T}}[(t_{k+1} - t)\Xi_8 - \Xi_{10}]\boldsymbol{\xi}(t) = 0 \tag{23}$$

Additionally, since $\Gamma\boldsymbol{\xi}(t) = 0$, there is a matrix $\boldsymbol{N}$ of suitable dimension that satisfies

$$\boldsymbol{\xi}^{\mathrm{T}}(t)(\boldsymbol{N}^{\mathrm{T}}\Gamma + \Gamma^{\mathrm{T}}\boldsymbol{N})\boldsymbol{\xi}(t) = 0 \tag{24}$$

Summarizing formulas (10) to (24),

$$\dot{V}(t)=\boldsymbol{\xi}^{\mathrm{T}}(t)\left[\frac{(t_{k+1}-t)}{h_k}\Theta_1(t_k)+\frac{(t-t_k)}{h_k}\Theta_2(t_k)\right]\xi(t) \quad (25)$$

where

$$\Theta_1(t_k)=\Pi_1+h_k\Pi_2+h_k\Delta_1$$
$$\Theta_2(t_k)=\Pi_1+h_k\Pi_3+h_k\Delta_2$$

Moreover,

$$\Delta_1=\boldsymbol{D}_1^{\mathrm{T}}\boldsymbol{R}_2^{-1}\boldsymbol{D}_1+\frac{h_2^2}{3}\boldsymbol{D}_2^{\mathrm{T}}\boldsymbol{R}_2^{-1}\boldsymbol{D}_2$$
$$\Delta_2=\boldsymbol{G}_1^{\mathrm{T}}\boldsymbol{R}_1^{-1}\boldsymbol{G}_1+\frac{h_2^2}{3}\boldsymbol{G}_2^{\mathrm{T}}\boldsymbol{R}_1^{-1}\boldsymbol{G}_2$$

From Schur's supplementary lemma, it is known that $\Theta_1(t_k)<0$ and $\Theta_2(t_k)<0$ are equivalent to linear matrix inequalities in formulas (11) and (12), respectively. If the conditions of formulas (11) and (12) are satisfied, then, according to **Lemma 1**, the sampled-data control system in formula (4) is asymptotically stable, and the proof is completed.

Note 3: In the process of deriving **Theorem 1**, the sampled-data control system in formula (4) is used to perform integral operations on the intervals of $[t_k,\ t)$ and $[t,\ t_{k+1})$. The integral states of $\alpha_1(t)$ and $\alpha_2(t)$ are introduced in formulas (22) and (23), respectively. Its connection with the system states is established by the free weight matrices of $\boldsymbol{Y}_1$ and $\boldsymbol{Y}_2$, that is, more system information is used. The zero-value formulas (22) and (23), thus, help reduce the conservativeness of the system further.

Note 4: Theorem 1 provides a stability criterion for a variable-period sampled-data control system. Obviously, if $h_1=h_2$ in **Theorem 1**, the same criterion applies to the constant-period sampled-data control system. Additionally, **Theorem 1** is also applicable to the sampled-data control systems of $\boldsymbol{A}$ and $\boldsymbol{B}_c$ with polyhedral uncertainties through appropriate transformations.

4.2 Example simulation

This subsection illustrates the effectiveness of the proposed method through two numerical examples.

Example 1: The sampled-data control system in formula (5) is assumed to satisfy the following matrix parameters

$$\boldsymbol{A}=\begin{bmatrix}-2 & 0\\ 0 & -0.9\end{bmatrix}\quad \boldsymbol{B}_c=\begin{bmatrix}-1 & 0\\ -1 & -1\end{bmatrix}$$

For the stable sampled-data control system, when $h_1=10^{-5}$, according to the stability criterion in **Theorem 1** and Refs. [16-17, 19, 22], the upper bound h_2 of the maximum sampling period is listed in Tab. 1. It can be seen that, **Theorem 1** obtains a larger sampling period compared with the results obtained in Refs. [16-17, 19, 22], indicating that the proposed method has a lower conservativeness, also verifying the effectiveness of **Note 1**.

Tab. 1 The maximum allowable upper bound of h_2 when $h_1=10^{-5}$ for aperiodic sampling

Methods	h_2
Ref. [16]	2.515 6
Ref. [17]	2.515 6
Ref. [22] ($Y_{1,2}=0$)	2.638 1
Ref. [19]	2.855 4
Theorem 1 ($Y_{1,2}=0$)	3.062 0
Theorem 1 ($Y_1=0$)	3.098 1
Theorem 1	3.109 2

Additionally, in **Theorem 1**, setting $\boldsymbol{Y}_1=0$, $\boldsymbol{Y}_2=0$ or $\boldsymbol{Y}_2=0$, respectively (that is, to eliminate the effect of formulas (22) or (23), respectively), and the upper bound h_2 of the maximum sampling interval for the stable sampled-data control system in formula (4) is calculated, as shown in Tab. 1. Tab. 1 shows that, formulas (18), and (19) are helpful in reducing the conservativeness of the system, verifying the effectiveness of **Note 3**.

Example 2: The sampled-data control system in formula (4) is assumed to satisfy the following matrix parameters

$$\boldsymbol{A}=\begin{bmatrix}0 & 1\\ -2 & 0.1\end{bmatrix}\quad \boldsymbol{B}_c=\begin{bmatrix}0 & 0\\ 1 & 0\end{bmatrix}$$

For the stable sampled-data control system in formula (4), when h_1 has different values, the maximum sampling interval is calculated using **Theorem 1** and Refs. [6, 8, 17-19]. The results are shown in Tab. 2.

Tab. 2 Intervals of allowable aperiodic samplings $[h_1,h_2]$

Methods	Sampling intervals $[h_1,h_2]$	Sampling intervals $[h_1,h_2]$
Ref. [17]	[0.4, 1.31]	[0.8, 1.56]
Ref. [6]	[0.4, 1.39]	[0.8, 1.61]
Ref. [8]	[0.4, 1.43]	[0.8, 1.58]
Ref. [19]	[0.4, 1.44]	[0.8, 1.73]
Ref. [18]	[0.4, 1.66]	[0.8, 1.86]
Theorem 1	[0.4, 1.75]	[0.8, 1.95]

It can be seen from Tab. 2 that the proposed method can obtain a larger sampling interval compared with [6,8,17-19], indicating that the proposed method is better and less conservative, further verifying the effectiveness of **Note 1**.

5 Application to electric power markets

A prerequisite for the reliable and stable operation of the grid is that the output and the load of the electric power (i.e., supply and demand) are balanced at all times. To maintain power balance, the scheme of load frequency control is usually adopted [26]. In recent years, researchers have proposed that the electric power market can use power prices to maintain the balance between supply and demand, which has attracted extensive attention [15, 27-29]. This section applies the proposed method to the stability analysis of the electric power market (mainly including modeling the electric power market) and theoretical calculation (for example simulation and discussion).

5.1 Models of the electric power market

A simple electric power market consists of power suppliers, power users, and real-time market.

(1) Market supply model.

The marginal production cost of power generation and the power supply satisfy the following relationship

$$\lambda_g = b_g + c_g P_g \tag{26}$$

Here, λ_g is the marginal cost, b_g is the fixed cost of the supplier, c_g is the fixed coefficient, and P_g is the amount of power supplied. If the power supplier observes that the power price on the market is higher than its production cost, the power supplier will expand production, and the degree of expansion will be proportional to the difference between the observed market power price and the actual production cost. Furthermore, additional costs will be incurred based on an oversupply of power at some point in the past. Therefore, the model of the electric power market supply can be described as

$$\tau_g \dot{P}_g = \lambda - b_g - c_g P_g - kE \tag{27}$$

where, τ_g is the time constant representing the response speed of the power output, λ is the observed power price on the market, k is the fixed coefficient, and E is the integral of the difference between the power supply and the demand.

(2) Market consumption model.

The marginal benefit and the demand for electrical power satisfy the following relationship

$$\lambda_d = b_d + c_d P_d \tag{28}$$

where, λ_d is the marginal income, b_d is the fixed income, c_d is the fixed coefficient, and P_d is the power demand. When marginal returns are higher than marginal prices, consumers will expand their consump- tion, and the speed of expansion is related to consumers. Therefore, the market consumption model can be described as

$$\tau_d \dot{P}_d = b_d + c_d P_d - \lambda \tag{29}$$

where, τ_d is the time constant representing the expansion speed of consumer demand.

(3) Unbalanced power supply and demand as well as market price response.

The imbalance between the supply and the demand can be expressed by the integral of the difference between the power supply and demand as

$$\dot{E} = P_g - P_d \tag{30}$$

The change in the power price on the market is determined by observing the demand and supply of the grid. This is given by

$$\tau_\lambda \dot{\lambda} = -E \tag{31}$$

where, τ_λ is the time constant representing the response speed to the power price at the time of market disturbance.

In the actual electric power market, the discrete power price signal received at a certain interval is equivalent to the market-clearing time, meaning the period of updating the power price [28]. According to formula (27) and formulas (29) to (31), the following linear control model based on sampled-data control is obtained.

$$\begin{cases} \tau_g \dot{P}_g(t) = \lambda(t_k) - b_g - c_g P_g(t) - kE(t) \\ \tau_d \dot{P}_d(t) = b_d + c_d P_d(t) - \lambda(t_k) \\ \dot{E}(t) = P_g(t) - P_d(t) \\ \tau_\lambda \dot{\lambda}(t) = -E(t) \end{cases} \tag{32}$$

Here, t_k is the clearing time for the power price and it satisfies

$$t_{k+1} - t_k = T_{mct_k} \quad T_{mct_k} \in [T_{mct_m}, T_{mct_M}] \tag{33}$$

where, T_{mct_k} and T_{mct_M} represent the update period of the k^{th} market power price and its maximum value, respectively.

5.2 Stability analysis of the electric power market

The electric power market model in formula (32) can be converted into

$$\dot{y}(t)=\boldsymbol{A}y(t)+\boldsymbol{B}_c y(t_k)+\boldsymbol{J} \tag{34}$$

where $T_{mct_m}<h_k=t_{k+1}-t_k<T_{mct_M}$

$$y(t)=\left[P_g^{\mathrm{T}}(t) \quad P_d^{\mathrm{T}}(t) \quad E^{\mathrm{T}}(t) \quad \lambda^{\mathrm{T}}(t)\right]^{\mathrm{T}}$$

$$\boldsymbol{J}=\left[-\frac{b_g}{\tau_g} \quad \frac{b_d}{\tau_d} \quad 0 \quad 0\right]^{\mathrm{T}}$$

$$\boldsymbol{A}=\begin{bmatrix} -\frac{c_g}{\tau_g} & 0 & -\frac{k}{\tau_g} & 0 \\ 0 & \frac{c_d}{\tau_d} & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -\frac{1}{\tau_\lambda} & 0 \end{bmatrix}$$

$$\boldsymbol{B}_c=\begin{bmatrix} 0 & 0 & 0 & \frac{1}{\tau_g} \\ 0 & 0 & 0 & -\frac{1}{\tau_d} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Assuming that $y^*=[y_1^*,y_2^*,\cdots,y_n^*]$ is the balance point of the system in formula (34), to facilitate the processing, the equilibrium point y^* is moved to the origin through coordinate transformation, and then the system in formula (34) can be transformed into

$$\dot{x}(t)=\boldsymbol{A}x(t)+\boldsymbol{B}_c x(t_k) \tag{35}$$

This system has been discussed in Section 4. The key parameters $\boldsymbol{A}$ and $\boldsymbol{B}_c$ of the system in formula (35) have the same physical meanings as those defined by the system in formula (34).

Therefore, the proposed stability analysis method of the sampled-data control system can also be used to discuss the stability of the electric power market.

5.3 Simulation verification

The model parameters of the system in formula (34) are $\tau_g=0.2, c_g=0.1, b_g=2, \tau_d=0.1, c_d=-0.2, b_d=10, \tau_\lambda=100$, and $k=0.1$.

For given parameters, $T_{mct_m}=10^{-5}$. According to the methods in **Theorem 1** and Refs. [14-15], the maximum update period T_{mct_M} (i.e., the sampling period of the price signal) for maintaining a stable and acceptable price signal in the electric power market can be obtained, as shown in Tab. 3. Tab. 3 shows that compared with Refs. [14-15], **Theorem 1** can obtain a larger update period T_{mct_M} of price signal, indicating a lower conservativeness.

Tab. 3 The maximum allowable updating period of power price signal T_{mct_M}

Methods	Ref. [14]	Ref. [15]	**Theorem 1**
T_{mct_M}	4.16	8.59	10.64

Additionally, when the initial conditions are set as $y(t)=[20,20,0,0]^{\mathrm{T}}$, and the sampling period of the electric power market price signal is $T_{mct}=10.64$, the stable balance point of the system in formula (34) is given by

$$y^*=[P_g^*,P_d^*,E^*,\lambda^*]^{\mathrm{T}}=[26.68,26.68,0,4.66]^{\mathrm{T}}$$

The state trajectory of the system is shown in Fig. 1. The figure shows that when the update period of the price signal is $T_{mct}=10.64$, the system is stable. The discrete control signal going through the zero-order holder (ZOH) is shown in Fig. 2.

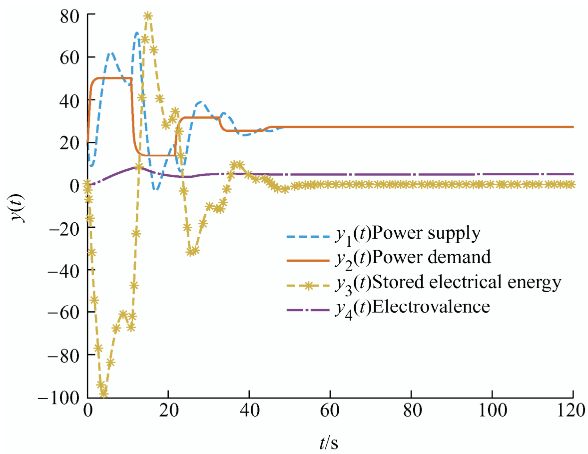

Fig. 1 State trajectories of the system (34) during $T_{mct}=10.64$

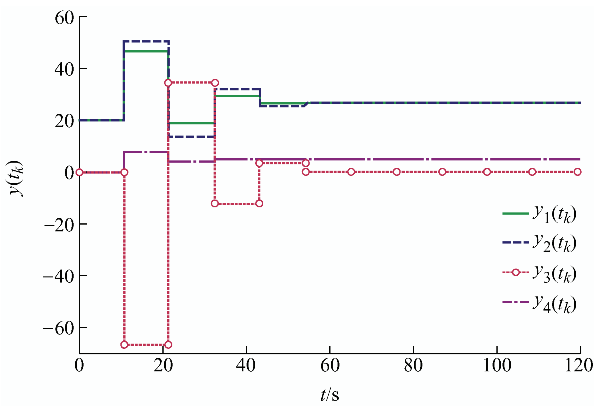

Fig. 2 Discrete control signals of the system (34) during $T_{mct}=10.64$

5.4 Significance of conservativeness reduction

The comparison and analysis were performed on the conservativeness between the proposed method and the existing methods by two numerical examples. This subsection will discuss the practical significance of reducing conservativeness through the example of the electric power market.

The maximum update period, T_{mct_M}, of the acceptable power price signal calculated by the stability criterion, meaning the sampling period in the sampled-data control system can be used as a reference for selecting a sampling device, such as the capacity and type of communication channel, and the sampling frequency of signal collector. Tab. 3 shows that the T_{mct_M} value obtained by **Theorem 1** is larger than that in Refs. [14-15], indicating that the conservativeness, and a lower requirement for hardware devices for the proposed method. That is, based on the conditions for maintaining the stability of the electric power market (i.e., the supply and demand balance of power), the sampling device can be selected with a lower sampling performance (and consequent lower price) through the proposed method compared to that used in Refs. [14-15], which also reduces cost.

It can be seen from Tab. 3 that both the methods in Ref. [15] and **Theorem 1** (the sampling frequency of the collector satisfies $f_0 \geqslant 1/8.59$), can maintain the stability of the electric power market. When $8.59 \leqslant T_{mct_M} \leqslant 10.64$, Ref. [15] cannot maintain the stability of the electric power market, while **Theorem 1** can still maintain the stability of the system, that is, for **Theorem 1**, the sampling frequency of $f_1 \geqslant 1/10.64$ is enough. Here, $f_1 < f_0$, highlighting that the proposed method allows for the selection of a signal collector with a lower frequency, hence reducing the cost.

6 Conclusions

This study provides a method for analyzing the stability of sampled-data control systems, and applies it to the electric power market. First, a new two-sided closed-loop Lyapunov functional is constructed by considering the information of the system over the entire sampling interval. Second, the modified free-matrix integral inequality is used to estimate the integral term in the functional derivative, and then derived the stability condition of the system. The effectiveness and low conservativeness of the proposed method are verified through two examples. Finally, through the examples, the proposed method is applied to analyze the stability of the electric power market, and to assess the practical significance of reducing the conservativeness.

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Tian Zhang received his B.S. degree in electrical engineering from Hunan University of Technology, Zhuzhou, China, from September 2014 to July 2018. He is currently a postgraduate student at Hunan University of Technology. His current research interest is robust control in power systems.

Shenping Xiao received the B.S. degree In automation from Northeastern University, Shenyang, China, in 1988, the M.S. degree in computer science from the Central South University of Forestry and Technology, Changsha, China, in 2002, and the Ph.D. degree in control theory and control engineering from Central South University, Changsha, in 2008. He has been with the Department of Electrical and Information Engineering, Hunan University of Technology, Zhuzhou, China, where he is currently a Professor of automatic control engineering. His current research interests are time-delay systems, networked control systems, and robust control in power systems.