Delay-dependent Robust Stability Analysis of Power Systems with PID Controller*

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Abstract: This study examines the robust stability of a power system, which is based on proportional-integral-derivative load frequency control and involves uncertain parameters and time delays. The model of the system is firstly established, following which the system is transformed into a closed-loop system with feedback control. On this basis, a new augmented Lyapunov-Krasovskii (LK) functional is established for using the new Bessel-Legendre inequality to estimate the derivative of the functional, which can provide a maximum lower bound. A stability criterion is then derived by employing the LK functional and Bessel-Legendre inequality. Finally, numerical examples are used to demonstrate the validity and superiority of the proposed method.

Keywords: Power systems, time delay systems, load frequency control, PID control, robust stability

1 Introduction

The development of an intelligent power grid requires that the power system frequency operate safely within a small range of its equilibrium value, and the load frequency control[1] (LFC) can meet this requirement. In the process of data transmission, time delays are inevitable and uncertain[2]. The existence of these factors is likely to affect the stability of the power system. Therefore, studying the robust stability of power systems with time delays and uncertain parameters is of tremendous value.

For time-delay systems, most research methods are based on the Lyapunov direct time-domain method[3-5]. Through the construction of the Lyapunov-Krasovskii (LK) functional, the stability of the system is analyzed with the aid of the Lyapunov stability theory, after which the stability criterion is derived. Finally, the stability margin of the system is obtained by employing the linear matrix inequality (LMI) toolbox. A new integral inequality containing the Jensen inequality was proposed in Ref. [6], which produced less conservative results. A generalized free-matrix-based integral inequality was presented in Ref. [7], and the inequality could deal with time-varying delay systems without using the reciprocal convexity lemma. An inverse convex inequality was introduced and a new augmented LK functional was proposed in Ref. [8], and the conservatism of existing results for time-delay systems was reduced considerably. The study in Ref. [9] provided an overview of recent developments in each step with an insightful understanding. The method of free-weight-matrix was proposed and introduced into the study of robust control for time-delay systems in Ref. [10]. The study in Ref. [11] examined the stability of time-delay power systems with small-signal and load frequency control. Upon application of the method discussed in Ref. [4], the delay upper bound was improved in Ref. [12]. The stability margin of the system was expanded by constructing a new LK functional and employing the Wirtinger integral inequality, Extended integral inequality, Convex combination approach, and Schur compliment to estimate the derivative of the functional in Ref. [13]. The Wirtinger inequality was used to process the integral term in the functional derivative, and the obtained stability criterion of power systems with time delays had certain limitations in Ref. [14]. The study in Ref. [15] analyzed the stability of multi-region time-delay power systems, but the stability criterion also had certain limitations. The processing method of uncertain parameters was given in Ref. [16].

This study aims to improve the upper bound of the time delay of the system by proposing an augmented LK functional and using the Bessel-Legendre inequality to estimate the derivative of the functional. This study assumes that the forward

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channel of the controller possesses time delays and the inertia time constant of both the prime mover and speed governor may have uncertain parameters in the system. A time-varying delay power system model is established with uncertain parameters based on proportional-integral-derivative (PID) load frequency control. An appropriate LK functional is constructed, and the robust stability of the system is then analyzed using the Lyapunov stability theory. Finally, the robust stability criterion of the system is obtained by employing the Bessel-Legendre inequality discussed in Ref. [9], and the stability margin that the system can withstand is solved using an LMI toolbox. The advantages and effectiveness of the proposed method are demonstrated by comparing the results of the proposed method with those of previous methods under the two controller gains.

The variables used in this study are defined as follows: $\boldsymbol{R}^n$ and $\boldsymbol{R}^{n\times m}$ denote n-dimensional vectors and n × m dimensional matrices in the real number domain, respectively; $\boldsymbol{R}^{\mathrm{T}}$ and $\boldsymbol{R}^{-1}$ represent the transpose and inverse of a matrix, respectively; $\boldsymbol{I}$ and $\boldsymbol{0}$ are identity and zero matrices, respectively; $\boldsymbol{P}>0$ means that the matrix $\boldsymbol{P}$ is symmetric and positive; Sym$\{\boldsymbol{X}\}=\boldsymbol{X}+\boldsymbol{X}^{\mathrm{T}}$; '*' represents symmetric terms in a symmetric matrix; and diag$\{\cdots\}$ denotes a diagonal matrix.

2 System model

To facilitate research, this study considered a simplified LFC model [11]. Suppose the time delays are generated by the forward channel of the controller.

A simple block diagram diagram of one area of the LFC structure is shown in Fig. 1.

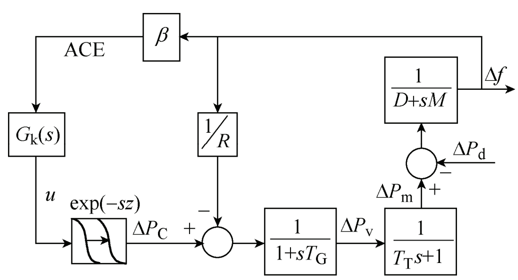

Fig. 1 Block diagram of one area of the LFC structure with time-delay

The state variable $\boldsymbol{x}_0(t)$ and the output variable $\boldsymbol{y}_0(t)$ are defined as follows

$$\boldsymbol{x}_0(t)=\begin{bmatrix}\Delta\boldsymbol{f} & \Delta\boldsymbol{P}_{\mathrm{m}} & \Delta\boldsymbol{P}_{\mathrm{v}}\end{bmatrix}^{\mathrm{T}} \quad \boldsymbol{y}_0(t)=\boldsymbol{ACE}(t)$$

where $\Delta\boldsymbol{f}$, $\Delta\boldsymbol{P}_{\mathrm{m}}$, and $\Delta\boldsymbol{P}_{\mathrm{v}}$ denote the variation in the system frequency, mechanical power, and control valve opening, respectively.

The following systematic mathematical model can then be obtained

$$\begin{cases}\dot{\boldsymbol{x}}_0(t)=\boldsymbol{A}\boldsymbol{x}_0(t)+\boldsymbol{B}\boldsymbol{u}(t-h(t))+\boldsymbol{F}\boldsymbol{\omega}(t)\\ \boldsymbol{y}_0(t)=\boldsymbol{C}\boldsymbol{x}_0(t)\end{cases} \tag{1}$$

where $h(t)$ represents a time-varying function that satisfies $0\leqslant h(t)\leqslant h$; h denotes the delay stability margin; $\boldsymbol{u}(t-h(t))=\Delta\boldsymbol{P}_{\mathrm{c}}(t)$; $\boldsymbol{\omega}(t)=\Delta\boldsymbol{P}_{\mathrm{d}}$; $\Delta\boldsymbol{P}_{\mathrm{d}}$ and $\Delta\boldsymbol{P}_{\mathrm{c}}$ represent changes in grid load and system control signals, respectively. In addition

$$\boldsymbol{A}=\begin{bmatrix}-\dfrac{\boldsymbol{D}}{\boldsymbol{M}} & \dfrac{\boldsymbol{D}}{\boldsymbol{M}} & \boldsymbol{0}\\ \boldsymbol{0} & -\dfrac{1}{\boldsymbol{T}_{\mathrm{T}}} & \dfrac{1}{\boldsymbol{T}_{\mathrm{T}}}\\ -\dfrac{1}{\boldsymbol{T}_{\mathrm{G}}\boldsymbol{R}} & \boldsymbol{0} & -\dfrac{1}{\boldsymbol{T}_{\mathrm{G}}}\end{bmatrix},\quad \boldsymbol{B}=\begin{bmatrix}\boldsymbol{0}\\ \boldsymbol{0}\\ \dfrac{1}{\boldsymbol{T}_{\mathrm{G}}}\end{bmatrix},$$

$$\boldsymbol{F}=\begin{bmatrix}-\dfrac{1}{\boldsymbol{M}} & \boldsymbol{0} & \boldsymbol{0}\end{bmatrix}^{\mathrm{T}}, \boldsymbol{C}=\begin{bmatrix}\boldsymbol{\beta} & \boldsymbol{0} & \boldsymbol{0}\end{bmatrix}$$

where $\boldsymbol{M}$ and $\boldsymbol{D}$ are the moment of inertia and damping coefficient of the generator, respectively; $\boldsymbol{T}_{\mathrm{T}}$ and $\boldsymbol{T}_{\mathrm{G}}$ are the inertia time of the governor of the steam turbine and unit, respectively; $\boldsymbol{R}$ is the speed drop coefficient of the governor; $\boldsymbol{\beta}$ is the frequency deviation factor; and $\boldsymbol{\beta}=1/\boldsymbol{R}+\boldsymbol{D}$.

PID control is such that

$$\boldsymbol{u}(t)=$$

$$\boldsymbol{K}_{\mathrm{P}}\boldsymbol{ACE}(t)+\boldsymbol{K}_{\mathrm{I}}\int\boldsymbol{ACE}(t)\mathrm{d}t+\boldsymbol{K}_{\mathrm{D}}\frac{\mathrm{d}\boldsymbol{ACE}(t)}{\mathrm{d}t} \tag{2}$$

Assume that the virtual state and output variables are as follows

$$\bar{\boldsymbol{x}}_0(t)=\begin{bmatrix}\boldsymbol{x}_0^{\mathrm{T}}(t) & \int\boldsymbol{y}_0^{\mathrm{T}}(t)\,\mathrm{d}t\end{bmatrix}^{\mathrm{T}} \tag{3}$$

$$\bar{\boldsymbol{y}}_0(t)=\begin{bmatrix}\boldsymbol{y}_0^{\mathrm{T}}(t) & \int\boldsymbol{y}_0^{\mathrm{T}}(t)\,\mathrm{d}t & \dfrac{\mathrm{d}\,\boldsymbol{y}_0^{\mathrm{T}}(t)}{\mathrm{d}t}\end{bmatrix}^{\mathrm{T}} \tag{4}$$

Because $\boldsymbol{CB}=\boldsymbol{0}$, system (1) and controller (2) can be changed into the following static output feedback control system

$$\begin{cases}\dot{\bar{\boldsymbol{x}}}_0(t)=\bar{\boldsymbol{A}}\bar{\boldsymbol{x}}_0(t)+\bar{\boldsymbol{B}}\boldsymbol{u}(t-h(t))+\bar{\boldsymbol{F}}\boldsymbol{\omega}(t)\\ \bar{\boldsymbol{y}}_0(t)=\bar{\boldsymbol{C}}\bar{\boldsymbol{x}}_0(t)+\bar{\boldsymbol{D}}_{\mathrm{w}}\boldsymbol{\omega}(t)\\ \boldsymbol{u}(t)=-\boldsymbol{K}\bar{\boldsymbol{y}}_0(t)\end{cases} \tag{5}$$

where $\bar{\boldsymbol{A}}=\begin{bmatrix}\boldsymbol{A} & \boldsymbol{0}\\ \boldsymbol{C} & \boldsymbol{0}\end{bmatrix}$, $\bar{\boldsymbol{B}}=\begin{bmatrix}\boldsymbol{B}\\ \boldsymbol{0}\end{bmatrix}$, $\bar{\boldsymbol{F}}=\begin{bmatrix}\boldsymbol{F}\\ \boldsymbol{0}\end{bmatrix}$,

$$\bar{C} = \begin{bmatrix} C & 0 \\ 0 & I \\ CA & 0 \end{bmatrix}, \quad \bar{D}_{\mathrm{w}} = \begin{bmatrix} 0 \\ 0 \\ CF \end{bmatrix}, \quad K = \begin{bmatrix} K_{\mathrm{P}} \\ K_{\mathrm{I}} \\ K_{\mathrm{D}} \end{bmatrix}^{\mathrm{T}}.$$

The aforementioned feedback control system (5) is further transformed into a linear system with state time-delay

$$\dot{\bar{x}}_0(t) = \bar{A}\bar{x}_0(t) + \bar{A}_{\mathrm{d}}\bar{x}_0(t-h(t)) + \bar{B}_{\mathrm{w}}\omega(t) \tag{6}$$

where $\bar{A}_{\mathrm{d}} = -\bar{B}K\bar{C}$, $\bar{B}_{\mathrm{w}} = \bar{F} - \bar{B}K\bar{D}_{\mathrm{w}}$.

If we assume that the operating equilibrium point of system (6) is $\bar{x}_0^*(t)$, then

$$0 = \bar{A}\bar{x}_0^*(t) + \bar{A}_{\mathrm{d}}x_0^*(t-h(t)) + \bar{B}_{\mathrm{w}}\omega(t) \tag{7}$$

Let us assume a new state variable $x(t) = \bar{x}_0(t) - \bar{x}_0^*(t)$. We then can derive the following function by subtracting formula (7) from formula (6)

$$\dot{x}(t) = \bar{A}x(t) + \bar{A}_{\mathrm{d}}x(t-h(t)), \tag{8}$$

where

$$\bar{A} = \begin{bmatrix} -\frac{D}{M} & \frac{1}{M} & 0 & 0 \\ 0 & -\frac{1}{T_{\mathrm{T}}} & \frac{1}{T_{\mathrm{T}}} & 0 \\ -\frac{1}{T_{\mathrm{G}}R} & 0 & -\frac{1}{T_{\mathrm{G}}} & 0 \\ \beta & 0 & 0 & 0 \end{bmatrix},$$

$$\bar{A}_{\mathrm{d}} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{\beta DK_{\mathrm{D}}}{MT_{\mathrm{G}}} - \frac{\beta K_{\mathrm{P}}}{T_{\mathrm{G}}} & -\frac{\beta K_{\mathrm{D}}}{MT_{\mathrm{G}}} & 0 & -\frac{K_{\mathrm{I}}}{T_{\mathrm{G}}} \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

However, the parameters of real power systems are inevitably disturbed. We can assume that the inertia time constant of the prime mover and that of the speed governor appear with a deviation of $\pm\alpha\%$, $\pm\gamma\%$, as follows

$$\begin{cases} T_{\mathrm{Ta}} \in \left[(1-\alpha\%)T_{\mathrm{T}}, (1+\alpha\%)T_{\mathrm{T}}\right] \\ T_{\mathrm{Ga}} \in \left[(1-\gamma\%)T_{\mathrm{G}}, (1+\gamma\%)T_{\mathrm{G}}\right] \end{cases} \tag{9}$$

where T_{Ta} and T_{Ga} are the actual inertia time constants.

The disturbance parameters in formula (9) can be expressed by the time-varying function:

$$\frac{1}{T_{\mathrm{Ta}}} = \frac{\alpha_1}{T_{\mathrm{T}}} + f_1(t)\frac{\alpha_2}{T_{\mathrm{T}}} \qquad \frac{1}{T_{\mathrm{Ga}}} = \frac{\gamma_1}{T_{\mathrm{G}}} + f_2(t)\frac{\gamma_2}{T_{\mathrm{G}}} \tag{10}$$

Where

$f_1(t), f_2(t) \in [-1, 1]$

$$\alpha_1 = \frac{1}{(1-\alpha\%)(1+\alpha\%)}, \quad \alpha_2 = \alpha\% \times \alpha_1$$

$$\gamma_1 = \frac{1}{(1-\gamma\%)(1+\gamma\%)}, \quad \gamma_2 = \gamma\% \times \gamma_1$$

Therefore, the system (8) can be transformed into a time-varying delay power system based on PID load frequency control with disturbance parameters

$$\dot{x}(t) = (A_0 + \Delta A_0)x(t) + (A_{\mathrm{d}} + \Delta A_{\mathrm{d}})x(t-h(t)), \tag{11}$$

where

$\begin{bmatrix} \Delta A_0 & \Delta A_{\mathrm{d}} \end{bmatrix} = HF(t)\begin{bmatrix} E_1 & E_2 \end{bmatrix} \quad H = I$

$F(t) = \mathrm{diag}\{0, f_1(t), f_2(t), 0\}$

$$A_0 = \begin{bmatrix} -\frac{D}{M} & \frac{1}{M} & 0 & 0 \\ 0 & -\frac{\alpha_1}{T_{\mathrm{T}}} & \frac{\alpha_1}{T_{\mathrm{T}}} & 0 \\ -\frac{\gamma_1}{T_{\mathrm{G}}R} & 0 & -\frac{\gamma_1}{T_{\mathrm{G}}} & 0 \\ \beta & 0 & 0 & 0 \end{bmatrix}$$

$$A_{\mathrm{d}} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{\gamma_1\beta DK_{\mathrm{D}}}{MT_{\mathrm{G}}} - \frac{\gamma_1\beta K_{\mathrm{P}}}{T_{\mathrm{G}}} & -\frac{\gamma_1\beta K_{\mathrm{D}}}{MT_{\mathrm{G}}} & 0 & -\frac{\gamma_1 K_{\mathrm{I}}}{T_{\mathrm{G}}} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$E_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -\frac{\alpha_2}{T_{\mathrm{T}}} & \frac{\alpha_2}{T_{\mathrm{T}}} & 0 \\ -\frac{\gamma_2}{T_{\mathrm{G}}R} & 0 & -\frac{\gamma_2}{T_{\mathrm{G}}} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$E_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{\gamma_2\beta DK_{\mathrm{D}}}{MT_{\mathrm{G}}} - \frac{\gamma_2\beta K_{\mathrm{P}}}{T_{\mathrm{G}}} & -\frac{\gamma_2\beta K_{\mathrm{D}}}{MT_{\mathrm{G}}} & 0 & -\frac{\gamma_2 K_{\mathrm{I}}}{T_{\mathrm{G}}} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Following a series of derivations and analyses, the analysis of the system (1) is changed to that of the system (11).

3 Robust stability criterion

To obtain the stability criterion of the system, the two following lemmas are considered:

Lemma 1[9]**.** Let ω be a differentiable functional: $[a, b] \to \mathbf{R}^{\mathrm{n}}$. For a real symmetric matrix $Z > 0$ and any real matrix M with appropriate dimensions, the following inequality holds:

$$-\int_a^b \dot{\boldsymbol{\omega}}^{\mathrm{T}}(s)\boldsymbol{Z}\dot{\boldsymbol{\omega}}(s)\mathrm{d}s \leqslant \varsigma^{\mathrm{T}}\left[\mathrm{Sym}\left\{\boldsymbol{\Pi}^{\mathrm{T}}\boldsymbol{M}\right\}+(b-a)\boldsymbol{M}^{\mathrm{T}}\tilde{\boldsymbol{Z}}\boldsymbol{M}\right]\varsigma \tag{12}$$

Where

$$\tilde{\boldsymbol{Z}}=\mathrm{diag}\left\{\boldsymbol{Z},3\boldsymbol{Z},5\boldsymbol{Z}\right\}$$

$$\varsigma=\left[\boldsymbol{\omega}(b)\quad \boldsymbol{\omega}(a)\quad \gamma_1\quad \gamma_2\right]^{\mathrm{T}}$$

$$\gamma_1=\int_a^b\frac{\boldsymbol{\omega}(s)\mathrm{d}s}{b-a}\qquad \gamma_2=\int_a^b\frac{(b-s)\boldsymbol{\omega}(s)\mathrm{d}s}{(b-a)^2}$$

$$\boldsymbol{\Pi}=\begin{bmatrix}\boldsymbol{I} & -\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{I} & \boldsymbol{I} & -2\boldsymbol{I} & \boldsymbol{0}\\ \boldsymbol{I} & -\boldsymbol{I} & -6\boldsymbol{I} & 12\boldsymbol{I}\end{bmatrix}$$

Lemma 2[16]**.** For any real matrices $\boldsymbol{Z}=\boldsymbol{Z}^{\mathrm{T}}$, $\boldsymbol{H}$, and $\boldsymbol{E}$ with appropriate dimensions, if the inequality $\boldsymbol{Z}+\boldsymbol{HFE}+\boldsymbol{E}^{\mathrm{T}}\boldsymbol{F}^{\mathrm{T}}\boldsymbol{H}^{\mathrm{T}}<0$ is true, for any $\boldsymbol{F}$ that satisfies $\boldsymbol{F}^{\mathrm{T}}\boldsymbol{F}\leqslant\boldsymbol{I}$, we can obtain the following inequality when a scalar $\lambda>0$ exists

$$\boldsymbol{Z}+\lambda\boldsymbol{H}\boldsymbol{H}^{\mathrm{T}}+\lambda^{-1}\boldsymbol{E}^{\mathrm{T}}\boldsymbol{E}<0. \tag{13}$$

For brevity, the following nomenclatures we require for vectors and matrices are given as

$$\tilde{h}(t)=h-h(t)\qquad \tilde{\boldsymbol{R}}=\mathrm{diag}\left\{\boldsymbol{R}\quad 3\boldsymbol{R}\quad 5\boldsymbol{R}\right\}$$

$$\boldsymbol{\theta}_0(s)=\left[\dot{\boldsymbol{x}}^{\mathrm{T}}(s)\quad \boldsymbol{x}^{\mathrm{T}}(s)\right]^{\mathrm{T}}$$

$$\boldsymbol{v}_i=\int_{t-h(t)}^{t}\frac{(t-s)^{i-1}\boldsymbol{x}^{\mathrm{T}}(s)}{h^i(t)}\mathrm{d}s$$

$$\boldsymbol{\varpi}_i=\int_{t-h}^{t-h(t)}\frac{\left[t-h(t)-s\right]^{i-1}\boldsymbol{x}^{\mathrm{T}}(s)}{\tilde{h}^i(t)}\mathrm{d}s\qquad i=1,2$$

$$\chi_1=\int_{t-h}^{t}\boldsymbol{x}^{\mathrm{T}}(s)\mathrm{d}s\qquad \chi_2=\int_{t-h}^{t}(t-s)\boldsymbol{x}^{\mathrm{T}}(s)\mathrm{d}s$$

$$\boldsymbol{\eta}_0(t)=\left[\boldsymbol{x}^{\mathrm{T}}(t)\quad \boldsymbol{x}^{\mathrm{T}}(t-h)\right]^{\mathrm{T}}$$

$$\boldsymbol{\eta}_1(t)=\left[\boldsymbol{x}^{\mathrm{T}}(t)\quad \boldsymbol{x}^{\mathrm{T}}(t-h(t))\quad \boldsymbol{x}^{\mathrm{T}}(t-h)\right]^{\mathrm{T}}$$

$$\boldsymbol{\eta}_2(t)=\left[\boldsymbol{v}_1\quad \boldsymbol{v}_2\quad \boldsymbol{\varpi}_1\quad \boldsymbol{\varpi}_2\right]^{\mathrm{T}}$$

$$\boldsymbol{\eta}_3(t)=\left[h(t)\boldsymbol{v}_1\quad h(t)\boldsymbol{v}_2\quad \tilde{h}(t)\boldsymbol{\varpi}_1\quad \tilde{h}(t)\boldsymbol{\varpi}_2\right]^{\mathrm{T}}$$

$$\boldsymbol{\eta}_4(t)=\left[\dot{\boldsymbol{x}}^{\mathrm{T}}(t-h)\quad \dot{\boldsymbol{x}}^{\mathrm{T}}(t)\right]^{\mathrm{T}}$$

$$\boldsymbol{\eta}(t)=\left[\boldsymbol{\eta}_0^{\mathrm{T}}(t)\quad \chi_1\quad \chi_2\right]^{\mathrm{T}}$$

$$\boldsymbol{\theta}(t,s)=\left[\boldsymbol{\theta}_0^{\mathrm{T}}(s)\quad \boldsymbol{\eta}_0^{\mathrm{T}}(t)\quad \int_{t-h}^{s}\boldsymbol{x}^{\mathrm{T}}(\theta)\mathrm{d}\theta\right]^{\mathrm{T}}$$

$$\boldsymbol{\xi}(t)=\left[\boldsymbol{\eta}_1^{\mathrm{T}}(t)\quad \boldsymbol{\eta}_2^{\mathrm{T}}(t)\quad \boldsymbol{\eta}_3^{\mathrm{T}}(t)\quad \boldsymbol{\eta}_4^{\mathrm{T}}(t)\right]^{\mathrm{T}}$$

$$\boldsymbol{e}_i=\left[\boldsymbol{0}_{n\times(i-1)n}\ \boldsymbol{I}_n\ \boldsymbol{0}_{n\times(13-i)n}\right]\quad i=1,2,\cdots,13$$

$$\boldsymbol{\Pi}_0=\left[h(t)(\boldsymbol{e}_9^{\mathrm{T}}+\boldsymbol{e}_{10}^{\mathrm{T}})+\tilde{h}(t)\boldsymbol{e}_{11}^{\mathrm{T}}\right]^{\mathrm{T}}$$

$$\boldsymbol{\Pi}_1=\left[\boldsymbol{e}_1^{\mathrm{T}}\quad \boldsymbol{e}_3^{\mathrm{T}}\quad \boldsymbol{e}_8^{\mathrm{T}}+\boldsymbol{e}_{10}^{\mathrm{T}}\quad \boldsymbol{\Pi}_0^{\mathrm{T}}\right]^{\mathrm{T}}$$

$$\boldsymbol{\Pi}_2=\left[\boldsymbol{e}_{13}^{\mathrm{T}}\quad \boldsymbol{e}_1^{\mathrm{T}}-\boldsymbol{e}_3^{\mathrm{T}}\quad \boldsymbol{e}_{12}^{\mathrm{T}}\quad \boldsymbol{e}_8^{\mathrm{T}}+\boldsymbol{e}_{10}^{\mathrm{T}}-h\boldsymbol{e}_3^{\mathrm{T}}\right]^{\mathrm{T}}$$

$$\boldsymbol{\Pi}_3=\left[\boldsymbol{e}_{13}^{\mathrm{T}}\quad \boldsymbol{e}_1^{\mathrm{T}}\quad \boldsymbol{e}_1^{\mathrm{T}}\quad \boldsymbol{e}_3^{\mathrm{T}}\quad \boldsymbol{e}_8^{\mathrm{T}}+\boldsymbol{e}_{10}^{\mathrm{T}}\right]^{\mathrm{T}}$$

$$\boldsymbol{\Pi}_4=\left[\boldsymbol{e}_{12}^{\mathrm{T}}\quad \boldsymbol{e}_3^{\mathrm{T}}\quad \boldsymbol{e}_1^{\mathrm{T}}\quad \boldsymbol{e}_3^{\mathrm{T}}\quad \boldsymbol{0}\right]^{\mathrm{T}}$$

$$\boldsymbol{\Pi}_5=\left[\boldsymbol{e}_1^{\mathrm{T}}-\boldsymbol{e}_3^{\mathrm{T}}\quad \boldsymbol{e}_8^{\mathrm{T}}+\boldsymbol{e}_{10}^{\mathrm{T}}\quad h\boldsymbol{e}_1^{\mathrm{T}}\quad h\boldsymbol{e}_3^{\mathrm{T}}\quad \boldsymbol{\Pi}_0^{\mathrm{T}}\right]^{\mathrm{T}}$$

$$\boldsymbol{\Pi}_6=\left[\boldsymbol{0}\quad \boldsymbol{0}\quad \boldsymbol{e}_{13}^{\mathrm{T}}\quad \boldsymbol{e}_{12}^{\mathrm{T}}\quad -\boldsymbol{e}_3^{\mathrm{T}}\right]^{\mathrm{T}}$$

$$\boldsymbol{\Pi}_{7a}=\left[\boldsymbol{e}_1^{\mathrm{T}}\quad \boldsymbol{e}_2^{\mathrm{T}}\quad \boldsymbol{e}_4^{\mathrm{T}}\quad \boldsymbol{e}_5^{\mathrm{T}}\right]^{\mathrm{T}}\qquad \boldsymbol{\Pi}_7=\boldsymbol{\Pi}\boldsymbol{\Pi}_{7a}$$

$$\boldsymbol{\Pi}_{8a}=\left[\boldsymbol{e}_2^{\mathrm{T}}\quad \boldsymbol{e}_3^{\mathrm{T}}\quad \boldsymbol{e}_6^{\mathrm{T}}\quad \boldsymbol{e}_7^{\mathrm{T}}\right]^{\mathrm{T}}\qquad \boldsymbol{\Pi}_8=\boldsymbol{\Pi}\boldsymbol{\Pi}_{8a}$$

$$\boldsymbol{\Pi}_{9a}=\left[\boldsymbol{e}_8^{\mathrm{T}}-h(t)\boldsymbol{e}_4^{\mathrm{T}}\quad \boldsymbol{e}_9^{\mathrm{T}}-h(t)\boldsymbol{e}_5^{\mathrm{T}}\right]^{\mathrm{T}}$$

$$\boldsymbol{\Pi}_{9b}=\left[\boldsymbol{e}_{10}^{\mathrm{T}}-\tilde{h}(t)\boldsymbol{e}_6^{\mathrm{T}}\quad \boldsymbol{e}_{11}^{\mathrm{T}}-\tilde{h}(t)\boldsymbol{e}_7^{\mathrm{T}}\right]^{\mathrm{T}}$$

$$\boldsymbol{\Pi}_9=\left[\boldsymbol{\Pi}_{9a}^{\mathrm{T}}\quad \boldsymbol{\Pi}_{9b}^{\mathrm{T}}\right]^{\mathrm{T}}\qquad \boldsymbol{\Pi}_{10}=\left[\boldsymbol{e}_{13}^{\mathrm{T}}\boldsymbol{X}_1+\boldsymbol{e}_1^{\mathrm{T}}\boldsymbol{X}_2+\boldsymbol{e}_2^{\mathrm{T}}\boldsymbol{X}_3\right]^{\mathrm{T}}$$

$$\boldsymbol{\Pi}_{11}=[\boldsymbol{e}_1^{\mathrm{T}}\boldsymbol{A}_0^{\mathrm{T}}+\boldsymbol{e}_2^{\mathrm{T}}\boldsymbol{A}_{\mathrm{d}}^{\mathrm{T}}-\boldsymbol{e}_{13}^{\mathrm{T}}]^{\mathrm{T}}\qquad \boldsymbol{\theta}_1=\left[\boldsymbol{\Pi}_{10}^{\mathrm{T}}\boldsymbol{H}\right]^{\mathrm{T}}$$

$$\boldsymbol{\theta}_2=\left[\boldsymbol{e}_1^{\mathrm{T}}\boldsymbol{E}_1^{\mathrm{T}}+\boldsymbol{e}_2^{\mathrm{T}}\boldsymbol{E}_2^{\mathrm{T}}\right]^{\mathrm{T}}\qquad \boldsymbol{\Phi}_0=\boldsymbol{\Phi}_1+\boldsymbol{\Phi}_2+\boldsymbol{\Phi}_3$$

$$\boldsymbol{\Phi}_1=\mathrm{Sym}\left\{\boldsymbol{\Pi}_1^{\mathrm{T}}\boldsymbol{P}\boldsymbol{\Pi}_2+\boldsymbol{\Pi}_5^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{\Pi}_6\right\}+\boldsymbol{\Pi}_3^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{\Pi}_3-\boldsymbol{\Pi}_4^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{\Pi}_4+h\dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{R}\dot{\boldsymbol{x}}(t)$$

$$\boldsymbol{\Phi}_2=\mathrm{Sym}\left\{\boldsymbol{\Pi}_{10}^{\mathrm{T}}\boldsymbol{\Pi}_{11}+\boldsymbol{N}\boldsymbol{\Pi}_9+\boldsymbol{\Pi}_7^{\mathrm{T}}\boldsymbol{Y}_1+\boldsymbol{\Pi}_8^{\mathrm{T}}\boldsymbol{Y}_2\right\}$$

$$\boldsymbol{\Phi}_3=h(t)\boldsymbol{Y}_1^{\mathrm{T}}\tilde{\boldsymbol{R}}^{-1}\boldsymbol{Y}_1+\tilde{h}(t)\boldsymbol{Y}_2^{\mathrm{T}}\tilde{\boldsymbol{R}}^{-1}\boldsymbol{Y}_2$$

Based on the Bessel-Legendre inequality (12) and the free weight matrix method, the stability criterion for the PID load frequency control system with time-varying delays is obtained. The following system stability criteria can be obtained.

Theorem 1. For a scalar $\lambda>0$, if there exist real symmetric matrices $\boldsymbol{P}$ ($\in\mathbf{R}^{4n\times4n}$)$>0$, $\boldsymbol{Q}$ ($\in\mathbf{R}^{5n\times5n}$)$>0$, $\boldsymbol{R}$ ($\in\mathbf{R}^{n\times n}$)$>0$, and any real matrices $\boldsymbol{Y}_1$, $\boldsymbol{Y}_2$, $\boldsymbol{X}_1$, $\boldsymbol{X}_2$, $\boldsymbol{X}_3$, and $\boldsymbol{N}$ with appropriate dimensions, the system (11) is stable if the following LMIs (14) and (15) are satisfied for $0\leqslant h(t)\leqslant h$

$$\begin{bmatrix}\boldsymbol{\Phi}(0) & \boldsymbol{\theta}_1^{\mathrm{T}} & h\boldsymbol{Y}_2^{\mathrm{T}}\\ * & -\lambda\boldsymbol{I} & \boldsymbol{0}\\ * & * & -h\tilde{\boldsymbol{R}}\end{bmatrix}<0 \tag{14}$$

$$\begin{bmatrix}\boldsymbol{\Phi}(h) & \boldsymbol{\theta}_1^{\mathrm{T}} & h\boldsymbol{Y}_1^{\mathrm{T}}\\ * & -\lambda\boldsymbol{I} & \boldsymbol{0}\\ * & * & -h\tilde{\boldsymbol{R}}\end{bmatrix}<0 \tag{15}$$

where $\boldsymbol{\Phi}(h(t))=\boldsymbol{\Phi}_1+\boldsymbol{\Phi}_2+\lambda\boldsymbol{\theta}_2^{\mathrm{T}}\boldsymbol{\theta}_2$.

Proof. According to the Lyapunov theory, a new LK functional is constructed as follows.

$$V(\boldsymbol{x}_t)=\boldsymbol{\eta}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{\eta}(t)+\int_{t-h}^{t}\boldsymbol{\theta}^{\mathrm{T}}(t,s)\boldsymbol{Q}\boldsymbol{\theta}(t,s)\mathrm{d}s+\int_{-h}^{0}\int_{t+\theta}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{R}\dot{\boldsymbol{x}}(s)\mathrm{d}s\mathrm{d}\theta \tag{16}$$

When $\boldsymbol{P}>0$, $\boldsymbol{Q}>0$, and $\boldsymbol{R}>0$, the LK functional is positive. Therefore, we should consider the derivative of the LK functional

$$\dot{V}(\boldsymbol{x}_t)=\boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{\Phi}_1\boldsymbol{\xi}(t)-\int_{t-h}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{R}\dot{\boldsymbol{x}}(s)\mathrm{d}s \tag{17}$$

Then

$$-\int_{t-h}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{R}\dot{\boldsymbol{x}}(s)\mathrm{d}s=-\int_{t-h}^{t-h(t)}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{R}\dot{\boldsymbol{x}}(s)\mathrm{d}s-\int_{t-h(t)}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{R}\dot{\boldsymbol{x}}(s)\mathrm{d}s \tag{18}$$

On applying Lemma 1, we can obtain

$$-\int_{t-h(t)}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{R}\dot{\boldsymbol{x}}(s)\mathrm{d}s\leqslant \boldsymbol{\xi}^{\mathrm{T}}(t)\left(\mathrm{Sym}\left\{\boldsymbol{\Pi}_7^{\mathrm{T}}\boldsymbol{Y}_1\right\}+h(t)\boldsymbol{Y}_1^{\mathrm{T}}\tilde{\boldsymbol{R}}^{-1}\boldsymbol{Y}_1\right)\boldsymbol{\xi}(t) \tag{19}$$

$$-\int_{t-h}^{t-h(t)}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)\boldsymbol{R}\dot{\boldsymbol{x}}(s)\mathrm{d}s\leqslant \boldsymbol{\xi}^{\mathrm{T}}(t)\left(\mathrm{Sym}\left\{\boldsymbol{\Pi}_8^{\mathrm{T}}\boldsymbol{Y}_2\right\}+\tilde{h}(t)\boldsymbol{Y}_2^{\mathrm{T}}\tilde{\boldsymbol{R}}^{-1}\boldsymbol{Y}_2\right)\boldsymbol{\xi}(t) \tag{20}$$

For any real matrix $\boldsymbol{N}$ with compatible dimensions, the following equation holds

$$2\boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{N}\boldsymbol{\Pi}_9\boldsymbol{\xi}(t)=0 \tag{21}$$

Considering any real matrices $\boldsymbol{X}_i$ (i=1, 2, 3) with compatible dimensions, the following equation holds

$$0=\begin{bmatrix}-\dot{\boldsymbol{x}}(t)+(\boldsymbol{A}_0+\Delta\boldsymbol{A}_0)\boldsymbol{x}(t)\\+(\boldsymbol{A}_{\mathrm{d}}+\Delta\boldsymbol{A}_{\mathrm{d}})\boldsymbol{x}^{\mathrm{T}}(t-h(t))\end{bmatrix}\times 2\left[\dot{\boldsymbol{x}}^{\mathrm{T}}(t)\boldsymbol{X}_1+\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{X}_2+\boldsymbol{x}^{\mathrm{T}}(t-h(t))\boldsymbol{X}_3\right] \tag{22}$$

From $\begin{bmatrix}\Delta\boldsymbol{A}_0 & \Delta\boldsymbol{A}_{\mathrm{d}}\end{bmatrix}=\boldsymbol{H}\boldsymbol{F}(t)\begin{bmatrix}\boldsymbol{E}_1 & \boldsymbol{E}_2\end{bmatrix}$, we can have formula (22) transformed into

$$\boldsymbol{\xi}^{\mathrm{T}}(t)\left(\mathrm{Sym}\left\{\boldsymbol{\Pi}_{10}^{\mathrm{T}}\boldsymbol{\Pi}_{11}+\boldsymbol{\theta}_1^{\mathrm{T}}\boldsymbol{F}(t)\boldsymbol{\theta}_2\right\}\right)\boldsymbol{\xi}(t)=0 \tag{23}$$

Combining formulas (17)-(23), we can easily obtain

$$\dot{V}(x_t)\leqslant\boldsymbol{\xi}^{\mathrm{T}}(t)\bar{\boldsymbol{\Phi}}\boldsymbol{\xi}(t) \tag{24}$$

where $\bar{\boldsymbol{\Phi}}=\boldsymbol{\Phi}_0+\boldsymbol{\theta}_1^{\mathrm{T}}\boldsymbol{F}(t)\boldsymbol{\theta}_2+\boldsymbol{\theta}_2^{\mathrm{T}}(t)F(t)\boldsymbol{\theta}_1$.

If $\boldsymbol{\Phi}_0+\boldsymbol{\theta}_1^{\mathrm{T}}\boldsymbol{F}(t)\boldsymbol{\theta}_2+\boldsymbol{\theta}_2^{\mathrm{T}}(t)\boldsymbol{F}(t)\boldsymbol{\theta}_1<0$ is true, and by employing Lemma 2 for a scalar $\lambda>0$, we can obtain

$$\boldsymbol{\Phi}_0+\lambda^{-1}\boldsymbol{\theta}_1^{\mathrm{T}}\boldsymbol{\theta}_1+\lambda\boldsymbol{\theta}_2^{\mathrm{T}}\boldsymbol{\theta}_2<0 \tag{25}$$

The inequality (25) is equivalent to the inequalities (14) and (15) by applying the Schur theorem. Therefore, if the inequalities (14) and (15) are true, we can hold $\dot{V}(t)<0$. Then, the system (11) is stable from Lyapunov stability theory.

Now, we can assume that the time-delay is constant, that is $\dot{h}(t)=0$. The system (11) is then transformed into the following system

$$\dot{\boldsymbol{x}}(t)=(\boldsymbol{A}_0+\Delta\boldsymbol{A}_0)\boldsymbol{x}(t)+(\boldsymbol{A}_{\mathrm{d}}+\Delta\boldsymbol{A}_{\mathrm{d}})\boldsymbol{x}(t-\bar{h}) \tag{26}$$

where $\bar{h}$ represents a constant time-delay that satisfies $0\leqslant\bar{h}\leqslant h$, and other parameters are the same as system (11).

We choose the LK functional to be

$$\bar{V}(t)=\bar{\boldsymbol{\eta}}^{\mathrm{T}}(t)\bar{\boldsymbol{P}}\bar{\boldsymbol{\eta}}(t)+\int_{t-h}^{t}\bar{\boldsymbol{\theta}}^{\mathrm{T}}(t,s)\bar{\boldsymbol{Q}}\bar{\boldsymbol{\theta}}(t,s)\mathrm{d}s+\int_{-h}^{0}\int_{t+\theta}^{t}\dot{\boldsymbol{x}}^{\mathrm{T}}(s)\bar{\boldsymbol{R}}\dot{\boldsymbol{x}}(s)\mathrm{d}s\mathrm{d}\theta \tag{27}$$

where

$$\bar{\boldsymbol{\eta}}(t)=\begin{bmatrix}\bar{\boldsymbol{e}}_1^{\mathrm{T}} & \chi_1 & \chi_2\end{bmatrix}^{\mathrm{T}}$$

$$\bar{\boldsymbol{\theta}}(t,s)=\begin{bmatrix}\dot{\boldsymbol{x}}^{\mathrm{T}}(s) & \boldsymbol{\eta}_0^{\mathrm{T}}(t) & \int_{t-h}^{s}\boldsymbol{x}^{\mathrm{T}}(\theta)\mathrm{d}\theta\end{bmatrix}^{\mathrm{T}}$$

$$\bar{\boldsymbol{\xi}}(t)=\begin{bmatrix}\boldsymbol{\eta}_0^{\mathrm{T}}(t) & \dfrac{\chi_1}{h} & \dfrac{\chi_2}{h^2} & \dot{\boldsymbol{x}}^{\mathrm{T}}(t-h) & \dot{\boldsymbol{x}}^{\mathrm{T}}(t)\end{bmatrix}^{\mathrm{T}}$$

$$\bar{\boldsymbol{e}}_i=\begin{bmatrix}\boldsymbol{0}_{n\times(i-1)n} & \boldsymbol{I}_n & \boldsymbol{0}_{n\times(6-i)n}\end{bmatrix}\quad i=1,\ 2,\ \cdots,\ 6$$

On applying the proposed method, we can derive the following criterion for this functional:

Corollary 1. For a scalar $\bar{\lambda}>0$, if there exist real symmetric matrices $\bar{\boldsymbol{P}}$ ($\in\mathbf{R}^{4n\times4n}$)>0, $\bar{\boldsymbol{Q}}$ ($\in\mathbf{R}^{4n\times4n}$)>0, $\bar{\boldsymbol{R}}$ ($\in\mathbf{R}^{n\times n}$)>0, and any real matrices $\bar{\boldsymbol{Y}}$, $\bar{\boldsymbol{X}}_1$, $\bar{\boldsymbol{X}}_2$, and $\bar{\boldsymbol{X}}_3$ with appropriate dimensions, the system (26) is stable if the following LMI (28) is satisfied for $0\leqslant\bar{h}\leqslant h$

$$\begin{bmatrix}\bar{\boldsymbol{\Phi}} & \bar{\boldsymbol{\theta}}_1^{\mathrm{T}} & h\bar{\boldsymbol{Y}}^{\mathrm{T}}\\ * & -\lambda\boldsymbol{I} & \boldsymbol{0}\\ * & * & -h\tilde{\tilde{\boldsymbol{R}}}\end{bmatrix}<0 \tag{28}$$

where

$$\tilde{\tilde{\boldsymbol{R}}}=\mathrm{diag}\left\{\bar{\boldsymbol{R}}\quad 3\bar{\boldsymbol{R}}\quad 5\bar{\boldsymbol{R}}\right\}\qquad \bar{\boldsymbol{\Phi}}=\bar{\boldsymbol{\Phi}}_1+\bar{\boldsymbol{\Phi}}_2$$

$$\bar{\boldsymbol{\Phi}}_1=\mathrm{Sym}\left\{\bar{\boldsymbol{\Pi}}_1^{\mathrm{T}}\bar{\boldsymbol{P}}\bar{\boldsymbol{\Pi}}_2+\bar{\boldsymbol{\Pi}}_5^{\mathrm{T}}\bar{\boldsymbol{Q}}\bar{\boldsymbol{\Pi}}_6+\bar{\boldsymbol{\Pi}}_7^{\mathrm{T}}\bar{\boldsymbol{Y}}+\bar{\boldsymbol{\Pi}}_8^{\mathrm{T}}\bar{\boldsymbol{\Pi}}_9\right\}$$

$$\bar{\boldsymbol{\Phi}}_2=\bar{\boldsymbol{\Pi}}_3^{\mathrm{T}}\bar{\boldsymbol{Q}}\bar{\boldsymbol{\Pi}}_3-\bar{\boldsymbol{\Pi}}_4^{\mathrm{T}}\bar{\boldsymbol{Q}}\bar{\boldsymbol{\Pi}}_4+h\dot{\boldsymbol{x}}^{\mathrm{T}}(t)\bar{\boldsymbol{R}}\dot{\boldsymbol{x}}(t)+\bar{\lambda}\bar{\boldsymbol{\theta}}_2^{\mathrm{T}}\bar{\boldsymbol{\theta}}_2$$

$$\bar{\boldsymbol{\Pi}}_1=\begin{bmatrix}\bar{\boldsymbol{e}}_1^{\mathrm{T}} & h\bar{\boldsymbol{e}}_3^{\mathrm{T}} & h^2\bar{\boldsymbol{e}}_4^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}$$

$$\bar{\boldsymbol{\Pi}}_2=\begin{bmatrix}\bar{\boldsymbol{e}}_6^{\mathrm{T}} & \bar{\boldsymbol{e}}_1^{\mathrm{T}}-\bar{\boldsymbol{e}}_2^{\mathrm{T}} & h\bar{\boldsymbol{e}}_3^{\mathrm{T}}-h\bar{\boldsymbol{e}}_2^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}$$

$$\bar{\boldsymbol{\Pi}}_3=\begin{bmatrix}\bar{\boldsymbol{e}}_6^{\mathrm{T}} & \bar{\boldsymbol{e}}_1^{\mathrm{T}} & \bar{\boldsymbol{e}}_2^{\mathrm{T}} & h\bar{\boldsymbol{e}}_3^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}$$

$$\bar{\boldsymbol{\Pi}}_4=\begin{bmatrix}\bar{\boldsymbol{e}}_5^{\mathrm{T}} & \bar{\boldsymbol{e}}_1^{\mathrm{T}} & \bar{\boldsymbol{e}}_2^{\mathrm{T}} & \boldsymbol{0}\end{bmatrix}^{\mathrm{T}}$$

$$\bar{\boldsymbol{\Pi}}_5=\begin{bmatrix}\bar{\boldsymbol{e}}_1^{\mathrm{T}}-\bar{\boldsymbol{e}}_2^{\mathrm{T}} & h\bar{\boldsymbol{e}}_1^{\mathrm{T}} & h\bar{\boldsymbol{e}}_2^{\mathrm{T}} & h^2\bar{\boldsymbol{e}}_4^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}$$

$$\bar{\boldsymbol{\Pi}}_6=\begin{bmatrix}\boldsymbol{0} & \bar{\boldsymbol{e}}_6^{\mathrm{T}} & \bar{\boldsymbol{e}}_5^{\mathrm{T}} & -\bar{\boldsymbol{e}}_2^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}$$

$$\bar{\boldsymbol{\Pi}}_{7a}=\begin{bmatrix}\bar{\boldsymbol{e}}_1^{\mathrm{T}} & \bar{\boldsymbol{e}}_2^{\mathrm{T}} & \bar{\boldsymbol{e}}_3^{\mathrm{T}} & \bar{\boldsymbol{e}}_4^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}} \quad \bar{\boldsymbol{\Pi}}_7=\boldsymbol{\Pi}\bar{\boldsymbol{\Pi}}_{7a}$$

$$\bar{\boldsymbol{\Pi}}_8=\begin{bmatrix}\bar{\boldsymbol{e}}_6^{\mathrm{T}}\bar{\boldsymbol{X}}_1+\bar{\boldsymbol{e}}_1^{\mathrm{T}}\bar{\boldsymbol{X}}_2+\bar{\boldsymbol{e}}_2^{\mathrm{T}}\bar{\boldsymbol{X}}_3\end{bmatrix}^{\mathrm{T}}$$

$$\bar{\boldsymbol{\Pi}}_9=[\bar{\boldsymbol{e}}_1^{\mathrm{T}}\boldsymbol{A}_0^{\mathrm{T}}+\bar{\boldsymbol{e}}_2^{\mathrm{T}}\boldsymbol{A}_{\mathrm{d}}^{\mathrm{T}}-\bar{\boldsymbol{e}}_6^{\mathrm{T}}]^{\mathrm{T}} \quad \bar{\boldsymbol{\theta}}_1=\begin{bmatrix}\bar{\boldsymbol{\Pi}}_8^{\mathrm{T}}\boldsymbol{H}\end{bmatrix}^{\mathrm{T}}$$

The remaining elements and the calculation process are the same as those of Theorem 1. Therefore, these have not been explicitly specified here again.

4 Case analysis

To verify the superiority of the method in this study, the LMI toolbox in MATLAB is used to solve the time-delay stability margins of the power system based on PID load frequency control under the conditions of random time-delay and uncertain parameters. System parameters are listed in Tab. 1.

Tab. 1 System parameters

T_{T}	T_{G}	R	D	M
0.3	0.1	0.05	1.0	10

Consider the two controller parameters $\boldsymbol{K}_1$: $[\boldsymbol{K}_{\mathrm{P}}, \boldsymbol{K}_{\mathrm{I}}, \boldsymbol{K}_{\mathrm{D}}]=[-0.100\,0, 0.066\,8, 0.053\,1]$ and $\boldsymbol{K}_2$: $[\boldsymbol{K}_{\mathrm{P}}, \boldsymbol{K}_{\mathrm{I}}, \boldsymbol{K}_{\mathrm{D}}]=[-0.403\,6, 0.635\,6, 0.183\,2]$.

When we set $\alpha = 2$ and $\gamma \in [0, 4]$, the random time-delay stability margins in different disturbances of different methods are as listed in Tab. 2.

Tab. 2 Stability margins for different methods

γ	Controller $\boldsymbol{K}_1$		Controller $\boldsymbol{K}_2$	
	Ref. [12]	Theorem 1	Ref. [12]	Theorem 1
0	2.777 2	3.386 7	0.607 5	0.627 2
0.5	2.579 7	3.152 5	0.580 9	0.601 7
1.0	2.067 5	2.510 8	0.492 8	0.533 7
1.5	1.477 9	1.632 3	0.357 4	0.392 1
2.0	1.024 1	1.068 6	0.265 2	0.288 8
2.5	0.712 2	0.736 9	0.197 5	0.213 0
3.0	0.481 5	0.494 1	0.141 1	0.150 6
3.5	0.285 4	0.293 7	0.083 4	0.087 9
4.0	0.033 0	0.033 0	—	—

Remark: — means that it cannot be solved.

Fig. 2 and Fig. 3 intuitively show the random time-delay stability margins of the perturbed state obtained by Theorem 1 under different controllers.

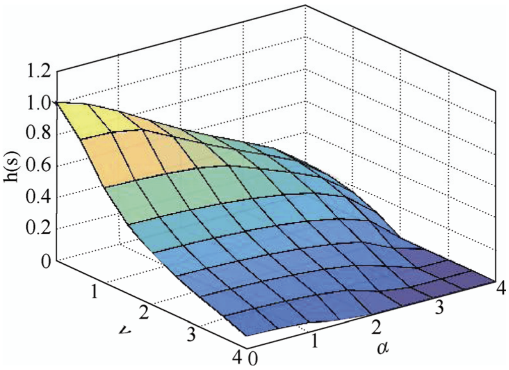

Fig. 2 Stability margin of controller $\boldsymbol{K}_1$

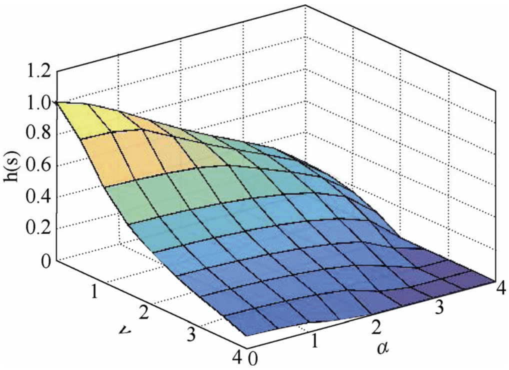

Fig. 3 Stability margin of controller $\boldsymbol{K}_2$

Through Tab. 2, Fig. 2 and Fig. 3, the following conclusions can be clearly obtained:

1) When the same method is used, the usage of different controllers gives rise to significantly different time-delay stability margins.

2) The system time-delay stability margin obtained by the proposed method is obviously superior to that in Ref. [12].

To further verify the correctness of the proposed method, Corollary 1 is applied to calculate the constant time-delay stability margin that the system can withstand when the controller is $\boldsymbol{K}_2$, corresponding to 1.290 6 s.

If we assume the load in the region increases by 0.01 pu at 10 s, i.e. $\Delta \boldsymbol{P}_{\mathrm{d}} = 0.01$ pu, then the response curve of the system with different time delays can be derived as shown in Fig. 4.

If the load increases at the 10 s, the following frequency deviation response is obtained, as shown in Fig. 4. When the time delay is not considered, the frequency deviation converts to zero by the primary frequency modulation of the speed regulating system and the secondary frequency modulation of LFC, and the grid frequency returns to the regulated value. When the system time delay is 1.29 s, the response time of the system frequency deviation increases, which indicates that the existence of a time delay has an impact on the system stability, although it tends to be stable. When the time delay of the system is 1.30 s, the system diverges and is no longer stable. Therefore, it can be concluded that the time-delay stability margin that the system can withstand is within the interval [1.29 s, 1.30 s], and the stability margin 1.290 6 s obtained by the proposed method is within this interval, indicating the correctness of the proposed method.

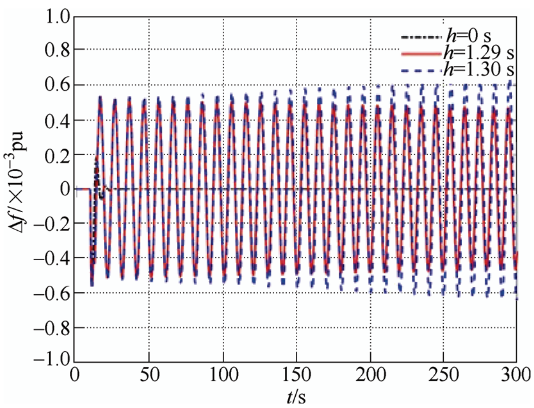

Fig. 4 Frequency deviation response of different time delays

5 Conclusions

Based on PID load frequency control, the time-delay robust stability of a power system with uncertain parameters was analyzed in this study. Through the construction of a new LK functional, a stability criterion was obtained using Lyapunov stability theory, and the time-delay stability margin of the system was derived using LMI. Through an analysis of the effects of different disturbances, different types of time delays, and different control parameters on the stability margin of the system, the effectiveness and superiority of the proposed method were demonstrated.

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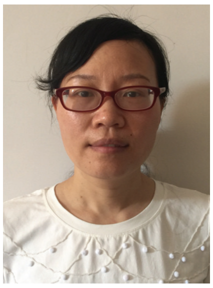

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