Magnetic Field Analytical Model for Magnetic Harmonic Gears Using the Fractional Linear Transformation Method

Yuejin Zhang[1*], *Junda Zhang*[1], *and Ronghui Liu*[1,2]

(1. School of Mechatronics Engineering and Automation, Shanghai University, Shanghai 200444, China;
2. College of Electric Engineering, Shanghai University of Electric Power, Shanghai 200090, China)

Abstract: Magnetic harmonic gears with high gear ratios exhibit high torque densities. However, the revolution and rotation of the eccentric rotor makes the magnetic field analysis complex. In this study, an analytical model of magnetic fields for magnetic harmonic gears is developed by using the fractional linear transformation method. The transformation formula is accurate in theory and suitable for the analysis of magnetic fields with large eccentricity. The rotor eccentricity region in the z-plane is mapped onto a uniform region in the w-plane. The magnetic field solutions are obtained by modulating the magnetic field distributions without rotor eccentricity with the relative permeance function derived from the effect of rotor eccentricity. The torque of magnetic harmonic gears is calculated from the radial and tangential components of the air-gap magnetic fields. Results of the finite element method and prototype test confirm the validity of the analytical prediction.

Keywords: Eccentric rotor, electromagnetic torque, fractional linear transformation, magnetic harmonic gears, relative permeance function.

1 Introduction

Magnetic gears have the advantages of being lubricant-free and having low noise, reduced maintenance, and inherent overload protection. Concentric magnetic gears with soft magnetic pole pieces that modulate the fields can exhibit torque densities of up to 100 kN • m/m³ [1]. It has been shown that eccentric magnetic harmonic gears with high gear ratios and pole pieces freely exhibit torque densities of as much as 150 kN • m/m³ [2-3]. Torque densities of as much as 291 kN • m/m³ can be achieved with a flux-focusing rotor topology [4].

A magnetic field analysis is complex because of the eccentric rotor revolution and rotation. Although the finite element (FE) method has high precision, rotation of the non-uniform wave generator makes automatic meshing more difficult and the computational process time-consuming. The analytic method has clear physical concepts, fast calculation, and free rotation without mesh restriction. Therefore, it is flexible even if the air gap lacks uniformity as a result of eccentric rotation of the rotor. In 1998, Kim and Lieu proposed the first solution of the magnetic field associated with rotor eccentricity by applying a perturbation method [5-6]. The boundary perturbation method, also known as the small parameter method, expands the function containing the eccentricity into an infinite series and eliminates the higher-order terms to obtain a concise expression. Therefore, the model itself has a truncation error. The conformal transformation method maps irregular regions into regular ones to obtain a magnetic field equation to be easily solved. Using Schwarz-Christoffel transformation, Zhu and Howe modeled the effect of stator slotting on the 2D magnetic field distribution and proposed an analytical method that considers stator slot openings by the application of a relative permeance function [7-8]. Zarko et al. introduced the complex relative air-gap permeance, which is calculated from the conformal transformation and from which a solution of both radial and tangential components of the flux density with the effect of slotting can be obtained [9]. Two groups of orthogonal circles representing the equipotential and magnetic force lines in the w-plane were obtained by using orthogonal coordinate transformation for the rotor eccentricity region in the z-plane. The radial air-gap relative permeance function with a unit magnetic potential difference in the w-plane was calculated to

correct the eccentric radial air-gap magnetic field [10]. This study develops an analytical model for magnetic harmonic gears using the fractional linear transformation (FLT) method. The method transforms two eccentric circles into two concentric circles. The formula is accurate in theory, free from the influence of eccentricity, and suitable for analyzing the magnetic fields of magnetic harmonic gears with large eccentricity. Given the unit magnetic potential difference between two concentric circles, the magnetic density distribution in the regular region can be calculated. The air-gap relative permeance function is obtained to correct the eccentric radial air-gap magnetic density distribution. The electromagnetic torque is then calculated based on the Maxwell stress tensor. The results of the analytical method are in good agreement with those of the FE method, and the prototype tests also verify the validity and effectiveness of the analytical method.

2 Basic structure and operation principles

The eccentric magnetic harmonic gear shown in Fig. 1 is composed of three parts: a high-speed rotor, which is equivalent to the wave generator in a mechanical harmonic gear; an eccentric low-speed rotor with p_r pole pairs of permanent magnets; and a stator with p_s (= p_r + 1) pole pairs of permanent magnets. The wave generator deforms the low-speed rotor using a sliding contact such that the low-speed rotor has both a rotational and an orbital motion. In addition, the magnetic fields are modulated by the variable air gap length.
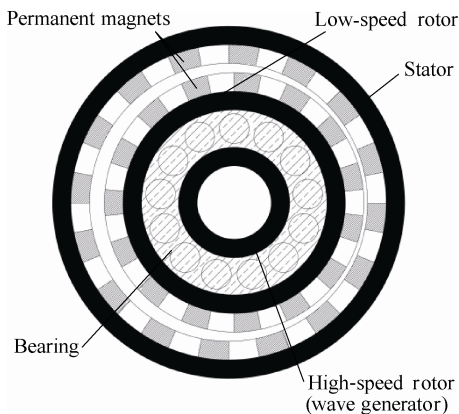

Fig. 1 Cross section of eccentric magnetic harmonic gears

We assume the permeability of the stator and rotor back-iron is infinite and that of permanent magnets is $\mu_r = 1$, while ignoring end effects in two-dimensional analysis.

Because of the circle circumference of the air-gap

length, the variation law of the air-gap length is assumed to be the cosine function of the space angle.

$$g = \frac{g_{max} + g_{min}}{2} + \frac{g_{max} - g_{min}}{2}\cos(\theta - \omega_h t) \qquad (1)$$

where g_{max} and g_{min} are the maximum and minimum air-gap lengths, respectively, and ω_h is the angular velocity of the high-speed rotor. To produce a constant torque, the arrangement between the pole pairs of the stator and rotor is given by

$$p_s = p_r + 1 \qquad (2)$$

where p_r and p_s are the numbers of pole pairs on the low-speed rotor and stator, respectively. The gear ratio is then given by [3]

$$G_r = -\frac{1}{p_r} \qquad (3)$$

Thus, a high gear ratio can be obtained by increasing the number of pole pairs on the low-speed rotor.

3 Analytical model with fractional linear transformation method

The high-precision solution of the magnetic field is proposed for the large eccentricity of magnetic harmonic gears based on the FLT method in the permanent magnet (PM) and the air gap regions where the linear superposition principle can be applied. The combined magnetic field is superposed by calculating the magnetic fields of the stator and rotor independently.

A two-dimensional analytical calculation of the magnetic field in a concentric smooth air gap can achieve satisfactory results with the scalar or vector magnetic potential model. In addition, this study employs the eccentricity permeance function based on the FLT method. Two non-concentric circles in the z-plane are mapped onto two concentric circles in the w-plane. Thus, the boundary condition problem can be easily solved in the regular field region.

The fractional linear transformation of mapping two eccentric circles onto two concentric circles can be expressed as

$$w = \frac{z - z_1}{z - z_2} \qquad (4)$$

where z_1, z_2 is the pair of symmetry points.

The circles C_1 (radius R_s) and C_2 (radius R_r) with eccentricity d in the z-plane are mapping of the concentric circles C'_1 (radius R_1) and C'_2 (radius R_2) in the w-plane as shown in Fig. 2.
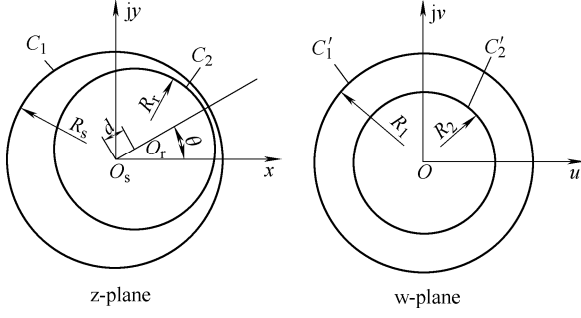
z-plane   w-plane

Fig. 2 Eccentric circles transformed into concentric circles

The relationships among the symmetry points, eccentricity, and radius of the circle are described as follows

$$\begin{aligned} |z_1||z_2| &= R_s^2 \\ (|z_1|-d)(|z_2|-d) &= R_r^2 \end{aligned} \qquad (5)$$

where $z_1 = x_1 + jy_1$, $z_2 = x_2 + jy_2$.

Let

$$r_i = |z_i| \quad i = 1,2 \qquad (6)$$

So that

$$x_i = r_i\cos\theta, \, y_i = r_i\sin\theta \quad i = 1,2 \qquad (7)$$

Therefore,

$$r_{1,2} = \frac{\left(R_1^2 - R_2^2 + d^2\right) \mp \sqrt{\left(R_1^2 - R_2^2 + d^2\right)^2 - 4d^2 R_2^2}}{2d} \qquad (8)$$

The radii of circles after the transformation are

$$\begin{cases} R_1 = \dfrac{\left|R_s e^{j\theta} - r_1 e^{j\theta}\right|}{\left|R_s e^{j\theta} - r_2 e^{j\theta}\right|} = \dfrac{|R_s - r_1|}{|R_s - r_2|} \\[4mm] R_2 = \dfrac{\left|(R_r + d)e^{j\theta} - r_1 e^{j\theta}\right|}{\left|(R_r + d)e^{j\theta} - r_2 e^{j\theta}\right|} = \dfrac{|R_r + d - r_1|}{|R_r + d - r_2|} \end{cases} \qquad (9)$$

The scalar magnetic potential in the w-plane is described by the Laplacian equation as

$$\frac{\partial^2 \varphi}{\partial u^2} + \frac{\partial^2 \varphi}{\partial v^2} = 0 \qquad (10)$$

The boundary condition is

$$\begin{cases} \varphi = \varphi_1, r = R_1 \\ \varphi = \varphi_2, r = R_2 \end{cases} \qquad (11)$$

The general solution in polar coordinates is

$$\varphi(u,v) = C_0 + A_0 \ln r \qquad (12)$$

where

$$A_0 = \frac{\varphi_2 - \varphi_1}{\ln\left(R_2 / R_1\right)}$$

$$C_0 = \frac{\varphi_1 \ln R_2 - \varphi_2 \ln R_1}{\ln\left(R_2 / R_1\right)}$$

Assuming $\varphi_1 = 1$, $\varphi_2 = 0$, we can obtain the distribution of the flux density with the unit magnetic difference potential. Thus, the magnetic field strength in the w-plane is

$$H_w = H_u + jH_v = -\frac{\partial \varphi}{\partial u} - j\frac{\partial \varphi}{\partial v} \qquad (13)$$

The magnetic induction is

$$\begin{cases} B_u = \mu H_u = -\mu_0 A_0 \dfrac{u}{u^2 + v^2} \\[4mm] B_v = \mu H_v = -\mu_0 A_0 \dfrac{v}{u^2 + v^2} \end{cases} \qquad (14)$$

Based on the Cauchy-Riemann equations, the magnetic field strength in the z-plane is

$$H_z = H_w \left(\frac{dw}{dz}\right)^* = H_u \frac{\partial u}{\partial x} + H_v \frac{\partial v}{\partial x} + j\left(H_v \frac{\partial u}{\partial x} - H_u \frac{\partial v}{\partial x}\right) \qquad (15)$$

where * indicates conjugate. Thus,

$$\begin{cases} H_x = H_u \dfrac{\partial u}{\partial x} + H_v \dfrac{\partial v}{\partial x} \\[4mm] H_y = H_v \dfrac{\partial u}{\partial x} - H_u \dfrac{\partial v}{\partial x} \end{cases} \qquad (16)$$

According to (4),

$$\begin{cases} u = \dfrac{(x-x_1)(x-x_2) + (y-y_1)(y-y_2)}{(x-x_2)^2 + (y-y_2)^2} \\[4mm] v = \dfrac{(x-x_2)(y-y_1) - (x-x_1)(y-y_2)}{(x-x_2)^2 + (y-y_2)^2} \end{cases} \qquad (17)$$

The partial derivatives can be easily calculated from (17).

The eccentric air-gap relative permeance of the radial direction is

$$\lambda_r = \frac{B_{wr}}{B_{br}} \qquad (18)$$

where B_{wr} and B_{br} are the radial and base radial components of the flux densities, respectively, and

$$B_{br} = \frac{\mu_0}{2r\ln(R_s / R_r)} \qquad (19)$$

Thus, the eccentric air-gap radial magnetic flux density produced by the permanent magnet is obtained as

$$B'_{rpm} = \lambda_r B_{rpm} \qquad (20)$$

where B_{rpm} is the radial component of the flux density generated by permanent magnets in the uniform air gap. The superscript ′ indicates the corrected air-gap magnetic flux density.

The electromagnetic torque of the magnetic

harmonic gear can be obtained from the radial and tangential air-gap magnetic flux densities.

$$T_{em} = \frac{L_{ef}}{\mu_0} \int_0^{2\pi} r^2 B_r B_\theta d\theta \tag{21}$$

where L_{ef} is the axial length of the magnetic harmonic gear.

4 Practical implementation

A practical realization of a magnetic harmonic gear has a ratio of 15:1 with 15 pole pairs in a low-speed rotor, the parameters for which are listed in Tab. 1.

Tab. 1 Parameters of the magnetic harmonic gear

Symbol	Parameter	Value
R_s	Stator yoke inner radius /mm	45.5
h_s	Stator yoke thickness /mm	6
R_r	Low-speed rotor yoke radius /mm	33.5
h_r	Rotor yoke thickness /mm	6
h_{sm}	Stator PM thickness /mm	3.5
h_{rm}	Low-speed rotor PM thickness /mm	3.5
d	Eccentricity /mm	4
P_s	Stator PM pole pairs	16
P_r	Rotor PM pole pairs	15
g_{min}	Min air-gap /mm	1
g_{max}	Max air-gap /mm	9
B_r	PM remanence /T	1.25
μ_r	PM relative permeability	1.0
G	Average air-gap length /mm	5
L_{ef}	Axial length /mm	20
α_p	Pole arc coefficient	1

The calculated waveform comparison of the FLT and FE methods for the air-gap magnetic flux density of the harmonic magnetic gear with maximum electromagnetic torque are shown in Fig. 3 and Fig. 4. It can be seen that two calculated results are in good agreement. Fig. 5 shows the lines of magnetic force distribution calculated by the FE method.
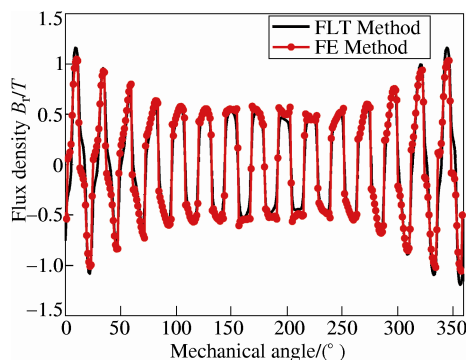

Fig. 3 Calculated distributions of the radial air gap magnetic flux densities by the FLT and FE methods
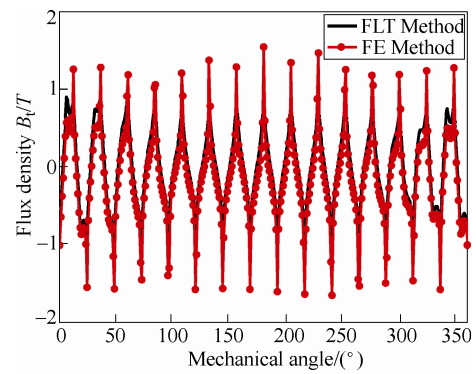

Fig.4 Calculated distributions of the tangential air gap magnetic flux densities by the FLT and FE methods
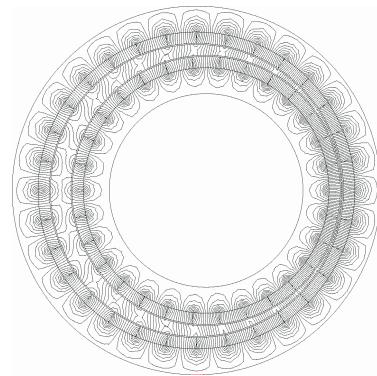

Fig. 5 Lines of magnetic force distribution of the eccentric magnetic harmonic gear

According to the Maxwell stress tensor method, the electromagnetic torque can be obtained from radial and tangential air gap magnetic density distribution by (21), and the result curves are shown in Fig. 6.
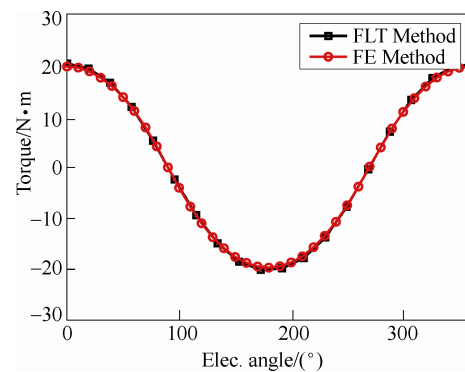

Fig. 6 Calculated magnetic torque curves of the eccentric magnetic harmonic gear by the FLT and FE methods

It can be seen that the torque calculated by the analytical method was slightly higher than that by the FE method. This is probably because the core permeability was assumed to be infinite in the analytical method, whereas the magnetization characteristics of the core as well as the core consumption of the magnetic potential were considered in the FE method.

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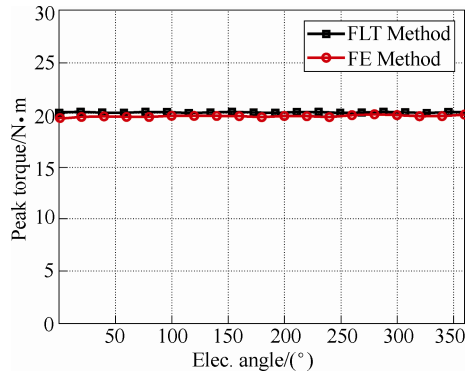Fig. 7 shows the torque output smoothness of the magnetic harmonic gear.

Fig. 7 Peak torque of the eccentric magnetic harmonic gear predicted by the FLT and FE methods

5 Experimental validation

To connect the concentric load shaft, a prototype dual-stage eccentric magnetic harmonic gear was built. The pole pairs of the two stages are listed in Tab. 2.

Tab. 2 Pole pairs of two stages and gear ratio

1st stage			2nd stage			Gear ratio
p_{r1}	p_{s1}	g_{r1}	p_{r2}	p_{s2}	g_{r2}	G_r
8	9	8	15	16	15	−18.286

The overall gear ratio is given as [3]

$$G_r = \frac{p_{r2}+1}{1-\dfrac{p_{r2}}{p_{r1}}} \qquad (22)$$

The rotor components of the prototype and test bench are shown in Fig. 8 and Fig. 9. The following measured data curves of the prototype are shown as in Fig. 10: ① pull-out torque versus outer rotor speed, in which the stable pull-out torque can attain 17.21 N • m and compares favorably with predicted peak torque of approximately 20 N • m; ② the efficiency measured curve. The efficiency in the measured power range was greater than 90%, where the maximum efficiency reached approximately 96.5%. When the input speed was 1 820 r/min, the outer rotor speed was 100 r/min. The average speed ratio was 18.31, which is basically consistent with the gear ratio of 18.286.
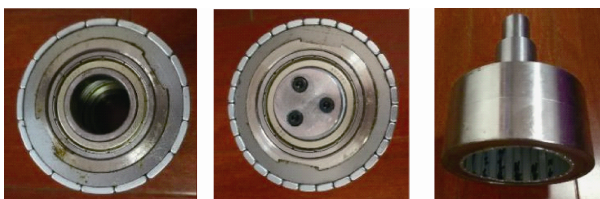
(a) 1st stage rotor   (b) 2nd stage rotor   (c) 2nd stage outer rotor
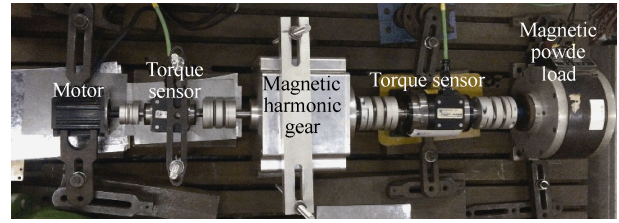
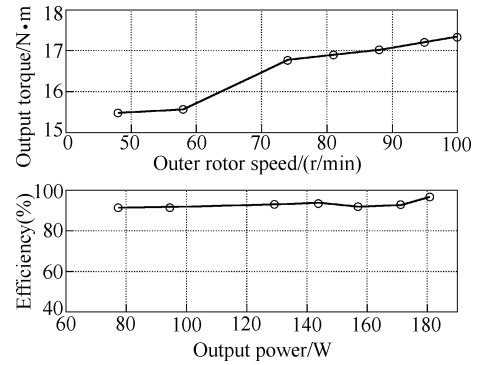Fig. 8 Prototype components

Fig. 9 Test bench of the prototype

Fig. 10 Measured data curves of the prototype

6 Conclusion

In this study, we introduced an analytical model for magnetic field analysis of eccentric magnetic harmonic gears. We showed that the process of calculating the relative permeance function of the eccentric magnetic field by the FLT method is concise and fast. The computed results of the air gap magnetic fields and torque agreed well with the FE method simulation. The prototype experimental results also verified the correct torque calculation and showed that the eccentric magnetic harmonic gear has high efficiency.

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Yuejin Zhang received his M.S. degree in electrical engineering from Shanghai University of Technology, Shanghai, China, in 1988, and his Ph.D. degree in electrical engineering from Shanghai University, Shanghai, China, in 2005.

He is a Professor at Shanghai University, Shanghai. His research interests include magnetic field analysis, design of magnetic gears, and permanent magnet brushless and direct-drive electrical machines.

Junda Zhang was born in Shanghai, China, in 1986. He received his B.S. degree in automation engineering from Tongji University, Shanghai, China, in 2009, and his M.S. degree in control engineering from Shanghai University, Shanghai, China, in 2015.

He is currently pursuing his Ph.D. degree in electric machinery at Shanghai University. His research interests include magnetic field analysis, design of electrical machines, and permanent magnet brushless motor control.

Ronghui Liu received her B.S. degree in computer engineering from Nanjing University of Aeronautics and Astronautics, Nanjing, China, in 2000 and her M.S. degree in electrical engineering from Southeast University, Nanjing, China, in 2003. She is currently pursuing her Ph.D. degree in electrical engineering at Shanghai University, Shanghai, China.

Since 2014, she has been a Vice Professor at the Electric Engineering College, Shanghai University of Electric Power, Shanghai, China. Her research interests include analytical magnetic field calculation of motors.