Remembering Prof. Valerian Tatarskii

Valery Zavorotny

University of Colorado Boulder/CIRES
216 UCB, Boulder, CO, 80309, USA (Retired)
E-mail: valery.zavorotny@colorado.edu

Abstract

Prof. Valerian Tatarskii is recognized in the field of radio science as one of the founders of the theory of wave propagation in random media. His outstanding seminal works made a global impact on studies of propagation and scattering of electromagnetic and acoustic waves in the atmosphere and the ocean. His papers and books remain a source of wisdom for scores of researchers in this field around the world. In this article, we pay special tribute to his exceptional career and contributions to science.

1. Introduction

Recently, in October of 2019, we celebrated Prof. Valerian Tatarskii's 90th birthday [1]. Unfortunately, on April 19, 2020, with deep sadness we learned about his passing in his home in Boulder, Colorado. V. I. Tatarskii received worldwide recognition as the founder of a new scientific direction, the theory of wave propagation in random media. The methods developed by him made it possible to solve a wide range of practical problems related to propagation and scattering of electromagnetic, sound, and seismic waves in the atmosphere, ionosphere, interplanetary and interstellar plasmas, the ocean, and the interior of the Earth. Prof. Tatarskii's innovative research was acknowledged as the authoritative and most widely referenced work in this field, making a global impact on physical science. He was one of the founders of the international journal *Waves in Random Media* which, together with Prof. Akira Ishimaru, laid the foundation for the current direction of this field. In this article, the main milestones of Prof. Tatarskii's life and scientific activities will be outlined, including the important publications and results.

2. Early Years of V. I. Tatarskii's Life

Valerian Tatarskii was born on October 13, 1929, in Kharkov, USSR, now Kharkiv, Ukraine. His father was an engineer who took part in the construction of various industrial facilities across the country in the years following the Russian Revolution. Valerian started school in Kharkov, but in1941 he and the Tatarskii family had to evacuate to the Siberian city of Krasnoyarsk, following the German Nazi invasion of the USSR. After the war, Valerian graduated from high school in Krasnoyarsk with a gold medal (the equivalent of valedictorian). In 1947, he was admitted to the Physics Department at Moscow State University (MSU).

3. V. I. Tatarskii's Education

While studying in the Physics Department at MSU (1947-1952), V. I. Tatarskii became interested by the problem of sound propagation in a turbulent atmosphere. Prof. V. A. Krasil'nikov, noticing the talented student's interest in scientific work, suggested that Valerian make this the basis of his master's thesis. It would be appropriate to mention here that V. A. Krasil'nikov was one of the first

Figure 1. Prof. Valerian Ill'ich Tatarskii in 2009.

Soviet scientists to study the phenomenon of light- and sound-wave propagation in the turbulent atmosphere. V. A. Krasil'nikov was the first to use the Geometrical Optics approximation in the interpretation of his experimental data on propagation of waves in random media [2-6]. Valerian Tatarskii, as his student, easily absorbed this new knowledge from his teacher. By the time of graduation, he successfully defended his thesis, which in turn became the basis of his first scientific publications [7, 8]. This set the direction for much of Tatarskii's subsequent research.

4. Beginning of the Scientific Career

In 1953, after graduating from MSU, Valerian Tatarskii joined the Laboratory for Turbulence Research at the Geophysical Institute of the USSR Academy of Sciences. There, under the leadership of A. M. Obukhov, V. I. Tatarskii continued to study the propagation and scattering of sound waves in a turbulent atmosphere. He later generalized this problem to the propagation of electromagnetic waves, which soon became his main scientific endeavor.

Here, it would be useful to briefly describe the state of this problem as it existed at that time. Early studies of light- and sound-wave propagation in random media in the Soviet Union were performed by V. A. Krasil'nikov and his colleagues. They used the Geometrical Optics approximation to explain their experimental data. This approximation works reasonably well when considering the propagation of waves in random media with characteristic scales that are much larger than the wavelength of the propagating waves and for relatively short paths [2-6]. In this case, wave scattering at large angles (let alone in the opposite direction) is extremely small. The medium that influences the forward propagation is therefore concentrated between the source and the observer points, along the geometrical-optics rays. This approach soon became very appealing, and was widely used for the interpretation of experiments performed over relatively short paths (see, e.g., [7-12] and elsewhere). Although decent agreement was observed between theoretical and experimental data for phase and angle-of-arrival fluctuations, there was no such agreement with respect to amplitude fluctuations.

In the 1940s and 1950s, Prof. Obukhov published two papers [14, 15], in which he applied the method of smooth perturbations to account for the diffraction effects in electromagnetic and acoustic wave propagation in a turbulent atmosphere. This method was independently developed by Prof. S. M. Rytov before WWII to describe the diffraction of light by ultrasound waves, and is today known as the Rytov approximation [16]. V. I. Tatarskii first met S. M. Rytov in 1955, and Rytov proved to be a significant and beneficial influence on Tatarskii's development as a scientist. The advances achieved by A. M. Obukhov and S. M. Rytov served as a starting point for Valerian Tatarskii to extend the analysis of wave propagation in random media beyond the Geometrical Optics approximation [17, 18]. Using Obukhov's spectral description of atmospheric turbulence and putting to wide use Rytov's method of smooth perturbations to allow for diffraction in the propagation of acoustic and light waves in a turbulent medium, he successfully solved a wide range of new, important, and practical problems. These results laid the basis for his PhD dissertation in 1957, and his first book, published in 1959 in Russian under the title *The Theory of Fluctuation Phenomena Upon Wave Propagation in a Turbulent Atmosphere*. In 1961, it was published abroad in English as *Wave Propagation in a Turbulent Medium* [19].

Continuing his work at the Institute of Atmospheric Physics – which split in 1956 from the Geophysical Institute – V. I. Tatarskii went on to explore more difficult problems of wave propagation in random media, and used increasingly powerful mathematical methods for their solution. V. I. Tatarskii recognized that applying methods developed in quantum field theory allowed him to go beyond the limits of the perturbation methods, and to obtain solutions applicable for a medium with strong fluctuations of the dielectric constant [20-22]. These developments became a valuable part of his DSci dissertation, which he defended in 1964, and for his second book, *Wave Propagation in a Turbulent Atmosphere*, published in 1967 [23]. This book also included other important results obtained by him at the IAP since the publication of his first book [19].

5. At the Frontier of Statistical Wave Propagation Theory

Both Geometrical Optics and Rytov's method of smooth perturbations are based on first-order perturbation theory. This limits their applicability to the regime of weak intensity fluctuations. However, electromagnetic waves can experience strong scintillations even in a medium with weak inhomogeneities of the dielectric constant, if it propagates over long enough paths. V. I. Tatarskii well understood the limitations of perturbation methods, and sought theories that would allow him to transcend the difficulty in describing the regime of strong scintillations. In the 1960s, he devoted significant efforts to constructing several such theories [24-26].

It should be mentioned that other researchers at that time were preoccupied with the same problem [27-30]. Strong and saturated scintillations in received radio signals had been already discussed for quite a long time in radio-astronomical and ionospheric studies. For the description of such scintillations, the model of a thin random phase screen was used with varied success. There were attempts made to employ this model for an extended random medium by replacing it with a equivalent phase screen. More successful was a numerical scheme in which the medium was replaced with a large number of phase screens. A similar approach,

with some assumptions about the correlation between the scattered field and fluctuations of the medium, led L. A. Chernov to the local method of small perturbations, which allowed him to obtain closed equations for field statistical moments [27].

Around that time, V. I. Tatarskii realized that this approach was equivalent to describing this phenomenon by means of Leontovich's parabolic equation [31]. Combining this with a Markov process for the description of the random media, i.e., assuming that inhomogeneities were delta-correlated along the nominal propagation path, he proposed a very elegant method for the derivation of equations for the mean field and the second-order coherence function. He was also able to derive equations of the Einstein-Fokker-Planck type for the characteristic functional of the wave field, and equations for higher statistical moments [26, 32, 33]. What is important is that all of these equations are applicable for the full range of wave intensity fluctuations, from weak to strong. In the following years, these equations were studied analytically and numerically by scientists across the world for atmospheric and oceanic applications. This was the beginning of a new and important line of research.

In 1970, when V. I. Tatarskii was invited to publish an English edition of his book [23], he augmented it with new material from several papers [26, 32, 33]. These changes had to do with the propagation of short waves in a medium with random inhomogeneities approximated by a Markov random process. This book was published in English in 1971 under the title *The Effects of the Turbulent Atmosphere on Wave Propagation* [34]. It soon gained great popularity among specialists worldwide and became one of the classics in its field. It firmly established Prof. Tatarskii as one of the foremost experts in the field of wave propagation and scattering in random media. Domestic and foreign researchers recognized the important role of his books in their mastering of this very complex discipline. For example, the two-volume monograph by A. D. "Bud" Wheelon [35, 36] is preceded by the words, "These volumes are dedicated to Valerian Tatarskii who taught us all."

In all of these years, despite being a theoretician, he was also interested in experimental studies of light propagation through atmospheric turbulence. Tatarskii's involvement in experimental research helped him better understand the specifics and limitations of sensors, and to more accurately formulate a theoretical description of the wave propagation phenomena. Many of Tatarskii's theoretical predictions were validated by experiments conducted at the Obukhov Institute of Atmospheric Physics. In particular, he collaborated closely with Dr. A. S. Gurvich in studying the statistical characteristics of light and sound affected by turbulence, and later investigated laser-beam propagation in a turbulent atmosphere [37-39]. He participated directly in measurements and in planning measurements of statistical characteristics of signals propagating in a turbulent atmosphere and their subsequent interpretation. Valerian Tatarskii also established a very productive collaboration with Dr. V. I. Klyatskin, with whom he developed the above-mentioned Markov approximation for wave propagation in random media [32, 33, 40]. These studies were covered in a review [41].

Deriving the equations for statistical moments of the propagating wave was a very important step, but to solve those equations was no less of an important task. The equations for the first two moments, the mean field and the mutual coherence function, can be solved in a general form for various cases of medium statistics. However, the fourth moment, which is used to describe intensity fluctuations, does not have a full analytical solution for all scattering regimes. Nevertheless, researchers began to look for asymptotic solutions of these differential equations at a regime of saturated scintillations [42, 43].

I was a graduate student when I met Prof. Tatarskii at the Institute of Atmospheric Physics in 1970, when I asked V. I. Tatarskii to be my master's advisor. This request was granted, and since then my life and work became closely involved with Prof. Tatarskii. Soon after graduating from the Gorky State University, I joined the group of scientists at the IAP led by V. I. Tatarskii who worked on wave propagation in a turbulent atmosphere. To address the problem of calculating moments in the regime of strong scintillations, we decided to pursue an approach similar to that of quantum field theory, namely, Feynmann path integrals, or functional integrals, which represent a formal solution to both the Helmholtz and the parabolic equations [33]. Using that approach, we managed to evaluate the asymptotic behavior of the statistical moments of the *n*th order in a regime of strong scintillations [44]. That result became a basis of my PhD dissertation, which I completed under the supervision of Prof. Tatarskii. The same asymptotic result for higher statistical moments was obtained by I. G. Yakushkin, using the method of Green's functions [45], and by R. Dashen, using the Feynmann path-integral method [46]. Various aspects of using the Feynmann path-integral method for studying the statistics of scintillations were covered in V. I. Tatarskii's series of review papers [47-49].

6. Diversity of Research Interests

The scientific interests of V. I. Tatarskii in the 1970s and 1980s included not only the problem of strong fluctuations of wave parameters in a turbulent medium, but also other important problems of wave-propagation theory and related application topics. He was interested in studying the propagation of partially coherent light beams in a turbulent atmosphere [50]. V. I. Tatarskii with A. Vinogradov and Yu. Kravtsov discovered the enhancement effect of backscattering by bodies placed in a medium with random inhomogeneities [51]. The problem of wave propagation through absorbing one-dimensional random media was considered by him in [52], where the fluctuations of the dielectric permittivity were described by three distinct

Figure 2. Participants of the meeting in Tallinn (Estonia), September 1988: (first row) Yu. A. Kravtsov, M. S. Tatarskaya, V. I. Tatarskii, U. Mullamaa, A. Ishimaru, Mrs. Lang, A. Orekhova, Mrs. Renelde Flatte, V. V. Varadan, S. Flatte, A. S. Gurvich; (second row) V. V. Vorob'ev, V. I. Shishov, M. Nieto-Vesperinas, R. Hill, R. Lang, I. Besieris, Y. Kuga; (third row) Yu. N. Barabanenkov, I. G. Yakushkin, A. I. Saichev, V. Freilikher, V. L. Brekhovskikh, E. Bakhar, Mrs. Bakhar, L. Tsang, K. C. Yeh, V. I. Klyatskin; (fourth row) I. G. Granberg, C. Rino, G. Brown, V. U. Zavorotny, V. E. Ostashev, J. C. Dainty, V. N. Sekistov.

random processes. He took part in developing the multiple forward-scattering method that makes it possible to remove the restrictions imposed by the parabolic-equation method in the Markov approximation, namely, that the angles of forward scattering should be small [53, 54]. V. I. Tatarskii obtained modified equations for arbitrary statistical moments of waves propagating in random media in which structure parameters exhibit intermittency [55].

V. I. Tatarskii always had the surprising ability, on the one hand, to fearlessly use analytical methods from quantum field theory and, on the other hand, to find simple qualitative illustrations for applied methods and with their help to lucidly explain the results obtained using simple examples understood even by a lay person. The outstanding pedagogical talent of V. I. Tatarskii fully came to life in the second volume of the book (written together with S. M. Rytov and Yu. A. Kravtsov), *The Introduction to Statistical Radiophysics. Random Fields*, issued in the USSR in 1987 [56]. When translated into English and edited in four volumes under the title *Principles of Statistical Radiophysics* [57], the book enjoyed worldwide popularity among physicists in very different fields. It remains important up to the present day.

The range of scientific interests of V. I. Tatarskii was unusually wide. Besides classical problems of wave propagation in inhomogeneous media, Tatarskii's interests extended beyond this well-established field of physics, showing his diverse choice of research topics. He published significant results in the field of adaptive optics [58, 59], quantum mechanics [60], mathematical and statistical physics [61, 63], the theory of turbulence [62], and quantum statistical optics [65, 66].

7. V. I. Tatarskii as a Leader and Mentor

V. I. Tatarskii was not a purely armchair scientist. He proved to be a talented organizer and leader of scientific research. The teams headed by him have always had an atmosphere of enthusiastic scientific research. From 1978 to 1990, V. I. Tatarskii headed the laboratory, and then the department, at the Institute of Atmospheric Physics at the USSR Academy of Sciences. In 1990, he was invited to head a department at the Astro Space Center, newly created by the academician N. S. Kardashev at the P. N. Lebedev Physical Institute at the USSR Academy of Sciences (FIAN).

V. I. Tatarskii's students highly appreciated him as a remarkable tutor and mentor for whom their successes and failures were also his successes and failures. He was always engaged in the discussion of the problems he stated and helped in their solution. At the same time, he taught his students to be independent, and demanded that they bring the quality of their scientific work to world standards. Among the former students and followers of V. I. Tatarskii, there were many who continued their successful research activities in various fields of physics and mentored their own students.

8. The International Cooperation

In September 1988 in Tallinn (Estonia), V. I. Tatarskii, together with Prof. Akira Ishimaru from Washington State University, organized a meeting regarding the problems of propagation and scattering of electromagnetic waves in randomly inhomogeneous media. Its organization was the result of great interest among the international scientific community regarding this problem. This conference was the first of its kind held in the USSR. About three dozen foreign scientists who held leading positions in the global scientific community in this field received an invitation to participate. One of the consequences of this event was a decision to create a new international journal called *Waves in Random Media*. Four years later, a similar conference ("Wave Propagation in Random Media (Scintillation)") was held in Seattle at the University of Washington, under the chairmanship of V. I. Tatarskii and A. Ishimaru. It was attended by over two hundred speakers from 13 countries, including 30 scientists from the former Soviet Union. The invited talks presented at the conference were later published as a separate book [48]. The Scintillation Conference was a success, and it deepened cooperation and exchange of ideas among scientists working on the problems of wave propagation in random media. It promoted interdisciplinary approaches to solving problems of statistical optics, radio science, and acoustics.

9. Moving to the USA

In the early 1990s, V. I. Tatarskii was invited to the Laboratory of Wave Propagation of the National Oceanic and Atmospheric Administration (Boulder, Colorado), USA by its director, Dr. S. F. Clifford. He worked there until he retired from NOAA in 2006. Afterwards, he carried out research work at the company Radio-Hydro-Physics, LLC. This period of Tatarskii's life in Boulder was exceptionally productive. First, he continued working in his traditional fields of research: wave scattering and propagation in random media, and the theory of turbulence. He published several works on these topics (see, e.g., [67-73]). At the same time, his scientific interests began to encompass problems occurring in the theory of radio wave scattering by a rough sea surface. He obtained and published some important results, applying both analytical [74-81] and numerical methods [82-85] for solving those problems.

10. V. I. Tatarskii's Public Activities and Recognitions

The scientific and public activities of V. I. Tatarskii included his duties as a member of the Editorial Board of the *Journal of Soviet Physics, Uspekhi* (1985-1996), Vice Editor-in-Chief of the international journal *Waves in Random Media* (1991-1998), and a member of the editorial board of the international *Journal of Electromagnetic Waves and Applications* (2001-2008). V. I. Tatarskii was a corresponding member of the Russian Academy of Sciences (since 1976), an honorary member of the Optical Society of America (since 1994), and a member of the National Academy of Engineering, USA (since 1994).

In 1990, for their scientific achievements, V I Tatarskii and his colleagues (A. S. Gurvich, V. I. Shishov, S. M. Rytov, V. I. Klyatskin, Yu. A. Kravtsov, and, posthumously, L. A. Chernov) were awarded the State Prize of the USSR "for the study of the basic laws of wave propagation through turbulent media." That same year, he received a prestigious award from the Optical Society of America, the Max Born Medal, "for his outstanding seminal contributions to the theory of wave propagation through random media, particularly optical propagation through atmospheric turbulence, as well as for his fundamental contributions to the fields of statistical and quantum optics."

11. He Will be Remembered

In the memories of people who knew V. I. Tatarskii, he was known to be surprisingly modest, gentle, and a deeply intelligent person. Although scientific creativity was his main passion, he found time for a variety of hobbies. He loved listening to classical music; was an avid tourist, spending summer holidays kayaking with family and friends; and enjoyed skiing in the winter. He deeply cared for his wife, son, and grandchildren. All his friends and colleagues will remember Prof. V. I. Tatarskii as a remarkable scientist, mentor, and person. He will remain in the hearts of all who were fortunate enough to work and interact with him.

12. References

1. G. Brown et al., "Professor Valerian Ill'ich Tatarskii: An Appreciation," *Waves in Random and Complex Media*, **30**, 1, 2020, pp. 1-4.

2 V. A. Krasil'nikov, "Some Experiments on Sound Propagation in a Turbulent Atmosphere," *Doklady Akademii Nauk SSSR*, *Seriya Geofizicheskaya*, **46**, 1945, pp. 108-110 (in Russian).

3. V. A. Krasil'nikov, "On Amplitude Fluctuations of Sound Propagating in a Turbulent Atmosphere," *Doklady*

Akademii Nauk SSSR, *Seriya Geofizicheskaya,* **58**, 1947, pp. 1353-1356 (in Russian).

4. V. A. Krasil'nikov, "On Fluctuations of the Angle-of-Arrival in the Phenomenon of Twinkling of Stars," *Doklady Akademii Nauk SSSR, Seriya Geofizicheskaya,* **65**, 1949, pp. 291-294 (in Russian).

5. V. A. Krasil'nikov, "On the Influence of Index-of-Refraction Pulsations in the Atmosphere on the Propagation of Ultrasound Radio Waves," *Izvestiya Akademii Nauk SSSR, Seriya Geographicheskayai Geofizicheskaya*, **13**, 1949, pp. 33-57 (in Russian).

6. V. A. Krasil'nikov, "On Phase Fluctuations of Ultrasonic Waves Propagating in the Layer of the Atmosphere Near the Earth," *Doklady Akademii Nauk SSSR, Seriya Geofizicheskaya,* **88,** 1953, pp. 657-660 (in Russian).

7. V. I. Tatarskii, "Theory of Sound Wave Propagation in Turbulent Flow," *Zhurnal Experimental'noii Teoreticheskoi Fiziki*, **25**, 1, 1953, pp. 74-83 (in Russian).

8. V. I. Tatarskii, "Criterion of Applicability of Geometric Optics to Wave Propagation in a Medium with Weak Refractive Index Inhomogeneities," *Zhurnal Experimental'noii Teoreticheskoi Fiziki*, **25**, 1, 1953, pp. 84-86 (in Russian).

9. P. G. Bergman, "Propagation of Radiation in a Medium with Random Inhomogeneities," *Physics Reviews*, **70**, 7/8, 1946, pp. 486-492.

10. T. H. Ellison, "The Propagation of Sound Waves Through a Medium with Very Small Random Variations in Refractive Index," *Journal of Atmospheric and Terrestrial Physics*, **2**, 1, 1951, pp. 14-21.

11. L. A. Chernov, "Correlation of the Amplitude and Phase Fluctuations in Wave Propagation in Media with Random Inhomogeneities," *Akusticheskii Zhurnal*, **1**, 1954, pp. 1-89 (in Russian).

12. R. B. Muchmore and A. D. Wheelon, "Line-of-Sight Propagation Phenomenon – I. Ray Treatment," *Proceedings of the IRE*, **43**, 10, 1955, pp. 1437-1449.

13. L. A. Chernov, *Wave Propagation in a Turbulent Medium*, New York, McGraw Hill, 1960.

14. A. M. Obukhov, "Scattering of Sound in Turbulent Flow," *Doklady Akademii Nauk*, **30**, 7, 1941, p. 611 (in Russian).

15. A. M. Obukhov, "On the Influence of Weak Atmospheric Inhomogeneities on the Propagation of Sound and Light," *Doklady Akademii Nauk SSSR, Seriya Geofizicheskaya*, **2**, 1953, pp. 155-165 (in Russian) [English translation: A. M. Obukhov," Sound and Light Propagation in a Weakly Inhomogeneous Atmosphere," *Waves in Random Media*, **4**, 1994, pp. 9-19].

16. S. M. Rytov, "Diffraction of Light on Ultrasonic Waves," *Izvestia Akademii Nauk SSSR, Seriya Fizicheskaya*, **2**, 1937, pp. 223-259 (in Russian).

17. V. I. Tatarskii, "Phase and Amplitude Fluctuations of a Wave Propagating in a Weakly Inhomogeneous Medium," *Doklady Akademii Nauk SSSR*, **107**, 2, 1956, p. 245 (in Russian).

18. V. I. Tatarskii, "Wave Propagation in a Locally Isotropic Turbulent Medium with Smoothly Varying Characteristics," *Doklady Akademii Nauk SSSR*, **120**, 2, 1958, p. 289 (in Russian).

19. V. I. Tatarskii, *Wave Propagation in a Turbulent Medium*, New York, McGraw Hill, 1961 (New York, Dover, 1966).

20. V. I. Tatarskii and M. E. Gertsenshtein, "Propagation of Waves in a Medium with Strong Fluctuation of the Refractive Index," *Soviet Physics – JETP*, **17**, 1963, pp. 458-463.

21. V. I. Tatarskii, "Propagation of Electromagnetic Waves in a Medium with Strong Dielectric Constant Fluctuations," *Soviet Physics – JETP*, **19**, 4, 1964, pp. 946-953.

22. Yu. A. Ryzhov, V. V. Tamoikin, and V. I. Tatarskii, "Spatial Dispersion of Inhomogeneous Media," *Soviet Physics – JETP*, **21**, 2, 1965, pp. 433-438.

23. V. I. Tatarskii, *Wave Propagation in the Turbulent Atmosphere*, Moscow, Nauka Publishing House, 1967 (in Russian).

24. V. I. Tatarskii, "Strong Fluctuations of Light Wave Parameters in a Turbulent Medium, "*Soviet Physics – JETP*, **22**, 5, 1966, pp. 1083-1088.

25. V. I. Tatarskii, "The Strong Amplitude Fluctuations of a Wave Traveling in a Medium with Weak Random Inhomogeneities," *Radiophysics and Quantum Electronics*, **10**, 1, 1967, pp. 48-58.

26. V. I. Tatarskii, "Light Propagation in a Medium with Random Refractive Index Inhomogeneities in the Markov Random Approximation," *Soviet Physics – JETP*, 29, 6, 1969, pp. 1133-1138.

27. L. A. Chernov, "The Local Method of Computing Strong Fluctuations of the Field in the Problem of Wave Propagation in a Medium with Random Inhomogeneities," Proceedings of 6th All-Union Acoustic Conference, 1968.

28. L. S. Dolin, "Equations for the Statistical Moments of the Field in a Randomly Inhomogeneous Medium,"

Radiophysics and Quantum Electronics, **11**, 6, 1968, pp. 486-491.

29. V. I. Shishov, "Theory of Wave Propagation in Random Media," *Radiophysics and Quantum Electronics*," **11**, 6, 1968, pp. 500-505.

30. M. J. Beran and T. L. Ho, "Propagation of the Fourth-Order Coherence Function in a Random Medium (a Non-Perturbative Formulation)," *Journal of the Optical Society of America*, **59**, 9, 1968, pp. 1134-1138.

31. M. A. Leontovich, "On the New Method of Solving of the Radio-Wave Propagation Problem," *Izvestia Akademii Nauk SSSR, Seriya Fizicheskaya*, **8**, 1944, pp. 16-22 (in Russian).

32. V. I. Klyatskin and V. I. Tatarskii, "Strong Fluctuations of a Plane Wave Moving in a Medium with Weak Random Inhomogeneities," *Soviet Physics – JETP*, **28**, 2, 1969, pp. 346-353.

33. V. I. Klyatskin and V. I. Tatarskii, "The Parabolic Equation Approximation for Propagation of Waves in a Medium with Random Inhomogeneities," *Soviet Physics – JETP*, **31**, 2, 1970, pp. 335-339.

34. V. I. Tatarskii, *The Effects of the Turbulent Atmosphere on Wave Propagation*, Springfield, VA, National Technical Information Service, 1971.

35. A. D. Wheelon, *Electromagnetic Scintillation, 1. Geometrical Optics*, Cambridge, UK, University Press, 2001.

36. A. D. Wheelon, *Electromagnetic Scintillation, 2. Weak Scattering*, Cambridge, UK, University Press, 2003.

37. A. S. Gurvich and V. I. Tatarskii, "Coherence and Intensity Fluctuations of Light in the Turbulent Atmosphere," *Radio Science*, **10**, 1, 1975, pp. 3-14.

38. A. S. Gurvich, V. Kan, V. I. Tatarskii, and V. U. Zavorotny, "Four-Point Field Coherence Function in a Turbulent Medium," *Optica Acta*, **26**, 5, 1979, pp. 543-553.

39. V. I. Tatarskii, A. S. Gurvich, B. S. Elepov, Vl. V. Pokasov, and K. K. Sabel'feld, "Space Structure of Strong Intensity Fluctuations of Light in a Turbulent Medium," *Optica Acta*, **26**, 5, 1979, pp. 531-542.

40. V. I. Klyatskin and V. I. Tatarskii, "A New Method of Successive Approximations in the Problem of Wave Propagation in a Medium Having Random Large-Scale Inhomogeneities," *Radiophysics and Quantum Electronics*, **14**, 9, 1971, pp. 1100-1111.

41. V. I. Klyatskin and V. I. Tatarskii, "Diffusive Random Process Approximation in Certain Nonstationary Statistical Problems of Physics," *Soviet Physics Uspekhi*, **16**, 4, 1974, pp. 494-511.

42. K. S. Gochelashvili and V. I. Shishov, "Laser Beam Scintillation Beyond a Turbulent Layer," *Optica Acta*, **18**, 4, 1971, pp. 313-320.

43. I. G. Yakushkin, "Asymptotic Calculation of Field-Intensity Fluctuations in a Turbulent Medium for Long Paths," *Radiophysics and Quantum Electronics*, **18**, 11, 1975, pp. 1224-1229.

44. V. U. Zavorotnyi, V. I. Klyatskin, and V. I. Tatarskii, "Strong Fluctuations of the Intensity of Electromagnetic Waves in Randomly Inhomogeneous Media," *Soviet Physics – JETP*, **46**, 2, 1977, pp. 252-260.

45. I. G. Yakushkin, "Moments of Field Propagating in Randomly Inhomogeneous Medium in the Limit of Saturated Fluctuations," *Radiophysics and Quantum Electronics*, **21**, 8, 1978, pp. 835-840.

46. R. Dashen, "Path Integrals for Waves in Random Media," *Journal of Mathematical Physics*, **20**, 5, 1979, pp. 894-920.

47. V. I. Tatarskii and V. U. Zavorotny, "Strong Fluctuations in Light Propagation in a Randomly Inhomogeneous Medium," in E. Wolf (ed.), *Progress in Optics*, Amsterdam, Netherlands, Elsevier Science Publishers B.V., 1980.

48. V. I. Tatarskii, A. Ishimaru, and V. U. Zavorotny (eds.), *Wave Propagation in Random Media (Scintillation)*, Bellingham WA, USA, SPIE Press and Bristol, UK, the Institute of Physics Publishing, 1993.

49. M. I. Charnotskii, J. Gozani, V. I. Tatarskii, and V. U. Zavorotny, "Wave Propagation Theories in Random Media Based on the Path Integral Approach," in E. Wolf (ed.), *Progress in Optics*, Amsterdam, Netherlands, Elsevier Science Publishers B.V., 1993.

50. A. I. Kon and V. I. Tatarskii, "On the Theory of the Propagation of Partially Coherent Light Beams in a Turbulent Atmosphere," *Radiophysics and Quantum Electronics*, **15**, 10, 1972, pp. 1187-1192.

51. A. G. Vinogradov, Yu. A. Kravtsov, and V. I. Tatarskii, "Amplification Effect of Backscattering by Bodies Placed in a Medium with Random Inhomogeneities," *Radiophysics and Quantum Electronics*, **16**, 7, 1973, pp. 818-823.

52. V. I. Klyatskin and V. I. Tatarskii, "Statistical Theory of Wave Propagation Through Random Layered Media," *Radiophysics and Quantum Electronics*, **20**, 7, 1977, pp. 720-729.

53. V. E. Ostashev and V. I. Tatarskii, "A Series Based on Multiple Backscattering Problems of Wave Propagation in Inhomogeneous Media," *Radiophysics and Quantum Electronics*, **21**, 5, 1978, pp. 504-513.

54. V. E. Ostashev and V. I. Tatarskii, "Multiple Forward Scattering Method in Media with Random Inhomogeneities," *Radiophysics and Quantum Electronics*, **22**, 2, 1979, pp. 116-122.

55. V. I. Tatarskii and V. U. Zavorotny, "Wave Propagation in Random Media with Fluctuating Turbulent Parameters," *Journal of the Optical Society of America A*, **2**, 12, 1985, pp. 2069-2076.

56. S. M. Rytov, Yu. A. Kravtsov and V. I. Tatarskii, *The Introduction to Statistical Radiophysics. Part II. Random Field*, Moscow, Nauka Publishing House, 1987 (in Russian).

57. S. M. Rytov, Y. A. Kravtsov, V. I. Tatarskii, *Principles of Statistical Radiophysics*, *Volumes 1-4*, Berlin, Springer-Verlag, 1989.

58. V. I. Tatarskii, "Adaptive Systems and Coherence, II, "*Radiophysics and Quantum Electronics*, **24**, 7, 1981, pp. 872-882.

59. V. I. Tatarskii, "Quantum Noise in Adaptive Optical Systems, II," *Radiophysics and Quantum Electronics*, **25**, 9, 1982, pp. 731-742.

60. V. I. Tatarskii, "The Wigner Representation of Quantum Mechanics," *Soviet Physics Uspekhi*, **26**, 4, 1983, pp. 311-327.

61. V. I. Tatarskii, "Example of the Description of Dissipative Processes in Terms of Reversible Dynamic Equations and Some Comments on the Fluctuation Dissipation Theorem," *Soviet Physics Uspekhi*, **30**, 2, 1987, pp. 134-152.

62. M. M. Dubovikov and V. I. Tatarskii, "Calculation of the Asymptotic Form of the Spectrum of Locally Isotropic Turbulence in the Viscous Range," *Soviet Physics – JETP*, **66**, 6, 1987, pp. 1136-1141.

63. V. I. Tatarskii, "Criteria for the Degree of Chaos," *Soviet Physics Uspekhi*, **30**, 5, 1989, pp. 450-451.

64. V. P. Bykov and V. I. Tatarskii, "Causality Violation in the Glauber Theory of Photodetection," *Physics letters A*, **136**, 1(2), 1989, pp. 77-80.

65. V. I. Tatarskii, "Corrections to the Theory of Photocounting," *Physics Letters A*., **144**, 8(9), 1990, pp. 491-499.

66. V. P. Bykov and V. I. Tatarskii, "Perturbation Theory for Resolvents as Applied to Problems in Radiation Theory," *Soviet Physics Uspekhi*, **34**, 2, 1991, 167-184.

67. V. I. Tatarskii and M. M. Dubovikov, A. A. Praskovskii, and M. Yu. Karyakin, "Temperature Fluctuation Spectrum in the Dissipation Range for Statistically Isotropic Turbulent Flow," *Journal of Fluid Mechanics*, **238**, 1992, pp. 683-698.

68. V. I. Tatarskii, "Markovian Non-Gaussian Approximation for Light Propagation in a Random Medium," *Waves in Random Media*, **4**, 1994, pp. 511-519.

69. V. I. Tatarskii, "Characteristic Functional for One Class of Non-Gaussian Random Functions," *Waves in Random Media*, **5**, 1995, pp. 243-252.

70. V. E. Ostashev and V. I. Tatarskii, "Representation of the Helmholtz Equation Solution in the Form of a Series Based on Backscattering Multiplicity," *Waves in Random and Complex Media*, **5**, 1995, pp. 125-135.

71. V. I. Tatarskii and A. Muschinski, "The Difference Between Doppler Velocity and Real Wind Velocity in Single Scattering from Refractive Index Fluctuations," *Radio Science*, **36**, 6, 2001, pp. 1405-1423.

72. V. I. Tatarskii, "Theory of Single Scattering by Random Distributed Scatterers," *IEEE Transactions on Antennas and Propagation*, **53**, 10, 2003, pp. 2806-2813.

73. V. I. Tatarskii, "Use of the 4/5 Kolmogorov Equation for Describing Some Characteristics of Fully Developed Turbulence," *Physics of Fluids*,17, 2005, pp. 035110.

74. V. I. Tatarskii, "The Expansion of the Solution of Rough-Surface Scattering Problem in Powers of Quasi-Slopes," *Waves in Random Media*, **3**, 1993, pp. 127-146.

75. M. M. Charnotskii and V. I. Tatarskii, "Tilt-Invariant Theory of Rough-Surface Scattering," *Waves in Random Media*, **5**, 1995, pp. 361-380.

76. V. I. Tatarskii, "Formulation of Rough-Surface Scattering Theory in Terms of Phase Factors and Approximate Solutions Based on This Formulation," *Waves in Random Media*, **7**, 1997, pp. 557-578.

77. V. I. Tatarskii and M. I. Charnotskii, "On the Universal Behavior of Scattering from a Rough Surface for Small Grazing Angles," *IEEE Transactions on Antennas and Propagation*, **46**, 1, 1998, pp. 67-72.

78. I. M. Fuks, V. I. Tatarskii, and D. E. Barrick, "Behaviour of Scattering from a Rough Surface at Small Grazing Angles," *Waves in Random Media*, **9**, 1999, pp. 295-305.

79. V. I. Tatarskii, "Phase Factors Representation for Electromagnetic Scattering from a Rough-Surface Perfect Conductor," *Waves in Random Media*, **10**, 2000, pp. 339-358.

80. V. I. Tatarskii, "Use of Semi-Inversion Method for the Dirichlet Problem in Rough Surface Scattering," *Progress in Electromagnetic Research*, **PIER54**, 2005, pp. 109-135.

81. V. I. Tatarskii, "Equations for the Description of Wave Scattering by Multi-Valued Random Surfaces," *Waves in Random and Complex Media*, **19**, 2009, pp. 480-500.

82. V. I. Tatarskii and V. V. Tatarskii, "Statistical Description of Rough-Surface Scattering Using the Quasi Small-Slope Approximation for Random Surfaces with a Gaussian Multivariate Probability Distribution," *Waves in Random Media*, **4**, 1994, pp. 191-214.

83. V. I. Tatarskii and V. V. Tatarskii, "Non-Gaussian Statistical Model of the Ocean Surface for Wave-Scattering Theories," *Waves in Random Media*, **6**, 1996, pp. 419-435.

84. V. I. Tatarskii and V. V. Tatarskii, "Statistical Non-Gaussian Model of Sea Surface with Anisotropic Spectrum for Wave Scattering Theory. Part I and II," *Progress in Electromagnetic Research*, **PIER22**, 1999, pp. 259-313.

85. V. I. Tatarskii, "Multi-Gaussian Representation of the Cox-Munk Distribution for Slopes of Wind-Driven Waves," *Journal of Atmospheric and Oceanic Technology*, **28**, 11, 2003, pp. 1697-1705.