Özgür Ergül
Department of Electrical and Electronics Engineering
Middle East Technical University
TR-06800, Ankara, Turkey
E-mail: ozergul@metu.edu.tr

SOLBOX-*15*

Özgür Eriş, Hande İbili, and Özgür Ergül

Department of Electrical and Electronics Engineering
Middle East Technical University
TR-06800, Ankara, Turkey
E-mail: ozergul@metu.edu.tr

1. Introduction

From computational point of view, frequency-selective structures are challenging to analyze since they involve resonating elements. In fact, resonances are essential for the operation of these structures, i.e., they generally provide the required responses (shadowing, focusing, full transmission, etc.) when their elements resonate. These resonances can be studied well via analytical approaches, e.g., by using circuit representations to model unit cells. However, as for other structures, numerical simulations – particularly those based on three-dimensional models – can be extremely useful for complete analyses of electromagnetic characteristics of complex frequency-selective structures before their realizations. The challenge is that when there are resonating elements, linear systems constructed in numerical simulations tend to become very ill-conditioned. For example, when using iterative solvers, iteration counts can be extremely large, while a convergence to a given error threshold may not guarantee an accurate result.

In this issue of Solution Box, three different frequency-selective structures are considered. The structures involve U-resonators that resonate at different frequencies, making the structures become opaque to block the transmission of electromagnetic waves. Sample solutions were obtained by using a frequency-domain integral-equation solver. In order to perform fast iterative solutions, the Multilevel Fast Multipole algorithm (MLFMA) was used, not only as an acceleration algorithm for the required matrix-vector multiplications, but also as a preconditioning tool to speed up iterative convergence. The solver used provided fast and accurate solutions of the three-dimensional models of the given frequency-selective structures, while it had drawbacks that may be mitigated by using other types of solvers. For example, using a frequency-domain approach, sampling frequency at discrete points may lead to loss of information, especially if resonances are sharp. Other types of solvers may also be helpful to explain why numerical resonances (at which iterative solutions become difficult to convergence) do not exactly coincide with physical resonances (at which electromagnetic responses abruptly change). As usual, we are looking forward to receiving alternative solutions to problems considered in this issue, as well as to all other problems presented in previous issues (SOLBOX-01 to SOLBOX-14).

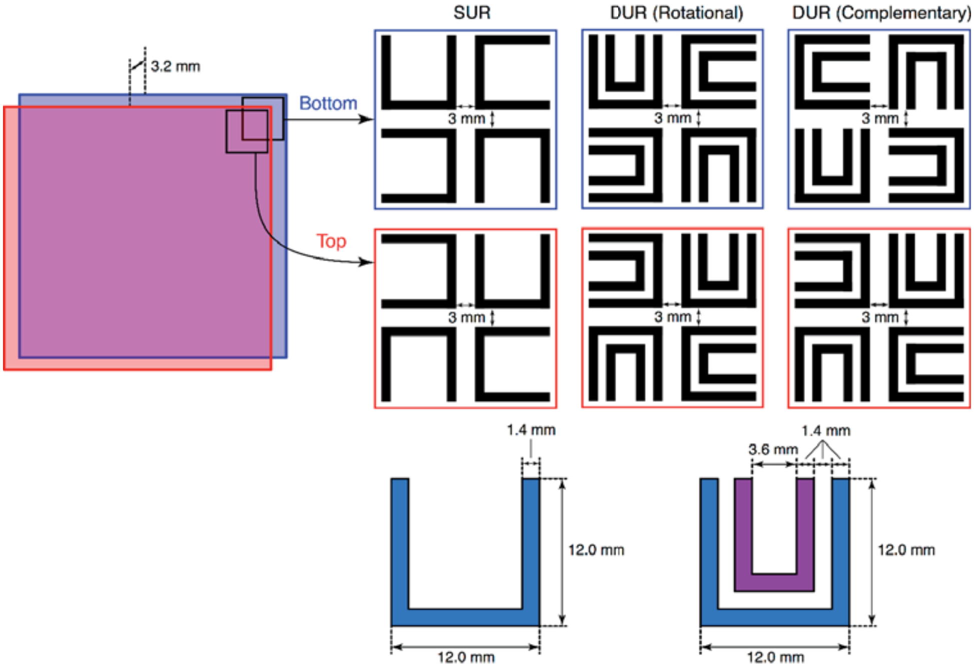

Figure 1. The three frequency-selective structures of SOLBOX-15. Each structure involved two layers with 3.2 mm distance between them, while each layer involved 9 × 9 unit cells. In the SUR design, each unit cell included four U-resonators. In the DUR designs, smaller U-resonators were added. In the rotational DUR design, the unit cell of the bottom layer was obtained from the unit cell of the top layer via three-dimensional rotation. In the complementary DUR design, resonators at the top and bottom layers complemented each other. All surfaces were assumed to have zero thicknesses.

2. Problems

2.1 Problem SOLBOX-15 (by Özgür Eriş, Hande İbili, and Özgür Ergül)

The problem SOLBOX-15 includes three different frequency-selective structures, each having two layers, as depicted in Figure 1. The layers consist of 9 × 9 unit cells, which are also detailed in the same figure. The design for the single-U-resonator (SUR) case is taken from [1]; each unit cell involves four U-resonators arranged differently at the bottom and top layers. Despite that there is a single type of resonator, the SUR design provides resonances at multiple frequencies (other than doubled frequencies). This design is upgraded by adding smaller U-resonators, leading to double-U-resonator (DUR) designs. While one may expect that the number of resonance frequencies directly increases by simply adding a new type of resonator [2], interactions between resonators lead to complex responses that may not be straightforwardly predicted. For example, using the template of the SUR design (leading to the rotational DUR design), the smaller resonators mostly act as parasitic elements, and indeed, they reduce the strength of the original resonances. In addition, their own resonances are extremely sharp, such that they are not easy to observe,

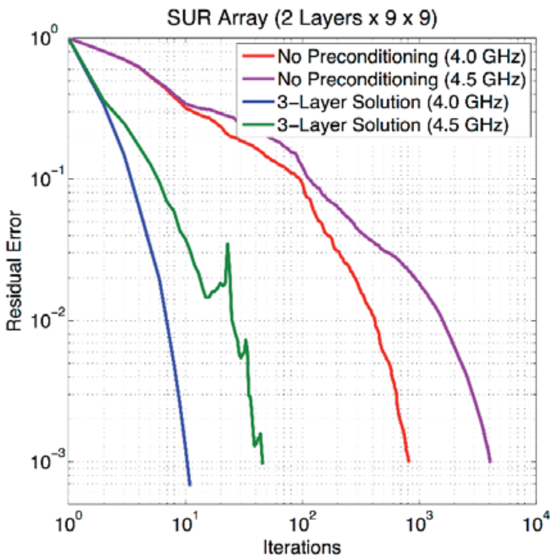

Figure 2. The iterative solutions of the SUR structure at two different frequencies. The structure was excited via a Hertzian dipole. Without preconditioning, the residual error was plotted with respect to GMRES iterations. However, in three-layer solutions the *x* axis represents the main iterations of the flexible variant of GMRES.

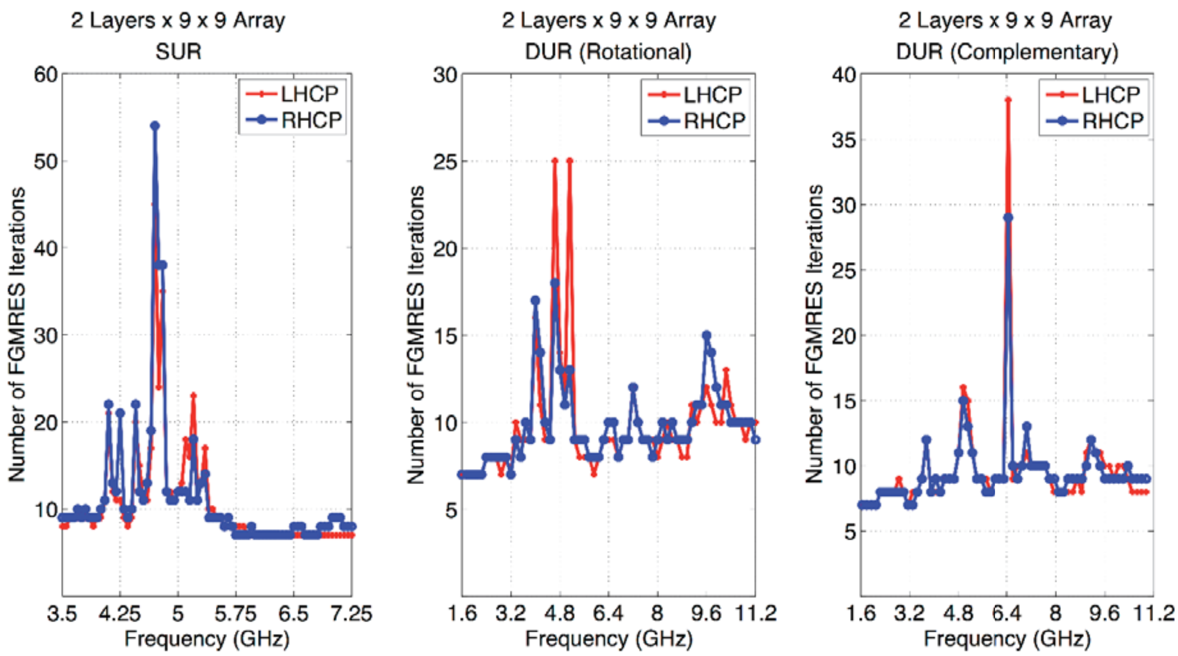

Figure 3. The iterative solutions of the frequency-selective structures, when they were illuminated by plane waves with LHCP and RHCP.

especially in frequency-domain numerical simulations. On the other hand, it is possible to design an effective structure by changing the arrangement of resonators at the bottom layer, leading to the complementary DUR design shown in Figure 1. As presented in the sample results, the resulting frequency-selective structure demonstrated strong resonances corresponding to both larger and smaller resonators.

Given the frequency-selective structures described above, the purpose was to find their electromagnetic responses at microwave frequencies, particularly from 1 GHz to 11 GHz when they were located in free space. In the sample solutions, only normal incidence was assumed, while both right-hand and left-hand circular polarizations were considered. We noted that the structures were expected to block transmission at resonances, while they were typically transparent at other microwave frequencies.

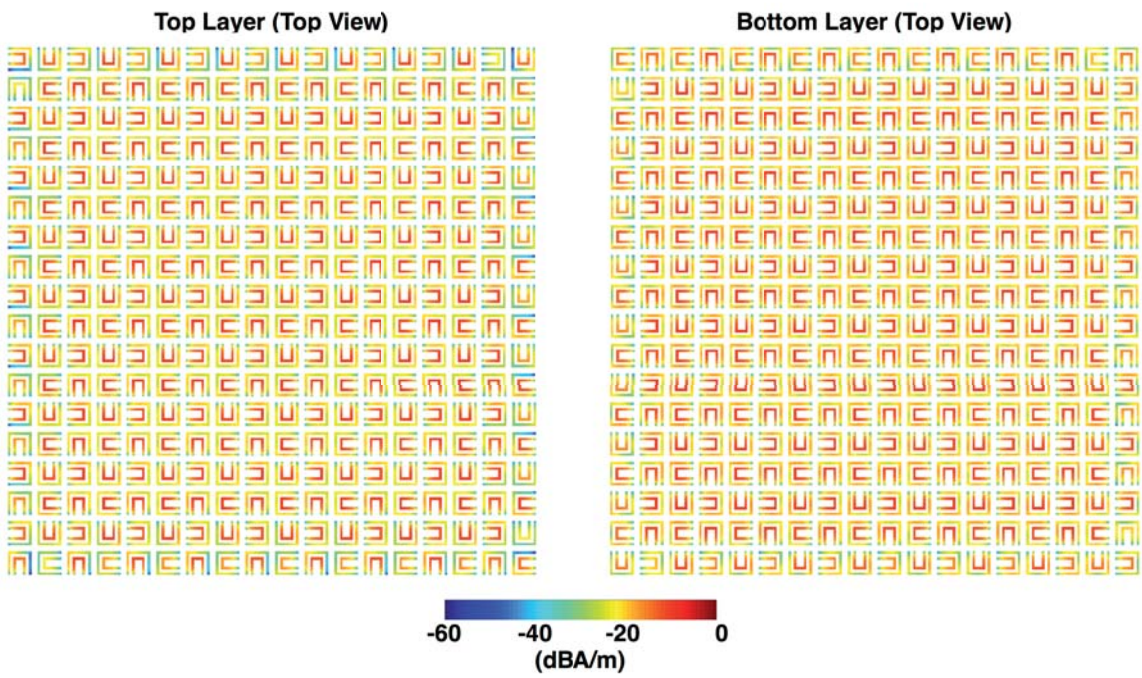

Figure 4. The electric-current density induced on the complementary DUR structure at 7.65 GHz when it was illuminated by a plane wave with LHCP. Smaller resonators were active at this frequency.

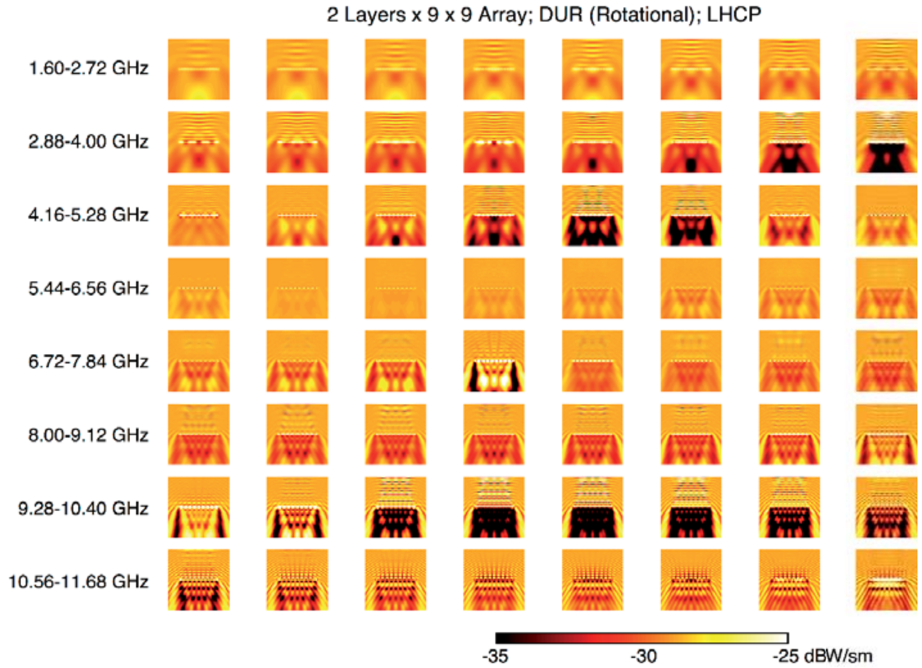

Figure 5. The near-zone power density in the vicinity of the rotational DUR structure at different frequencies when it was excited by plane waves with LHCP.

3. Solution to Problem SOLBOX-15

3.1 Solution Summary

Solver type (e.g., Noncommercial, commercial): Noncommercial research-based code developed at CEMMETU, Ankara, Turkey

Solution core algorithm or method: Frequency-domain MLFMA

Programming language or environment (if applicable): *MATLAB + MEX*

Computer properties and resources used: 2.5 GHz Intel Xeon E5-2680v3 processors (using 1 core)

Total time required to produce the results shown (categories: < 1 sec, < 10 sec, < 1 min, < 10 min, < 1 hour, < 10 hours, < 1 day, < 10 days, > 10 days) <1 hour for each solution (per frequency)

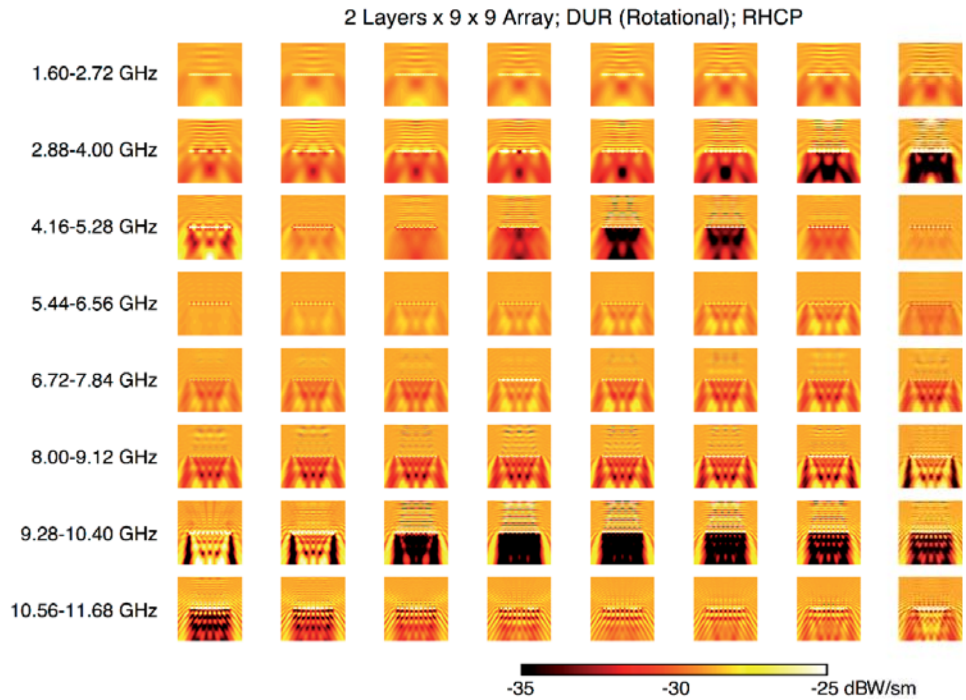

Figure 6. The near-zone power density in the vicinity of the rotational DUR structure at different frequencies when it was excited by plane waves with RHCP.

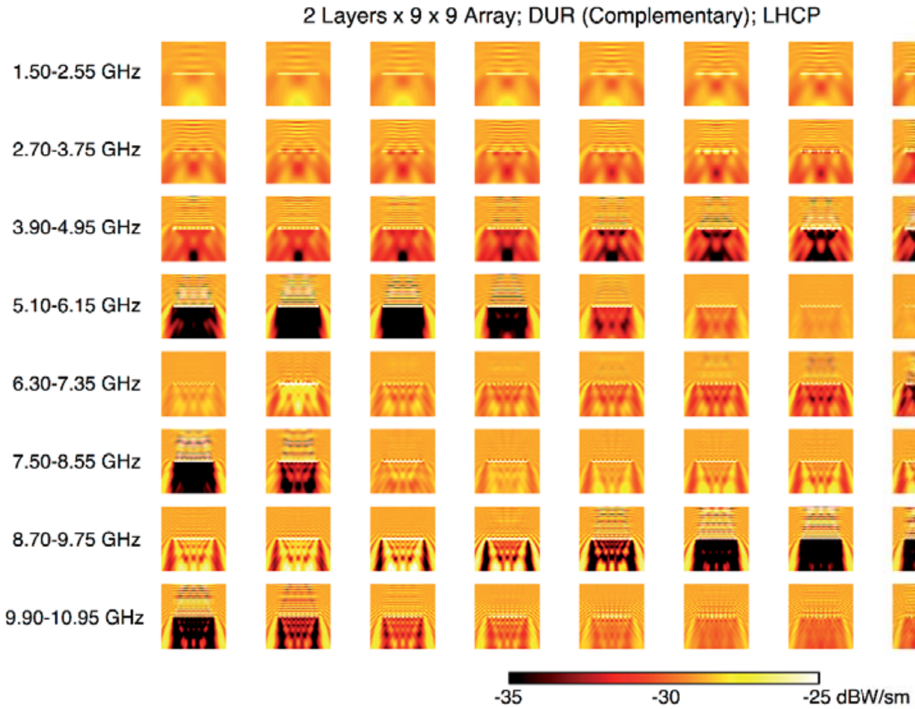

Figure 7. The near-zone power density in the vicinity of the complementary DUR structure at different frequencies when it was excited by plane waves with LHCP.

3.2 Short Description of the Numerical Solutions

The frequency-selective structures described in SOLBOX-15 were solved by using a frequency-domain solver based on the conventional MLFMA [3]. Assuming perfectly conducting surfaces with zero thicknesses, the well-known electric-field integral equation was employed as the formulation. Each structure was discretized with nearly 30,000 triangles, while Rao-Wilton-Glisson functions were used to expand the electric-current density induced on surfaces. The frequency was sampled with 150 MHz or 160 MHz intervals. Iterative solutions were performed by using the Generalized Minimal

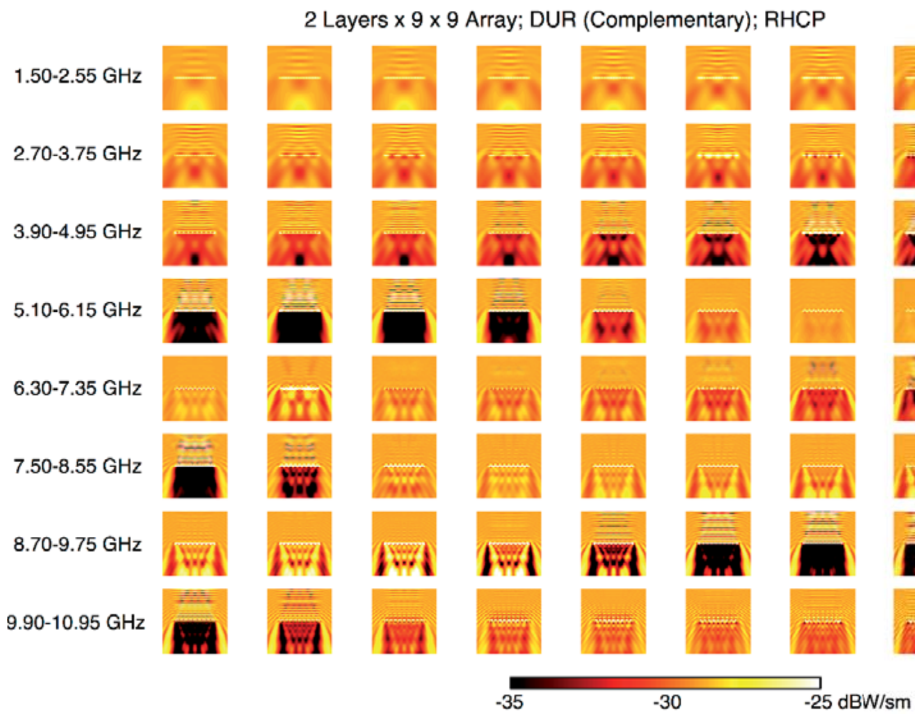

Figure 8. The near-zone power density in the vicinity of the complementary DUR structure at different frequencies when it was excited by plane waves with RHCP.

Residual (GMRES) algorithm. However, even when using no-restart GMRES, solutions are quite challenging without preconditioning (or with simple preconditioners). Therefore, multilayer solutions involving recursive application of MLFMA and its approximate forms [4] were carried out to reach results in reasonable processing times. For example, Figure 2 presents iterative solutions of the SUR design at 4.0 GHz and 4.5 GHz. The residual error was plotted with respect to iterations of the flexible GMRES (that allowed for multilayer solutions) when three-layer solutions were performed, in addition to those without preconditioning. At 4.0 GHz, the number of iterations to reach 0.001 error was 818 without preconditioning, while it could be reduced to only 11 via three-layer solutions. At the more challenging frequency of 4.5 GHz, the reduction in the number of iterations was from 4086 to 41. We noted that these numbers did not directly correspond to processing times. However, it was clear that a complete analysis of the frequency-selective structures of SOLBOX-15 required rigorous acceleration techniques if iterative methods were to be used. In the following results, all solutions were performed via the three-layer mechanism.

3.3 Results

Figure 3 presents iterative solutions of the three frequency-selective structures of SOLBOX-15 when they were illuminated by plane waves with left-hand-circular polarization (LHCP) and right-hand-circular polarization (RHCP). For each structure, the number of flexible GMRES iterations was plotted with respect to frequency when three-layer solutions were performed. We observed relatively small numbers of iterations, except at specific frequencies. However, even at these frequencies iteration counts did not exceed 60. It was remarkable that challenging frequencies in terms of iterative solutions did not exactly coincide with frequencies at which element resonances were observed. These types of shifts (numerical versus physical resonances) have occasionally been observed in the literature of metamaterials and frequency-selective structures [5]. As an example, Figure 4 depicts the electric-current density induced on the complementary DUR design at 7.65 GHz when it was illuminated by a plane wave with LHCP. At this frequency, smaller U-resonators became active and made the structure opaque. On the other hand, this frequency was not among the most challenging frequencies in terms of iterative solutions, as depicted in Figure 3. Similarly, at the numerically challenging frequency of 6.4 GHz for the same structure, the electromagnetic response was not very strong, i.e., no shadowing occurred (see Figure 7).

Figures 5 and 6 present the power density (in dBW/sm) in the vicinity of the frequency-selective structure with the rotational DUR design from 1.6 GHz to 11.68 GHz. Power density values were plotted on the *z*-*x* plane, assuming that the structure was located on the *x*-*y* plane (side view). Plane waves with LHCP (Figure 5) and RHCP (Figure 6) illuminated the structure from the top, while the transmission region was located at the bottom of the plots. In the frequency range considered, we observed three important resonance effects: one located at around 4.0 GHz, another at around 4.8 GHz, and the third one centered at approximately 9.8 GHz. Although not shown here, a close examination revealed that all these three resonances were related to the larger resonators, while resonances of the smaller resonators were not visible, at least with the sampling of the frequency used. As a related observation, a rapid change in the near-zone characteristics at 7.2 GHz for the LHCP excitation, which might be caused by small resonators, was remarkable.

Near-zone characteristics of the complementary DUR structure are shown in Figures 7 and 8, where we again considered excitations with LHCP and RHCP, respectively. In this case, the frequency was sampled at 150 MHz intervals from 1.5 GHz to 10.95 GHz. In this frequency range, there were three main resonance regions. The first region became strong at around 5.25 GHz, while it started to be visible even at 3.75 GHz. This resonance was caused by the larger resonators. The second resonance, which was related to the smaller resonators, occurred at approximately 7.5 GHz. Finally, the third resonance that was also induced by the larger resonators was centered at around 9.6 GHz. It was remarkable that as opposed to the rotational DUR design, this frequency-selective structure provided very similar responses to LHCP and RHCP excitations, which may be preferred in real-life applications.

4. References

1. Z. Li, et al., "Chiral Metamaterials with Negative Refractive Index Based on four 'U' Split Ring Resonators," *Appl. Phys. Lett.*, **97**, August 2010, p. , 081901.

2. O. Turkmen, E. Ekmekci, and G. Turhan-Sayan, "Nested U-Ring Resonators: A Novel Multi-Band Metamaterial Design in Microwave Region," *IET Microwaves, Antennas & Propagation*, **6**, 10, August 2012, pp. 1102-1108.

3. Ö. Ergül and L. Gürel, *The Multilevel Fast Multipole Algorithm (MLFMA) for Solving Large-Scale Computational Electromagnetics Problems*, New York, Wiley-IEEE, 2014.

4. C. Önol, A. Üçüncü, and Ö. Ergül, "Efficient Multilayer Iterative Solutions of Electromagnetic Problems Using Approximate Forms of the Multilevel Fast Multipole Algorithm," *IEEE Antennas and Wireless Propagation Letters*, **16**, 2017, pp. 3253-3256.

5. L. Gürel, et al., "Fast and Accurate Analysis of Large Metamaterial Structures Using the Multilevel Fast Multipole Algorithm," *Prog. Electromagn. Res.*, **95**, 2009, pp. 179-198.