Application of the Optimization Method to the Point-to-Point Radio Wave Ray-Tracing Problem

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Abstract

Point-to-point ray tracing is an important problem in many fields of science. In direct variational methods, some trajectory is transformed to an optimal trajectory. While these methods are routinely used in calculations of pathways of seismic waves, chemical reactions, diffusion processes, etc., these approaches are not widely known in ionospheric point-to-point ray tracing. A two-dimensional representation of the optical path functional is developed, and used to gain insight into the fundamental difference between high and low ionospheric rays. We conclude that high and low rays are minima and saddle points of the optical path functional, respectively. An optimization method for point-to-point ionospheric ray tracing, based on the direct variational principle for the optical path, is proposed. This is applied to calculations of high, trans-ionospheric, and multi-hop rays.
Keywords: point-to-point ray tracing; ionospheric radio; Fermat's principle; nudged elastic-band method

1. Introduction

The point-to-point radio wave ray-tracing problem essentially involves two steps. The first step is related to the choice of the environment model describing the ionospheric parameters. Another important issue concerns the methods for the ionospheric ray tracing, where the positions of the receiver and transmitter are fixed (Figure 1). The accuracy of both steps has a direct impact on the agreement between modeled and experimental oblique-sounding ionograms.

There are essentially two approaches for radio wave point-to-point ray tracing. The most traditional approach is the numerical solution of the eikonal equation, combined with the shooting method. This is also known as the homing-in approach [1-12], where the directions at which the rays are sent out are iteratively refined so as to achieve the desired landing point. The shooting method is widely used to calculate radio-wave paths in the ionosphere, although it

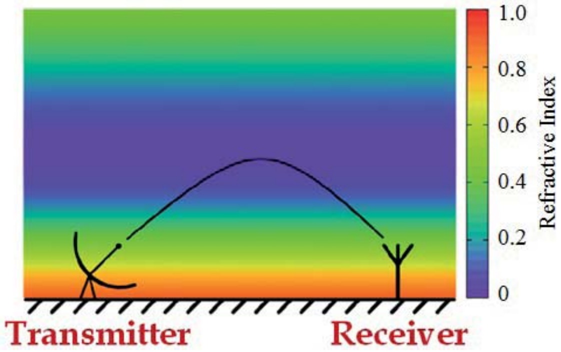

Figure 1. A schematic representation of the ray-tracing problem with boundary conditions. A radio ray emitted from the transmitter to the receiver is shown with a black line.

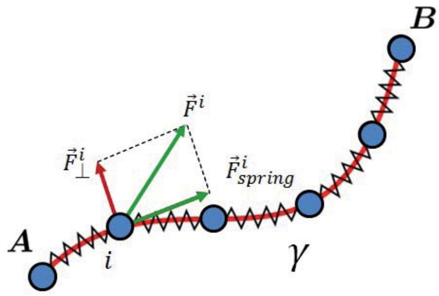

Figure 2. The principles of the nudged elastic-band method. The red line represents a trajectory discretized by a number of points (blue dots). Force vectors acting at each ray point are shown with red and green arrows.

has some disadvantages [13]. Another approach is based on optimization of the optical path (Fermat's principle) [14]. In this approach, the initially defined radio-wave trajectory is transformed to an optimal trajectory, while its endpoints are kept fixed according to the boundary conditions.

In this paper, we discuss the following issues: 1) The use of an optimization method for finding radio-wave paths with fixed endpoints based on the direct variational principle for the optical path; 2) Application of the optimization method to both low and high ionospheric radio-wave trajectories; 3) Advantages of the optimization method in comparison to the shooting method.

2. Optimization Method

The optimization method for point-to-point ray tracing based on the direct variational principle for the optical path is widely used in seismology, as previously mentioned [15, 16]. There, it is known as the bending method [17] and the pseudo-bending method [18- 20]. However, it is hardly known in ionospheric radiophysics. The direct variational method for point-to-point ionospheric ray tracing was proposed by [21], which derived ordinary differential equations for the radio ray and solved them using a Galerkin technique. Coleman [3] developed an alternative approach, involving discretization of the optical-path functional. However, the direct minimization method fails to converge on the low rays [3]. The low rays do not satisfy the Jacobi test for a minimum of the optical-path functional, and therefore cannot be found by a direct minimization procedure. This problem can be solved with the Newton-Raphson method, as advocated by Coleman [3]. However, the Newton-Raphson method converges to any stationary point of an object function, and does not discriminate among minima, maxima, and saddle points of all orders.

2.1 Fermat's Principle

In an isotropic medium, the optical path of the radio-wave ray is defined by the following equation:

$$S[\gamma] = \int_A^B n(\bar{r})\, dl \,. \tag{1}$$

Integration is performed along the curve γ , which joins boundary points A and B. $n(\bar{r})$ is the refractive index at point $\bar{r} = (x, y, z)$. dl is the length element along γ . According to Fermat's principle, the optical path of the radio wave satisfies the equation

$$\delta S = 0 \,. \tag{2}$$

This problem of the calculus of variations can be transformed to the optimization problem in multidimensional space by representing the curve by a polygonal line connecting N points, and using the trapezoidal rule or Simpson's rule to compute the integral in Equation (1). The set of points corresponding to the minimum of the optical path gives the discrete representation of the radio-wave trajectory. The negative gradient of the optical path with respect to $\bar{r}$, which has the meaning of the force acting on the point in the multidimensional configuration space, can be used to guide the minimization:

$$F = -\nabla S = \left(\bar{F}^2, \bar{F}^3, \ldots, \bar{F}^{N-1}\right). \tag{3}$$

2.2 Nudged Elastic-Band Method

In our investigations [22-24], the nudged elastic-band (NEB) method is applied to a point-to-point ionospheric ray-tracing problem. The method was originally developed to identify minimum-energy paths of chemical reactions [25], and it is widely used in various fields of science [26-28]. The use of the force defined in Equation (3) in the minimization procedure, such as the steepest-descent method or the conjugate-gradient method, can lead to a problem connected with the discrete representation of the path γ. The minimum of the optical path can correspond to a highly nonuniform distribution of the points where there are several localization centers with very low density of points in between them. As a result, the information about the radio wave's trajectory in some critical regions may be lost (see, for example, [22]). The remedy to this problem lies in force projection and the inclusion of elastic forces, which is the basis of the nudged elastic-band (NEB) method. According to the nudged elastic-band method, the force acting on each point, i, on the path γ is defined as

$$\overline{F}^i = \overline{F}^i_{\perp} + \overline{F}^i_{spring} . \tag{4}$$

Here, $\overline{F}^i_{\perp}$ is a transverse component of $-\nabla S$, while $\overline{F}^i_{spring}$ is the parallel component of the artificial spring force acting between the points. We previously proposed a method of transverse displacements using only projected forces, $\overline{F}^i_{\perp}$ for the isotropic medium [22]. $\overline{F}^i_{\perp}$ defines the transverse displacement of the path, while the spring force controls the distribution of the points along the path. If the same value of the spring constant is used for all springs connecting the points, the method ensures the uniform distribution of the points along the path after convergence has been achieved, providing good resolution of all parts of the path [23]. Since the spring force acts only along the path, it does not affect the position of the path in space. A summary of the nudged elastic-band method is schematically presented in Figure 2.

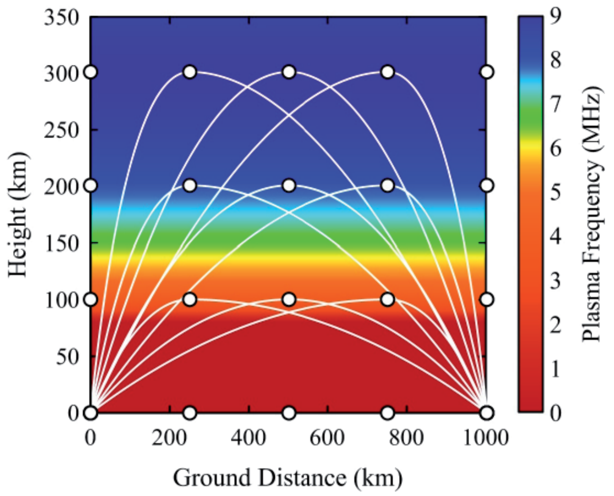

Figure 3. The three-point representation of the radio wave's trajectory. Two points are fixed according to the boundary conditions, and the third point defines the position of the apex (white dots), with spline interpolation in between the points (white solid lines).

3. New Results

3.1 Analysis of Low and High Rays: Optical Path Maps

We develop a two-dimensional representation of the optical path as a function of control parameters defining the radio-wave trajectory, and we use it to gain a deeper insight into the problem of determining the high and low rays. The optical path given by the discretized functional (see Equation (1)) is a function of many variables defining the position of each vertex of the polygonal representation of the radio wave's trajectory. In order to visualize this as a two-dimensional map, we use a reduced description of the model in terms of only two essential variables. This is accomplished by choosing a three-point representation of the radio wave's trajectory, where two points are fixed

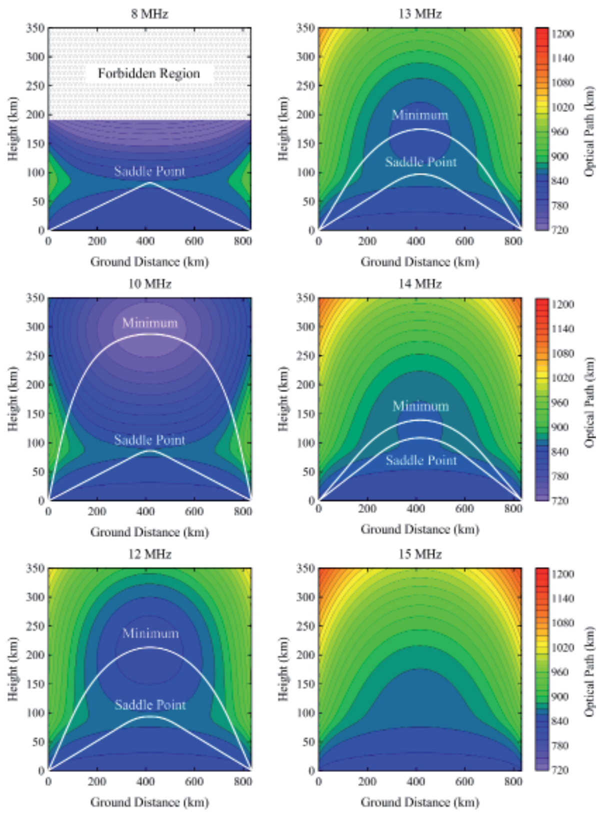

Figure 4. Contour maps of the optical path for various frequencies in a range of 8 MHz to 15 MHz in the parabolic-layer ionosphere. The high ray, obtained with the nudged elastic-band method, and the low ray, obtained by the numerical solution of the Euler-Lagrange equation, are shown with white solid lines.

according to the boundary conditions and the third point defines the position of the apex. Spline interpolation is performed in between the points (see Figure 3). With this representation, the radio wave's trajectory is completely defined by two variables, the coordinates of the apex point, and a contour map of the optical path can be constructed. These two-dimensional maps of the optical path are consistent with detailed calculations of the trajectories using the full set of variables, and they give a qualitative insight into the distribution of the optical path.

The resulting contour map of the optical path obtained for six values of the frequency in a range of 8 MHz to 15 MHz is shown in Figure 4. Radio-wave trajectories are also shown, where the high rays were obtained with the nudged elastic-band method, and the low rays were obtained by solving the Euler-Lagrange equation. Our results showed that the high rays corresponded to minima of the optical path, which led to the robust determination of the rays with the minimization approach. However, the low rays corresponded to saddle points of the optical-path functional, which were difficult to locate. The difficulty arises from the need to minimize the optical path with respect to all but one degree of freedom, for which a maximization should be carried out. It is not known a priori which degree of freedom should be treated differently. In particular, implementation of the nudged elastic-band method and the dimer method [29] will extend the applicability of the optimization approach in the field of HF radio-wave trajectory calculations.

3.2 Identification of High, Trans-Ionospheric, and Multi-Hop Rays

Point-to-point ionospheric ray tracing reduces to the identification of all high and low rays connecting the receiver and transmitter. As an example, we here focus on the radio-wave ray tracing in the region between Kaliningrad and Tromsö at 8.7 MHz. The electron density was given by the IRI-2007 model [30] for 12:00 UT on June 22, 2014, with settings simplified for traveling ionospheric disturbances (TIDs). The results obtained with the homing-in approach [10, 11] are presented in Figure 5.

Five radio-wave rays were found: two high rays, two low rays, and one multi-hop ray, which are consistent with a well-defined two-layer structure in the vertical electron-density profile. These solutions served as a reference for the nudged elastic-band calculations. Both high rays could be calculated with the nudged elastic-band method by setting the initial guesses for the radio-wave trajectory at the altitudes of the F2- and E-layer peaks.

As an example, we also obtained the trans-ionospheric ray that is shown in Figure 5. The results were in good agreement with the solutions given by the homing-in approach. However, the direct minimization method failed to converge on the low and multi-hop rays. The low rays

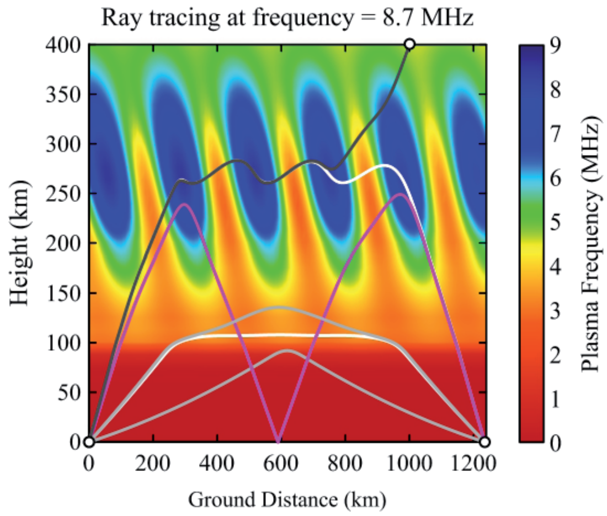

Figure 5. The results of point-to-point ray tracing calculations using the nudged elastic-band method at a frequency of 8.7 MHz between Kaliningrad (54.57°N, 20°E) and Tromsö (65.65°N, 18.57°E) for daytime summer solstice on 22.06.2014. The white solid lines are for the high ray. The grey solid line is for the low ray. The black solid and dashed line is for the trans-ionospheric ray. The pink solid line is for the multi-hop ray. The electron density given by the IRI-2007 model was perturbed by TIDs.

corresponded to a saddle point of the optical-path functional, and therefore could not be found by a direct minimization procedure. The low rays as well as the multi-hop ray could still be found with the proposed technique if the trajectory was divided at the apex, and separate calculations were performed for each segment of the radio ray. However, this scheme is only possible if the position of the apex is known.

4. Discussion

The most traditional approach for radio-wave point-to-point ray tracing is a homing-in approach. However, such an approach may suffer from convergence problems when applied to a realistic three-dimensional ionosphere [13]. The direct variational method has advantages compared to the homing-in approach, since it automatically satisfies the boundary conditions. High, trans-ionospheric, and multi-hop rays were calculated for the ionospheric medium predicted by the International Reference Ionosphere (IRI) model, where the electron density was perturbed by traveling ionospheric disturbances (TIDs). The results obtained with the nudged elastic-band method were in good agreement with those given by the homing-in approach.

However, low rays needed special treatment, since they corresponded to a saddle point of the optical-path functional [24], and therefore could not be found by direct minimization of the optical-path functional [3]. The problem of low-ray identification by the direct variational method was also discussed. For that reason, a two-dimensional representation of the optical path surface was introduced and used to gain insight into the nature of low rays, which are particularly difficult to calculate, and to discuss a scheme for their identification. This problem can be solved with the Newton-Raphson method, as advocated by Coleman [3]. However, the Newton-Raphson method converges to any stationary point of an object function, and does not discriminate among minima, maxima, and saddle points of all orders.

However, our preliminary analysis suggests that the definite identification of the low rays is equivalent to the first-order saddle-point search, for which several methods have been developed. The method that is very efficient and commonly used is actually the nudged elastic-band method. Originally, the nudged elastic-band method was introduced to calculate lowest-lying paths between minima of a multidimensional surface. A saddle point is extracted from the position of maxima along such paths. Therefore, the low ionospheric rays can be found by applying the nudged elastic-band method in its original context. An optimal path needs to be found in a space of radio-ray trajectories. The final, relaxed path obtained from a nudged elastic-band calculation lies lowermost on the multidimensional optical-path surface so that the maximum along the path is precisely a saddle point corresponding to a low ray. Calculation of the low rays using this approach is a subject of future research.

5. Summary

In this paper, we applied the nudged elastic-band method to a point-to-point ionospheric ray-tracing problem. Although the method was originally developed for calculations of mechanisms and pathways of chemical reactions, it proved to be well-suited for the identification of radio-ray trajectories in realistic ionospheric media, especially when the positions of the receiver and transmitter were fixed. All high rays could be found, given that some sampling of the initial conditions for the radio-wave trajectory was performed.

Care needs to be taken when calculating the low rays. Although both high and low rays are stationary radio-wave trajectories, our analysis showed that the former correspond to the minima of the optical-path functional, while the latter correspond to the saddle points, which are difficult to locate. A better strategy is to again exploit the nudged elastic-band method, but in a different context. The nudged elastic-band method was originally designed to identify saddle points on a multidimensional surface. In order to locate a low ionospheric ray, the nudged elastic-band method needs to be applied to a path in a space of radio-ray trajectories. The final, relaxed path lies lowermost on the multidimensional optical-path surface, so that the maximum along the path is precisely a saddle point corresponding to a low ray. Formulation of new methods for finding low rays can be based on conclusions drawn from the present work, and will be addressed in a future study.

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