Corrections

Corrections to "Vector Cross-Product Direction-Finding' With an Electromagnetic Vector-Sensor of Six Orthogonally Oriented But Spatially Noncollocating Dipoles/Loops"

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In [1], written by the second and the third authors of this correction, the Monte Carlo simulations are incorrectly conducted in Section V and in Figs. 4–5, for the case of *arbitrarily* oriented array-axes. This erratum presents a correct algorithm and the correct simulation results, for the case of *arbitrarily* oriented array-axes. Also presented here will be some other miscellaneous corrections. The second and the third authors apologize for these errors in [1].

I. A New Algorithm For Arbitrarily Oriented Array-Axes

Referring to the notation in [1],

$$\mathbf{q} = e^{j\frac{2\pi}{\lambda}(x_h u + y_h v + z_h w)} \begin{bmatrix} u\, e^{-j\frac{2\pi}{\lambda}\left[(2\Delta_{x,y}+\Delta_{y,z})(\tilde{u}u+\tilde{v}v+\tilde{w}w)\right]} \\ v\, e^{-j\frac{2\pi}{\lambda}\left[(\Delta_{x,y}+\Delta_{y,z})(\tilde{u}u+\tilde{v}v+\tilde{w}w)\right]} \\ w\, e^{-j\frac{2\pi}{\lambda}\left[\Delta_{x,y}(\tilde{u}u+\tilde{v}v+\tilde{w}w)\right]} \end{bmatrix},$$

which is the first unnumbered equation on p. 163 of [1]. Re-write the above as in (1), where ρ_x, ρ_y, ρ_z in (2)–(4) symbolize the *non*-Cartesian direction cosines [see the equations at the bottom of the page].

A. To Estimate the Non-Cartesian Direction Cosines (ρ_x, ρ_y, ρ_z)

From (1), ρ_x may be estimated in two complementary ways in parallel:

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i) For a sparse spacing of $\frac{\frac{2\Delta_{x,y}+\Delta_{y,z}}{\lambda}}{} > \frac{1}{2}$, a many-to-one relationship links ρ_x to $e^{j2\pi\frac{2\Delta_{x,y}+\Delta_{y,z}}{\lambda}\rho_x}$. Therefore,

$$\begin{aligned}\hat{\rho}_{x,\text{phs}} &= \frac{1}{2\pi}\frac{\lambda}{2\Delta_{x,y}+\Delta_{y,z}}\angle\left\{s_u[\mathbf{q}]_1\right\} \\ &= m\frac{\lambda}{2\Delta_{x,y}+\Delta_{y,z}} + \rho_x, \end{aligned} \tag{5}$$

estimates ρ_x (ambiguously) to within some integer-multiple $(m\times)$ of $\pm\frac{\lambda}{2\Delta_{x,y}+\Delta_{y,z}}$, where m refers to a to-be-determined integer and $s_u \in \{+1, -1\}$ symbolizes the sign of u.

ii) Substitute the magnitudes $\hat{u}_{\text{mag}} = |[\mathbf{q}]_1|, \hat{v}_{\text{mag}} = |[\mathbf{q}]_2|, \hat{w}_{\text{mag}} = |[\mathbf{q}]_3|$ into (2), to estimate ρ_x (also ambiguously) to within three $\pm$ signs (i.e., $s_u, s_v, s_w \in \{+1, -1\}$).

$$\begin{aligned}\hat{\rho}_{x,\text{mag}}^{(s_u,s_v,s_w)} = &\left(-\tilde{u} + \frac{x_h}{2\Delta_{x,y}+\Delta_{y,z}}\right)s_u\hat{u}_{\text{mag}} \\ &+ \left(-\tilde{v} + \frac{y_h}{2\Delta_{x,y}+\Delta_{y,z}}\right)s_v\hat{v}_{\text{mag}} \\ &+ \left(-\tilde{w} + \frac{z_h}{2\Delta_{x,y}+\Delta_{y,z}}\right)s_w\hat{w}_{\text{mag}}. \end{aligned} \tag{6}$$

Next, estimate the aforementioned integer

$$m \in \left\{\left\lceil \frac{1}{2} - \frac{2\Delta_{x,y}+\Delta_{y,z}}{\lambda}\right\rceil, \ldots, \left\lfloor \frac{1}{2} + \frac{2\Delta_{x,y}+\Delta_{y,z}}{\lambda}\right\rfloor\right\},$$

by substituting $\hat{\rho}_{x,\text{mag}}^{(s_u,s_v,s_w)}$ of (6) for ρ_x in (5). I.e., $\hat{m}_x^{(s_u,s_v,s_w)} \stackrel{\text{def}}{=}$ [see (7) at the bottom of the next page]. There exist altogether eight possible candidates for this $\hat{m}^{(s_u,s_v,s_w)}$, because of the three ambiguous signs in s_u, s_v, s_w. To choose among these eight candidates, needed will be $\epsilon_x(m), \epsilon_y(m)$, and $\epsilon_z(m)$—with the latter two obtainable analogously as in (5)–(7) but now for ρ_y and for ρ_z. To choose among the eight candidates,

$$(s_u^o, s_v^o, s_w^o) \stackrel{\text{def}}{=} \underset{(s_u,s_v,s_w)}{\arg\min} \sum_{\xi=x,y,z} \epsilon_\xi^2\left(\hat{m}_\xi^{(s_u,s_v,s_w)}\right). \tag{8}$$

$$\mathbf{q} = \begin{bmatrix} u\, e^{j\frac{2\pi}{\lambda}(2\Delta_{x,y}+\Delta_{y,z})\left[\left(-\tilde{u}+\frac{x_h}{2\Delta_{x,y}+\Delta_{y,z}}\right)u+\left(-\tilde{v}+\frac{y_h}{2\Delta_{x,y}+\Delta_{y,z}}\right)v+\left(-\tilde{w}+\frac{z_h}{2\Delta_{x,y}+\Delta_{y,z}}\right)w\right]} \\ v\, e^{j\frac{2\pi}{\lambda}(\Delta_{x,y}+\Delta_{y,z})\left[\left(-\tilde{u}+\frac{x_h}{\Delta_{x,y}+\Delta_{y,z}}\right)u+\left(-\tilde{v}+\frac{y_h}{\Delta_{x,y}+\Delta_{y,z}}\right)v+\left(-\tilde{w}+\frac{z_h}{\Delta_{x,y}+\Delta_{y,z}}\right)w\right]} \\ w\, e^{j\frac{2\pi}{\lambda}\Delta_{x,y}\left[\left(-\tilde{u}+\frac{x_h}{\Delta_{x,y}}\right)u+\left(-\tilde{v}+\frac{y_h}{\Delta_{x,y}}\right)v+\left(-\tilde{w}+\frac{z_h}{\Delta_{x,y}}\right)w\right]} \end{bmatrix} = \begin{bmatrix} u\, e^{j\frac{2\pi}{\lambda}(2\Delta_{x,y}+\Delta_{y,z})\rho_x} \\ v\, e^{j\frac{2\pi}{\lambda}(\Delta_{x,y}+\Delta_{y,z})\rho_y} \\ w\, e^{j\frac{2\pi}{\lambda}\Delta_{x,y}\rho_z} \end{bmatrix}. \tag{1}$$

$$\rho_x \stackrel{\text{def}}{=} \left(-\tilde{u} + \frac{x_h}{2\Delta_{x,y}+\Delta_{y,z}}\right)u + \left(-\tilde{v} + \frac{y_h}{2\Delta_{x,y}+\Delta_{y,z}}\right)v + \left(-\tilde{w} + \frac{z_h}{2\Delta_{x,y}+\Delta_{y,z}}\right)w, \tag{2}$$

$$\rho_y \stackrel{\text{def}}{=} \left(-\tilde{u} + \frac{x_h}{\Delta_{x,y}+\Delta_{y,z}}\right)u + \left(-\tilde{v} + \frac{y_h}{\Delta_{x,y}+\Delta_{y,z}}\right)v + \left(-\tilde{w} + \frac{z_h}{\Delta_{x,y}+\Delta_{y,z}}\right)w, \tag{3}$$

$$\rho_z \stackrel{\text{def}}{=} \left(-\tilde{u} + \frac{x_h}{\Delta_{x,y}}\right)u + \left(-\tilde{v} + \frac{y_h}{\Delta_{x,y}}\right)v + \left(-\tilde{w} + \frac{z_h}{\Delta_{x,y}}\right)w, \tag{4}$$

This allows ρ_x, ρ_y and ρ_z to be *un*ambiguously estimated as[1]

$$\hat{\rho}_x = \left(\hat{m}_x^{(s_u^o, s_v^o, s_w^o)} + \frac{1}{2\pi} \angle s_u^o [\mathbf{q}]_1 \right) \frac{\lambda}{2\Delta_{x,y} + \Delta_{y,z}}, \tag{9}$$

$$\hat{\rho}_y = \left(\hat{m}_y^{(s_u^o, s_v^o, s_w^o)} + \frac{1}{2\pi} \angle s_v^o [\mathbf{q}]_2 \right) \frac{\lambda}{\Delta_{x,y} + \Delta_{y,z}}, \tag{10}$$

$$\hat{\rho}_z = \left(\hat{m}_z^{(s_u^o, s_v^o, s_w^o)} + \frac{1}{2\pi} \angle s_w^o [\mathbf{q}]_3 \right) \frac{\lambda}{\Delta_{x,y}}. \tag{11}$$

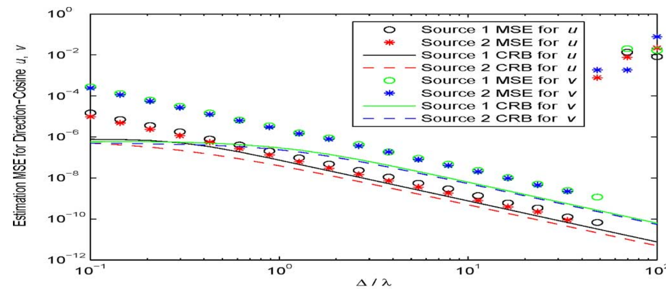
B. To Estimate the Cartesian Direction-Cosines u, v, w

Rewrite (2)–(4) as (12) in matrix form, shown at the bottom of the page. Denote the i row of the 3×3 matrix $\mathbf{M}$ in (12) as $\mathbf{r}_i$. Then, $\mathbf{r}_1(2\Delta_{x,y} + \Delta_{y,z}) = \mathbf{r}_2(\Delta_{x,y} + \Delta_{y,z}) + \mathbf{r}_3 \Delta_{x,y}$; hence, $\mathrm{rank}\,(\mathbf{M}) \leq 2$. If $\mathrm{rank}\,(\mathbf{M}) = 1$, estimate (u, v) from only (6), but not from (9)–(11) because of an insufficient of number constraints to obtain any "fine" estimate of u and v. Iff $\mathrm{rank}(\mathbf{M}) = 2$, express (12) as (13), shown at the bottom of the page, whose quadratic equations may be solved for two sets of candidates of $(\hat{u}, \hat{v})$. Select the candidate-set closest in magnitude to $(\hat{u}_{\mathrm{mag}}, \hat{v}_{\mathrm{mag}})$ as $(\hat{u}, \hat{v})$.

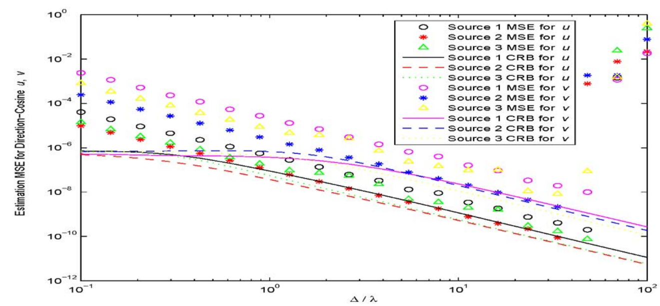
II. Monte Carlo Simulation

Fig. 1(a) shows a two-source scenario, whereas Fig. 1(b) shows a three-source scenario. Both graphs verify the proposed algorithm's efficacy to improve the accuracy of each individual Cartesian direction cosine. Each graph plots the composite mean-square-error of each incident source's three Cartesian direction-cosine estimates, versus the inter-antenna spacing parameter $\frac{\Delta}{\lambda} = \ell$. Each data-point on each graph consists of 1000 statistically independent Monte Carlo trials, each with 400 temporal snapshots. All sources have unity power. Each source's signal-to-noise ratio equals 30 dB. The "spatially spread" vector-sensor is oriented not along any Cartesian axis, but at $\tilde{\theta} = 45^\circ$ and $\tilde{\phi} = 45^\circ$. Specifically, referring to the array geometric symbols defined in Figures 1–2 in [1]: the coordinates of loop H_x are $(x_h, y_h, z_h) = \ell\lambda(2, 1, \sqrt{2})$, with $\frac{\Delta_{x,y}}{\lambda} = \frac{\Delta_{y,z}}{\lambda} = \ell$, and λ referring to the shortest wavelength among all incident sources' wavelengths.

[1]In [1], each direction-cosine's sign was assumed prior known, e.g., Section III-A-1) there assumes that $u \geq 0$. This errata's method makes no such presumption of such priori knowledge. Hence, the disambiguation requires a simultaneous determination of all three direction-cosines' signs.



a



b

Fig. 1. (a) The composite mean-square-error of the Cartesian direction-cosines estimates, for *two* incident sources, at digital frequencies $f_1' = 0.1$ and $f_2' = 0.1265$, respectively with $(\theta_1, \phi_1, \gamma_1, \eta_1) = (56^\circ, 55^\circ, 45^\circ, -90^\circ)$ and $(\theta_2, \phi_2, \gamma_2, \eta_2) = (54^\circ, 57^\circ, 45^\circ, 90^\circ)$. (b) The composite mean-square-error of the Cartesian direction-cosines estimates, for *three* incident sources, at digital frequencies $f_1' = 0.1, f_2' = 0.1265$, and $f_3' = 0.1165$, respectively with $(\theta_1, \phi_1, \gamma_1, \eta_1) = (35^\circ, 42^\circ, 45^\circ, -90^\circ), (\theta_2, \phi_2, \gamma_2, \eta_2) = (43^\circ, 35^\circ, 45^\circ, 90^\circ)$, and $(\theta_3, \phi_3, \gamma_3, \eta_3) = (52^\circ, 52^\circ, 45^\circ, -90^\circ)$.

III. Other Miscellaneous Corrections of [1]

1) The right column on p. 163 of [1] should have $m_{x,y}$ and $m_{x,z}$:

$$m_{x,y} \in \left\{ \left\lceil \frac{\Delta_{x,y}}{\lambda}(-1 - \hat{u}_{\mathrm{fine},1}) \right\rceil, \left\lfloor \frac{\Delta_{x,y}}{\lambda}(1 - \hat{u}_{\mathrm{fine},1}) \right\rfloor \right\},$$

$$m_{x,z} \in \left\{ \left\lceil \frac{\Delta_{x,y} + \Delta_{y,z}}{\lambda}(-1 - \hat{u}_{\mathrm{fine},2}) \right\rceil, \left\lfloor \frac{\Delta_{x,y} + \Delta_{y,z}}{\lambda}(1 - \hat{u}_{\mathrm{fine},2}) \right\rfloor \right\}.$$

$$\arg\min_{m} \underbrace{\left| \left\{ \left(\overbrace{\frac{1}{2\pi} \frac{\lambda}{2\Delta_{x,y} + \Delta_{y,z}} \angle (s_u [\mathbf{q}]_1)}^{=\hat{\rho}_{x,\mathrm{phs}}} - m \frac{\lambda}{2\Delta_{x,y} + \Delta_{y,z}} \right) - \hat{\rho}_{x,\mathrm{mag}}^{(s_u, s_v, s_w)} \right\} \right|}_{\stackrel{\mathrm{def}}{=} \epsilon_x(m)}. \tag{7}$$

$$\begin{bmatrix} \hat{\rho}_x \\ \hat{\rho}_y \\ \hat{\rho}_z \end{bmatrix} = \underbrace{\begin{bmatrix} -\tilde{u} + \frac{x_h}{2\Delta_{x,y} + \Delta_{y,z}}, & -\tilde{v} + \frac{y_h}{2\Delta_{x,y} + \Delta_{y,z}}, & -\tilde{w} + \frac{z_h}{2\Delta_{x,y} + \Delta_{y,z}} \\ -\tilde{u} + \frac{x_h}{\Delta_{x,y} + \Delta_{y,z}}, & -\tilde{v} + \frac{y_h}{\Delta_{x,y} + \Delta_{y,z}}, & -\tilde{w} + \frac{z_h}{\Delta_{x,y} + \Delta_{y,z}} \\ -\tilde{u} + \frac{x_h}{\Delta_{x,y}}, & -\tilde{v} + \frac{y_h}{\Delta_{x,y}}, & -\tilde{w} + \frac{z_h}{\Delta_{x,y}} \end{bmatrix}}_{\stackrel{\mathrm{def}}{=} \mathbf{M}} \begin{bmatrix} \hat{u} \\ \hat{v} \\ \hat{w} \end{bmatrix}. \tag{12}$$

$$\begin{bmatrix} \hat{\rho}_x \\ \hat{\rho}_y \end{bmatrix} = \begin{bmatrix} -\tilde{u} + \frac{x_h}{2\Delta_{x,y} + \Delta_{y,z}} & -\tilde{v} + \frac{y_h}{2\Delta_{x,y} + \Delta_{y,z}} & -\tilde{w} + \frac{z_h}{2\Delta_{x,y} + \Delta_{y,z}} \\ -\tilde{u} + \frac{x_h}{\Delta_{x,y} + \Delta_{y,z}} & -\tilde{v} + \frac{y_h}{\Delta_{x,y} + \Delta_{y,z}} & -\tilde{w} + \frac{z_h}{\Delta_{x,y} + \Delta_{y,z}} \end{bmatrix} \begin{bmatrix} \hat{u} \\ \hat{v} \\ s_w^o \sqrt{1 - \hat{u}^2 - \hat{v}^2} \end{bmatrix} \tag{13}$$

A similar correction is needed in the left column on page 164, for the subscripts of $\hat{v}$ as for the subscripts of $\hat{u}$ above. Similarly, at the bottom of 164 and at the top of page 165, $m_{x,z}$ and $m_{y,z}$ should be

$$m_{x,z} \in \left\{ \left\lceil \frac{\Delta_{x,y} + \Delta_{y,z}}{\lambda}(-1 - \hat{w}_{\text{fine},1}) \right\rceil, \left\lfloor \frac{\Delta_{x,y} + \Delta_{y,z}}{\lambda}(1 - \hat{w}_{\text{fine},1}) \right\rfloor \right\},$$
$$m_{y,z} \in \left\{ \left\lceil \frac{\Delta_{y,z}}{\lambda}(-1 - \hat{w}_{\text{fine},2}) \right\rceil, \left\lfloor \frac{\Delta_{y,z}}{\lambda}(1 - \hat{w}_{\text{fine},2}) \right\rfloor \right\}.$$

2) The first vector on the far right side of (3) of [1] should have $\tilde{w}w$ instead of $\tilde{v}w$.
3) For $e^{j\frac{2\pi}{\lambda}x_h u}$ in the unnumbered equation above (5) in [1], for $e^{j\frac{2\pi}{\lambda}y_h v}$ in the unnumbered equation above (10), and for $e^{j\frac{2\pi}{\lambda}z_h w}$ in (14)—all should be $e^{j\frac{2\pi}{\lambda}(x_h u + y_h v + z_h w)}$.
4) Right below (13) of [1], it should be $\Delta_{x,y} > \lambda$.
5) Step $\{2b.\}$ on page 165 of [1] should have $(\mathbf{E}_1^H\mathbf{E}_1)^{-1}(\mathbf{E}_1^H\mathbf{E}_2)$, instead of $(\mathbf{E}_1^H\mathbf{E}_1)^{-1}(\mathbf{E}_1^H\mathbf{E}_1)$.
6) Footnote 10 in [1] should have $\mathbf{d}(\hat{\theta}_k, \hat{\phi}_k) \cdot \mathbf{\Theta}(\hat{\theta}_k, \hat{\phi}_k)$, instead of $\mathbf{d}(\hat{\theta}_k, \hat{\phi}_k)\mathbf{\Theta}(\hat{\theta}_k, \hat{\phi}_k)$.
7) On line 9 in the right column on page 167 of [1], $6 \times N$ should be $6N \times 1$.
8) Reference [23] was published in November 2004.

References

[1] K. T. Wong and X. Yuan, "'Vector cross-product direction-finding' with an electromagnetic vector-sensor of six orthogonally oriented but spatially non-collocating dipoles/loops," *IEEE Trans. Signal Process.*, vol. 59, no. 1, pp. 160–171, Jan. 2011.