# Corrections to "Semiparametric CRB and Slepian-Bangs Formulas for Complex Elliptically Symmetric Distributions"

Stefano Fortunati, Fulvio Gini, *Fellow, IEEE*, Maria S. Greco, *Fellow, IEEE*, Abdelhak M. Zoubir, *Life Fellow, IEEE*, and Muralidhar Rangaswamy, *Fellow, IEEE*

Errors in [1] are corrected below.

1. In Eq. (17), $\text{vecs}(\boldsymbol{\Sigma}_0)$ should be $\text{vec}(\boldsymbol{\Sigma}_0)$. Specifically, the correct version of Eq. (17) is:

$$\mathbf{s}_{\boldsymbol{\phi}_0} \triangleq \nabla_{\boldsymbol{\phi}} \ln p_Z(\mathbf{z};\boldsymbol{\phi}_0,h_0) = [\mathbf{s}^T_{\boldsymbol{\mu}_0}, \mathbf{s}^T_{\boldsymbol{\mu}_0^*}, \mathbf{s}^T_{\text{vec}(\boldsymbol{\Sigma}_0)}]^T. \tag{17}$$

2. In the first line after Eq. (18), $\mathbf{s}^T_{\text{vecs}(\boldsymbol{\Sigma}_0)}$ should be $\mathbf{s}^T_{\text{vec}(\boldsymbol{\Sigma}_0)}$.
3. A minus "-" is missing in front of the right-hand side of Eq. (25). The correct equation is:

$$\begin{aligned}\bar{\mathbf{s}}_{\text{vec}(\boldsymbol{\Sigma}_0)} =_d & -\mathcal{Q}\psi_0(\mathcal{Q})\times \\ & \times (\boldsymbol{\Sigma}_0^{-*/2} \otimes \boldsymbol{\Sigma}_0^{-1/2}\text{vec}(\mathbf{u}\mathbf{u}^H) - N^{-1}\text{vec}(\boldsymbol{\Sigma}_0^{-1})).\end{aligned} \tag{25}$$

4. A minus "-" is missing in front of $\text{tr}(\mathbf{P}_i^0)$ in Eqs. (38), (40), (41), (42). The correct equations are:

$$\begin{aligned}[\mathbf{s}_{\boldsymbol{\theta}_0}]_i &\triangleq \left.\frac{\partial \ln p_Z(\mathbf{z};\boldsymbol{\theta},h_0)}{\partial\theta_i}\right|_{\boldsymbol{\theta}=\boldsymbol{\theta}_0} \\ &= -\text{tr}(\mathbf{P}_i^0) + \psi_0(Q_0)\frac{\partial Q_0}{\partial\theta_i},\end{aligned} \tag{38}$$

$$\begin{aligned}[\mathbf{s}_{\boldsymbol{\theta}_0}]_i = & -\text{tr}\left(\mathbf{P}_i^0\right) - \psi_0(Q_0) \\ & \times \left(2\text{Re}\left[(\mathbf{z}-\boldsymbol{\mu}_0)^H\boldsymbol{\Sigma}_0^{-1}\boldsymbol{\mu}_i^0\right] + \right. \\ & \left. +(\mathbf{z}-\boldsymbol{\mu}_0)^H\mathbf{S}_i^0(\mathbf{z}-\boldsymbol{\mu}_0)\right),\ i=1,\ldots,d.\end{aligned} \tag{40}$$

$$\begin{aligned}[\mathbf{s}_{\boldsymbol{\theta}_0}]_i =_d & -\psi_0(\mathcal{Q})\left(2\sqrt{\mathcal{Q}}\text{Re}\left[\mathbf{u}^H\boldsymbol{\Sigma}_0^{H/2}\boldsymbol{\Sigma}_0^{-1}\boldsymbol{\mu}_i^0\right] + \right. \\ & \left. +\mathcal{Q}\mathbf{u}^H\boldsymbol{\Sigma}_0^{H/2}\mathbf{S}_i^0\boldsymbol{\Sigma}_0^{1/2}\mathbf{u}\right) - \text{tr}\left(\mathbf{P}_i^0\right) \\ = & -\psi_0(\mathcal{Q})\left(2\sqrt{\mathcal{Q}}\text{Re}\left[\mathbf{u}^H\boldsymbol{\Sigma}_0^{-1/2}\boldsymbol{\mu}_i^0\right]\right. \\ & \left. +\ \mathcal{Q}\mathbf{u}^H\mathbf{P}_i^0\mathbf{u}\right) + \\ & -\text{tr}\left(\mathbf{P}_i^0\right),\ i=1,\ldots,d.\end{aligned} \tag{41}$$

$$\begin{aligned}[\Pi(\mathbf{s}_{\boldsymbol{\theta}_0}|\mathcal{T}_{h_0})]_i &= E_{0|\sqrt{\mathcal{Q}}}\{[\mathbf{s}_{\boldsymbol{\theta}_0}]_i|\sqrt{\mathcal{Q}}\} \\ &=_d -\text{tr}(\mathbf{P}_i^0) - 2\sqrt{\mathcal{Q}}\psi_0(\mathcal{Q})\text{Re}\left[E\{\mathbf{u}\}^H\boldsymbol{\Sigma}_0^{-1/2}\boldsymbol{\mu}_i^0\right] \\ &\quad - \mathcal{Q}\psi_0(\mathcal{Q})\text{tr}\left(\mathbf{P}_i^0E\{\mathbf{u}\mathbf{u}^H\}\right) \\ &= -\text{tr}(\mathbf{P}_i^0) - N^{-1}\mathcal{Q}\psi_0(\mathcal{Q})\text{tr}\left(\mathbf{P}_i^0\right),\ i=1,\ldots,d.\end{aligned} \tag{42}$$

## REFERENCES

[1] S. Fortunati, F. Gini, M. S. Greco, A. M. Zoubir, and M. Rangaswamy, "Semiparametric CRB and slepian-bangs formulas for complex elliptically symmetric distributions," *IEEE Trans. Signal Process.*, vol. 67, no. 20, pp. 5352–5364, Oct. 2019, doi: 10.1109/TSP.2019.2939084.

Manuscript received 11 January 2024; accepted 11 January 2024. Date of current version 29 January 2024. *(Corresponding author: Stefano Fortunati.)*

S. Fortunati was with the Dipartimento di Ingegneria dell'Informazione, Università di Pisa, Pisa 56122, Italy, was also with the Signal Processing Group, Technische Universität Darmstadt, Darmstadt 64283, Germany. He is now with Université Paris-Saclay, CNRS, CentraleSupélec, Laboratoire des signaux et systèmes, 91190, Gif-sur-Yvette & DR2I-IPSA, 94200 Ivry sur Seine, France (e-mail: stefano.fortunati@centralesupelec.fr).

F. Gini and M. S. Greco are with the Dipartimento di Ingegneria dell'Informazione, Università di Pisa, Pisa 56122, Italy (e-mail: f.gini,m.greco@iet.unipi.it; m.greco@iet.unipi.it).

A. M. Zoubir is with the Signal Processing Group, Technische Universität Darmstadt, Darmstadt 64283, Germany (e-mail: zoubir@spg.tu-darmstadt.de).

M. Rangaswamy is with the U.S. Air Force Research Laboratory, Sensors Directorate, Wright-Patterson AFB, OH 45433 USA (e-mail: muralidhar.rangaswamy@us.af.mil).

Digital Object Identifier 10.1109/TSP.2024.3353889