

# Performance Model for 4G/5G Networks Taking Into Account Intra- and Inter-Cell Mobility of Users

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**Abstract**—In this paper, we investigate the influence of intra- and inter-cell mobility of users on performance of 4G/5G cellular networks, such as LTE and LTE-A. To this end, we develop a multi-class PS queue model that captures mobility of users between zones of a cell and between cells, through a simple mobility model, that is decoupled from the cell model itself, enabling to directly apply the approach to more realistic mobility patterns. We first show that this model is consistent with known analytical bounds corresponding to a system with either static users or users having an infinite speed. We then compare our model to simulations for more realistic speeds, and show that it provides user and cell performance with a very good accuracy. The outcomes of our model confirm that mobility may improve both users and cells performance, and enable to quantify the gain as a function of users speed.

## I. INTRODUCTION

In LTE systems modulation and/or coding can change with time for a given user depending on its location and its channel quality. This is especially true when users are mobile, highlighting the influence of mobility on user and cell performance. The resulting variations in the transmission rates is exploited by opportunistic schedulers to increase overall throughput of data transmissions [1], [2], [3]. But even schedulers with fair resource sharing strategy can take advantage of users mobility.

The fact that mobility may improve performance has already been observed in the literature (e.g., in [4], [5], [6], ). These papers mainly present theoretical properties and performance bounds. For example, authors in [4] identify two limit regimes of infinitely fast and infinitely low channel variations, and show that these limit regimes provide simple bounds on performance at a flow-level. [5] also develop lower and upper bounds for the flow-level performance measures, showing that mobility tends to increase the overall capacity of the network.

The most related work is [7], in which authors assess the impact of users mobility on cell performance, under a

fair and an opportunistic scheduling scheme. They show that under both scheduling policies, mobility improves throughput performance at cell edge. But as the Markovian process associated with their model is no longer reversible in the case where mobile users fairly share resources, they can only develop closed-form expressions in two limiting cases, namely when users are static and when users have an infinite speed.

In this paper we develop a multi-class Processor Sharing (PS) queue model, that captures mobility of users through the distribution of the time a given user physically stays in the different coding zones of cells. Contrarily to previous studies, our model does not rely on the numerical analysis of complex Markov chains, or on limiting assumptions such as infinite speed of users, and as such is one of the first tractable and accurate approximations for 4G/5G cellular networks with mobile users. The originality of the approach is to decouple the mobility model from the cell model itself, by relating input parameters of the PS queue to physical mobility parameters. We show that our model is consistent with aforementioned analytical bounds for realistic speeds of users. Thanks to our model, we quantify the gain of speed on both the performance of the cell and the end-to-end performance of users, and investigate the influence of intra- and inter-cell mobility.

The paper is organized as follows. Section II presents system and traffic assumptions used in the model. Section III develops the PS queue model and all performance parameters of interest. The model is validated through simulation in Section IV that also investigates the impact of users speed on performance. Finally, Section V concludes the paper.

## II. SYSTEM AND TRAFFIC ASSUMPTIONS

We consider a LTE macrocell with a round robin scheduling discipline. For a given number of active users, Resource Blocks are equally divided among users. A user that is alone in the cell will have different bit rates if he is close to the base station, compared to the case where he is far from it. The cell can thus be divided into  $J$  zones of equal radio conditions, or

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classes, each characterized by an achievable throughput, i.e., a throughput that can be achieved by a user when scheduled by the base station [8]. A user of class  $j$ ,  $j = 1, \dots, J$ , i.e., currently present in zone  $j$ , will obtain a throughput  $C_j$ , if he is alone in the whole cell. We can assume as, e.g., in [9], that the  $J$  zones form concentric circles of radius  $R_j$ , where zone 1 is the central zone and zone  $J$  is the peripheral zone.

We consider that connection demands arrive to the cell according to a Poisson process of rate  $\Lambda$ . We assume that a user that carries a new connection demand has a probability  $p_j$  to start its download in zone  $j$ . As a result, new connection demands arrive in zone  $j$  according to a Poisson process with a rate  $\Lambda_j = p_j \Lambda$ . Contrarily to previous classical works on PS queue models (e.g., [8]), we assume that users are mobile, and can thus change zone during their download or leave the cell before completing their transfer. The mobility model we consider is the following. It is important to emphasize that it is a physical mobility model of users that is decoupled from the utilization of the resources by users. We denote by  $\Theta_j$  the physical sojourn time of a user in zone  $j$  at each visit of the zone, that is, the time duration he physically stays in zone  $j$  at each visit before moving to another zone or another cell. We then define  $P_{jk}$ , the probability that a user that physically exists in zone  $j$  (active or not) move to a neighboring zone  $k$ . If the outside of the cell is labelled 0,  $P_{j0}$  is the probability that a user exits the cell from zone  $j$ , and  $P_{0j}$  is the probability that a user enters the cell by zone  $j$ .

We assume that each new connection demand (regardless of its initial class) brings an identically distributed volume  $\Sigma$  of data to be downloaded. As soon as a new request arrives, it triggers the start of a new data transmission (in the zone where the request appears). This transmission ends either because the user has completed its transfer before leaving the cell (whatever the number of zones he has visited meanwhile), or because he has left the cell before completing its transfer. As a result, the volume actually transferred by a given user in the cell is, in average, less than  $\Sigma$ .

We consider in this paper that random variables  $\Sigma$  and  $\Theta_j$  are exponentially distributed. The first assumption is necessary for the derivations presented below. On the other hand, assuming exponential sojourn time in each zone is only made for simplification purposes, and other distributions can be considered.

We now consider the special case where the  $J$  zones form concentric circles of radius  $R_j$ , and see how we can estimate the traffic and mobility parameters. First, the surface of zone  $j$  is  $S_j = \pi(R_j^2 - R_{j-1}^2)$ , for  $j > 1$ , and  $S_1 = \pi R_1^2$ . If arrivals of new connection demands are uniformly distributed over the whole surface of the cell, the probability  $p_j$  is proportional to the surface of zone  $j$  as show in equation 1. Concerning the mean sojourn time in zone  $j$ ,  $\mathbb{E}(\Theta_j)$ , we can reasonably

assume that it is proportional to the square root of the surface of the zone and inversely proportional to the speed  $V$  of users. If we denote by  $K$  the proportionality coefficient, the mean sojourn time in zone  $j$ , can be expressed as in equation (1). When zones form concentric circles, a user that physically exists in zone  $j$ ,  $j = 2, \dots, J - 1$ , has a probability  $P_{j,j-1}$  to move to zone  $j - 1$ , and a probability  $P_{j,j+1}$  to move to zone  $j + 1$  (with, of course,  $P_{j,j-1} + P_{j,j+1} = 1$ ). For zone 1, obviously,  $P_{12} = 1$ . And from zone  $J$ , a user can either move back to zone  $J - 1$  with a probability  $P_{J,J-1}$ , or exit the cell with a probability  $P_{J0}$ . All these probabilities clearly depend both on the radius  $R_j$  of zones and on the real mobility of users. However, without additional assumptions on physical mobility of users, we can use the following approximation:

$$p_j = \frac{S_j}{\pi R_J^2}, \quad \mathbb{E}(\Theta_j) = K \frac{\sqrt{S_j}}{V} \quad \text{and} \quad P_{j,j-1} = \frac{R_{j-1}}{2R_j}, \quad j > 1. \quad (1)$$

Note that probabilities  $p_j$  and  $P_{ij}$ , as well as mean sojourn times  $\mathbb{E}(\Theta_j)$ , are input parameters for our PS queue model. Any alternative expressions, resulting from a realistic physical mobility model of users, can be alternately used without changing the development presented below.

### III. MODEL

The cell occupancy can be represented by a multi-class Processor Sharing queue with  $J$  classes. Each class corresponds to a zone of the cell. Customers of class  $j$  arrive to the queue according to Poisson process of rate  $\lambda_j$ ,  $j = 1, \dots, J$ . It is important to note that, contrarily to [10],  $\lambda_j$  is different from the rate  $\Lambda_j$  of new connection requests that appear in zone  $j$ , as it must include the arrival of users moving from other zones while still being active. If we denote by  $\Lambda_{ij}$  the rate of active users moving from zone  $i$  to zone  $j$  and by  $\Lambda_{0J}$  the rate of active users making a handover from the outside, then:

$$\begin{cases} \lambda_1 = \Lambda_1 + \Lambda_{21} \\ \lambda_j = \Lambda_j + \Lambda_{j-1,j} + \Lambda_{j+1,j}, \quad j = 2, \dots, J - 1 \\ \lambda_J = \Lambda_J + \Lambda_{J-1,J} + \Lambda_{0J} \end{cases} \quad (2)$$

Class- $j$  rate can in turn be expressed as in [10]:

$$\mu_j = \frac{C_j}{\bar{x}_j}, \quad (3)$$

where  $\bar{x}_j$  is defined as the average number of bits transferred by an active user in zone  $j$ , for each visit of the zone.

As a result, we are left to estimate all the input parameters of the PS queue, namely  $\lambda_j$  and  $\mu_j$ , for  $j = 1, \dots, J$ , or more precisely all  $\Lambda_{ij}$ ,  $\bar{x}_j$  and  $\Lambda_{0J}$ . If we denote by  $h_j$  the handover probability from zone  $j$ , i.e., the probability that an active user in zone  $j$  leaves the zone without having finished its transfer, we can express  $\Lambda_{j,j+1}$  and  $\Lambda_{j,j-1}$  as:

$$\begin{cases} \Lambda_{j,j+1} = \lambda_j h_j P_{j,j+1}, \quad j = 1, \dots, J - 1 \\ \Lambda_{j,j-1} = \lambda_j h_j P_{j,j-1}, \quad j = 2, \dots, J \end{cases} \quad (4)$$

To estimate the incoming handover  $\Lambda_{0,J}$ , we assume that the considered cell is involved in a network of statistically equivalent cells. If this is true, the incoming and outgoing handover rates in the considered cell are equal, i.e.,  $\Lambda_{0,J} = \Lambda_{J,0} = \lambda_J h_J P_{J,0}$ . As a result, to take into account a cell involved in a network of equivalent cells, the last equation of system (2) must be replaced by:  $\lambda_J = \frac{1}{1-h_J P_{J,0}} (\Lambda_J + \Lambda_{J-1,J})$ .

As shown in [7], the stability condition of this system is independent of the speed  $V$  of users and is equivalent to the stability condition of a system where users have an infinite speed. It can be expressed as:  $\Lambda < \frac{C_\infty}{\mathbb{E}(\Sigma)}$ , where  $C_\infty$  is the equivalent capacity of the system where users have an infinite speed.

Standard results for the stationary multi-class Processor Sharing queues can be readily applied to calculate the average throughputs  $\bar{\gamma}_j$  obtained by users in zone  $j$  during their transfer:

$$\bar{\gamma}_j = C_j(1 - \rho), \quad (5)$$

where  $\rho = \sum_{j=1}^J \rho_j$  and  $\rho_j = \frac{\lambda_j}{\mu_j}$ . Therefore, applying the methodology developed in [10] to each zone individually,  $\bar{x}_j$  and  $h_j$  can be expressed as:

$$\bar{x}_j = \frac{\mathbb{E}(\Sigma)\mathbb{E}(\Theta_j)\bar{\gamma}_j}{\mathbb{E}(\Sigma) + \mathbb{E}(\Theta_j)\bar{\gamma}_j} \quad \text{and} \quad h_j = \frac{\mathbb{E}(\Sigma)}{\mathbb{E}(\Sigma) + \mathbb{E}(\Theta_j)\bar{\gamma}_j}. \quad (6)$$

We finally end up with a system of 5 dependent equations (2-6) that will be solved using a fixed-point iterative technique.

We now see how we can derive from the model the performance of an active user in the considered cell. Let us denote by  $\lambda$  the total arrival rate of connection demands in a cell.

From classical results of PS queues, we can calculate the average number of active users in each zone, denote by  $Q_j$  and the probability  $q_j$  that an active users is in zone  $j$ :

$$\bar{Q}_j = \frac{\rho_j}{1 - \rho} \quad \text{and} \quad q_j = \frac{\bar{Q}_j}{\sum_{i=1}^J \bar{Q}_i}. \quad (7)$$

From Little's law, we can express the average time  $\bar{R}$  spent by an active user in the cell. From  $\bar{\gamma}_j$  and  $q_j$ , we estimate the average throughput  $\bar{\gamma}$  obtained by an active user during its whole sojourn in the cell. We finally estimate the global handover probability  $H$ , i.e., the probability that an active user leaves the cell before completing its transfers, whatever the number of zones he has visited meanwhile.  $H$  is calculated as the ratio between the average number of active users leaving the cell by unit of time (from zone  $J$ ) and the total number of new active users that appear in the cell by unit of time:

$$\bar{R} = \frac{\sum_{j=1}^J \bar{Q}_j}{\lambda}, \quad \bar{\gamma} = \sum_{j=1}^J q_j \bar{\gamma}_j, \quad \text{and} \quad H = \frac{\bar{Q}_J P_{J,0}}{\lambda}. \quad (8)$$

We now see how to derive the end to end performance of users in case of network of statically equivalent cells. Because the size  $\Sigma$  of data to be transferred by users is supposed to be exponentially distributed, the remaining volume to be transferred by a user after a handover has the same distribution as the original one. As a result, the number  $n_h$  of handovers a user has to make and the number  $n_c$  of cells a user has to visit, before the completion of its transfer, are both geometrically distributed with parameter  $H$  (starting from 0 for the first one and from 1 for the second one), with means given by:

$$\bar{n}_h = \frac{H}{1-H} \quad \text{and} \quad \bar{n}_c = \frac{1}{1-H}. \quad (9)$$

We denote by  $t_h$  the duration of a handover procedure, i.e., the duration of the service interruption when a user change cell. We can estimate with relation (10), the average end to end transfer time  $\bar{T}$  of user, defined as the average time for a user to complete a full transfer, whatever the number of cells and the number of zones in each cell the user has visited during its transfer. Finally, we obtain the average end to end throughput  $\bar{\Gamma}$  a user obtains during its full transfer as:

$$\bar{T} = \bar{n}_c \bar{R} + \bar{n}_h t_h \quad \text{and} \quad \bar{\Gamma} = \frac{\mathbb{E}(\Sigma)}{\bar{T}}. \quad (10)$$

#### IV. PERFORMANCE RESULTS

We compare the results of the proposed model with those delivered by a home-made discrete-event simulator developed in Matlab. We reproduce the traffic assumptions and the mobility model described in Section II. We assume that the cell uses a number of 100 Resource Blocks for the downlink channel and offers to users four MCS (28, 23, 16, 6). This results in four transmission zones with corresponding capacity  $C_1 = 75$  Mbit/s,  $C_2 = 51$  Mbit/s,  $C_3 = 31$  Mbit/s and  $C_4 = 10$  Mbit/s [11]. We set the constant  $K = \frac{1}{\sqrt{\pi}}$  and we take  $\mathbb{E}(\Sigma) = 10$  MB for the mean data volume to be transferred by all users. We use the following radius corresponding to the concentric circles model of the cell:  $R_1 = 100$  m,  $R_2 = 150$  m,  $R_3 = 200$  m and  $R_4 = 250$  m,  $R_4$  corresponds approximately to the operating range of LTE antenna in urban environment. The mean sojourn time in each zone  $\mathbb{E}(\Theta_j)$  is given by equation (1). According to the estimations given in Section II, resulting probabilities are reported in Table I.

TABLE I: Parameters of the cells

parameters	zone 1	zone 2	zone 3	zone 4
new connection probabilities $p_j$	0.16	0.20	0.28	0.36
moving probabilities $P_{i,j}$	$P_{1,2} = 1$	$P_{2,1} = 0.33$ $P_{2,3} = 0.67$	$P_{3,2} = 0.37$ $P_{3,4} = 0.63$	$P_{4,3} = 0.40$ $P_{4,0} = 0.60$

##### A. Validation

Figures 1 and 2 respectively show comparison of users' throughput and handover probability in the considered cell,

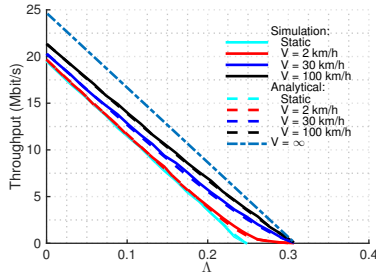


Fig. 1: Throughput obtained by active users in considered cell as a function of the total arrival rate of new connection demands.

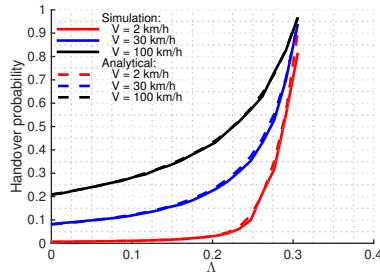


Fig. 2: Handover probability as a function of the total arrival rate of new connection demands.

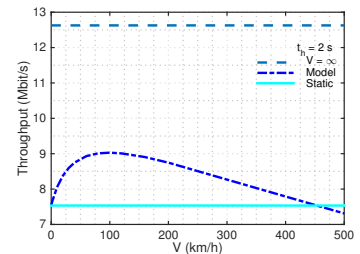


Fig. 3: Average Throughput obtain by active users as a function of users' speed taking into account the service interruption due to handover procedure when  $\Lambda = 0.15$ .

as a function of the total arrival rate of new connection demands  $\Lambda$ . We chose different values of speed  $V$ . We observe that the predicted model is very close to simulation results. The average relative error between performances predicted by the model and those obtained by simulation is about 8% in average. Mobile users' throughput (Figure 1) is bounded by the throughput obtained when all users are static (lower bound) and the throughput obtained with infinite speed (upper bound). As expected, users mobility improves the capacity, the throughput and stability of the system, the latter being  $\Lambda < 0.24$  for the system with static users and  $\Lambda < 0.30$  when users are mobile. The reason of this improvement is that, even if a user that physically exits a zone is more likely to move to a zone with poorer channel conditions, users in a favorable conditions have a better chance to complete their transfers before leaving the zone. Thereby, the average number of active users moving from zone with good channel conditions to zones with poor channel conditions by unit of time is lower than the average number of active users moving in the opposite direction. This improvement comes with an increase in the handover rate (see Figure 2). The handover  $H$  starts from an initial value that depends on speed and increases with  $\Lambda$ .  $H$  converges to 1 when  $\Lambda \rightarrow 0.30$ .

### B. Impact of inter-cell mobility

Figure 3 presents, for a total arrival rate of new connexion demands  $\Lambda = 0.15$ , the end to end throughput as a function of users' speed with consideration of the duration of service interruption due to handover procedure, which is set to 2s. This curve shows how handover rate counterbalances the flow-level performance improvement and proves that mobility gain is a non-monotonic function of users' speed.

## V. CONCLUSION

We have developed a PS queue model for performance evaluation of data cellular networks with a round-robin policy,

taking into account intra-and inter-cell mobility of users. We have shown that this model is consistent with known analytical bounds corresponding to static users or infinite speed, and provides a very good accuracy for more general speeds. Our model confirms that mobility may improve performance of users in a given cell, and enables to quantify the gain. It also provides end-to-end performance of users among a network of statistically equivalent cells, and shows that performance is not anymore a monotonic function of the speed.

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