Mechanical sensors

Nonlinearity in the Dynamic Response of Flexural Ultrasonic Transducers

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Abstract—Recent studies of the electro-mechanical behavior of flexural ultrasonic transducers have shown that their response can be considered as three distinct characteristic regions, the first building towards a steady state, followed by oscillation at the driving frequency in the steady state, before an exponential decay from the steady state at the transducer's dominant resonance frequency, once the driving force is removed. Despite the widespread industrial use of these transducers as ultrasonic proximity sensors, there is little published information on their vibration characteristics under different operating conditions. Flexual transducers are composed of a piezoelectric ceramic disc bonded to the inner surface of a metallic cap, the membrane of which bends in response to the high-frequency ceramic vibrations of the ceramic. Piezoelectric devices can be subject to nonlinear behavior, but there is no reported detail of the nonlinearity in flexural transducers. Experimental investigation through laser Doppler vibrometry shows strong nonlinearity in the vibration response, where resonance frequency reduces with increasing vibration amplitude.

Index Terms—Mechanical senors, flexural transducer, ultrasound, nonlinear behavior.

I. INTRODUCTION

The flexural ultrasonic transducer (FUT) is commonly applied as an air-coupled sensor, for example, in non-destructive evaluation or proximity sensing applications [1]. FUTs are typically composed of a metallic cap, usually aluminium, which encloses a piezoelectric ceramic disc bonded to the underside of the cap. A commercial FUT is depicted in Fig. 1, showing composition and nominal dimensions.

For a FUT, high frequency vibrations of the piezoelectric ceramic generate a bending of the compliant metallic membrane. The transducer membrane can be considered as a constrained plate, whose vibration modes are defined as the mode shapes of the transducer. FUTs are commonly operated in the $(0, 0)$ or $(1, 0)$ axisymmetric modes of vibration [1], enabling multiple modes of vibration to be selected for a single device. Despite the prevalence of FUTs, their electro-mechanical behaviour has only recently been reported [2]. A phenomenon which can occur in the vibration response of piezoelectric-based ultrasonic transducers is dynamic nonlinearity, where there is a change in device vibration frequency characteristics in response to a change in the operating amplitude.

Nonlinearity in piezoelectric devices originates from a range of sources, including structural configuration, strain-dependent material properties, epoxy bond layers, such as those found in FUTs, and within piezoelectric materials themselves [3]. Specifically regarding the piezoelectric material, localised thermal and stress concentrations are caused by the internal energy dissipation, which increases with excitation voltage [3]. These thermal and stress concentrations contribute to the dielectric loss and the quality factor of the piezoelectric material, thereby creating nonlinearity in the vibration response of the transducer. There has been no research reported on the nonlinear response of FUTs, but it is essential for understanding their operation, and how to optimize them for applications requiring controlled resonance at a range of excitation voltages and amplitudes. The stability of the resonance frequency with respect to amplitude of an FUT around resonance is demonstrated in this study.

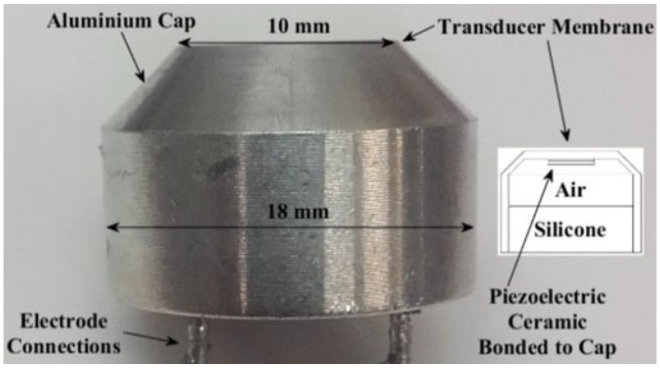

Fig. 1. Flexural ultrasonic transducer of the type used in this study.

The change of the resonance frequency of a transducer or resonator with respect to oscillation amplitude can be expressed in terms of spring softening or hardening [4]. These behaviors can be considered as nonlinear dynamic responses, since a change in frequency is generated through an increased oscillation amplitude. Many ultrasonic devices are designed to be operated at high power and, therefore, high vibration amplitude levels. At high drive voltages or vibration amplitudes, ultrasonic transducers can exhibit nonlinear effects. Schematics of these dynamic nonlinearity phenomena which have been observed in other types of commercial ultrasonic transducers are shown in Fig. 2, where the deviation of resonance frequency with amplitude from a nominal centre resonance frequency f_N takes place. We will show that the FUTs used in these experiments have exhibited a nonlinear softening behavior. We have not seen any evidence for Duffing-type nonlinear behavior in the FUTs used, sometimes referred to as a jump phenomenon, where the resonance frequency of the system can shift

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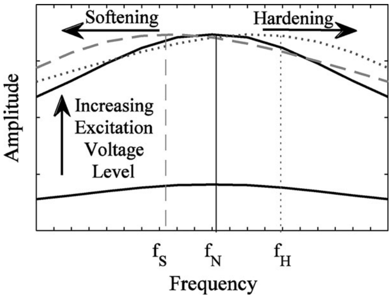

Fig. 2. Qualitative softening (dashed) and hardening (dotted) nonlinear behavior, compared to linearity (solid).

from one stable mode to another [3], [5], [6]. The nonlinearity we have observed is consistent with a softening or continuous decrease in the resonance frequency of the FUT with increasing vibration amplitude.

In this article, the decrease and increase in resonance frequency as a function of vibration amplitude adheres to accepted terminology in the form of softening and hardening nonlinearities, respectively [4], [6]. A softening nonlinearity occurs where resonance frequency decreases with increasing amplitude [4], [6], shown by the dashed curve, where the nominal resonance frequency of the linear amplitude-frequency response f_N, approaches a nonlinear softening resonance frequency, f_S. Conversely, the increase in resonance frequency, termed a hardening nonlinearity [4], [6], is exhibited by the dotted curve, where f_N approaches f_H, the nonlinear hardening resonance frequency.

This investigation first analyses the electrical characteristics of a set of five commercially-available FUTs, where dynamic properties including electro-mechanical coupling factor, resonance frequency, and quality factor, are measured. This information is used to configure the experimental setup for the measurement of dynamic nonlinearity in the vibration response of all five FUTs, which is undertaken using single-point laser Doppler vibrometry.

II. EXPERIMENTAL RESULTS

A. Electrical Characterization

The FUT shown in Fig. 1 is a commercial-type (Multicomp) device. The set of five nominally identical FUTs are arbitrarily labelled as FUT 1–5. Prior to nonlinear characterization, the electrical characteristics of each FUT, all with a nominal resonance frequency of around 40 kHz, have been measured to provide information regarding the resonance of the FUTs which can be correlated with the nonlinear characterisation measurements, and also the quality factor of the devices. Electrical impedance analysis (Agilent 4294A) is an efficient method of determining the resonance properties of ultrasonic transducers. Each FUT needs to be driven at a range of frequencies around resonance in order to capture nonlinear behavior with sufficient resolution. The results of the electrical impedance analysis for the FUTs are shown in Fig. 3.

The FUTs studied in this research are in general fabricated with frequencies of their (0, 0) mode of 39.5 ± 1.5 kHz. The results in Fig. 3 show that each transducer centre frequency falls within this range, where the FUT resonance frequency is that of lowest impedance in the spectra shown. Despite the fact FUTs are all designed to resonate in the (0, 0) mode at a nominal frequency of 40 kHz, minor variations in cap geometry and fabrication can cause disparities. Measurements of the susceptance and conductance of the FUTs can be used to generate admittance loops, which, together with electrical impedance and phase measurements, can be used to determine electrical properties of the transducers, including the coupling coefficient k^2, and the mechanical quality factor Q_M [7], [8]. The admittance loops for the FUTs are exhibited in Fig. 4 and provide an immediate indication of the relative efficiency and quality of the FUTs.

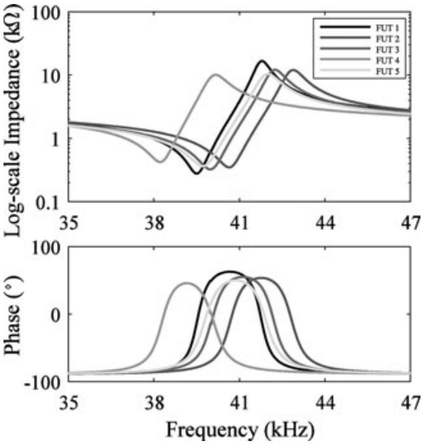

Fig. 3. Resonance characterisation of the FUTs.

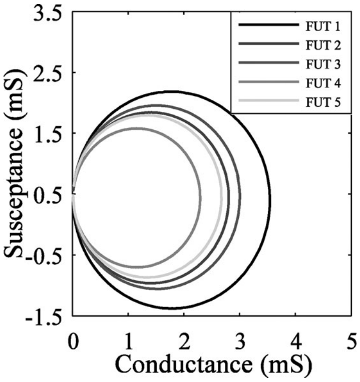

Fig. 4. Admittance loops for the FUTs.

A larger admittance loop signifies generally a more resonant device [8], and so the technique enables a practical method of device selection. The magnitudes of the k^2 and Q_M properties are frequency dependent, and are calculated from the measured data for the FUTs using standard relationships [8], shown below in (1) and (2), where f_a and f_r are the anti-resonance and resonance frequencies, respectively; f_{rG} is the frequency of maximum conductance, and f_{1G} and f_{2G} are the half-maximum frequencies, representing the bandwidth.

$$k^2 = 1 - \left(\frac{f_r}{f_a}\right)^2 \tag{1}$$

$$Q_M = \frac{f_{rG}}{f_{2G} - f_{1G}}. \tag{2}$$

The electrical impedance information has been used to determine the frequency required to drive the FUTs at resonance. The calculated k^2 and Q_M parameter magnitudes from the electrical characterisation are provided for reference in Table 1.

Table 1. Dynamic Properties of the FUTs From Electrical Characterization and Laser Doppler Vibrometry, Where All Frequencies are in kHz.

FUT	k^2	Q_M	f_r	LDV f_r, nom. $4\,V_{p-p}$	LDV $f_N - f_S$ (Hz), nom. $4 - 20\,V_{p-p}$
1	0.33	71.01	39.51	40.00	300
2	0.32	56.13	40.64	41.00	200
3	0.33	56.71	39.97	40.40	200
4	0.31	54.17	38.23	37.90	200
5	0.32	49.75	39.72	40.10	200
Mean (St. Dev.)	0.322 (0.007)	57.55 (7.16)	39.59 (0.88)	39.66 (0.90)	220 (40)

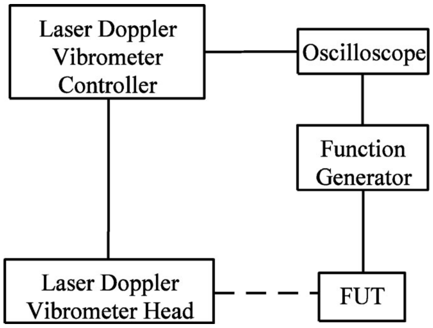

Fig. 5. Laser Doppler vibrometry setup.

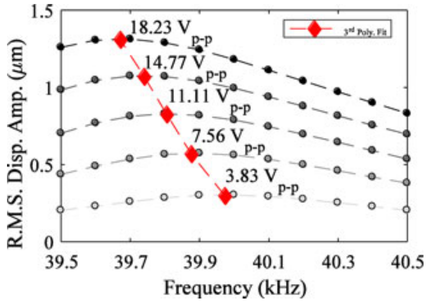

Fig. 6. The softening nonlinearity of FUT 1 at steady-state.

B. Laser Doppler Vibrometry

The center resonance frequency measured for each FUT from their respective electrical properties is used to define the measurement bandwidth for the nonlinear characterisation. To identify the nonlinearity in the amplitude-frequency response of the FUTs, a laser Doppler vibrometer (LDV, Polytec OFV-5000) was used to measure the velocity amplitude of the FUT membrane. The laser beam was directed perpendicular to the membrane surface and focused at its centre, for a range of drive frequencies around resonance, over a range of 1 kHz, and in 100 Hz steps, in order to capture the vibration behavior with sufficient resolution. This was performed for multiple excitation voltages, which produces different levels of FUT output amplitude. In each case, a burst sinusoidal wave signal was applied per voltage level for 150 cycles, used to minimize the influence of increased temperatures within the piezoelectric ceramic [9]. The experimental setup is shown in Fig. 5.

The vibration response of the FUTs were measured at excitation voltage levels from $3.83\,V_{p-p}$ to $18.23\,V_{p-p}$, in nominal $4\,V_{p-p}$ increments, where the time-dependent velocity signal output from the FUT was recorded, stored as an output voltage from the LDV system. To convert this data into amplitude-frequency spectra, the root mean square (R.M.S.) method was applied, to provide an indication of resonance frequency. The time-dependent amplitude response of a FUT has been shown to be a superposition of steady-state and natural decay, or ring-down [2], where steady-state occurs due to a driving force applied to the FUT, and ring-down occurs upon the removal of the driving force. Based on this, the dynamic response of a FUT at different excitation voltage levels can be identified in the steady-state and ring-down regions separately. The displacement amplitude-frequency spectra for different voltage levels and drive frequencies are displayed in Fig. 6 for one of the FUTs, i.e., FUT 1, for its steady-state region response. A third-order polynomial fit has also been applied, which was calculated to both emphasize the trend in resonance frequency change, as well as provide an indication of the true maximum displacement amplitude for each response spectrum.

An excitation voltage of $3.83\,V_{p-p}$ has produced an amplitude-frequency spectrum with a centre frequency of approximately 40.0 kHz. This result is a discrepancy of 1.23% compared to the centre frequency measured using electrical impedance analysis, shown in Fig. 3, a close correlation. An explanation for the discrepancy is that the vibration response of the FUT membrane is measured optically through LDV, which is not the case with electrical impedance analysis, and since the vibration of the FUT membrane is not instantaneously sinusoidal [2], there will be a discrepancy between the resonance frequency detected by different measurement systems.

It is evident from the amplitude-frequency spectra shown in Fig. 6, that as the excitation voltage level is increased, the vibration response of the FUT exhibits a softening nonlinear response. The dashed trendlines have been included for clarity in the illustration of the reduction in measured resonance frequency. For an excitation voltage of $3.83\,V_{p-p}$, the resonance frequency of FUT 1 was measured to be approximately 40.0 kHz. Upon the application of a $18.23\,V_{p-p}$ excitation voltage, the measured resonance frequency had dropped to around 39.7 kHz, which is a reduction of around 300 Hz and 0.75%, and has significant implications in relation to how FUTs are operated in practice. Many applications utilising ultrasonic transducers require operation at a particular frequency, but also at a range of excitation voltages. This result shows that the resonance frequency, or the FUT driving for a given application, cannot be considered to be constant for different amplitudes of vibration.

However, the R.M.S. method cannot be used for the ring-down region, since displacement amplitude is not constant with time for natural decay. For ring-down, it is known that a FUT decays at resonance [2]. To identify any nonlinearity, the exponential signal was removed through curve-fitting, and a zero-crossing method was applied to obtain the frequency change. The results for this analysis for FUT 1 close to resonance, after an excitation signal of 40 kHz has been

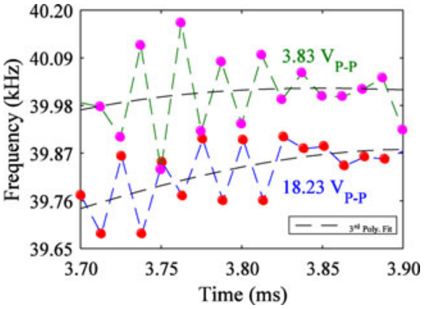

Fig. 7. Responses of the ring-down region of FUT 1 for two voltages at 40 kHz, immediately after stopping the drive signal.

switched off, are shown in Fig. 7, for excitation voltages of 3.83 V_{p-p} and 18.23 V_{p-p}. The larger drive voltage results in a larger displacement of the FUT membrane from the start of the decay region once the drive voltage is switched off. Third-order polynomial fits were applied to provide a line of best fit to the data and a clear indication of the frequency change as a function of time. If the ring-down response was linear, the frequency would be relatively unchanged as the FUT resonance decays.

The results show evidence of nonlinearity for both excitation cases. However, the effect is smaller for the lower excitation voltage. In general, the nonlinear effect exists in the ring-down region, but is smaller than for at steady-state. A reduction of around 300 Hz was measured at steady-state as the excitation voltage was increased from 3.83 V_{p-p} to 18.23 V_{p-p}, but this was only measured to be approximately 100 Hz for the ring-down region when the excitation voltage had been 18.23 V_{p-p}. Since there exists nonlinearity at ring-down, in the absence of an excitation voltage, the voltage level is not likely the sole cause of the nonlinearity. It is postulated that nonlinearity in FUTs also originates from physical displacement and deformation of the FUT cap membrane, based on its physical material properties, at increasing amplitudes. This is supported by observations from the literature [3].

Data for all five FUTs are shown in Table 1, summarizing the measured k^2, Q_M, resonance frequency from electrical impedance analysis, and reduction in resonance frequency from nominal excitation voltages of 4 V_{p-p} to 20 V_{p-p} for the entire response signal from each FUT. The results demonstrate that nonlinear behavior is consistent across multiple FUTs, even when the dynamic properties are slightly different, thus making consideration of this phenomenon important. FUT 4 shows the smallest admittance loop in Fig. 4 but not the lowest Q_M magnitude. However, both the k^2 and Q_M parameters together dictate the admittance loop size [8], in addition to the transducer f_r magnitude. The results show how the resonance frequency of a FUT is sensitive to the vibration amplitude, where nonlinearity in the amplitude-frequency spectrum has been identified.

III. CONCLUSION

This study has demonstrated the dynamic nonlinearity in the vibration response of FUTs. Using dynamic properties obtained through electrical characterisation, LDV was used to measure the dynamic nonlinearity of the FUTs. Dynamic nonlinearity is exhibited as a softening nonlinearity for each, differing in magnitude between steady-state and ring-down response regions. There are significant implications of nonlinearity on the efficient operation of FUTs, many of which require different vibration amplitudes. However, dynamic nonlinearity related to the transducer assembly configuration or the material properties can be investigated in more detail, for improved performance in practical application.

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Experimental data can be accessed at www2.warwick.ac.uk/fac/sci/physics/research/ultra/research/N1.zip. This work was supported by the EPSRC under Grant EP/N025393/1.

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