A Hybrid Sensorless Observer for the Robust Global Asymptotic Flux Reconstruction of Permanent Magnet Synchronous Machines

Alessandro Bosso, *Member, IEEE*, Andrea Tilli, and Christian Conficoni

***Abstract*—We propose a hybrid sensorless observer for permanent magnet synchronous machines with global asymptotic stability guarantees. Exploiting the constraint of the rotor flux on a circle of unknown radius, we design an integrator system with periodic jumps triggered by a clock to generate a linear regression containing the flux estimation error. Then, a normalized projected gradient descent identifier provides the observer estimates. For the closed-loop system, it is shown that there exists a robustly globally asymptotically stable compact attractor, which, additionally, ensures zero estimation error if appropriate Persistency of Excitation (PE) conditions are satisfied. In this respect, sufficient conditions ensuring PE are provided for the angular speed and the clock period.**

***Index Terms*—Permanent magnet machines, sensorless observers, stability of hybrid systems.**

I. Introduction

PERMANENT Magnet Synchronous Machines (PMSMs) are employed in numerous fields, ranging from industrial applications to hybrid-electric vehicles and unmanned aerial vehicles. Usually, field-oriented control strategies are needed to achieve accurate speed and torque regulation. While ensuring high performance and efficiency, these techniques require accurate flux amplitude and angular configuration information, usually recovered through sensors (e.g., encoders). Nowadays, the trend is to remove such components and adopt the so-called sensorless strategies, where no direct knowledge of the machines' angular position and speed is available. These approaches are beneficial in terms of availability and reduced costs, and they unlock accurate regulation in applications where mechanical sensors cannot be installed.

A vast literature has been dedicated to sensorless control, with monographs [1], [2], and reviews [3] collecting the main research results. Without the ambition to be exhaustive, we report some recently presented solutions, adhering to the control theory-oriented literature. In [4]–[6], an observation approach based on a continuous-time gradient descent algorithm has been proposed and applied to sensorless position reconstruction of PMSMs. The first version [4], which assumed known constant rotor flux amplitude, has been enhanced in [5] to achieve global convergence, and then in [6] to reconstruct the rotor flux amplitude. Besides formal convergence properties, a peculiar feature of this method is that no knowledge about the mechanical dynamics is required since it exploits an algebraic constraint on the electromagnetic variables. On the other hand, the approach proposed in [7] is based on the combination of a high-gain observer and an attitude estimator of the PMSM angular position represented as an element of the unit circle, leading to a structure ready to be combined with current controllers for torque regulation [8]. Finally, a meaningful line of work related to gradient descent algorithms is presented in [9]–[12]. The essential idea behind these methods is to adopt a suitable representation of the electromagnetic dynamics, combined with filtering, to translate the observer design problem into one of parameter identification from a linear regression. The original work [9] ensured global convergence under mild Persistency of Excitation (PE) conditions, without a priori information on the mechanical model. In [10], the approach has been extended via Dynamic Regressor Extension and Mixing [13] to improve observer performance and weaken the excitation conditions required for convergence.

This letter proposes a novel observer for PMSMs closely related to the parameter identification framework. The design is based on a linear regression form, involving continuous-time open-loop integration to highlight the flux estimation error, that is derived from the constraint of the rotor flux on a circle with constant radius. The same geometric constraint has been exploited, e.g., in [5], [6]. However, different from all previously mentioned solutions, we design an observer exploiting the hybrid systems formalism [14].

The proposed architecture is based on the following key elements. Firstly, the mentioned continuous-time open-loop integration, which is inherently fragile to a fairly general class of perturbations, is combined with periodic jumps triggered

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The authors are with the Department of Electrical, Electronic, and Information Engineering, University of Bologna, 40136 Bologna, Italy (e-mail: alessandro.bosso@unibo.it; andrea.tilli@unibo.it; christian.conficoni3@unibo.it).

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by a clock. This mechanism has the advantage of ensuring desirable robust stability properties while preserving the previous linear regression structure. Then, a hybrid identifier is introduced to reconstruct the flux estimation error. In particular, given the time delays in the linear regression, adaptation is naturally formulated in discrete time, exploiting the clock dynamics. The proposed identifier, in particular, involves a discrete-time normalized gradient descent algorithm and a continuous-time dead-zone-based projection mechanism. Defining an autonomous exosystem modeling the PMSM dynamics and its input signals, we exploit the properties of ω-limit sets of well-posed hybrid dynamical systems to prove that the closed-loop system admits a compact attractor that is robustly globally asymptotically stable, regardless of the estimates behavior. This relevant result is ensured by the integrator resets and the parameter projection mechanism. Furthermore, under appropriate discrete-time PE conditions, the asymptotically stable attractor ensures zero estimation error in all its points. In general, PE for this observer depends on both the PMSM trajectories and the clock period, thus it may be hard to verify. For this reason, we dedicate part of this letter to providing sufficient conditions for PE involving the continuous-time behavior of the PMSM angular speed and an upper bound on the clock period.

This letter is organized as follows. In Section II, we recall the PMSM model and formalize the observation problem. Section III presents the observer architecture and its stability properties, while Section IV provides a sufficient condition for PE. Then, in Section V, we validate the features of the proposed solution through a numerical simulation. Finally, Section VI concludes this letter.

Notation: $\mathbb{N}$ is the set of natural numbers including zero and $\mathbb{R}$ is the set of real numbers. The transpose of real-valued matrices is denoted by $(\cdot)^\top$. Given column vectors v and w, the notation (v, w) denotes the concatenated vector $[v^\top\ w^\top]^\top$. For any positive integer n, I_n is the identity matrix of dimension n, while we define $J := \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$. For any square matrix M we denote by $\mathrm{msv}(M)$ its minimum singular value. For convenience in the representation of the angular position (see [7]), we employ the unit circle $\mathbb{S}^1 := \{x \in \mathbb{R}^2 : |x| = 1\}$. We define $\mathrm{atan2}(v)$, with $v = (v_1, v_2) \in \mathbb{R}^2$, as the argument of the complex number $v_1 + \mathrm{i}v_2$. Finally, we refer to [14] for the notation, main definitions, and stability results for hybrid dynamical systems. In particular, we recall that x^+ denotes the value of variable x after an instantaneous change (jump).

II. Problem Statement

A. PMSM Model

Under typical simplifying assumptions, the PMSM electromagnetic model can be expressed in a stationary two-phase reference frame as

$$\dot{\phi} = -\frac{R}{L}(\phi - \varphi\zeta) + u, \qquad \dot{\zeta} = \omega J\zeta, \tag{1}$$

where $\phi \in \mathbb{R}^2$ is the stator flux, $\zeta \in \mathbb{S}^1$ is the rotor position, φ is the rotor flux linkage, and R and L are the stator resistance and inductance, respectively. Additionally, $u \in \mathbb{R}^2$ is the applied stator voltage and $\omega \in \mathbb{R}$ is the rotor electric speed. Finally, the stator current $i \in \mathbb{R}^2$ is defined as

$$i := \frac{\phi - \varphi\zeta}{L}. \tag{2}$$

Focusing on the properties of system (1) we note that, for any initial condition $(\phi(t_0), \zeta(t_0)) \in \mathbb{R}^2 \times \mathbb{S}^1$, existence and uniqueness of solutions is ensured for all $[t_0, \infty)$ by assuming that the input signals $u(\cdot)$ and $\omega(\cdot)$ are piecewise continuous (see, e.g., [15, Ch. 3] for basic existence and uniqueness results). In this letter, we additionally require that vector (u, ω) is the output of a differential inclusion of the form

$$\begin{aligned} \dot{w} &\in F_w(w, \phi, \zeta), \qquad w \in \mathbb{R}^n \\ (u, \omega) &= h_w(w), \end{aligned} \tag{3}$$

where h_w is a continuous function, while F_w is a set-valued map that we assume outer semicontinuous, locally bounded relative to $\mathbb{R}^{n+2} \times \mathbb{S}^1$, and convex for all (w, ϕ, ζ).

For the characterization of the observer properties in terms of global asymptotic stability of a compact set, we impose the following assumption.

Assumption 1: For the interconnection among systems (1) and (3), having states $(w, \phi, \zeta) \in \mathbb{R}^{n+2} \times \mathbb{S}^1$, there exists a compact set $\mathcal{W} \subset \mathbb{R}^{n+2} \times \mathbb{S}^1$ that is strongly forward invariant.

Given Assumption 1, we will restrict the solutions of (1), (3) to $\mathcal{W}$. This choice is reasonable for the representation of most PMSM working conditions, especially due to technological constraints, boundedness of external inputs, and control saturations. Define $x := (\phi, \zeta)$. Combining (1) and (3), we obtain the following constrained differential inclusion:

$$\left.\begin{aligned} \dot{w} &\in F_w(w, x) \\ \dot{x} &= f_x(h_w(w), x) \end{aligned}\right. \quad (w, x) \in \mathcal{W}, \tag{4}$$

where f_x compactly denotes the vector field in (1). Observe that, due to the properties of F_w, f_x, and h_w, system (4) satisfies the so-called hybrid basic assumptions [14, Assumption 6.5].

B. Robust Observer Design Problem

We assume that the following information is available.

Assumption 2: u and i are available from measurements and parameters R and L are known. On the other hand, ζ, ϕ, ω, and parameter φ are unknown. Finally, a positive scalar φ_M is known such that $0 < \varphi \leq \varphi_M$.

The objective of this letter is to design a robust observer of states $x := (\phi, \zeta)$ that ensures global boundedness of the closed-loop trajectories and, under appropriate PE conditions, global asymptotic convergence of the estimates. To this aim, we seek for a hybrid dynamical system, having state $z \in \mathcal{Z}$, of the form

$$\begin{aligned} &\begin{cases} \dot{z} &= f_z(z, u, i) \quad (z, u, i) \in C_z \times \mathbb{R}^4 \\ z^+ &= g_z(z, u, i) \quad (z, u, i) \in D_z \times \mathbb{R}^4 \end{cases} \\ &\hat{x} := h_z(z, u, i) := (\hat{\phi},\ \hat{\zeta}), \end{aligned} \tag{5}$$

where C_z and D_z are the flow and jump sets satisfying $\mathcal{Z} = C_z \cup D_z$, while the output $\hat{x}$ is the estimate of x in (1).

The interconnection among systems (4) and (5) leads to the following closed-loop dynamics

$$\begin{cases} \begin{bmatrix} \dot{w} \\ \dot{x} \\ \dot{z} \end{bmatrix} \in \begin{bmatrix} F_w(w,x) \\ f_x(h_w(w),x) \\ f_z(z,u,i) \end{bmatrix}, & (w,x,z) \in \mathcal{W} \times C_z \\ \begin{bmatrix} w^+ \\ x^+ \\ z^+ \end{bmatrix} = \begin{bmatrix} w \\ x \\ g_z(z,u,i) \end{bmatrix}, & (w,x,z) \in \mathcal{W} \times D_z. \end{cases} \tag{6}$$

This way, the observation objective $\hat{x} \to x$ can be expressed more conveniently in terms of ensuring suitable stability and attractivity properties for a compact subset of $\mathcal{W} \times \mathcal{Z}$ which, under appropriate conditions for the solutions of (6), is a subset of the following *observation set*:

$$\mathcal{O} := \{(w,x,z) \in \mathcal{W} \times \mathcal{Z} : \hat{x} - x = h_z(z,u,i) - x = 0\}. \tag{7}$$

In this context, our aim is to find an algorithm (5) whose data satisfy the hybrid basic conditions. This way, global asymptotic stability implies robust stability as shown in [14, Sec. 7.3]. We summarize this discussion in a formal statement.

Problem 1: Under Assumptions 1 and 2, design a hybrid observer in the form of (5) such that for system (6) there exists a compact set $\mathcal{A}$ that is robustly globally $\mathcal{KL}$ asymptotically stable in the sense of [14, Definition 7.18]. Furthermore, define sufficient PE conditions for the trajectories of (6) such that $\mathcal{A} \subset \mathcal{O}$, with $\mathcal{O}$ defined as in (7).

III. Hybrid Observer Design

A. Continuous-Time Linear Regression

Our strategy involves manipulating (1), (2) to exploit, in view of $\zeta \in \mathbb{S}^1$, the constant magnitude of the rotor flux $\varphi\zeta$. Specifically, we exploit the property

$$\frac{d}{dt}\left(|\varphi\zeta(t)|^2\right) = 0, \quad t \geq 0. \tag{8}$$

Given any positive scalar δ, (8) also implies that, for all $t \geq \delta$:

$$|\varphi\zeta(t)|^2 - |\varphi\zeta(t-\delta)|^2 = 0. \tag{9}$$

To leverage (9), we can introduce a dynamical system of the form

$$\dot{\psi} = u - Ri, \tag{10}$$

then define

$$\lambda := \phi - \psi, \quad \chi := \psi - Li = \varphi\zeta - \lambda. \tag{11}$$

From (1), (10), it holds that $\dot{\lambda} = \dot{\phi} - \dot{\psi} = 0$, so that (9) can be rewritten in terms of χ and λ as

$$|\chi(t) + \lambda|^2 - |\chi(t-\delta) + \lambda|^2 = 0, \tag{12}$$

leading to a continuous-time linear regression with unknown parameter λ:

$$|\chi(t)|^2 - |\chi(t-\delta)|^2 = 2(\chi(t-\delta) - \chi(t))^\top \lambda. \tag{13}$$

It follows that the problem of observation of $x := (\phi, \zeta)$ could be solved by implementing (10) and then reconstructing λ from (13) given the available quantities u, i, and ψ. The estimates of the PMSM states are thus computed from:

$$\hat{x} := (\hat{\phi},\ \hat{\zeta}) := \left(\psi + \hat{\lambda}, \quad \begin{bmatrix} \cos(\operatorname{atan2}(\chi + \hat{\lambda})) \\ \sin(\operatorname{atan2}(\chi + \hat{\lambda})) \end{bmatrix}\right). \tag{14}$$

A continuous-time strategy of this form is related to the parameter identification approach of [16]. In particular, λ can be viewed as the initial condition mismatch $\phi(0) - \psi(0)$. However, implementing (10) involves an inherently fragile open-loop integration operation, which cannot ensure the robust asymptotic stability objective of Problem 1. Indeed, even in case of arbitrarily small perturbations for (10), the solution $\psi(t)$ is in general not contained in a compact set.

B. Proposed Architecture

To remove the mentioned robustness issues and obtain a simple implementation of the delayed signals as per (13), we augment (10) with a reset mechanism based on a clock:

$$\begin{cases} \dot{\psi} = u - Ri \\ \dot{\rho} = 1 \end{cases} \quad (\psi,\rho,u,i) \in \mathbb{R}^2 \times [0,\tau] \times \mathbb{R}^4$$
$$\begin{cases} \psi^+ = Li \\ \rho^+ = 0 \end{cases} \quad (\psi,\rho,u,i) \in \mathbb{R}^2 \times \{\tau\} \times \mathbb{R}^4, \tag{15}$$

where ρ is the clock state and τ is a positive gain.

Remark 1: From Assumption 1, we have that the interconnection among systems (4) and (15) yields an autonomous hybrid system whose solutions are bounded. Indeed, ρ is bounded by construction, while ψ is bounded by

$$|\psi| \leq \max\{|\psi(0,0)|, \max_{(w,x)\in\mathcal{W}} L|i|\} + \max_{(w,x)\in\mathcal{W}} \tau|u - Ri|. \tag{16}$$

Define again $\lambda := \phi - \psi$ and $\chi := \psi - Li$. Then, (15) yields $\dot{\lambda} = 0$ along flows, whereas along jumps we have

$$\lambda^+ = \phi - Li = \lambda + \chi, \qquad \chi^+ = 0. \tag{17}$$

Therefore, λ is no longer the constant $\phi(0) - \psi(0)$, but it is reset after each jump (this kind of dynamics will be crucial, along with other elements, to guarantee asymptotic stability and the related robustness). Employing (15), regression (13) can reformulated in the hybrid setting by letting $\delta = \tau$ and evaluating it in discrete time during jumps. Namely, consider any solution ξ of (4), (15), with hybrid time domain $\operatorname{dom}\xi := \bigcup_{j\in\mathbb{N}} [t_j, t_{j+1}] \times \{j\}$. Then, by (17) we have, for all $j \geq 1$:

$$|\chi(t_{j+1},j)|^2 = -2\chi(t_{j+1},j)^\top \lambda(t_{j+1},j). \tag{18}$$

The above regression suggests that an identifier for λ is naturally defined in discrete time. In particular, we propose an observer comprising (15) and a projected normalized gradient descent estimator for λ. The overall architecture, having state $z := (\psi, \rho, \lambda) \in \mathcal{Z} := \mathbb{R}^2 \times [0,\tau] \times \mathbb{R}^2$, is:

$$\begin{cases} \dot{\psi} = u - Ri \\ \dot{\rho} = 1 \\ \dot{\hat{\lambda}} = -\sigma\, \mathrm{dz}(\hat{\lambda}) \end{cases} \quad (z,u,i) \in C_z \times \mathbb{R}^4,$$
$$\begin{cases} \psi^+ = Li \\ \rho^+ = 0 \\ \hat{\lambda}^+ = \hat{\lambda} + \chi - \frac{\gamma\chi(|\chi|^2 + 2\chi^\top\hat{\lambda})}{1 + 2\gamma|\chi|^2} \end{cases} \quad (z,u,i) \in D_z \times \mathbb{R}^4,$$

$$\hat{x} := (\hat{\phi}, \hat{\zeta}) := \left(\psi + \hat{\lambda}, \quad \begin{bmatrix} \cos(\mathrm{atan2}(\chi + \hat{\lambda})) \\ \sin(\mathrm{atan2}(\chi + \hat{\lambda})) \end{bmatrix}\right), \chi := \psi - Li, \tag{19}$$

where $C_z := \mathcal{Z}$ and $D_z := \{z \in \mathcal{Z} : \rho = \tau\}$ are the flow set and jump set, respectively, σ, γ, and τ are positive gains, while $\mathrm{dz}(\cdot) : \mathbb{R}^2 \to \mathbb{R}^2$ is a dead-zone function defined as

$$\mathrm{dz}(\mu) := \mu - \frac{r}{\max\{r, |\mu|\}}\mu, \tag{20}$$

with radius $r > \varphi_M$. By construction, it holds that

$$(\mu - \lambda)^\top \mathrm{dz}(\mu) \geq 0, \quad \text{if } |\lambda| \leq \varphi_M, \tag{21}$$

while for all $(\mu, \lambda) \in \mathbb{R}^4$ satisfying $|\mu| \geq \max\{r, |\lambda|\}$:

$$(\mu - \lambda)^\top \mathrm{dz}(\mu) \geq (|\mu| - \max\{r, |\lambda|\})^2. \tag{22}$$

C. Closed-Loop Stability Analysis

Define $\tilde{\lambda} := \hat{\lambda} - \lambda$. Along flows, it holds that $\dot{\tilde{\lambda}} = -\sigma\, \mathrm{dz}(\hat{\lambda})$. Moreover, by (18), for all jumps after the first one we have

$$\tilde{\lambda}^+ = \left(I_2 - \frac{2\gamma\chi\chi^\top}{1 + 2\gamma|\chi|^2}\right)\tilde{\lambda}, \tag{23}$$

which displays the typical error dynamics of discrete-time gradient descent algorithms. The next result confirms that (19) ensures the stability properties stated in Problem 1.

Theorem 1: For the interconnection among system (4) and observer (19), there exists a compact set $\mathcal{A}$ that is robustly globally $\mathcal{KL}$ asymptotically stable.

Proof: By Assumption 1, (w, x) is contained in the compact invariant set $\mathcal{W}$. From the arguments of Remark 1, it holds that ψ and ρ are bounded. Therefore, $|\lambda| \leq \lambda_M$, for some $\lambda_M > 0$ depending on the initial conditions $(w(0, 0), x(0, 0), \psi(0, 0))$. Consider the Lyapunov function:

$$V(w, x, z) := \frac{1}{2}|\tilde{\lambda}|^2 := \frac{1}{2}|\hat{\lambda} - \phi + \psi|^2. \tag{24}$$

Along the flows, by (22), it holds that

$$|\hat{\lambda}| > \max\{r, \lambda_M\} \implies \dot{V} = -\sigma(\hat{\lambda} - \lambda)^\top \mathrm{dz}(\hat{\lambda}) < 0. \tag{25}$$

When the first jump occurs, since χ and λ are bounded, (25) and $\hat{\lambda}^+$ in (19) imply that $V(w(t_1, 1), x(t_1, 1), z(t_1, 1))$ is bounded, with bounds depending on the initial conditions. Then, after the first jump, from (23) it holds that

$$\begin{aligned}
\Delta V &= \frac{1}{2}\left|\tilde{\lambda} - \frac{2\gamma\chi\chi^\top\tilde{\lambda}}{1 + 2\gamma|\chi|^2}\right|^2 - \frac{1}{2}|\tilde{\lambda}|^2 \\
&= \frac{2\gamma^2\tilde{\lambda}^\top\chi\chi^\top\chi\chi^\top\tilde{\lambda}}{(1 + 2\gamma|\chi|^2)^2} - \frac{2\gamma\tilde{\lambda}^\top\chi\chi^\top\tilde{\lambda}}{1 + 2\gamma|\chi|^2} \\
&= \frac{2\gamma^2 e^2|\chi|^2}{(1 + 2\gamma|\chi|^2)^2} - \frac{2\gamma e^2}{1 + 2\gamma|\chi|^2} \\
&= -\frac{2\gamma e^2}{1 + 2\gamma|\chi|^2}\left(1 - \frac{\gamma|\chi|^2}{1 + 2\gamma|\chi|^2}\right) \\
&\leq -\frac{\gamma e^2}{1 + 2\gamma|\chi|^2} \leq 0,
\end{aligned} \tag{26}$$

where $\Delta V := V(w^+, x^+, z^+) - V(w, x, z)$ and $e := \chi^\top\tilde{\lambda}$. From (25), (26), we conclude that $\hat{\lambda}$ is bounded. By [14, Proposition 6.10], it holds that the solutions are forward complete, thus they are precompact. Identities $\lambda = \varphi\zeta - \chi$, $\chi^+ = 0$, and $|\varphi\zeta| \leq \varphi_M$ imply that, after the first jump, $|\lambda| \leq \varphi_M < r$ and

$$|\psi| \leq \psi_M := \max_{(w,x)\in\mathcal{W}} L|i| + \max_{(w,x)\in\mathcal{W}} \tau|u - Ri|. \tag{27}$$

From (21), (22), and (26), after the first jump we have that $\Delta V \leq 0$ and

$$\begin{aligned}
&\dot{V} \leq -\sigma\tilde{\lambda}^\top \mathrm{dz}(\hat{\lambda}) = 0, && \text{if } |\hat{\lambda}| \leq r \\
&\dot{V} \leq -\sigma(|\hat{\lambda}| - r)^2 < 0, && \text{if } |\hat{\lambda}| > r.
\end{aligned} \tag{28}$$

In particular, for $|\tilde{\lambda}| = \sqrt{2V} \geq 2r + \tilde{r}$ with $\tilde{r} > 0$ arbitrary, we can show from (28), $|\lambda| < r$, $|\tilde{\lambda} + \lambda| - r \geq |\tilde{\lambda}| - |\lambda| - r$, and straightforward computations that:

$$\dot{V} \leq -\sigma(|\tilde{\lambda}| - 2r)^2 \leq -\sigma\frac{\tilde{r}}{4r + \tilde{r}}(2V - 4r^2) < 0. \tag{29}$$

Therefore, due to the persistence of flow intervals ensured by the clock dynamics, we conclude from (29) and $\Delta V \leq 0$ that the solutions converge exponentially to the compact set

$$\mathcal{K}_M := \Big\{(w, x, z) \in \mathcal{W} \times \mathcal{Z} : \\ |\psi| \leq \psi_M, |\lambda| \leq \varphi_M, |\hat{\lambda} - \lambda| \leq 2r + \tilde{r}\Big\}. \tag{30}$$

For any set of initial conditions $\mathcal{K}_\varepsilon := \mathcal{K}_M + \varepsilon\mathbb{B}$, where $\varepsilon\mathbb{B}$ is a closed ball of radius $\varepsilon > 0$ to be added to any point of $\mathcal{K}_M$, it holds that $\mathcal{A} := \Omega(\mathcal{K}_\varepsilon) \subset \mathcal{K}_M \subset \mathrm{Int}(\Omega_\varepsilon)$, where $\Omega(\mathcal{K}_\varepsilon)$ is the ω-limit set of $\mathcal{K}_\varepsilon$. Therefore, $\mathcal{A}$ is asymptotically stable by [14, Corollary 7.7] and globally attractive from the previous arguments, thus it is UGAS. Since the closed-loop system (4), (19) satisfies the hybrid basic assumptions, we conclude robust $\mathcal{KL}$ asymptotic stability from [14, Th. 7.21]. ■

By [14, Corollary 8.4] and the arguments in the proof of Theorem 1, the solutions approach the largest weakly invariant subset of $\mathcal{K}_M$ satisfying $e := \chi^\top\tilde{\lambda} = 0$. This condition, however, is not sufficient to ensure $\tilde{\lambda} = 0$ and, thus, $\hat{x} = x$. In order to guarantee estimate convergence, we provide a sufficient condition based on persistency of excitation of χ.

Proposition 1: Consider a solution ξ of (4), (19), with domain $\mathrm{dom}\,\xi := \bigcup_{j\in\mathbb{N}}[t_j, t_{j+1}] \times \{j\}$. If there exist $\alpha > 0$, $N \geq 1$ such that, for all $j \in \mathbb{N}$:

$$\sum_{i=j}^{j+N} \chi(t_{i+1}, i)\chi(t_{i+1}, i)^\top \geq \alpha I_2, \tag{31}$$

then attractor $\mathcal{A}$ in Theorem 1 is a subset of $\mathcal{O}$ in (7), i.e., all points in $\mathcal{A}$ satisfy $\hat{x} = x$.

Proof: If (31) holds, [17, Th. 1] implies that $\mathcal{A}_\chi := \{(\tilde{\lambda}, s, k) \in \mathbb{R}^2 \times \mathrm{dom}\,\chi : \tilde{\lambda} = 0\}$, with $\mathrm{dom}\,\chi = \mathrm{dom}\,\xi$, is uniformly exponentially stable for the auxiliary system

$$\begin{cases}
\begin{bmatrix} \dot{\tilde{\lambda}} \\ \dot{s} \\ \dot{k} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, & (\tilde{\lambda}, s, k) \in C_\chi \\
\begin{bmatrix} \tilde{\lambda}^+ \\ s^+ \\ k^+ \end{bmatrix} = \begin{bmatrix} \left(I_2 - \frac{2\gamma\chi(s,k)\chi(s,k)^\top}{1+2\gamma|\chi(s,k)|^2}\right)\tilde{\lambda} \\ s \\ k + 1 \end{bmatrix}, & (\tilde{\lambda}, s, k) \in D_\chi,
\end{cases}$$

where $D_\chi := \{(\tilde{\lambda}, s, k) \in \mathbb{R}^2 \times \operatorname{dom}\chi : (s, k+1) \in \operatorname{dom}\chi\}$ and $C_\chi := (\mathbb{R}^2 \times \operatorname{dom}\chi)\backslash D_\chi$. In particular, V of (24) is employed in the proof of [17, Th. 1]. This fact, combined with (28), implies strict decrease of $|\tilde{\lambda}|$ along the solutions of (4), (19) after the first jump, thus proving $\mathcal{A} \subset \mathcal{O}$. ∎

IV. A Sufficient Condition for Persistency of Excitation

In this section, we investigate observer convergence by providing a sufficient condition, involving the continuous-time signals of (4) and the selection of the clock period τ of (19), such that the persistency of excitation condition (31) is satisfied by all solutions ξ of (4), (19).

From $\chi = \varphi\zeta - \lambda$, $\chi^+ = 0$, and $\dot{\lambda} = 0$, we have that:

$$\begin{aligned}\chi(t_{j+1}, j) &= \chi(t_{j+1}, j) - \chi(t_j, j)\\ &= \varphi\zeta(t_{j+1}, j) - \lambda(t_{j+1}, j) - \varphi\zeta(t_j, j) + \lambda(t_j, j)\\ &= \varphi(\zeta(t_{j+1}, j) - \zeta(t_j, j-1)),\end{aligned} \tag{32}$$

for all $j \geq 1$ and all $(t_{j+1}, j) \in \operatorname{dom}\xi$. Therefore, χ is the difference between discrete samples of the rotor flux $\varphi\zeta$. For this reason, we endow the rotor angular speed ω with the following regularity and richness properties.

Assumption 3: The solutions of system (4) are such that:

1) ω is continuously differentiable, for all $t \in [0, \infty)$;
2) there exists a positive scalar ω_M^d such that $|\dot{\omega}(t)| \leq \omega_M^d$ for all $t \in [0, \infty)$;
3) there exist positive scalars c and T such that:

$$\int_t^{t+T} \omega(s)^2 ds > c, \qquad \forall t \in [0, \infty). \tag{33}$$

Remark 2: In addition to the boundedness of Assumption 3-(2), from Assumption 1 we also have that there exists a positive scalar ω_M such that $|\omega(t)| \leq \omega_M$, for all $t \in [0, \infty)$.

Remark 3: Assumption 3-(3), also adopted in [9], is a continuous-time persistency of excitation condition, which is closely connected to the well-known non-permanently-zero speed requirement that ensures PMSM observability [18].

From (32), we observe that $\chi/(\varphi\tau)$ can be interpreted as a discrete-time estimate of $\dot{\zeta}$. Therefore, noting that $\zeta^\top\dot{\zeta} = 0$, we have that (31) is ensured if ζ "rotates sufficiently" along $\mathbb{S}^1$ as per Assumption 3-(3), while τ is selected sufficiently small to ensure $\chi/(\varphi\tau)$ and $\dot{\zeta}$ are sufficiently close. This intuition is confirmed in the following result.

Theorem 2: Under Assumption 3, there exists $\tau^* > 0$ such that, for all τ satisfying $0 < \tau \leq \tau^*$, (31) holds for all solutions of (4), (19).

Proof: Applying the mean value theorem to the integral (33), for any $t \geq 0$, there exists $s^* \in [t, t+T]$ such that $|\omega(s^*)| \geq \sqrt{c/T} =: c_1$. Since $|\dot{\omega}|$ is bounded, it holds that $|\omega(s)| \geq c_1/2$, for some $s \in [s^* - \Delta, s^* + \Delta] \cap [t, t+T]$, where Δ is defined as [9, Lemma 2]:

$$\Delta := \frac{1}{2}\min\left\{\frac{c_1}{\omega_M^d}, T\right\}. \tag{34}$$

Pick $\nu \in (0, 1)$, consider ω_M from Remark 2, then define

$$\tau^* := \min\left\{\frac{\Delta}{2}, \nu\frac{\pi}{\omega_M}\right\} > 0. \tag{35}$$

Let $N \geq 2$ be the minimum integer satisfying $N\tau \geq T$. For system (4), (19) with $\tau \in (0, \tau^*]$, consider any solution ξ, with domain $\operatorname{dom}\xi = \bigcup_{j\in\mathbb{N}} [t_j, t_{j+1}] \times \{j\}$. To simplify the notation, in the following we will omit the discrete-time argument of signals that do not jump. Since $2\tau \leq \Delta$, for all $j \in \mathbb{N}$ there exists an integer $k \geq j+1$ such that

$$[t_k, t_{k+2}] \subset [t_{j+1}, t_{j+1} + T], \quad |\omega(s)| \geq c_1/2, \forall s \in [t_k, t_{k+2}].$$

It follows that, for $l \in \{k, k+1\}$:

$$\frac{c_1\tau}{2} \leq \left|\int_{t_l}^{t_{l+1}} \omega(s)ds\right| = \int_{t_l}^{t_{l+1}} |\omega(s)|ds \leq \omega_M\tau \leq \nu\pi. \tag{36}$$

Define:

$$\vartheta_l := \int_{t_l}^{t_{l+1}} \omega(s)\mathrm{d}s, \qquad l \in \{k, k+1\}. \tag{37}$$

From (37), it is possible to compute the evolution of ζ from t_l to t_{l+1}. In particular, for $l \in \{k, k+1\}$, it holds that $\zeta(t_{l+1}) = (\cos(\vartheta_l)I_2 + \sin(\vartheta_l)J)\zeta(t_l)$. It follows that

$$\begin{aligned}\zeta(t_{k+1})^\top(\zeta(t_{k+1}) - \zeta(t_k)) &= 1 - \cos(\vartheta_k)\\ \zeta(t_{k+1})^\top(\zeta(t_{k+2}) - \zeta(t_{k+1})) &= \cos(\vartheta_{k+1}) - 1\\ \zeta(t_{k+1})^\top J^\top(\zeta(t_{k+1}) - \zeta(t_k)) &= \sin(\vartheta_k)\\ \zeta(t_{k+1})^\top J^\top(\zeta(t_{k+2}) - \zeta(t_{k+1})) &= \sin(\vartheta_{k+1}).\end{aligned} \tag{38}$$

From (36), (38) for $l \in \{k, k+1\}$, we obtain $|\chi(t_{l+1}, l)|^2/\varphi^2 = |\zeta(t_{l+1}) - \zeta(t_l)|^2 \geq a > 0$, where $a := 2(1 - \cos(c_1\tau/2))$. Denote by η_k, $\eta_{k+1} \in [0, \pi/2]$ the angles between $J\zeta(t_{k+1})$ and $\chi(t_{k+1}, k) = \varphi(\zeta(t_{k+1}) - \zeta(t_k))$ and $\chi(t_{k+2}, k+1) = \varphi(\zeta(t_{k+2}) - \zeta(t_{k+1}))$. By definition of inner product:

$$\cos(\eta_l) = \frac{\sin(\vartheta_l)}{\sqrt{2(1 - \cos(\vartheta_l))}} = \cos\left(\frac{\vartheta_l}{2}\right), \quad l \in \{k, k+1\}.$$

Let $p \in \{k, k+1\}$, $q \in \{k, k+1\}$ be such that $\eta_p \leq \eta_q$. Then, by monotonicity of $\cos(\cdot)$ in $[0, \pi]$ we apply the bounds:

$$\cos(\vartheta_q) = \cos(2\eta_q) \leq \cos(\eta_p + \eta_q) \leq \cos(2\eta_p) = \cos(\vartheta_p).$$

From (36), we have that $\beta_1 := \cos(\eta_k + \eta_{k+1}) \in (-b, b)$, $b < 1$, where $\eta_k + \eta_{k+1}$ is the angle between $\chi(t_{k+1}, k)$ and $\chi(t_{k+2}, k+1)$. Consider the orthogonal matrix $U := \left[\frac{\chi(t_{k+1},k)}{|\chi(t_{k+1},k)|} \quad J\frac{\chi(t_{k+1},k)}{|\chi(t_{k+1},k)|}\right]$. It follows that

$$\begin{aligned}U^\top\chi(t_{k+1}, k) &= \varphi\sqrt{2(1 - \cos(\vartheta_k))}\begin{bmatrix}1 & 0\end{bmatrix}^\top\\ U^\top\chi(t_{k+2}, k+1) &= \varphi\sqrt{2(1 - \cos(\vartheta_{k+1}))}\begin{bmatrix}\beta_1 & \beta_2\end{bmatrix}^\top,\end{aligned}$$

where β_2 is such that $\beta_2^2 = 1 - \beta_1^2$. Define $A := \chi(t_{k+1}, k)\chi(t_{k+1}, k)^\top + \chi(t_{k+2}, k+1)\chi(t_{k+2}, k+1)^\top$ and note that $\operatorname{msv}(A) = \operatorname{msv}(A_U)$, where $A_U := U^\top AU$. Finally, define $v_k := 2\varphi^2(1 - \cos(\vartheta_k))$, $v_{k+1} := 2\varphi^2(1 - \cos(\vartheta_{k+1}))$, then we obtain

$$A_U = v_k\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix} + v_{k+1}\begin{bmatrix}\beta_1^2 & \beta_1\beta_2\\ \beta_1\beta_2 & \beta_2^2\end{bmatrix},$$

which is positive definite as $\det(A_U) = v_k v_{k+1}\beta_2^2 \geq \varphi^4 a^2(1 - b^2) > 0$. Finally, from the characteristic polynomial $\varsigma^2 - \varsigma(v_k + v_{k+1}) + v_k v_{k+1}\beta_2^2$, we obtain $\operatorname{msv}(A) \geq (\frac{1}{v_k} + \frac{1}{v_{k+1}})^{-1}\beta_2^2 \geq \frac{a\varphi^2}{2}(1 - b^2)$, from which we conclude the existence of α in (31). ∎

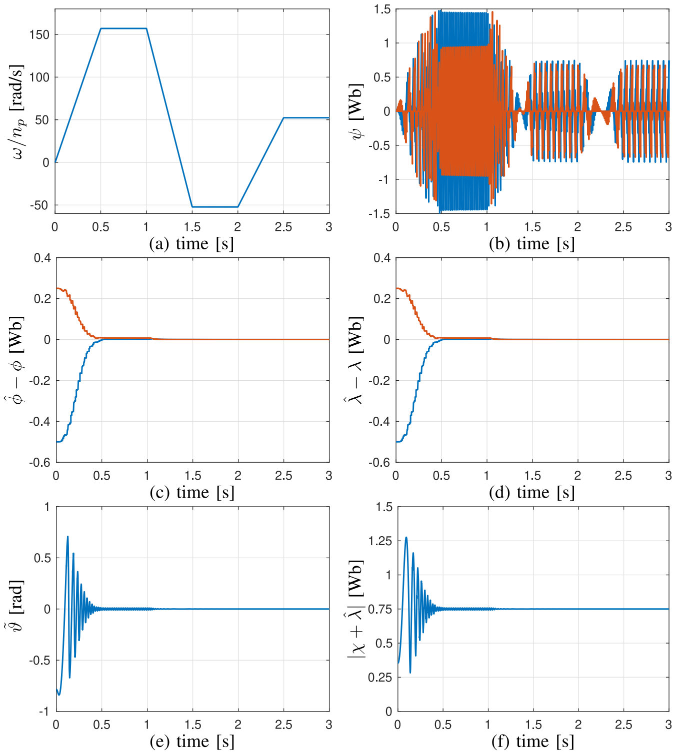

Fig. 1. PMSM observer performance. (a): rotor mechanical speed ω/n_p. (b): behavior of state ψ. (c): ϕ estimation error. (d): estimation error of parameters λ. (e): angular position estimation error $\tilde{\vartheta} := \text{atan2}(\zeta) - \text{atan2}(\hat{\zeta})$. (f): norm of the estimated rotor flux.

Remark 4: The sufficient conditions in Theorem 2 are far from necessity, as non-consecutive samples of χ may also satisfy a non-collinearity condition. Thus, a small τ is not restrictive for the effectiveness of the proposed design.

V. Numerical Results

In this section, we report simulation results to validate the properties of the proposed estimation scheme. We considered a PMSM with nominal flux $\varphi = 0.75$ Wb, number of pole pairs $n_p = 2$, stator resistance $R = 0.15\ \Omega$, and stator inductance $L = 0.6$ mH. The PMSM was interconnected with a standard cascade control solution with the current loop based on the field-orientation principle. To highlight the observer's features, we present results without employing the estimates in the control loop, i.e., the controller is fed by the measured angular position.

We validated the proposed observer under the trapezoidal speed profile of Fig. 1-(a). For the tuning of (19), we selected $\sigma = 10$, $\gamma = 0.1$, $r = 3\varphi$ (see (20)), and clock period $\tau = 10$ ms. In Fig. 1 we report a simulation with $\hat{\lambda}$ initialized in $(0.25, 0.25)$ Wb, showing that the PMSM states are correctly estimated. In particular, the estimation performance for ζ is highlighted in Fig. 1-(e) through the angular error $\tilde{\vartheta} := \text{atan2}(\zeta) - \text{atan2}(\hat{\zeta})$. Finally, in Fig. 1-(f) we report the norm of the estimated flux, which converges to the rotor flux value φ.

VI. Conclusion

We developed a hybrid sensorless observer for the flux reconstruction of PMSMs. The proposed architecture involves an integrator with periodic resets triggered by a clock and a gradient descent identifier. Employing the properties of ω-limit sets of well-posed hybrid systems, we proved global robust asymptotic stability of a compact set ensuring zero estimation error under appropriate persistency of excitation conditions. Moreover, to enhance the applicability of the proposed scheme, we provided a sufficient condition for persistency of excitation involving the rotor angular speed and a suitable selection of the clock period. Future work will focus on including estimation of the stator resistance and extending the design to other classes of electric machines.

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