A Simple Architecture for Arbitrary Interpolation of State Feedback

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***Abstract*—Stability of a linear system under fast switching or blending of a set of controllers can be ensured by an appropriate observer-based state-space realization. In this letter, the more specific problem is considered of arbitrary interpolation of a set of state feedback gains based on an initial static state feedback. First, the dynamic augmentation generating this parameterization is derived as well as the associated parameters for local recovery of predefined static controllers. By further simplification, a simple and intuitive structure is obtained with only a single design matrix. We propose to exploit this remaining degree of freedom to maximize robustness in terms of coprime factor uncertainty. The resulting parameterization is comparatively simple to implement in both continuous and discrete time. The robotics problem of active variable impedance control serves to illustrate utility of this parameterization.**

***Index Terms*—Switched systems, robotics, control system architecture.**

I. Introduction

CERTAIN control systems demand for switching or interpolation between state feedback. For example, active variable impedance control, a current robotics research topic [1], requires a robot manipulator to achieve by regulation some desired stiffness/damping characteristics, respectively, a mechanical impedance. Active *variable* impedance, however, implies varying feedback gains. It is well-known [2] that standard linear analyses, *e.g.,* assessment of the closed-loop eigenvalues over time, are in general not sufficient to conclude stability of the resulting closed loop system. This concern is nonetheless frequently ignored in favor of simplicity in implementation: it is mitigated by using demonstrations to bias towards admissible behavior [3], [4]. High damping on hardware and slow variations as in [5] and [6] in practice further alleviate the potential for instabilities caused by ad-hoc gain interpolation. Current solutions which consider stability are tailored specifically to the robotics problem domain [7], [8] or do not provide a synthesis method [9].

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With these issues in mind, we report an interpolation scheme for state feedback controllers addressing the following requirements.

R1) The closed loop must be stable under arbitrary interpolation or switching, and the state feedback controllers must be recovered in the design points.

R2) The scheme must be based on a static state feedback controller and use the terminal connections, *i.e.*, the initial state feedback controller cannot be replaced.[1]

R3) The interpolated controller must be simple to implement and allow for a transparent interpretation of its components.

A. State-Space Realizations for Arbitrary Switching

The concept to switch or blend controllers is common in practice, particularly if the plant is nonlinear and the controller needs to be adapted along the operating conditions. Research on gain scheduling correspondingly has a long history, see [2], [10] and the references therein. Switching or interpolation of controllers can lead to instability even if each controller separately stabilizes the system [11]. The interpolation of the family of locally linear controllers is therefore a crucial step in gain scheduling and diverse approaches exist to ensure stability. One way is to exploit the state-space realization of each controller as a design degree of freedom [12].

A specific realization useful for interpolation is by construction within the class of all stabilizing controllers,[2] *i.e.*, in terms of a stable parameter system Q. Stabilizing controller interpolation then reduces to the simpler task of interpolating stable systems [15]. In similar spirit, the so-called J-Q-interpolation constitutes an important stability preserving scheme for gain scheduling [16]. Hespanha and Morse [17] prove that a similarity transformation of the parameters Q is sufficient to construct a Common Quadratic Lyapunov Function (CQLF), ensuring stability under *arbitrary* switching between linear controllers. Blanchini *et al.* [18] extend this result later to the case when also the plant is switching and similar results exist for polytopic Linear Parameter Varying (LPV) systems [19]. Stability under switching yet does not ensure performance. This drawback is resolved by

[1] In particular, the user interface of some robotic hardware only allows to modify the control torque by adding to some ever-present state feedback.

[2] For the classic observer-based Youla-Kučera (YK) parameterization, see [13, Ch. 12] or [14] and the references therein.

the parameterization of Hencey and Alleyne [20] that preserves suboptimal $\mathcal{H}_\infty$ performance and suppresses transient peaks. All these parameterizations require a specific initial stabilizing controller. If, however, an existing controller must remain implemented, parameterizations can also be constructed upon only the terminal connections [21]. Several structures to implement switching controllers based on the Youla-Kučera (YK) parameterization are compared in [22].

B. Contributions and Outline

One common drawback of the parameterizations reviewed above is a potentially high dynamic order. In addition, the complexity of the schemes seems to restrict the scope of applications. This letter is to provide a complementary parameterization, *i.e.*, an architecture fulfilling the requirements R1–R3. To this end, the general YK switching framework is specialized to the state feedback case and the dynamic parameters Q are derived that recover state feedback behavior in the design points. In contrast to many works that do not provide guidelines how to systematically exploit the freedom in the choice of the coprime factorization, [14]–[17], [19], [21]–[22], we propose a design for the free parameter matrix to simplify the structure and to allow for a transparent and intuitive interpretation of the scheme. Formulae for all design steps involved are provided in both continuous- and discrete-time domains. In summary, this letter is to make the parameterization approach easily accessible for interpolated control problems where state information is available.

After giving a formal problem statement in Section II, the parameterization and the choice of parameters are discussed (Section III). As an example, we return to the active variable impedance control problem in Section IV and discuss real-world implementation issues by means of a robotic system. Some technical details are left for the Appendix.

Notation: In the formulae that apply to both the continuous- and discrete-time domains, the symbol δ represents the derivative operator $\frac{\partial}{\partial t}$ in continuous-time and the one-step shift operator $\delta \boldsymbol{x}(t) = \boldsymbol{x}(t+1)$ in the discrete-time setting, and $\mathcal{I}$ denotes the corresponding integration. Accordingly, by stability of a matrix $\boldsymbol{A}$, in continuous time we refer to a Hurwitz matrix $\boldsymbol{A}$ with strictly negative real part of all eigenvalues, *i.e.*, $\mathrm{Re}(\lambda_i) < 0, \boldsymbol{\lambda} = \mathrm{eig}(\boldsymbol{A})$; respectively, a Schur matrix $\boldsymbol{A}$ with strictly negative spectral radius $\rho(\boldsymbol{A}) < 1$ in the discrete-time case. The corresponding set of proper and real rational stable systems is $\mathcal{RH}_\infty$. Two systems G_1 and G_2 are input-output equivalent, denoted $G_1 \sim G_2$, if $\|G_1 - G_2\| = 0$ for an induced norm $\|\cdot\|$. Assuming well-posedness, the lower linear fractional transformation [13, Ch. 10] of a suitably partitioned 2×2 operator G by Φ is denoted $\mathcal{F}_\ell(G, \Phi) \triangleq G_{11} + G_{12}\Phi(I - G_{22}\Phi)^{-1}G_{21}$.

II. Definitions and Problem Setting

Consider the nominal system G with measurable state $\boldsymbol{x} \in \mathbb{R}^n$, control input $\boldsymbol{u} \in \mathbb{R}^{n_\mathrm{u}}$ and output $\boldsymbol{y} \in \mathbb{R}^{n_\mathrm{y}}$

$$G : \begin{cases} \delta \boldsymbol{x} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}_u\boldsymbol{u}, & \boldsymbol{x}(0) = \boldsymbol{x}_0, \\ \boldsymbol{z} = \boldsymbol{C}_z\boldsymbol{x} + \boldsymbol{D}_{zu}\boldsymbol{u}, & \boldsymbol{y} = \boldsymbol{x}, \end{cases} \tag{1}$$

such that some desired performance is expressed by the quantities $\boldsymbol{z} \in \mathbb{R}^{n_\mathrm{z}}$. Assuming stabilizability of $(\boldsymbol{A}, \boldsymbol{B}_u)$, let a set

$$\mathcal{K} = \{K_1, K_2, \ldots, K_{N_\mathrm{K}}\} \tag{2}$$

of static state feedback controllers $K_i : \boldsymbol{u} = \boldsymbol{D}_{\mathrm{K},i}\boldsymbol{x}$, $\boldsymbol{D}_{\mathrm{K},i} \in \mathbb{R}^{n_\mathrm{u} \times n}$ be given such that $\boldsymbol{A} + \boldsymbol{B}_u\boldsymbol{D}_{\mathrm{K},i}$ is stable for all $i = 1, \ldots, N_\mathrm{K}$.

For the interpolated controller $K(\boldsymbol{\alpha}(t))$, let the variable $\boldsymbol{\alpha}(t) \in \mathbb{R}^{N_\mathrm{K}}$ describe the extent of how much each controller of the family $\mathcal{K}$ contributes at instant t. We allow for the set of piecewise continuous *arbitrary interpolation signals* [23]

$$\mathcal{A} = \left\{\boldsymbol{\alpha}(t) : \mathbb{R}_0^+ \mapsto [0, 1]^{N_\mathrm{K}} \;\middle|\; \textstyle\sum_{i=1}^{N_\mathrm{K}} \alpha_i = 1,\ \alpha_i \geqslant 0\right\}, \tag{3}$$

covering both arbitrarily fast switching and blending of controllers. The controller interpolation criteria from [23] are adopted accordingly to characterize admissible controllers.

Definition 1 (Admissible Interpolated State Feedback): Given a set $\mathcal{K}$ of local state feedback controllers, an admissible interpolated controller $K(\boldsymbol{\alpha})$ satisfies the following controller interpolation criteria [23]:

1) $K(\boldsymbol{\alpha})$ is stabilizing for all $\boldsymbol{\alpha} \in \mathcal{A}$.
2) $K(\boldsymbol{\alpha}) \sim K_i$ for constant $\alpha_i = 1$, $\forall i = 1, \ldots, N_\mathrm{K}$.
3) $K(\boldsymbol{\alpha})$ is a continuous function of $\boldsymbol{\alpha}(t)$.

We are now ready to give the formal problem statement for arbitrary interpolation of state feedback controllers subject to requirements R1–R3.

Problem 1 (Arbitrary Interpolation of State Feedback): Consider an initial controller $K_0 : \boldsymbol{u} = \boldsymbol{D}_{\mathrm{K},0}\boldsymbol{x}$ such that $\boldsymbol{A} + \boldsymbol{B}_u\boldsymbol{D}_{\mathrm{K},0}$ is stable. Given a set $\mathcal{K}$ of static state feedback controllers, construct a dynamic augmentation of K_0 such that $K(\boldsymbol{\alpha})$ is an admissible interpolated state feedback controller and $\boldsymbol{D}_{\mathrm{K},0}$ is the only free design parameter.

III. State-Feedback Interpolation Scheme

In general, under arbitrary interpolation of the set of feedback gains (2) by

$$K(\boldsymbol{\alpha}) : \boldsymbol{u} = \left(\textstyle\sum_{i=1}^{N_\mathrm{K}} \alpha_i \boldsymbol{D}_{\mathrm{K},i}\right)\boldsymbol{x}, \tag{4}$$

stability cannot be guaranteed. In order to solve Prob. 1, first the realization of an admissible interpolated state feedback controller is derived in Section III-A before we discuss the choice of parameters in Section III-B to further simplify the scheme.

A. Parameterization for Arbitrary Interpolation

Theorem 1 (Interpolation of State Feedback Controllers): Consider an LTI plant (1), a stabilizing state feedback controller $\boldsymbol{u} = \boldsymbol{D}_{\mathrm{K},0}\boldsymbol{x}$, a set of stabilizing static controllers (2), and a matrix $\boldsymbol{F} \in \mathbb{R}^{n_\mathrm{u} \times n}$ such that $\boldsymbol{A} + \boldsymbol{B}_u\boldsymbol{F}$ is stable. An admissible interpolated controller $K(\boldsymbol{\alpha})$ is given by interconnecting

$$J : \begin{cases} \delta \boldsymbol{x}_\mathrm{J} = (\boldsymbol{A} + \boldsymbol{B}_u\boldsymbol{F})\boldsymbol{x}_\mathrm{J} + \boldsymbol{B}_u\boldsymbol{s}, & \boldsymbol{x}_\mathrm{J}(0) = \boldsymbol{0}, \\ \boldsymbol{u} = \boldsymbol{D}_{\mathrm{K},0}\boldsymbol{x} + (\boldsymbol{F} - \boldsymbol{D}_{\mathrm{K},0})\boldsymbol{x}_\mathrm{J} + \boldsymbol{s}, \\ \boldsymbol{r} = \boldsymbol{x} - \boldsymbol{x}_\mathrm{J} \end{cases}$$
$$\text{with} \quad \boldsymbol{s} = Q(\boldsymbol{\alpha})\boldsymbol{r}, \tag{5}$$

where the interpolated system $Q(\boldsymbol{\alpha})$ of parameters $\mathcal{Q} = \{Q_i \,|\, Q_i \in \mathcal{RH}_\infty, i = 1, \ldots, N_\mathrm{K}\}$ is realized such that all $Q \in \mathcal{Q}$ share a CQLF, *i.e.*,

$$\exists\, \boldsymbol{P}_\mathrm{Q} \in \mathbb{R}^{n \times n},\ \boldsymbol{P}_\mathrm{Q} = \boldsymbol{P}_\mathrm{Q}^\top \succ 0:$$
$$\forall \boldsymbol{x} \neq 0 : V(\boldsymbol{x}) = \boldsymbol{x}^\top\boldsymbol{P}_\mathrm{Q}\boldsymbol{x} > 0 \text{ and } \forall \boldsymbol{\alpha} \in \mathcal{A}: \tag{6}$$

$$\text{(continuous-time)}\ \ \boldsymbol{P}_\mathrm{Q}\boldsymbol{A}_\mathrm{Q}(\boldsymbol{\alpha}) + \boldsymbol{A}_\mathrm{Q}^\top(\boldsymbol{\alpha})\boldsymbol{P}_\mathrm{Q} \prec 0, \tag{7a}$$

$$\text{(discrete-time)}\ \ \boldsymbol{A}_\mathrm{Q}(\boldsymbol{\alpha})^\top\boldsymbol{P}_\mathrm{Q}\boldsymbol{A}_\mathrm{Q}(\boldsymbol{\alpha}) - \boldsymbol{P}_\mathrm{Q} \prec 0. \tag{7b}$$

An interpolated $Q(\boldsymbol{\alpha})$ can be realized to satisfy (6)–(7) by the standard procedures reported in the Appendix, where the dynamic parameters Q_i corresponding to the local state feedback controllers K_i are given by

$$Q_i : \begin{cases} \delta \boldsymbol{x}_{\mathrm{Q}} = \left(\boldsymbol{A} + \boldsymbol{B}_u \boldsymbol{D}_{\mathrm{K},i}\right)\boldsymbol{x}_{\mathrm{Q}} + \boldsymbol{B}_u\left(\boldsymbol{D}_{\mathrm{K},0} - \boldsymbol{D}_{\mathrm{K},i}\right)\boldsymbol{r}, \\ \boldsymbol{x}_{\mathrm{Q}}(0) = \boldsymbol{0}, \\ \boldsymbol{s}_i = \left(\boldsymbol{F} - \boldsymbol{D}_{\mathrm{K},i}\right)\boldsymbol{x}_{\mathrm{Q}} + \left(\boldsymbol{D}_{\mathrm{K},i} - \boldsymbol{D}_{\mathrm{K},0}\right)\boldsymbol{r}. \end{cases} \tag{8}$$

Proof: The steps in the derivation are similar to the gain scheduling literature exploiting the general Youla parameterization [19]. Consider coprime factorizations of the nominal plant $G_{yu} = \tilde{M}_0^{-1}\tilde{N}_0 = N_0 M_0^{-1}$, respectively of the nominal controller $K_0 = \tilde{V}_0^{-1}\tilde{U}_0 = U_0 V_0^{-1}$ such that the double Bezout identity holds, *i.e.*,

$$\begin{bmatrix} \tilde{V}_0 & -\tilde{U}_0 \\ -\tilde{N}_0 & \tilde{M}_0 \end{bmatrix}\begin{bmatrix} M_0 & U_0 \\ N_0 & V_0 \end{bmatrix} = \begin{bmatrix} M_0 & U_0 \\ N_0 & V_0 \end{bmatrix}\begin{bmatrix} \tilde{V}_0 & -\tilde{U}_0 \\ -\tilde{N}_0 & \tilde{M}_0 \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}. \tag{9}$$

According to the Q-parameterization theorem, see for example [13], [14], all internally stabilizing controllers for G_{yu} can be written in terms of a parameter system $Q \in \mathcal{RH}_\infty$ in a right stable fractional form as

$$K(Q) = (U_0 + M_0 Q)(V_0 + N_0 Q)^{-1}. \tag{10}$$

In the controlled channel of (1), all states are measurable without direct feedthrough, *i.e.*, $G_{yu} : \delta\boldsymbol{x} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}_u\boldsymbol{u},\ \boldsymbol{y} = \boldsymbol{x}$. Therefore, by $\boldsymbol{C}_y = \boldsymbol{I}$ and $\boldsymbol{D}_{yu} = \boldsymbol{0}$ and for static state feedback $\boldsymbol{u} = \boldsymbol{D}_{\mathrm{K},i}\boldsymbol{x}$, the state-space realizations of the general coprime factorization given in [24] specialize to

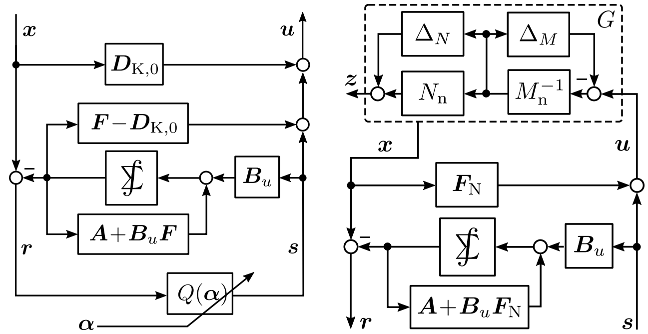
$$\begin{bmatrix} M_i & U_i \\ N_i & V_i \end{bmatrix} : \left[\begin{array}{c|cc} \boldsymbol{A}+\boldsymbol{B}_u\boldsymbol{F} & \boldsymbol{B}_u & \boldsymbol{0} \\ \hline \boldsymbol{F} & \boldsymbol{I} & \boldsymbol{D}_{\mathrm{K},i} \\ \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{I} \end{array}\right], \begin{bmatrix} \tilde{V}_i & -\tilde{U}_i \\ -\tilde{N}_i & \tilde{M}_i \end{bmatrix} : \left[\begin{array}{c|cc} \boldsymbol{A}+\boldsymbol{B}_u\boldsymbol{D}_{\mathrm{K},i} & -\boldsymbol{B}_u & \boldsymbol{B}_u\boldsymbol{D}_{\mathrm{K},i} \\ \hline \boldsymbol{F}-\boldsymbol{D}_{\mathrm{K},i} & \boldsymbol{I} & -\boldsymbol{D}_{\mathrm{K},i} \\ \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{I} \end{array}\right]. \tag{11}$$

The parameterized controller (10) can be reformulated [14, p. 43] using (9) as lower fractional transformation $K = \mathcal{F}_\ell(J, Q)$ with the central system $J = \begin{bmatrix} U_0 V_0^{-1} & \tilde{V}_0^{-1} \\ V_0^{-1} & -V_0^{-1}N_0 \end{bmatrix}$. Employing the realizations of the coprime factors from (11) and reducing to a minimal realization, the central system (5) is obtained. In order to calculate the set $\mathcal{Q}$ corresponding to $\mathcal{K}$, both the nominal controller K_0 as well as controllers $K_i \in \mathcal{K}$ are expressed by means of coprime factors (11). Thus, coprime factorizations are constructed for all the plant/controller interconnection pairs. One may then work with the coprime factors in order to consider the differences between the loops. The Q_i corresponding to controller K_i, given a factorization of the central controller K_0, can be calculated by $Q_i = (-\tilde{U}_0 + \tilde{V}_0 K_i)(\tilde{M}_0 - \tilde{N}_0 K_i)^{-1}$ [14, Ch. 8.3]. One can use

$$Q_i = \tilde{U}_i V_0 - \tilde{V}_i U_0 = \tilde{V}_i(K_i - K_0)V_0 \tag{12}$$

from [15, Th.1] alternatively. Plugging in the factors (11), the state-space construction of parameters Q_i follows as (8) after removal of one unobservable state. Recovery of the local controllers can be shown by interconnecting (5) with (8). Removing 5 uncontrollable and 1 unobservable states, it follows indeed $\mathcal{F}_\ell(J, Q_i) = K_i$. Stability under arbitrary interpolation, however, is not yet ensured by (5) and (8). To see this, consider the closed loop $\mathcal{F}_\ell(G, \mathcal{F}_\ell(J, Q))$ that can be reduced to dynamics with state matrix

$$\boldsymbol{A}_{\mathrm{cl}} = \begin{bmatrix} \boldsymbol{A}+\boldsymbol{B}_u\boldsymbol{D}_{\mathrm{K},0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{A}+\boldsymbol{B}_u\boldsymbol{F} & \boldsymbol{B}_u\boldsymbol{C}_{\mathrm{Q}}(\boldsymbol{\alpha}) & \boldsymbol{B}_u\boldsymbol{D}_{\mathrm{Q}}(\boldsymbol{\alpha}) \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{A}_{\mathrm{Q}}(\boldsymbol{\alpha}) & \boldsymbol{B}_{\mathrm{Q}}(\boldsymbol{\alpha}) \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{A}+\boldsymbol{B}_u\boldsymbol{D}_{\mathrm{K},0} \end{bmatrix},$$



(a) Structure of interpolation based on static state feedback controller $\boldsymbol{D}_{\mathrm{K},0}$

(b) Simplified structure and model of the nominal closed state feedback loop contained in J

Fig. 1. Proposed parameterization for arbitrary interpolation of state feedback controllers.

where $\boldsymbol{A}_{\mathrm{Q}}(\boldsymbol{\alpha}), \boldsymbol{B}_{\mathrm{Q}}(\boldsymbol{\alpha}), \boldsymbol{C}_{\mathrm{Q}}(\boldsymbol{\alpha}), \boldsymbol{D}_{\mathrm{Q}}(\boldsymbol{\alpha})$ define the realization of the plug-in filter $Q(\boldsymbol{\alpha})$. Given the block-diagonal respectively block-triangular structure of $\boldsymbol{A}_{\mathrm{cl}}$, there exists a CQLF if there exists one for each sub-block on the diagonal. Hence, (7) is enforced for $\boldsymbol{A}_{\mathrm{Q}}$, the matrices $\boldsymbol{A} + \boldsymbol{B}_u\boldsymbol{D}_{\mathrm{K},0}$ and $\boldsymbol{A} + \boldsymbol{B}_u\boldsymbol{F}$ are stable by construction. As pointed out in [17], there is always a transformation to realize $\mathcal{Q}$ such that a CQLF (6) exists. ■

In order to construct an admissible $\boldsymbol{P}_{\mathrm{Q}}$ for (6), all Q_i in (8) have to be stable. Therefore, the requirement that all $\boldsymbol{D}_{\mathrm{K},i}$ must stabilize the plant model cannot be relaxed. The order of the resulting controller is $2n$ or $(N_{\mathrm{K}} + 1)n$, depending on the scheme to implement the interpolated $Q(\boldsymbol{\alpha})$ according to (17) or (16), respectively.

Requirement R1 is fulfilled directly given Thm. 1. As the original controller $\boldsymbol{D}_{\mathrm{K},0}$ stays in place and the dynamic augmentation to realize J only requires to measure $\boldsymbol{x}$ and to add to $\boldsymbol{u}$, R2 is satisfied as well. The structure of the interpolation scheme is depicted in Fig. 1a.

B. Choice of Parameters

Free parameters in the parameterization (5)–(8) are the initial static controller $\boldsymbol{D}_{\mathrm{K},0}$ and the virtual gain $\boldsymbol{F}$ which is used to construct the coprime factorizations (11) of the plant and controllers. In general, these gains affect the transient behavior under varying $\boldsymbol{\alpha}$ and are a degree of freedom to be chosen by the designer.

One specific choice is particularly beneficial and key to satisfy requirement R3. By choosing the virtual gain $\boldsymbol{F}$ equal to the static initial controller,

$$\boldsymbol{F} = \boldsymbol{D}_{\mathrm{K},0}, \tag{13}$$

the structure of the generator system (5) is further simplified. The control input is then simply $\boldsymbol{u} = \boldsymbol{D}_{\mathrm{K},0}\boldsymbol{x} + \boldsymbol{s}$. Note that the dynamic augmentation in (5) models the effect of the filtered signal $\boldsymbol{s}$ on the *closed* controlled nominal loop. Therefore, the principle of function of the parameterization in Fig. 1 becomes very transparent.

It remains to design the gain $\boldsymbol{D}_{\mathrm{K},0}$. The most obvious choice is to take some $\boldsymbol{D}_{\mathrm{K},0} \in \mathcal{K}$. However, we propose to employ a specific LQR gain.

Corollary 1 (Choice of Initial Gain): Consider the solution $\boldsymbol{F}_{\mathrm{N}}$ to the standard Linear Quadratic Regulator (LQR) problem of designing $\boldsymbol{u} = \boldsymbol{F}_{\mathrm{N}}\boldsymbol{x}$ such that the continuous- or the discrete-time cost functional $\int_0^\infty \boldsymbol{z}^\top\boldsymbol{z} + \boldsymbol{u}^\top\boldsymbol{u}\,\mathrm{d}t$ respectively

$\sum_0^\infty (z^\top z + u^\top u)$ is minimized. Then, choosing $D_{K,0} = F_N$ yields a central controller which is maximally robust w.r.t. normalized coprime factor uncertainty $\Delta_N, \Delta_M \in \mathcal{RH}_\infty$, *i.e.*, the robust stability margin of the nominal loop is maximized.

Proof: The central controller of the parameterization in Thm. 1 is $\mathcal{F}_\ell(J, Q = 0)$. By construction, F is stabilizing; hence, (5) reduces to $\mathcal{F}_\ell(J, 0) = D_{K,0}$. The finding of $u = F_N x$ being a maximally robust state feedback controller for the plant $G = (N_n + \Delta_N)(M_n + \Delta_M)^{-1}$ with normalized right coprime factor uncertainty is due to [25]. ■

The resulting generator system of the parameterization proposed in this letter is visualized in Fig. 1b.

C. Relation to Other Parameterizations

Some remarks are in order so as to put the proposed parameterization in context to the literature.

Given R2, parameterization (5)–(8) can be seen as an instance of those in [21] specialized to state feedback. Compared to the classic parameterization based on a state-estimate controller [13], [14], the order of the interpolated controller $K(\alpha)$ according to Thm. 1 is lower, as the order of the elements in $\mathcal{Q}$ for controller recovery of a family $\mathcal{K}$ of desired *static* controllers is higher than n when using an observer-based central controller. By (13), the system J in (5) reduces to the generator systems utilized in [26] and [27] for purposes other than interpolation. In particular, [26] is relevant w.r.t. achievable robustness as detailed in the following remark.

Remark 1 (On Robustness): The generator system J of (5) can be used to construct the set of $\mathcal{H}_2$-optimal controllers—in this case, some specific $D_{K,0} = F$ and certain $Q \in \mathcal{RH}_2$ come into place [26, Th. 1]. By centering the parameterization on F_N according to Corollary 1 as depicted in Fig. 1b, one might similarly parameterize a set of sub-optimal coprime factor uncertainty robust controllers. To this end, just as in the more general output feedback case [28], an appropriate restriction of $\|Q_i\|_\infty$ would have to be enforced. The requirement R1 of local controller recovery conflicts, however, as $D_{K,i}$ is only assumed to be stabilizing and consequently $\|Q_i\|_\infty$ becomes arbitrarily large.

D. Two Degrees-of-Freedom Controllers

We also outline a two-degree-of-freedom control design for enhanced reference tracking. A stable reference model $T_r : \left[\begin{array}{c|c} A_r & B_r \\ \hline C_r & D_r \end{array}\right] \in \mathcal{RH}_\infty$ is introduced with the states x_r accessible. The augmented plant with exogenous $w \in \mathbb{R}^{n_w}$ is

$$G_{\text{aug}} : \begin{cases} \delta \begin{bmatrix} x \\ x_r \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & A_r \end{bmatrix} \begin{bmatrix} x \\ x_r \end{bmatrix} + \begin{bmatrix} B_u \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ B_r \end{bmatrix} w \\ z = C_z x - C_r x_r + D_{zu} u - D_r w \\ y = \left[x^\top, x_r^\top\right]^\top. \end{cases} \quad (14)$$

With G_{yu} taken non-minimal, Corollary 1 now yields $F_N = [D_{\text{fb},0}, D_{\text{ff},0}]$ and Thm. 1 turns into a simple parameterization for arbitrary interpolation of two-degrees-of-freedom state feedback controllers.

IV. Illustrative Study

A. Variable Impedance Control of a Point Mass

To illustrate the utility of the parameterization, let us revisit the variable impedance control problem from [7] and [9] with a single translational degree of freedom. The system consists of a mass M of nominally $M_{\text{nom}} = 10\,\text{kg}$ in free motion. For brevity, all quantities are normalized to SI units in what follows. The initial position is $x(0) = 10$ with $\dot{x}(0) = 0$, the virtual reference trajectory is $x_d(t) = 10\sin(0.1t)$ and the desired variable stiffness and damping are

$$K_P(t) = 12 + 10\sin(t) \triangleq K_c + K'(t), \quad K_D(t) = 1. \quad (15)$$

A simulation of the system with standard impedance control, *i.e.*, feedback of position and velocity errors with direct interpolation (4) of the gains, results in the trajectory shown in Fig. 2 for the first 100 s. The increasing magnitude of the oscillation results only from the hidden coupling induced by time-varying $K'(t)$, as the error system reduces to an exponentially stable LTI system for all constant $K'(t) = K' \in [-10, 10]$.

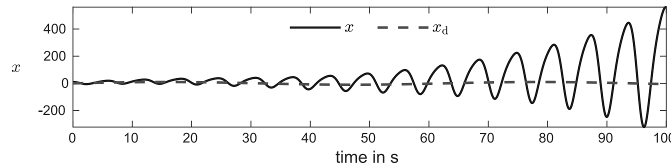


Fig. 2. Simulation of a mass under standard implementation of impedance control with variable gains.

TABLE I
CONTROLLER CONFIGURATIONS

Label	$D_{K,0}$	$\|Q_1\|_\infty$	$\|Q_2\|_\infty$	$\|S\|_\infty$ for M/M_{nom}		
				0.1	1	5
A	$D_{K,1}$	0	894.7	1.203	0	1.808
B	$D_{K,2}$	270.4	0	2.404	0	5.060
C	$\bar{D}_K$	68.16	226.6	1.347	0	2.270
D	F_N	317.0	18.97	0.399	0	1.267

This problem can be circumvented by the architecture of Section III; for simplicity, the one degree-of-freedom setup is used for design while the state error is used during implementation. First, the desired gains are reformulated as a convex combination of the two matrices $D_{K,1} \triangleq -\max[K_P(t), K_D(t)] = -[22, 1]$ and $D_{K,2} \triangleq -\min[K_P(t), K_D(t)] = -[2, 1]$. The interpolation signal such that $-[K_P(t), K_D(t)] = \alpha_1 D_{K,1} + \alpha_2 D_{K,2}$ is then given by $\alpha_1 = \frac{1}{2}(\sin(0.1t) + 1)$ and $\alpha_2 = 1 - \alpha_1$. As a plant model, we simply have $G_{yu}(s) = {}^1\!/_{10}\, s^{-2}$ and a minimal realization with $A = \left[\begin{smallmatrix} 0 & 1 \\ 0 & 0 \end{smallmatrix}\right]$, $B_u = [0, 0.1]^\top$ is taken. Next, one needs to design the initial gain $D_{K,0}$. It may seem natural to choose $D_{K,i} \in \mathcal{K}$ or average $\bar{D}_K \triangleq \frac{D_{K,1}+D_{K,2}}{2} = -[12, 1]$. As by Corollary 1, we also calculate $D_{K,0} = F_N$ by simply taking $C_z = I$ and $D_{zu} = 0$, *i.e.*, the performance variable is $z = x$ and equally weighs position and velocity. This results in $D_{K,0} \approx [-1.00, -4.58]$. The dynamic parameters Q_1 and Q_2 for controller recovery of $D_{K,1}$ and $D_{K,2}$ can now be calculated by (8). The resulting controllers are labeled A–D as summarized in Table I. In order to realize the interpolated $Q(\alpha)$ to satisfy (6), both (16) and (17) with $P_Q = I$ are considered, yielding $Q_{\text{LQN}}(\alpha)$ of order 4 and $Q_{\text{LPV}}(\alpha)$ of order 2, respectively.

The simulation results are shown in Fig. 3. Controllers A–D achieve asymptotic stabilization of the error system, effectively avoiding the scheduling-induced instability of Fig. 2. Opposed to [7], the simple controller of this letter does not result in

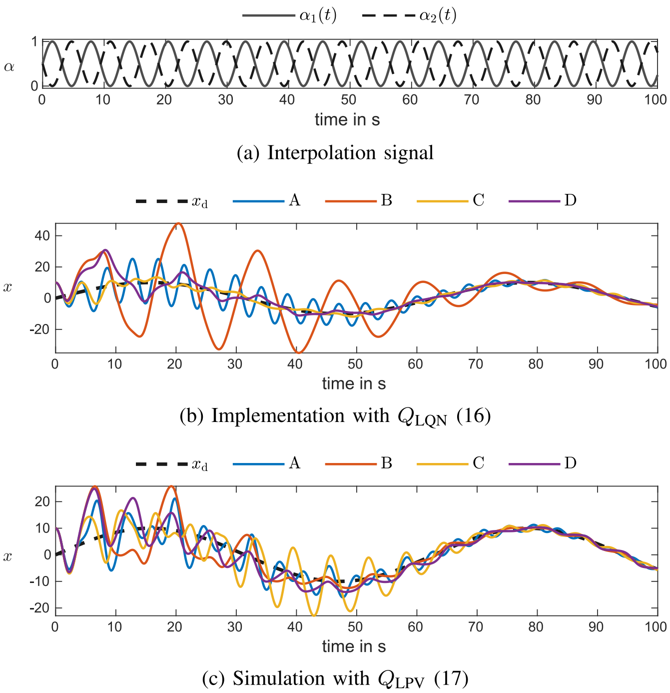


Fig. 3. Simulation of the mass-spring damper system with varying impedance, implemented by the controller of Fig. 1 with single gain (13). Labels A–D refer to the settings according to Table I.

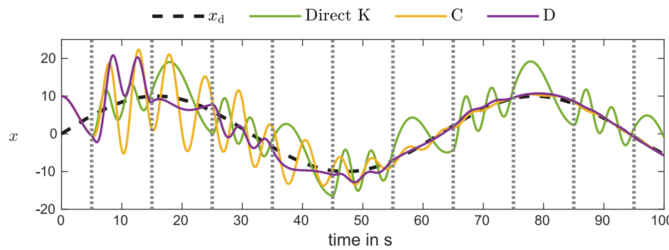


Fig. 4. Transient behavior under switching. The switching instants are marked by vertical dotted lines. Direct K refers to (4) and labels C–D to the controller architecture with $\boldsymbol{D}_{\mathrm{K},0}$ according to Table I, implemented with Q_LQN.

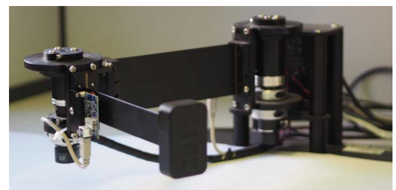


Fig. 5. The interpolation architecture has been implemented for a tracking control experiment on the depicted flexible-link robot platform.

high-frequency chattering of the control input $\boldsymbol{u}$. The analysis of [9] in turn only allows to check if an impedance profile is suitable for ad-hoc implementation by (4). Indeed, the stiffness profile (15) is disqualified in [9], whereas the parameterization approach reported here constitutes a straightforward synthesis method.

B. Additional Discussion and Implementation Aspects

Recall that $K(\boldsymbol{\alpha})$ reduces to the corresponding static feedback for frozen $\boldsymbol{\alpha}$. Therefore, the effect of the stabilizing parameterization becomes discernible only when $\boldsymbol{\alpha}$ is varying. The central controller $\boldsymbol{D}_{\mathrm{K},0}$, the coprime factor stabilizing gain $\boldsymbol{F}$, and the realization of interpolated $Q(\boldsymbol{\alpha})$ all affect the transient behavior.

It depends on the particular application if the implementation by either Q_LQN or Q_LPV is preferable. In our simulation studies, we could not observe a general advantage of the higher-order Q_LQN over Q_LPV except for a more straightforward implementation. However, the state dimensionality of Q_LQN may become an issue particularly if $|\mathcal{K}|$ is large.

Due to the parameterization architecture, the difference of the feedback controllers w. r. t. the nominal loop is separated by (12) into the corresponding plug-in filters $\mathcal{Q}$. Therefore, one could aim to keep $\max\limits_{Q_i \in \mathcal{Q}} \|Q_i\|_\infty$ low by choice of the parameters. With equal gains (13), the only decisive factor in (8) is $(\boldsymbol{D}_{\mathrm{K},i} - \boldsymbol{D}_{\mathrm{K},0})$. This point of view suggests to use $\boldsymbol{D}_{\mathrm{K},0} = \bar{\boldsymbol{D}}_\mathrm{K}$ (controller C), instead of $\boldsymbol{D}_{\mathrm{K},0} \in \mathcal{K}$ (controllers A and B).

Given the requirement of local controller recovery, in general no assertions can be made concerning robustness— the maximum coprime factor uncertainty margin of the central gain $\boldsymbol{F}_\mathrm{N}$ is lost once the local recovery filters Q_1 and Q_2 are employed, *see* Remark 1. Nonetheless, controller D constitutes a reasonable compromise of transient performance (see Figs. 3 and 4) and robustness: in order to ensure the effectiveness of the parameterization, the dual Youla operator S [14] characterizing the model mismatch should be stable and build a stable feedback loop with Q. As summarized in Table I, if for example the mass M is varied, the setup with gain $\boldsymbol{F}_\mathrm{N}$ yields a lower $\mathcal{H}_\infty$ norm to the dual Youla parameter computed according to [29, Th. 3.1]. Guaranteed robust arbitrary interpolation *and* controller recovery, however, would require that the controllers individually ensure robust stability w. r. t. the dual Youla parameter. In this case, a robust arbitrary switching scheme comes in reach as recently shown in a SISO setting [30].

When switching controllers, peaks occur in the proposed scheme just as in the standard YK parameterization. Recall that controller D is based on the LQR criterion of Corollary 1, hence with an implicit penalty on the performance variables $\boldsymbol{z}$ respectively the control input $\boldsymbol{u}$. An example of good transient performance achievable with the scheme is shown in Fig. 4 for the mass-spring damper system. However, if the transient peaks are of primary concern and must be suppressed, requirement R3 should be lifted in favor of the full dynamic $\mathcal{H}_\infty$ interpolation scheme of [20]. In this case, notably more engineering effort is required to tune the weighting filters involved in the $\mathcal{H}_\infty$ design; moreover, the dynamic order of the resulting controller is considerably higher.

Finally, we briefly report our experience with the two-degrees-of-freedom interpolation scheme of Section III-D implemented on a robotic manipulator testbed. The robot (Fig. 5) consists of two serial flexible links and is controlled to track a reference trajectory. The linearized model of each joint/flexible link stage requires 4 states, resulting in an interpolated controller $K(\boldsymbol{\alpha})$ of order $2(2 \cdot 4) = 16$. The sampling rate is 500 Hz in our experiments, yet it is essential to use the discrete-time formulation, particularly if a controller with noticeable derivative action K_D is contained in $\mathcal{K}$. Although the model is bound to be imprecise, the controller with $\boldsymbol{D}_{\mathrm{K},0}$ designed according to Corollary 1 works effectively. Given the strict limits on achievable torque, it is also sensible to add an anti-windup scheme. We use a full-order compensator design by the conditioning technique based on coprime factorization, given in discrete-time in [31, Fig. 2 and Th. 1].

V. Conclusion

The parameterization approach to assure stability when switching or interpolating controllers has received much attention theoretically, yet there is a lack of dissemination to a wider range of application domains. We have therefore formulated requirements from a practitioner's point of view. Consequently, a simple architecture has been proposed based on state feedback. Its utility has been demonstrated by the variable impedance control problem. The additional discussion is meant to assist in resolving issues of practical implementation and in the selection of a more advanced scheme where necessary.

Appendix

Two approaches are predominant in the literature to implement the interpolated system $Q(\boldsymbol{\alpha})$ subject to (6)–(7).

1) *Local Q-Network:* It is straightforward to interpolate the output of stable parallel systems $Q_i \in \mathcal{RH}_\infty$, *i.e.,* $Q_{\mathrm{LQN}}(\boldsymbol{\alpha}) = \sum_{i=1}^{N_{\mathrm{K}}} \alpha_i Q_i \in \mathcal{RH}_\infty$. This is sometimes termed Local Q-Network (LQN) [20]. Formally, a realization is

$$\boldsymbol{A}_{\mathrm{Q}} = \mathrm{diag}(\boldsymbol{A}_{\mathrm{Q},1}, \dots, \boldsymbol{A}_{\mathrm{Q},N_{\mathrm{K}}}), \boldsymbol{B}_{\mathrm{Q}} = \left[\boldsymbol{B}_{\mathrm{Q},1}^\top, \dots, \boldsymbol{B}_{\mathrm{Q},N_{\mathrm{K}}}^\top\right]^\top, \quad (16a)$$

$$\boldsymbol{C}_{\mathrm{Q}} = \left[\alpha_1 \boldsymbol{C}_{\mathrm{Q},1}, \dots, \alpha_{N_{\mathrm{K}}} \boldsymbol{C}_{\mathrm{Q},N_{\mathrm{K}}}\right], \boldsymbol{D}_{\mathrm{Q}} = \textstyle\sum_{k=1}^{N_{\mathrm{K}}} \alpha_i \boldsymbol{D}_{\mathrm{Q},i} \quad (16b)$$

yielding an admissible $\boldsymbol{P}_{\mathrm{Q}}$ in (6) since $\boldsymbol{A}_{\mathrm{Q}}$ is a constant block diagonal matrix of stability matrices.

2) *Implementation With Shared States:* The interpolated controller can also be implemented as a polytopic LPV system. By a similarity transformation of all $\boldsymbol{A}_{\mathrm{Q},i}$, a common Lyapunov matrix $\boldsymbol{P}_{\mathrm{Q}} = \boldsymbol{S}_{\mathrm{Q}}^\top \boldsymbol{S}_{\mathrm{Q}} \in \mathbb{R}^{n \times n}$ exists as first shown in the switching literature [17, Appendix A.1]. As $Q_i \in \mathcal{RH}_\infty$ by construction, there exist associated $\boldsymbol{P}_{\mathrm{Q},i}$ obtained by solving (7) with $\alpha_i = 1, \forall i = 1, \dots, N_{\mathrm{K}}$. Denote by nonsingular $\boldsymbol{S}_{\mathrm{Q},i}$ a Cholesky factorization such that $\boldsymbol{S}_{\mathrm{Q},i}^\top \boldsymbol{S}_{\mathrm{Q},i} = \boldsymbol{P}_{\mathrm{Q},i}$. Then, $Q(\boldsymbol{\alpha})$ can be realized as a system of order n as

$$Q_{\mathrm{LPV}}(\boldsymbol{\alpha}) : \left[\begin{array}{c|c} \bar{\boldsymbol{A}}_{\mathrm{Q}}(\boldsymbol{\alpha}) & \bar{\boldsymbol{B}}_{\mathrm{Q}}(\boldsymbol{\alpha}) \\ \hline \bar{\boldsymbol{C}}_{\mathrm{Q}}(\boldsymbol{\alpha}) & \bar{\boldsymbol{D}}_{\mathrm{Q}}(\boldsymbol{\alpha}) \end{array}\right], \quad (17)$$

where $\bar{\boldsymbol{A}}_{\mathrm{Q}}(\boldsymbol{\alpha}) = \sum_{k=1}^{N_{\mathrm{K}}} \alpha_i \bar{\boldsymbol{A}}_{\mathrm{Q},i}$, $\bar{\boldsymbol{B}}_{\mathrm{Q}}(\boldsymbol{\alpha}) = \sum_{k=1}^{N_{\mathrm{K}}} \alpha_i \bar{\boldsymbol{B}}_{\mathrm{Q},i}$, $\bar{\boldsymbol{C}}_{\mathrm{Q}}(\boldsymbol{\alpha}) = \sum_{k=1}^{N_{\mathrm{K}}} \alpha_i \bar{\boldsymbol{C}}_{\mathrm{Q},i}$, $\bar{\boldsymbol{D}}_{\mathrm{Q}}(\boldsymbol{\alpha}) = \sum_{k=1}^{N_{\mathrm{K}}} \alpha_i \bar{\boldsymbol{D}}_{\mathrm{Q},i}$, and $\bar{\boldsymbol{A}}_{\mathrm{Q},i} \triangleq \boldsymbol{S}_{\mathrm{Q}}^{-1} \boldsymbol{S}_{\mathrm{Q},i} \boldsymbol{A}_{\mathrm{Q},i} \boldsymbol{S}_{\mathrm{Q},i}^{-1} \boldsymbol{S}_{\mathrm{Q}}$, $\bar{\boldsymbol{B}}_{\mathrm{Q},i} \triangleq \boldsymbol{S}_{\mathrm{Q}}^{-1} \boldsymbol{S}_{\mathrm{Q},i} \boldsymbol{B}_{\mathrm{Q},i}$, $\bar{\boldsymbol{C}}_{\mathrm{Q},i} \triangleq \boldsymbol{C}_{\mathrm{Q},i} \boldsymbol{S}_{\mathrm{Q},i}^{-1} \boldsymbol{S}_{\mathrm{Q}}$ and $\bar{\boldsymbol{D}}_{\mathrm{Q},i} \triangleq \boldsymbol{D}_{\mathrm{Q},i}$.

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