"Weak" Control for Human-in-the-Loop Systems

Masaki Inoue, *Member, IEEE*, and Vijay Gupta, *Senior Member, IEEE*

***Abstract*—We propose a control framework for human-in-the-loop systems, in which many human decision makers are involved in the feedback loop composed of a plant and a controller. The novelty of the framework is that the decision makers are *weakly controlled*; in other words, they receive a set of admissible control actions from the controller and choose one of them in accordance with their private preferences. For example, the decision makers can decide their actions to minimize their own costs or by simply relying on their experience and intuition. A class of controllers which output set-valued signals is designed such that the overall control system is stable independently of the decisions made by the humans. Finally, a learning algorithm is applied to the controller that updates the controller parameters to reduce the achievable minimal costs for the decision makers. Effective use of the algorithm is demonstrated in a numerical experiment.**

***Index Terms*—Human-in-the-loop system, stability, optimization, internal model control, robust control.**

I. Introduction

THIS letter is devoted to constructing a control framework for human-in-the-loop (HIL) systems, in which decision makers are involved in the feedback loop composed of a plant and a controller. In the last five decades, the HIL concept has been realized and developed significantly in the literature. Most works focus on cooperative operation of the human and autonomous robots. There have been a variety of frameworks for the analysis and design of such human-robots interaction (see the pioneering works and survey papers [1]–[3] and recent trials [4]–[8]).

Applications of HIL systems are now being proposed beyond such human-robots systems, where cooperation between human and robot is the key. Potential applications of HIL systems include, for example, demand response in power grids involving humans decisions [9], air traffic management that must include human factors for pilots and control centers [10], incentive-based control of intelligent transportation systems relying on humans smart decisions [11], and so on. In such systems, the priorities of the humans in the loop may be unknown to and misaligned with those of the system designer. To realize such systems and to further broaden the applications, a broader control framework for HIL systems is necessary.

Some works have tried to construct more general control frameworks for HIL systems in, e.g., [5]–[7] and [12]–[15]. In [14] and [15], humans are modeled as uncertainties or constraints, and various methods of compensating their negative actions are proposed. In [5]–[7], [12], and [13], humans are positively involved in the feedback loop of the controlled systems. In [5]–[7], humans are modeled as reference generators for autonomous controlled robots. This can be viewed as human decision-making being involved in the outer feedback loop of the overall control system. The cooperation of the human and inner controller is achieved by model predictive control scheme or using passivity-properties. In the problem setting of [12] and [13], humans are involved in the inner feedback loop. In particular, humans handle both actuation and measurement of the plant based on the request by the controller. In the control problem of [12] and [13], humans are characterized by the intermittency of their control actions or measurements and their spatial mobility. "Controlling humans" as is done in [12] and [13] is desirable for system managers, however, it may not always be acceptable for human operators or users. Infrastructure systems such as power grids [9] and transportation systems [11] must pursue human comfort, and any participation in a control scheme must be voluntary rather than through forcefully controlling humans. In such systems, system management while allowing some degree of freedom (DOF) to the human operators or users is needed. One approach for this is game theoretic, where the design of economic or behavioral incentives is pursued to nudge the behavior of the human users. However, this approach usually requires utility functions of humans to be known to the system designer, which is a strong assumption.

In this letter, we propose a novel modeling and control framework for HIL systems. In this framework, humans are interpreted as decision makers and are involved in the inner feedback loop of a plant and a controller. The humans handle the actuation at a plant based on the request by the controller. We aim to realize "weak control" of the HIL system, in which the controller does not impose "too severe" requests for the decision makers that completely consume the DOF of their decisions. Instead, the controller provides a set of admissible control actions to enable the decision makers to pursue their

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M. Inoue is with the Department of Applied Physics and Physico-Informatics, Keio University, Yokohama 223-8522, Japan (e-mail: minoue@appi.keio.ac.jp).

V. Gupta is with the Department of Electrical Engineering, University of Notre Dame, Notre Dame, IN 46556 USA (e-mail: vgupta2@nd.edu).

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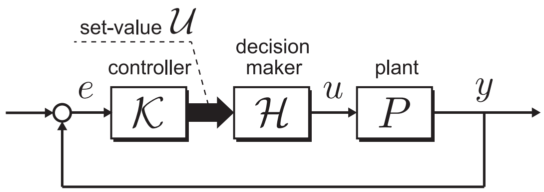

Fig. 1. A framework of weak control for human-in-the-loop system. The overall control system is composed of the plant P, controller $\mathcal{K}$, and decision maker (e.g., humans) $\mathcal{H}$. A set-valued signal $\mathcal{U}$ is generated by $\mathcal{K}$ and is provided to $\mathcal{H}$, and a signal u is chosen as $u \in \mathcal{U}$ by $\mathcal{H}$ to actuate P.

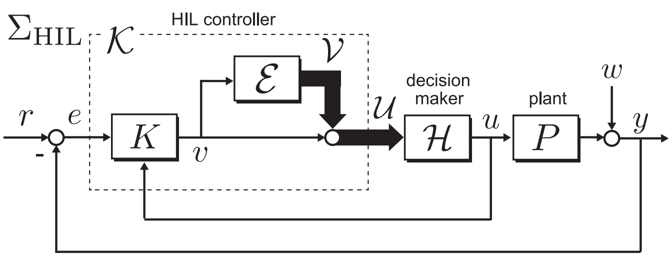

Fig. 2. HIL controller and overall control system $\Sigma_{\rm HIL}$.

own aims by utilizing the remaining DOF. The basic idea of weak control comes from study on low rank interaction [16] for hierarchical multi-agent systems.

In the rest of this letter, first, the problem of weak control for HIL systems is formulated, in which the human decision makers choose one control action u from a given set of admissible actions $\mathcal{U}$ as illustrated in Fig. 1. Then, the solution is derived based on the idea of internal model control (IMC, [17]). The resulting controller generates a set-valued signal, and it is shown that the overall control system is stable independently of the decisions made by the humans. Finally, a learning algorithm is applied to the controller that updates the controller parameters in order to reduce the achievable cost for the decision makers. Effective use of the framework is demonstrated in a numerical experiment.

Notation: Letting v and $\mathcal{V}$ be a signal and set-valued signal, respectively, their sum is defined as $v+\mathcal{V} := \{v+\tilde{v} \mid \tilde{v} \in \mathcal{V}\}$. The symbol I represents the identity operator. For a given set $\mathcal{V}$, the symbol $\mathcal{S}(\mathcal{V})$ represents an element of $\mathcal{V}$, i.e., $\mathcal{S}(\mathcal{V}) \in \mathcal{V}$ holds. For an input-output system Σ, $\|\Sigma\|$ represents some performance criterion of interest.

II. Human-in-the-Loop Control Systems

A. Problem Setting: Weak Control

In this section, we formulate and solve the problem of weak control for the HIL systems. The control structure for the HIL systems is illustrated in Fig. 2, which is a specialization of the conceptual diagram illustrated in Fig. 1. In Fig. 2, the plant P, decision maker $\mathcal{H}$, and controller $\mathcal{K}$ are connected to construct the overall control system $\Sigma_{\rm HIL}$.

The system description is given as follows. The signals r and w are called the reference and the disturbance, respectively. The plant P is a dynamical system that generates the output $y \in \mathbb{R}^\ell$ depending on the control input $u \in \mathbb{R}^m$. The model of P is described by

$$P : y = P(u) + w,$$

where $P(\cdot)$ is an operator. The decision maker $\mathcal{H}$ is a static system that generates $u(t)$ from a given input candidate $\mathcal{U}(t) \subset \mathbb{R}^m$ for all t.[1] The model of $\mathcal{H}$ is described by

$$\mathcal{H} : u(t) = \mathcal{S}(\mathcal{U}(t)), \tag{1}$$

or equivalently by $\mathcal{H} : u(t) \in \mathcal{U}(t)$. The operator $\mathcal{S}$ represents the decision by $\mathcal{H}$. The controller $\mathcal{K}$ is a dynamical system that generates $\mathcal{U}$ based on the error $e := r - y$ and u. The controller $\mathcal{K}$ is composed of an internal controller K and an *expander* $\mathcal{E}$. The signal v is generated by K and is *expanded* to a set-valued signal $\mathcal{V}$ by $\mathcal{E}$. The sum of v and $\mathcal{V}$ becomes the input candidate $\mathcal{U}$. The model of $\mathcal{K}$ is described by

$$\mathcal{K} : \begin{cases} v = K(e, u), \\ \mathcal{V} = \mathcal{E}(v), \\ \mathcal{U} = v + \mathcal{V}, \end{cases}$$

where $K(\cdot, \cdot)$ and $\mathcal{E}(\cdot)$ are operators.

The main characteristics of the proposed HIL system are the existence of a *set-valued* signal in the feedback loop. Due to this set-valued signal $\mathcal{U}$, we say that the HIL system is *weakly controlled*. This weak control framework allows us to express the case that decision makers can freely choose their own actions to some extent. Thus, they can pursue their own benefits or simply rely on their experience and intuition for their choices. This freedom can be a useful feature in many problems involving humans in smart infrastructure systems, where the priorities of the humans may be private information or misaligned with those of the system operator, yet the system operator should give the human users sufficient freedom to choose from among a set of possible actions.

The HIL control problem addressed in this letter is summarized in the following problem.

Problem 1 (HIL Control Problem): Find K and $\mathcal{E}$ such that $\Sigma_{\rm HIL}$ is input-output stable for all decisions by $\mathcal{H}$.

Note again that any strategy or model of $\mathcal{H}$ is unavailable for the design of K and $\mathcal{E}$ in the general problem setting. Only the rule (1) is known and available to the designer.

B. Signal Expander

Examples of signal expanders $\mathcal{E}$ are given in this subsection.

Example 1: An example of the expander is given by the following *rectangular prism* $\mathcal{E}_1$:

$$\mathcal{E}_1(v) = \left\{ {\rm diag}(\delta_1, \ldots, \delta_m)v \,\middle|\, \delta_i \in [-\gamma_i, \gamma_i] \right\},$$

where $\gamma_i, i \in \{1, 2, \ldots, m\}$ are positive constants. Equivalently, this $\mathcal{E}_1$ is written as

$$\mathcal{E}_1(v) = \left\{ \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_m \end{bmatrix} \,\middle|\, \varepsilon_i \in [-\gamma_i v_i, \gamma_i v_i] \right\}.$$

[1] It is implicitly assumed that the decision in $\mathcal{H}$ is fast enough compared with the dynamic behavior of P. Therefore, $\mathcal{H}$ is modeled as a static system.

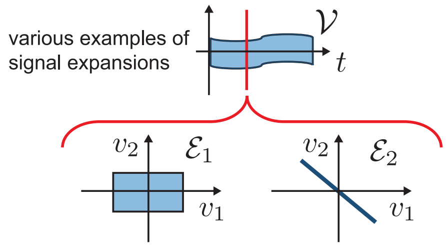

Fig. 3. Examples of signal expansions.

Example 2: The expander $\mathcal{E}_1$ is generalized to $\mathcal{E}_2$ with some coordinate transformation as:

$$\mathcal{E}_2(v) = \left\{ E_{\rm L}{\rm diag}(\delta_1, \ldots, \delta_p)E_{\rm R}^\top v \,\middle|\, \delta_i \in [-\gamma_i, \gamma_i] \right\},$$

where $p \leq m$ is a natural number, $E_{\rm L} \in \mathbb{R}^{m\times p}$ and $E_{\rm R} \in \mathbb{R}^{m\times p}$ are matrices of full column ranks. By the introduction of E_L and E_R, the signal v is expanded more flexibly than $\mathcal{E}_1(v)$. Let us consider a simple example of $\mathcal{E}_2$. We define

$$E_{\rm L} = E_{\rm R} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

Then, $\mathcal{E}_2(v)$ is reduced to

$$\mathcal{E}_2(v) = \left\{ \frac{1}{2}\begin{bmatrix} \varepsilon \\ -\varepsilon \end{bmatrix} \,\middle|\, \varepsilon \in [-\gamma_2|v_1 - v_2|, \gamma_2|v_1 - v_2|] \right\}.$$

We see that this $\mathcal{E}_2(v)$ expands the signal v such that the sum of the elements is invariant.

The set-valued signal $\mathcal{V}$ generated by $\mathcal{E}_i$, $i \in \{1, 2\}$ is illustrated in Fig. 3. The resulting $\mathcal{U} := v + \mathcal{V}$ must be a constraint for $\mathcal{H}$ of decision making.

Remark 1: Consider here that multiple decision makers $\mathcal{H}_i$, $i \in \{1, \ldots, m\}$ are included in $\mathcal{H}$ and choose their actions u_i, $i \in \{1, \ldots, m\}$, respectively by pursuing their own aims. We note that $\mathcal{U}$ generated by $\mathcal{E}_2$ implicitly requires *coordination* between $\mathcal{H}_i$, $i \in \{1, \ldots, m\}$ for their decision-making, while $\mathcal{E}_1$ does not. The decision makers $\mathcal{H}_i$, $i \in \{1, \ldots, m\}$ must cooperate or negotiate each other to determine their actions satisfying the *constraint* $u \in \mathcal{U}$ for the case $\mathcal{E}_2$.

C. Weak Control: IMC-Based Approach

In this subsection, we give a general solution to the HIL control problem, which is formulated in Problem 1.

First, the HIL control problem is reduced to a robust control problem as follows. Noting that $\mathcal{U} = v + \mathcal{V}$, the behavior of $\mathcal{H}$ is equivalently expressed as

$$\mathcal{H} : \begin{cases} d(t) = \mathcal{S}(\mathcal{V}(t)), \\ u(t) = v(t) + d(t). \end{cases}$$

This transformation is illustrated in Fig. 4. Letting Δ be

$$\Delta : d(t) = \mathcal{S}(\mathcal{E}(v(t))), \tag{2}$$

or more simply $\Delta : d(t) \in \mathcal{E}(v(t))$ as illustrated in Fig. 4(b), we reduce the overall control system $\Sigma_{\rm HIL}$ to the system illustrated in Fig. 5. The system illustrated in Fig. 5 represents a control system addressed in a robust control problem with the time-varying uncertainty Δ.

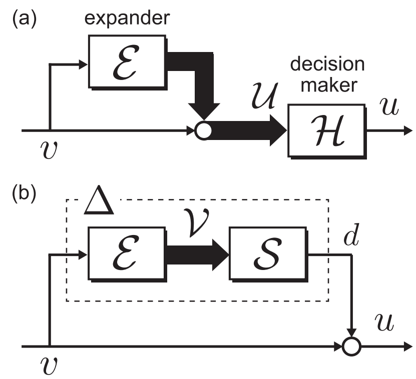

Fig. 4. Transformation of expander $\mathcal{E}$ and decision maker $\mathcal{H}$.

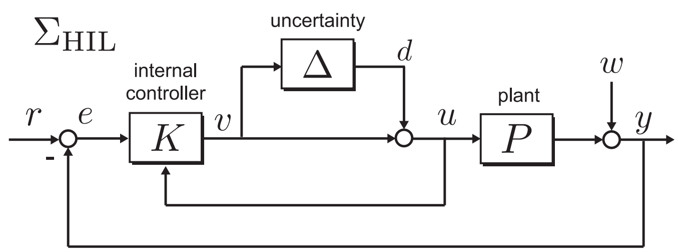

Fig. 5. Transformation of overall control system $\Sigma_{\rm HIL}$.

This transformation implies that the HIL control problem is essentially a robust control problem. Still, there are some practical differences between the traditional robust control problems and here. The HIL control *positively* utilizes the uncertainty for the signal expansion, which brings some benefit to $\mathcal{H}$. On the other hand, robust control focuses mainly on the *negative effect* of the uncertainty. In addition, the uncertainty in the HIL control is *designable* to achieve some aims, while that in robust control literature is not. Details of design examples and applications are given in Section II-B and Section III.

Next, we design the internal controller K, which is also depicted as a block in Fig. 5. In particular, we propose a special structure in K to guarantee the stability of the overall system $\Sigma_{\rm HIL}$ independently of the decisions made by $\mathcal{H}$.

To this end, the internal controller K is given by

$$K : v = K_e(e + P(u)), \tag{3}$$

where $K_e(\cdot)$ is an operator. Note that the controller structure in (3) involves the plant model P. The structure is based on internal model control (IMC, [17]), which plays a role of opening the feedback loop and extracting model uncertainties. By applying (3) to $\Sigma_{\rm HIL}$, we obtain the following theorem.

Theorem 1: Suppose that $K(e, u)$ is given by (3). Then, if P and K_e are L_2-stable, $\Sigma_{\rm HIL}$ is L_2-stable for all decisions of $\mathcal{H}$.

Proof: We recall that

$$\begin{aligned} y &= P(u) + w, \\ u &= v + \Delta(v), \\ v &= K_e(r - y + P(u)) \end{aligned}$$

hold, where Δ is the operator that represents the input-output map (2). By summarizing the equations, we obtain the expression

$$\Sigma_{\mathrm{HIL}} : y = P(K_e(r-w) + \Delta(K_e(r-w))) + w. \tag{4}$$

From the cascaded and parallel structure in (4), we see that the statement of the theorem holds. ■

As stated in the proof of the theorem, the implementation of the IMC-based controller (3) results in the cascaded and parallel structure in Σ_{HIL}. The structure contributes to the stability guarantee independently of the expander $\mathcal{E}$ and the decision in $\mathcal{H}$, which is described by $\mathcal{S}$ in (2). In addition, the structure enables us to easily evaluate the performance of Σ_{HIL} as follows. For simplicity, let us consider a linear regulation control problem; it is assumed that $r = 0$ and that both P and K_e are linear. Then, $P(x)$ and $K_e(x)$ are simply written as Px and $K_e x$, respectively. This reduces the expression (4) to

$$\Sigma_{\mathrm{HIL}} : y = (I - P(I+\Delta)K_e)w. \tag{5}$$

Supposing $\Delta = 0$, i.e., v is not expanded in $\mathcal{E}$ or $u \equiv v$ is chosen in $\mathcal{H}$, we can evaluate the nominal performance $\|I - PK_e\|$ in some criterion such as the L_2 gain. We emphasize that the performance $\|I - P(I+\Delta)K_e\|$ is *continuously and linearly* deteriorated from the nominal one with the increase of $\|\Delta\|$. This enables us to simply evaluate the bound of $\|I - P(I+\Delta)K_e\|$. The continuity of the performance deterioration is called *persistence* and analyzed for general uncertain systems in [18].

Remark 2: A design strategy of K_e and $\mathcal{E}$ is given in this remark. First, we design K_e such that the desired nominal performance is achieved; for example, minimize the performance as $\min \|I - PK_e\| =: \rho$. Then, determine the degree of the expansion in $\mathcal{E}$, which is characterized by, e.g., γ_i of $\mathcal{E}_i$, $i \in \{1, 2\}$. We design $\mathcal{E}$ such that the performance deterioration is admissible for the designer, who is responsible for the overall control system; for example, for a given $\Delta\rho > 0$, find or maximize γ such that

$$\|I - P(I+\Delta)K_e\| \le \rho + \Delta\rho \tag{6}$$

holds for all decisions in $\mathcal{H}$ satisfying (1).

III. Learning of Human Preferences for Updating the Controller

In the general problem formulated in Section II, no assumption is imposed on the decision maker $\mathcal{H}$ except for the rule (1). In this section, it is assumed that $\mathcal{H}$ is rational and determines the control action u based on an optimization; a cost function is minimized under the constraint (1). Then, we design and implement a mechanism of learning a part of the model in $\mathcal{H}$ and of updating the expander $\mathcal{E}$ online.

A. Problem Setting

The models of the plant P and controller $\mathcal{K}$ are specialized in the following discussion. For simplicity, we consider a linear regulation problem under the step disturbance; $r = 0$, w is the unit step signal, both P and K_e are linear, and the overall control system is expressed by (5). The following discussion can be extended to other practical cases, e.g., tracking control with $r \neq 0$, persistent disturbance to w, nonlinear plant systems, and so on, with some modification. In addition, the structure of $\mathcal{E}$ is fixed at $\mathcal{E}_2$, which is defined in Example 2. In addition, $\mathcal{E}$ has only one dimensional degree of freedom; letting E_{L} and E_{R} be vectors in $\mathbb{R}^m$, $\mathcal{E}$ is described by

$$\mathcal{E}(v) = \mathcal{E}_2(v) = \left\{ \delta E_{\mathrm{L}} E_{\mathrm{R}}^\top v \,\middle|\, \delta \in [-\gamma, \gamma] \right\}, \tag{7}$$

where γ is a positive constant. Note here that E_{L} represents the *direction* of the expansion, while $\gamma|E_{\mathrm{R}}^\top v|$ represents the degree of the expansion.

We consider that the following optimization algorithm is implemented in $\mathcal{H}$.

$$\mathcal{H} : \begin{cases} \min & f(u) \\ \text{subject to} & u \in \mathcal{U}. \end{cases} \tag{8}$$

The global minimizer of the *unconstrained* optimization, simply $\min f(u)$, is denoted by u^*, while that of the constrained one, described by (8), is denoted by $u^\dagger$. Trivially, $f(u^*) \le f(u^\dagger)$ holds. Note that the achievable minimum cost $f(u^\dagger)$ depends on $\mathcal{U}$, and therefore, it depends on the designed $\mathcal{E}$. The aim of this section is to find $\mathcal{E}$ that minimizes the achievable minimum cost $f(u^\dagger)$ subject to some performance specification on Σ_{HIL}.

To formulate the problem in a clearer manner, we define a specific set of expanders $\mathcal{E}$, which is essentially the same as a set of triplets $\{E_{\mathrm{L}}, E_{\mathrm{R}}, \gamma\}$, as follows. Let ρ be the nominal performance $\rho := \|I - PK_e\|$.

Notation 1: For a given positive constant $\Delta\rho$, the symbol $\{\mathcal{E}\}_{\Delta\rho}$ represents the set of the expanders $\mathcal{E}$ such that for any element in $\{\mathcal{E}\}_{\Delta\rho}$, the inequality in (6) holds for all decisions by $\mathcal{H}$, i.e., all realizations of Δ. In addition, $\{\mathcal{U}(v)\}_{\Delta\rho} := \{v + \mathcal{E}(v) \mid \mathcal{E}(v) \in \{\mathcal{E}(v)\}_{\Delta\rho}\}$, which represents the set of all input candidates $\mathcal{U}$ generated by $\mathcal{E}(v) \in \{\mathcal{E}(v)\}_{\Delta\rho}$.

The problem addressed in the rest of this section is formulated as follows.

Problem 2: For a given $\Delta\rho$, find $\mathcal{E} \in \{\mathcal{E}\}_{\Delta\rho}$ that minimizes $f(u^\dagger)$ at the steady state.

In the next subsection, the solution method by updating $\mathcal{E} \in \{\mathcal{E}\}_{\Delta\rho}$ is given.

B. Learning Algorithm for Updating Expander

The graphical interpretation of u^*, $u^\dagger$, v, $\{\mathcal{U}(v)\}_{\Delta\rho}$, and $f(u)$ is illustrated in Fig. 6. We see that the generated $\mathcal{U}(v) \in \{\mathcal{U}(v)\}_{\Delta\rho}$ illustrated in Fig. 6 (b) is more beneficial for $\mathcal{H}$ than Fig. 6 (a); the achievable cost $f(u^\dagger)$ is reduced by the update of $\mathcal{E}$. We aim to find the best $\mathcal{E} \in \{\mathcal{E}\}_{\Delta\rho}$ in this sense.

For updating $\mathcal{E}$, we first estimate u^* by using some data set $\{v_k, u_k^\dagger\}$, where k is the discrete time. Let $E_{\mathrm{L}0}, E_{\mathrm{L}1}, \cdots, E_{\mathrm{L}k}$ be the sequence of the updated E_{L}. We suppose that

$$|u_k^\dagger - v_k| < \gamma |E_{\mathrm{L}k} E_{\mathrm{R}}^\top v_k| \tag{9}$$

holds, which implies that $u_k^\dagger$ is located on the interior of $\mathcal{U}(v_k)$ as illustrated in Fig. 6 (a). Then, it follows that u^* is located on the hyperplane described by $E_{\mathrm{L}k}^\top (u^* - u_k^\dagger) = 0$, which is graphically shown in Fig. 6 (a). The set of the hyperplanes is expressed by the vector form

$$[E_{\mathrm{L}0}\, E_{\mathrm{L}1}\, \ldots\, E_{\mathrm{L}k}]^\top u^* - [E_{\mathrm{L}0}^\top u_0^\dagger\, E_{\mathrm{L}1}^\top u_1^\dagger\, \ldots\, E_{\mathrm{L}k}^\top u_k^\dagger]^\top = 0.$$

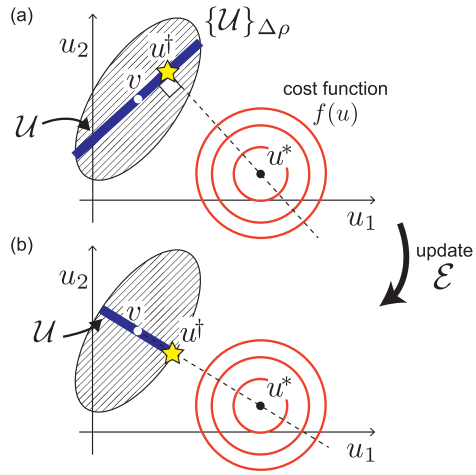

Fig. 6. Graphical interpretation of the input candidate $\mathcal{U}$ and decision by $\mathcal{H}$. If $\mathcal{U} \in \{\mathcal{U}\}_{\Delta\rho}$ is provided by the controller $\mathcal{K}$, rational $\mathcal{H}$ chooses $u^\dagger$, which is the minimizer of the constrained optimization problem (8).

Algorithm 1 Updating Expander $\mathcal{E}$

Initialization: $E_{\mathrm{L}k}, E_{\mathrm{R}k}, \gamma_k$ at $k = 0$
repeat
  get data $\{v_k, u_k^\dagger\}$ that satisfies (9)
  if $E_{\mathrm{ex}k}$ is of full row rank **then**
    $E_{\mathrm{L}k+1} \leftarrow v_k - u^*$, where u^* is given by (10)
  else
    $E_{\mathrm{L}k+1} \leftarrow E_{\mathrm{L}k} + \delta$, where δ is a small perturbation
  end if
  find $E_{\mathrm{R}k+1}, \gamma_{k+1}$ maximizing $\gamma_{k+1}|E_{\mathrm{R}k+1}^\top v_k|$ subject to $\mathcal{E} \in \{\mathcal{E}\}_{\Delta\rho}$
  return $E_{\mathrm{L}k+1}, E_{\mathrm{R}k+1}, \gamma_{k+1}$
  $k \leftarrow k+1$
until $E_{\mathrm{L}k}, E_{\mathrm{R}k}, \gamma_k$ converge

If $E_{\mathrm{ex}k} := [E_{\mathrm{L}0}\, E_{\mathrm{L}1}\, \dots\, E_{\mathrm{L}k}]$ is of full row rank, we obtain the estimate of u^* as

$$u^* = (E_{\mathrm{ex}k}E_{\mathrm{ex}k}^\top)^{-1}E_{\mathrm{ex}k}[E_{\mathrm{L}0}^\top u_0^\dagger\ E_{\mathrm{L}1}^\top u_1^\dagger\ \cdots\ E_{\mathrm{L}k}^\top u_k^\dagger]^\top. \quad (10)$$

The estimate of u^* is utilized for updating $\mathcal{E}$. The algorithm for the update is briefly stated as follows.

In the algorithm above, it is assumed that $u_k^\dagger$ is available for updating $\mathcal{E}$. We justify the assumption as follows. We emphasize that the update can bring benefits only to the *decision maker* $\mathcal{H}$, not to the system manager or controller designer who is responsible for the performance of Σ_{HIL}. The benefits for $\mathcal{H}$ can be incentive to disclose some information of $\mathcal{H}$. It is thus natural to assume that the result of the decision, denoted by $u^\dagger$, is disclosed and available for the update of $\mathcal{E}$.

IV. Numerical Experiment

The plant P, decision maker $\mathcal{H}$, and controller $\mathcal{K}$ are given as follows. The plant P is the linear dynamical system described by

$$P: \begin{cases} \dot{x} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -0.5 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} w + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} u, \\ y = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} x. \end{cases}$$

The step disturbance is injected to w to drive the plant system. The transfer matrices from u and w to y are denoted by $\bar{P}_u(s)$ and $\bar{P}_w(s)$, $s \in \mathbb{C}$, respectively. Then, the DC gains of $\bar{P}_u(s)$ and $\bar{P}_w(s)$ are given by

$$\bar{P}_u(0) = \begin{bmatrix} 1 & 0.5 & 2 \end{bmatrix}, \quad \bar{P}_w(0) = 3.5,$$

respectively. In the decision maker $\mathcal{H}$, the following optimization algorithm is implemented.

$$\mathcal{H}: \begin{cases} \min & f(u) := 2u^\top u - \begin{bmatrix} 1 & 0 & 4 \end{bmatrix} u, \\ \text{subject to} & u \in \mathcal{U}. \end{cases}$$

This optimization model is blind for the design of the controller. The controller $\mathcal{K}$ is described by

$$\mathcal{K}: \begin{cases} v = K_e(r - y + P_u u), \\ \mathcal{U} = v + \mathcal{E}(v), \end{cases}$$

where K_e is a static system, i.e., it is simply a constant matrix, and P_u is the operator representation of $\bar{P}_u(s)$. In this section, we demonstrate the design procedure of K_e and $\mathcal{E}$.

The performance criterion for Σ_{HIL} is the DC gain, which represents the disturbance suppression performance $|y(t)|$ as $t \to \infty$ corresponding to the unit step disturbance $w(t)$. The performance criterion for $\mathcal{H}$ is the value of $f(u^\dagger(t))$. We aim to minimize $f(u^\dagger(t))$ as $t \to \infty$ subject to the specification $|y(t)| \le 0.2$ as $t \to \infty$, denoted by $|\Sigma_{\mathrm{HIL}}|_{\mathrm{dc}} \le 0.2$.

First, K_e is designed as

$$K_e = \frac{1}{6}\begin{bmatrix} 2 & 4 & 1 \end{bmatrix},$$

which achieves $|y(t)| \to 0$ as $t \to \infty$ in the *nominal* situation; in other words, if the expander $\mathcal{E}$ is inactive, the step disturbance $w(t)$ does not propagate to $y(t)$ as $t \to \infty$. We see this fact as follows. Note that $(1 - \bar{P}_u(s)K_e)\bar{P}_w(s)$ represents the transfer function of Σ_{HIL} when $\mathcal{E}(v) \equiv 0$. The above K_e guarantees that $(1 - \bar{P}_u(0)K_e)\bar{P}_w(0) = 0$ holds.

Next, the structure of $\mathcal{E}$ is fixed as (7). The *initial condition* of E_{L} and E_{R} is given by

$$E_{\mathrm{L}0} := \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^\top, \quad E_{\mathrm{R}0} := \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}.$$

The value of γ is determined such that the DC gain specification $|\Sigma_{\mathrm{HIL}}|_{\mathrm{dc}} \le 0.2$ holds. The specification is expressed as

$$\begin{aligned} &|(1 - \bar{P}_u(0)(I_3 + \delta E_{\mathrm{L}}E_{\mathrm{R}}^\top)K_e)\bar{P}_w(0)| \\ &\quad = \delta|\bar{P}_u(0)E_{\mathrm{L}}E_{\mathrm{R}}^\top K_e \bar{P}_w(0)| \le 0.2 \end{aligned}$$

holds for all $\delta \in [-\gamma, \gamma]$. By maximizing γ under the inequality, we obtain the initial value of γ as

$$\gamma_0 = 0.2/|\bar{P}_u(0)E_{\mathrm{L}0}E_{\mathrm{R}0}^\top K_e \bar{P}_w(0)| = 0.0848.$$

Then, the updating algorithm proposed in Section III is applied to update E_{L} and γ, while E_{R} is fixed at $E_{\mathrm{R}0}$.

The numerical experiments are performed for the following four cases; 1) no feedback controller is applied, 2) the standard IMC is applied without the expander $\mathcal{E}$, 3) the controller $\mathcal{K}$ is applied with fixed $\mathcal{E}$, i.e., $\mathcal{E}$ is composed of $E_{\mathrm{L}} = E_{\mathrm{L}0}$, $E_{\mathrm{R}} = E_{\mathrm{R}0}$, and $\gamma = \gamma_0$, and 4) the controller $\mathcal{K}$ is applied with updating $\mathcal{E}$. The experiment, the time step is fixed at 1 sec, and the continuous models in P and $\mathcal{K}$ are discretized.

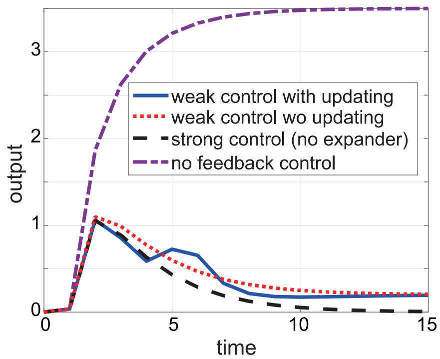

Fig. 7. Trajectory of regulated output $y(t)$. The blue solid, red dotted, and black dashed lines represent the costs achieved by the weak control with the learning mechanism, weak control without any learning mechanism, and strong control, i.e., control without any expander, respectively. The purple dot-dash line represents the case with no feedback control.

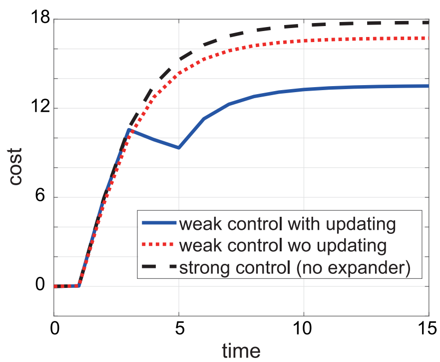

Fig. 8. Trajectory of cost $f(u^\dagger(t))$ for $\mathcal{H}$. The blue solid, red dotted, and black dashed lines represent the costs achieved by the weak control with the learning mechanism, weak control without any learning mechanism, and strong control, i.e., control without any expander, respectively.

At each time step, the optimization problem in $\mathcal{H}$ is solved, and the expander $\mathcal{E}$ is updated.

The trajectories $y(t)$ for all cases are illustrated in Fig. 7. We see that the feedback control effectively suppresses the disturbance effects in $y(t)$. The control in Case 2 results in the best performance, while the weak control in Cases 3 and 4 satisfies the specification, $|y(t)| \leq 0.2$ at a large t.

The values of the cost $f(u^\dagger(t))$ for all cases are illustrated in Fig. 8. We see that the costs achieved by the weak control in Cases 3 and 4 are smaller than that by the standard IMC in Case 2. This demonstrates that the expander $\mathcal{E}$ brings smaller costs for decision makers $\mathcal{H}$. Furthermore, we compare Cases 3 and 4 to show the effectiveness of the updating algorithm. The weak control with updating $\mathcal{E}$ in Case 4 contributes to reducing the cost compared with no updating case in Case 3 as illustrated in Fig. 8. It should be emphasized that Case 4 further reduces the cost while keeping the same DC gain performance in Σ_{HIL} as illustrated in Fig. 7.

V. Conclusion

We proposed a framework for *weak control* for HIL systems. In this framework, a signal *expander* is embedded in the controller and generates admissible control actions with some DOF. The DOF allows the human decision-makers to pursue their own aims, while guaranteeing the stability and the specified performance in the overall control system. A simple algorithm of updating the expander was also given, which was beneficial to human decision makers. Finally, a numerical experiment was performed, where the weak control was compared only with the standard IMC. No comparison with previous HIL approaches is because the problem setting of the weak control is completely different from that of other HIL approaches.

There are a variety of future works for the weak control. A limitation of the proposed weak control is in the assumption that the result of decision-making, denoted by u, is available for control. The assumption does not always hold for real-world HIL control problems such as demand response in power grids [9]. Relaxation of the assumption needs to be addressed.

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