A Low-Complexity SCMA Detector for AWGN Channel Based on Solving Overdetermined Systems of Linear Equations

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***Abstract*—Sparse code multiple access (SCMA) is one of proposed non-orthogonal multiple access schemes for Fifth generation (5G) wireless communication standard. Maximum likelihood (ML) algorithm for SCMA detection has very high complexity and can not be implemented in real-time applications. Message Passing Algorithm (MPA) has smaller complexity, but it is still high. In this paper, we propose a new SCMA detector for AWGN channel which complexity is smaller than that of simplified log-domain MPA. This algorithm is based on antipodal symmetry of codewords in codebooks. Because of this property decoding process is reduced to the solution of a small number of overdetermined systems of linear equations. The performance of the proposed algorithm is equivalent to Max-Log-MPA with 2–3 iterations. The implementation of the algorithm is well parallelized, calculations of inverse matrices are performed in advance and obtained results are stored in memory, i. e. operations with very high computational complexity need not be performed in real-time application.**

I. Introduction

Fifth generation (5G) wireless communication standard requires higher spectral efficiency, massive connectivity and lower latency. 5G is expected to be commercially deployed in 2020, therefore currently a lot of research is being carried out. One of the main applications of this technology is the Internet of Things (IoT). 5G systems should support 100 billion connections, data rate of several tens of megabits per second for thousands of users and 1 ms latency [1]. Non-orthogonal multiple access (NOMA) schemes [2] are possible solutions to increase the number of users inside a given time-frequency resource. Unlike conventional orthogonal multiple access techniques such as frequency division, time division and code division multiple access, NOMA introduces some controllable interference to implement overloading at the cost of increased receiver complexity. As a result, higher spectral efficiency and massive connectivity can be achieved [3]. NOMA is divided into two types: power-domain multiplexing and code-domain multiplexing. SCMA [4] is a scheme of the second type and a possible candidate of NOMA for 5G. This system is improved Low Density Signature (LDS) [5] scheme. The main advantage over LDS is some potential gain of multi-dimensional constellation shaping [2]. Another advantage of SCMA over many uplink schemes (UL) is the ability to provide grant-free UL data transmission that increases spectral efficiency of the system [6].

The detection of SCMA can be implemented using Maximum Likelihood (ML) algorithm or simplified Message Passing Algorithm (MPA) in log-domain with different logarithmic approximations, but even the latter has a high computational complexity.

In this paper, we proposed a new algorithm with low computational complexity for SCMA detection in AWGN channel based on codebook structure and solution of overdetermined systems of linear equations.

II. SCMA Description

A. SCMA Encoding

An SCMA encoding procedure is defined as a mapping from m bits to an K-dimensional complex codebook of size M, where $M = 2^m$ [4]. K-dimensional complex codewords consist of $N < K$ non-zero elements. Each user j has a unique codebook from the set of J codebooks, i. e. J users (usually called layers) can transmit information over K orthogonal resources simultaneously. The overloading factor is defined as $\lambda = J/K$. An example of codebook set for $J = 6$ and $K = 4$ is presented below [7]:

$$\mathbf{CB}_1 = \begin{bmatrix} 0 & -0.1815 - 0.1318j & 0 & 0.7851 \\ 0 & -0.6351 - 0.4615j & 0 & -0.2243 \\ 0 & 0.6351 + 0.4615j & 0 & 0.2243 \\ 0 & 0.1815 + 0.1318j & 0 & -0.7851 \end{bmatrix}^{\mathrm{T}},$$

$$\mathbf{CB}_2 = \begin{bmatrix} 0.7851 & 0 & -0.1815 - 0.1318j & 0 \\ -0.2243 & 0 & -0.6351 - 0.4615j & 0 \\ 0.2243 & 0 & 0.6351 + 0.4615j & 0 \\ -0.7851 & 0 & 0.1815 + 0.1318j & 0 \end{bmatrix}^{\mathrm{T}},$$

$$\mathbf{CB}_3 = \begin{bmatrix} -0.6351 + 0.4615j & 0.1392 - 0.1759j & 0 & 0 \\ 0.1815 - 0.1318j & 0.4873 - 0.6156j & 0 & 0 \\ -0.1815 + 0.1318j & -0.4873 + 0.6156j & 0 & 0 \\ 0.6351 - 0.4615j & -0.1392 + 0.1759j & 0 & 0 \end{bmatrix}^{\mathrm{T}},$$

$$\mathbf{CB}_4 = \begin{bmatrix} 0 & 0 & 0.7851 & -0.0055-0.2242j \\ 0 & 0 & -0.2243 & -0.0193-0.7848j \\ 0 & 0 & 0.2243 & 0.0193+0.7848j \\ 0 & 0 & -0.7851 & 0.0055+0.2242j \end{bmatrix}^{\mathrm{T}},$$

$$\mathbf{CB}_5 = \begin{bmatrix} -0.0055-0.2242j & 0 & 0 & -0.6351+0.4615j \\ -0.0193-0.7848j & 0 & 0 & 0.1815-0.1318j \\ 0.0193+0.7848j & 0 & 0 & -0.1815+0.1318j \\ 0.0055+0.2242j & 0 & 0 & 0.6351-0.4615j \end{bmatrix}^{\mathrm{T}},$$

$$\mathbf{CB}_6 = \begin{bmatrix} 0 & 0.7851 & 0.1392-0.1759j & 0 \\ 0 & -0.2243 & 0.4873-0.6156j & 0 \\ 0 & 0.2243 & -0.4873+0.6156j & 0 \\ 0 & -0.7851 & -0.1392+0.1759j & 0 \end{bmatrix}^{\mathrm{T}},$$

where $\mathbf{CB}_j$ is a codebook for user j.

The columns of codebooks are codewords, thus every user maps $m = 2$ bits to one of $M = 4$ four-dimensional codewords.

SCMA codewords are transmitted over K shared resource elements (RE), e. g. orthogonal frequency division multiple access subcarriers. Users' placement on REs (i. e., codebook sparsity) can be described by a factor graph (Fig. 1) [7]. This structure is equivalent to the structure of low density parity check codes (LDPC). Circles correspond to users, while rectangles correspond to REs.

B. SCMA Detection

After transmitting over channel, received signal is expressed by the following equation:

$$\mathbf{y} = \sum_{j=1}^{J} \operatorname{diag}(\mathbf{h}_j)\mathbf{x}_j + \mathbf{n}, \tag{1}$$

where $\mathbf{x}_j = (x_{1j}, \ldots, x_{Kj})^{\mathrm{T}}$ is the SCMA codeword of user j, $\mathbf{h}_j = (h_{1j}, \ldots, h_{Kj})^{\mathrm{T}}$ is a channel coefficients vector of user j and $\mathbf{n}$ is a complex additive white Gaussian noise with zero mean and σ^2 variance, i. e. $\sigma^2/2$ per in-phase and quadrature components. In AWGN channel, $h_{kj} = 1$.

The signal (1) can be detected by ML algorithm, but it has very large complexity, $O(M^J)$, that increases exponentially with the number of users J and polynomially with the codebook size M [4]. For many users and/or large codebook size, ML detection is not feasible in real-time applications. Fortunately, there is an iterative suboptimal algorithm with a lower computational complexity. MPA has complexity $O\left(M^{d_f}\right)$ per RE per iteration, where d_f is the number of users contributing to every RE [4], however, the energy costs in the receiver are still significant. The procedure of detection is similar to decoding of LDPC codes. The detailed description of the algorithms can be found, for example, in [8]. The detail analysis of computational complexity of algorithms is presented in Section IV.

III. Detector Based on Solving Overdetermined Systems of Linear Equations

The proposed algorithm is based on codebook structure and solution of overdetermined systems of linear equations.

A. Codebook Structure

Each codebook $\mathbf{CB}_j$ contains M codewords. There are two pairs of antipodal codewords (1st–4th and 2nd–3rd columns), i. e. each pair has linearly dependent vectors with coefficient -1. We introduce the matrix $\mathbf{C}$ with size $K \times (M/2)J$, which contains only linearly independent codewords from all codebooks. For example, for presented codebooks $\mathbf{CB}_j$ matrix $\mathbf{C}$ contains 12 codewords (1st and 2nd columns from all codebooks). Thus received signal (1) after AWGN channel can be written as

$$\mathbf{y} = \mathbf{C}\mathbf{a} + \mathbf{n}, \tag{2}$$

where $\mathbf{a}$ is a vector containing $(M/2)J$ ternary elements equal to 0, $+1$ or -1.

The system (2) has J unknowns and K equations, thus it is a underdetermined system of linear equations. This system either has an infinite number of solutions or no solutions at all, but we can transform it to an overdetermined system of equations.

Let vector $\mathbf{g}$ be a column vector containing J non-zero elements of $\mathbf{a}$, and matrix $\mathbf{S}$ be a $K \times J$ matrix containing codewords from $\mathbf{C}$, which corresponds to non-zero elements of $\mathbf{a}$. Each combination of non-zero elements in vector $\mathbf{a}$ has its own matrix $\mathbf{S}$ and vector $\mathbf{g}$. There are $(M/2)^J$ such combinations. Below is an example of one of them.

Example: Vector $\mathbf{a}$ for 1st, 2nd, 3rd, 4th, 3rd, 1st codewords of 1st–6th users is shown as

$$\mathbf{a} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & -1 & -1 & 0 & 0 & -1 & 1 & 0 \end{bmatrix}^{\mathrm{T}},$$

and vector $\mathbf{g}$ is

$$\mathbf{g} = \begin{bmatrix} 1 & 1 & -1 & -1 & -1 & 1 \end{bmatrix}^{\mathrm{T}}.$$

The matrix $\mathbf{S}$ (shown values are rounded just to fit matrix into page width):

$$\mathbf{S} = \begin{bmatrix} 0 & -.182-.132j & 0 & .785 \\ -.224 & 0 & -.635-.462j & 0 \\ .182-.132j & .487-.616j & 0 & 0 \\ 0 & 0 & .785 & -.006-.224j \\ -.019-.785j & 0 & 0 & .182-.132j \\ 0 & .785 & .139-.176j & 0 \end{bmatrix}^{\mathrm{T}}.$$

B. Overdetermined Systems of Linear Equations

Matrix $\mathbf{S}$ and vectors $\mathbf{g}$ and $\mathbf{n}$ contain complex values. We can replace each equation with complex variables by two equations with real-valued variables. So (2) will be transformed as

$$\mathbf{y}_{\text{real}} = \mathbf{S}_{\text{real}}\mathbf{g} + \mathbf{n}_{\text{real}}, \tag{3}$$

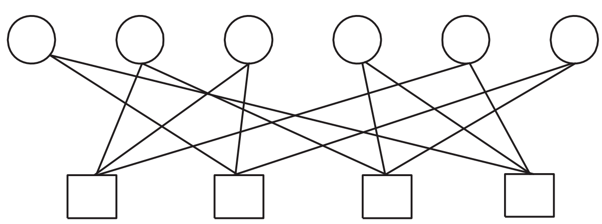

Fig. 1. Factor graph ($J = 6$, $K = 4$, $\lambda = 1.5$)

where $\mathbf{y}_{\text{real}}$ and $\mathbf{n}_{\text{real}}$ are column vectors with $2K$ elements, and $\mathbf{S}_{\text{real}}$ has size $2K \times J$.

The equation (3) is an overdetermined system of linear equations. This system can be solved in the least square sense:

$$\hat{\mathbf{g}} = \left(\mathbf{S}_{\text{real}}^T \mathbf{S}_{\text{real}}\right)^{-1} \mathbf{S}_{\text{real}}^T \mathbf{y}_{\text{real}}. \tag{4}$$

The estimate of received codewords combination is obtained as a product of codeword matrix $\mathbf{S}$ and the signs of the elements from solution vector:

$$\tilde{\mathbf{y}} = \mathbf{S}\,\text{sign}(\hat{\mathbf{g}}). \tag{5}$$

The estimate (5) corresponds to a combination of transmitted bits.

While solving the system of equations (4), we do not know locations of non-zero elements in vector $\mathbf{a}$. There are $(M/2)^J$ combinations of non-zero elements allocation (each combination has its own matrix $\mathbf{S}_{\text{real}}$). In this way we must solve $(M/2)^J$ systems of equations (4) with all possible combinations of non-zero elements allocation and choose the decision according to a particular criteria.

C. Decision Criteria

After solving all the equations (4) we have $(M/2)^J$ vectors $\hat{\mathbf{g}}_i$. There are several criteria for selecting the decision.

1) Minimum of residual: The L^1-norms based on residuals are calculated for all $(M/2)^J$ systems:

$$r_i = |\mathbf{S}_{i\text{real}} \hat{\mathbf{g}}_i - \mathbf{y}_{\text{real}}|,$$

where i is a number of system of equations.

The decision is a vector $\text{sign}(\hat{\mathbf{g}}_i)$ corresponding to the minimum residual r_i.

2) Minimum Euclidean norm of solution quantization error: The Euclidean distances between solutions and their quantized versions are calculated for all $(M/2)^J$ vectors $\hat{\mathbf{g}}_i$:

$$d_i = \|\hat{\mathbf{g}}_i - \text{sign}(\hat{\mathbf{g}}_i)\|.$$

The decision is a vector $\text{sign}(\hat{\mathbf{g}}_i)$ corresponding to the minimum Euclidean norm d_i.

3) Minimum Euclidean norm of estimation error: The Euclidean distances between estimate (5) and received signal are calculated for all $(M/2)^J$ systems of equations:

$$e_i = \|\mathbf{S}_i\,\text{sign}(\hat{\mathbf{g}}_i) - \mathbf{y}\|. \tag{6}$$

The decision is a vector $\text{sign}(\hat{\mathbf{g}}_i)$ corresponding to the minimum Euclidean norm e_i.

D. Algorithm

The algorithm for hard detection of SCMA with decision criterion 3 in AWGN channel is presented as the Algorithm 1. This criterion is chosen because as will be seen in Section V it provides the best performance.

Algorithm 1 Proposed algorithm (Decision criterion 3)

0: Calculate and store $(M/2)^J$ products $\left(\mathbf{S}_{\text{real}}^T \mathbf{S}_{\text{real}}\right)^{-1} \mathbf{S}_{\text{real}}^T$ (for each combination of non-zero elements in $\mathbf{a}$);
1: Solve $(M/2)^J$ overdetermined systems of linear equations (3) and obtain $(M/2)^J$ vectors (4);
2: Calculate Euclidean distances (6);
3: Choose vector $\text{sign}(\hat{\mathbf{g}}_i)$ corresponding to the minimum Euclidean distance (6);
4: Obtain hard decisions for received bits according to the quantized values $\text{sign}(\hat{\mathbf{g}}_i)$.

IV. Computational Complexity

A comparison of the computational complexity of three algorithms is presented below. We compared two common algorithms (ML and MPA) with proposed algorithm. In all cases, operations are counted as real-valued computations.

A. Maximum Likelihood Algorithm

ML algorithm requires Euclidean distance calculations. Square root is not calculated to reduce complexity. Thus for calculating square of Euclidean distance between received signal (3) and one of M^J possible codeword combinations $2K$ multiplications and $(4K-1)$ additions are required. For all codeword combinations, $2KM^J$ multiplications and $(4K-1)M^J$ additions are required. Furthermore, M^J comparisons are required to choose the minimum Euclidean distance.

It should be noted that the implementation of the algorithm needs to save in memory all M^J complex codeword combinations.

B. Message Passing Algorithm

We consider a MPA in log-domain, because Max-Log-MPA algorithm is used in practice having a modest computational complexity. According to [9], in AWGN channel Max-Log-MPA needs $3M^{d_f}Kd_f N_{\text{iter}}$ multiplications and $3M^{d_f}Kd_f^2 N_{\text{iter}}$ additions, where N_{iter} is a number of iterations. It also needs $M^{d_f}Kd_f N_{\text{iter}} + (N-2)MKd_f N_{\text{iter}}$ comparisons.

C. Proposed Algorithm

The proposed algorithm needs $2KJ$ multiplications and $(2K-1)J$ additions for solution of each system of equations (4). It also needs $2K(M/2)^J$ multiplications, $(4K-1)(M/2)^J$ additions and $(M/2)^J$ comparisons to choose decision by criteria 3 from Section III. In (5), it is required KJ trivial multiplications by ± 1 (they will not be taken into account) and $K(J-1)$ additions. The total number of required operations is $2K(J+1)(M/2)^J$ multiplications, $(3KJ+3K-J-1)(M/2)^J$ additions and $(M/2)^J$ comparisons.

Matrix products $\left(\mathbf{S}_{\text{real}}^T \mathbf{S}_{\text{real}}\right)^{-1} \mathbf{S}_{\text{real}}^T$ for $(M/2)^J$ systems of equations (4) are calculated in advance and stored in memory.

Table I shows the computational complexity of ML, Max-Log-MPA and proposed algorithm.

The comparisons of computational complexity are shown in Fig. 2–Fig. 4 for ML, Max-Log-MPA (labelled as MPA) with 1–3 iterations and proposed algorithm. The proposed algorithm has 22% and 48% complexity reduction in the number of multiplications compared with Max-Log-MPA with 2 and 3 iterations, respectively, about 64% and 76% complexity reduction in the number of additions, and 96% and 97% complexity reduction in the number of comparisons.

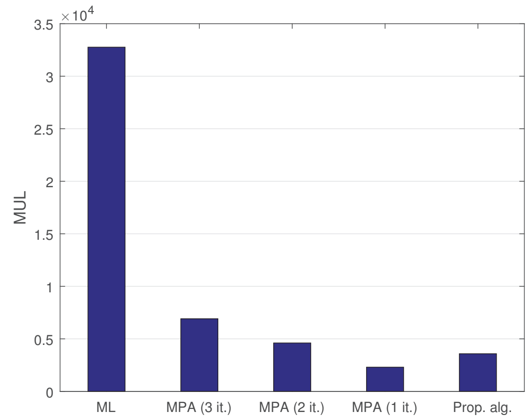

Fig. 2. Computational complexity comparison: number of multiplications

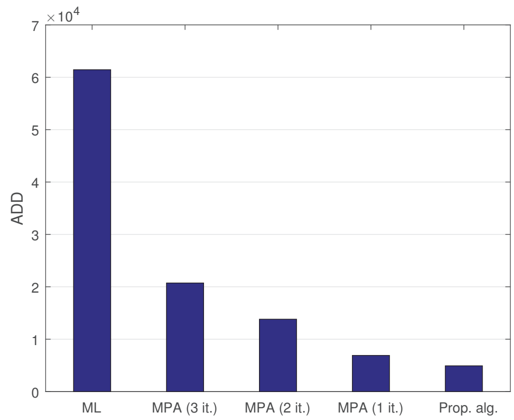

Fig. 3. Computational complexity comparison: number of additions

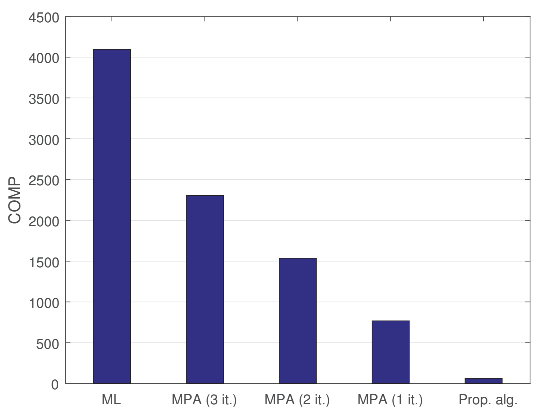

Fig. 4. Computational complexity comparison: number of comparisons

TABLE I
COMPLEXITY OF DETECTION ALGORITHMS

Alg.	ML	Max-Log-MPA	Prop. alg.
MUL	$2KM^J$	$3M^{d_f}Kd_fN_{\text{iter}}$	$2K(J+1) \times (M/2)^J$
ADD	$(4K-1)M^J$	$3M^{d_f}Kd_f^2N_{\text{iter}}$	$(3KJ+3K - J-1) \times (M/2)^J$
COMP	M^J	$M^{d_f}Kd_fN_{\text{iter}} + (N-2)MKd_fN_{\text{iter}}$	$(M/2)^J$

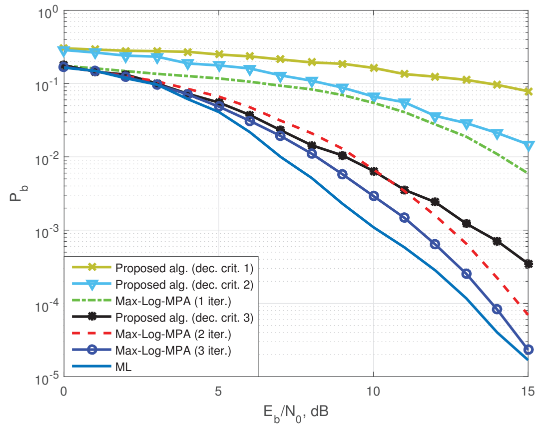

Fig. 5. Bit error probability vs. E_b/N_0

V. SIMULATION RESULTS

Computer simulation was carried out for uncoded SCMA in AWGN channel. Perfect time, frequency and phase synchronization was assumed. We also assumed that all users always transmit data, i. e. they are always active. The problem of active users detection is not considered. As a measure of signal-to-noise ratio (SNR), we used SNR per bit (E_b/N_0) for a single user:

$$E_b/N_0 = \text{SNR} - 10\log_{10}(3) \text{ dB},$$

where SNR is a power signal-to-noise ratio, and 3 bits per orthogonal resource is a spectral efficiency (12 bits per 4 REs). ML, Max-Log MPA with 2 and 3 iterations and proposed algorithm were simulated. The simulation was executed until reaching either 1000 errors or 10^7 processed bits (for every user).

The dependence of BER on E_b/N_0 is shown in Fig. 5. At $E_b/N_0 < 7.5$ dB the proposed algorithm has the same performance as the Max-Log-MPA with 3 iterations. At $E_b/N_0 < 11$ dB the proposed algorithm outperforms Max-Log-MPA with 2 iterations.

VI. CONCLUSION

We proposed new algorithm for detection of SCMA in AWGN channel, based on solving overdetermined systems of linear equations. In terms of bit error probability, the algorithm performance is equivalent to the Max-Log-MPA with 2–3 iterations, but it has lower computational complexity. Furthermore, linear equation systems solving, candidate codewords coding, and final codeword selection can be implemented in parallel.

The proposed algorithm can be used for SCMA detection in downlink channel with fading after channel estimation. The investigation of algorithm performance at perfect channel estimation and with different channel estimation methods is a topic for future research.

Another possible direction of future work is obtaining soft decisions for received bits.

Acknowledgment

The authors would like to acknowledge the financial support of this work by the Ministry of the Education and Science of the Russian Federation (basic state assignment No. 2014/187, project code 1031).

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