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A Game of Ages for Slotted ALOHA With Capture

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Abstract—Within a recent line of research, age of information is supported as an alternate network performance metric with respect to throughput or delay, to evaluate the performance of medium access techniques, especially for remote sensing applications. Analytical investigations based on game theory have shown how selfish players can behave efficiently in random access systems if they are driven by AoI-based objectives. We extend this kind of reasoning to the case of a slotted ALOHA system with capture. We present a fully analytical derivation of the general framework and its main results. We provide a quantitative characterization for the strength of capture in relation to the efficiency of the resulting Nash equilibrium, which provides extremely useful insights for a distributed system management. We apply our analysis to some scenarios of interest, in particular the case of exponentially distributed powers, for which we obtain a closed-form relationship. We highlight the impact of the system parameters, specifically the cost coefficient and the capture threshold, towards achieving an efficient allocation that represents an equilibrium for the network management. It is ultimately shown that, when capture is strong, as quantified through precise conditions (the system is driven towards a Nash equilibrium achieving near-optimal performance).

Index Terms—Age of Information; Game theory; Capture effect; Slotted ALOHA; Internet of Things.

1 Introduction

In many sensing and monitoring applications for remote process control, freshness of the data exchanged may be more important than their sheer amount [1], [2]. For this reason, the concept of *age of information* (AoI) [3] is gaining momentum in analytical investigations of medium access. The idea is to offer a quantitative performance metric beyond standard indicators such as throughput or delay.

AoI evaluations can be framed in closed form, along the lines of throughput investigations, but often with unexpected conclusions. This is especially true for ALOHA-like protocols [4], [5], which are typical whenever the nodes are dense in number, heterogeneous in nature, and limited in computation and energy resources. A prime example is machine-to-machine communication in the Internet of Things (IoT), where centralized access control is impractical. Scenarios of distributed random access can also benefit from investigations based on *game theory* [6], [7]. Such a mathematical tool can model the intelligence of the nodes as guided by an individual utility, which is realistic in massive access scenarios and can capture the system performance to identify practical solutions for efficient distributed control.

Some investigations of random access protocols have been performed under the lens of game theory, but mostly focusing on throughput as the main performance metric [8], [9]. Characterizations of the opportunistic behavior of selfish nodes acting under the objective of minimizing the AoI are seldom found in the literature, with few notable exceptions [10], [11], [12], [13]. Since game theoretic investigations have shown that even uncoordinated contention-based access protocols may perform well to some extent, an efficient Nash equilibrium (NE) is possible, when the individual objectives of the players include their local AoI.

Most such investigations, including our own previous work [11], only focus on simple protocols such as slotted ALOHA, which is well known to achieve low access efficiency due to collisions. However, in [14] we made a fully analytical investigation of the impact of the so-called *capture effect* on a slotted ALOHA system. This corresponds to the ability of the receiver to detect the strongest signals even in the presence of collisions, i.e., interference by other signals. Standard slotted ALOHA assumes that two or more packets transmitted in the same time slot collide and are considered lost. The only successful transmissions are those of packets transmitted alone in a given time slot. The capture effect is the ability of the receiver to successfully decode a signal even if it is overlapping with other ones, e.g., by exploiting differences in the received signal power, or coding, or other techniques [15]. In particular, we will assume that a signal is captured if its received power is higher than the product of a constant (called capture threshold) and the sum of the received powers of all the colliding transmissions. This results in a higher transmission success rate compared to traditional slotted ALOHA. A graphical representation of this scenario is shown in Fig. 1.

As a result, the probability of successful transmission is improved thanks to the strongest signals surviving collisions and being correctly decoded in spite of the interference of other signals. Notably, the analysis in [14] did *not* involve game theory. In addition, it only considered throughput.

The contribution of the present paper is to combine these three elements of (i) slotted ALOHA with capture, (ii) AoI,

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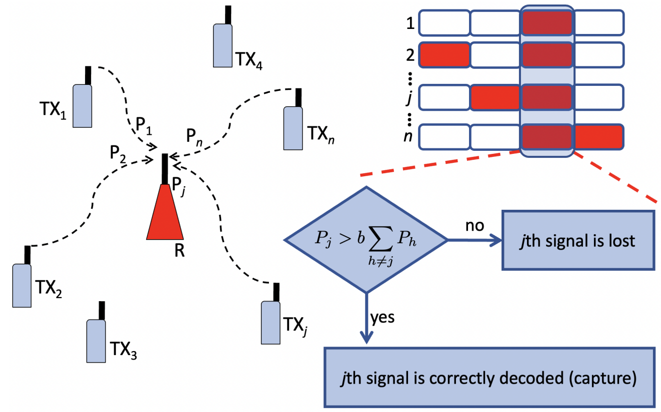

Fig. 1. Graphical display of a slotted ALOHA network with capture.

and (iii) game theory in an original way. We obtain a game theoretic investigation of a network where access is based on slotted ALOHA with capture and where nodes have the objective of obtaining a low AoI of their data, taking into account a transmission cost to dampen the aggressive behavior of the nodes driven by selfish objectives.

While our investigation is analytical and with exact derivations, and thus entirely general, we will instantiate it in some specific cases, with the purpose of drawing design guidelines for practical systems. We will show the role of the capture probability of one specific terminal in the case of simultaneous transmissions by N concurrent nodes, and we apply it to sample cases of common statistical models for the received powers (e.g., path loss or shadowing). In particular, we will consider the case of exponential received power values, which corresponds to a Rayleigh fast fading model and allows for a simple yet substantial derivation in closed form, where all the relevant parameters are included.

For this case, we will also show how the capture threshold b affects the relationship between the transmission cost c and the system equilibrium, so that the impact of the values of b, c, and the number of terminals N can be better understood. We can also gain useful insights on network control by means of a distributed management, where nodes are just incentivized to follow individual objectives. In this case, the system may be assumed to work at a NE, and our numerical results show that, when capture is strong enough, the equilibrium performance is very close to that of an optimal allocation from a global standpoint; as a result, network management can be greatly simplified, while still achieving near-optimal performance [16].

The rest of this paper is organized as follows. In Section 2, we discuss the previous contributions on the subjects of game theory and random access with multi-packet reception, and explain how the contribution of the present paper fills a gap in the related literature. Also, we summarize the key ingredients, based on existing results, of our analysis. These elements are developed for our analysis, where we first discuss in Section 3 the AoI evaluation with capture, also discussing possible examples of practical scenarios, and then in Section 4 apply game theory to derive the possible NEs of the resulting systems and their connection to the technical parameters. We will combine these together in practical contexts and show numerical results in Section 5. Finally, we will conclude the paper in Section 6.

2 Background

2.1 Related work

Several papers in the literature apply game theory to network systems. The most common reference scenario is for security issues in adversarial contexts at the different layers of the protocol stack, such as denial-of-service or jamming [17], [18]. In [19], game theory is applied to investigate security issues in an AoI-based scenario. The authors of [20] apply instead game theory to discuss the tradeoff between the objectives of minimizing AoI and maximizing throughput.

There is also a line of research for medium access seen as a game played by selfish agents, where ALOHA-like techniques are a reference case investigated by many classic contributions [6], [8], [9], [21]; however, these papers mostly consider throughput-based objectives for the players, since the popularity of AoI as a performance metric is relatively recent. From a game theoretic standpoint, one can generally conclude that the NE of such systems is less efficient than a globally optimum operating point [22]. However, in a recent contribution [11], we prove that this conclusion is mitigated for systems where the players aim at minimizing their AoI, since when the cost parameter is above a given threshold, a better NE arises. Moreover, a main criterion often adopted in the literature to represent the medium access is that collisions result in lost packets, and the so-called *capture effect* is rarely considered. Our contribution in the present paper is to extend the investigation to this case, leveraging the analysis of [14].

We note that there has been a recent flourishing of papers focusing on AoI evaluations, especially for remote sensing in IoT. While slotted ALOHA is already considered in [3] as a reference scenario, and there have been some recent investigations along this line [23], [24], [25], [26], the field is relatively unexplored for what concerns datalink layer aspects such as modeling the medium access and/or the capture effect. At the same time, it is also still uncommon to find AoI employed within the utility functions of game theoretic approaches. A notable reference is [27], where the authors consider a game based on AoI, but the access model is just based on an abstract assumption that collisions lead to losing the packets. Another relevant paper is [28], where two transmitter/receiver pairs share access over an interference channel, but the end goal is related to the achievable capacity at the physical layer, without specific considerations on medium access control.

2.2 Summary of Preliminary Results

We now highlight the starting point of our analysis, to better frame the contribution of the present paper. In [11], we applied game theory to AoI in slotted ALOHA. For random-based medium access, the AoI follows from all nodes' transmission probabilities $\mathbf{t} = (t_1, t_2, \dots, t_N)$, chosen independently by the nodes seen as distributed agents. Thus, a standard minimization of the AoI can be derived. It can be proven, following [8], [29], that the minimization

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is achieved in a *symmetric* point where all transmission probabilities are equal to an optimal value t^*, which is the same for all the nodes, i.e., $t_j = t^*$ for all $j \in \{1, \ldots, N\}$.

The reason for this symmetry lies in the common approach for medium access investigations to consider nodes having an identical prior to the characteristics of the other terminals compared to their own. For example, if power control is applied, transmission powers will compensate for the path loss, but the received signal powers will still be affected by independent and identically distributed random variations due to, e.g., Rayleigh fading, whose per-user distribution (not the instantaneous values) are characteristic of the scenario and therefore identical for all nodes [14]. If there is no power control, the received signal power will still be randomly distributed according to the positions of the nodes around the receiver, which are different but statistically equivalent.

There may be scenarios where the received signal power distribution is not the same for all nodes, which may be the case of uneven positioning. Still, for this to translate into actual different choices of the transmission probabilities, one must also make the additional assumption that the nodes are aware of such differences, which is far from trivial [18]. Gaining such information is likely possible only in very simple cases, such as a partition in two regions, with the nodes belonging to each region being aware not just of their position but also of the region of everyone else. Nevertheless, such an asymmetric case can be developed along the same lines of what follows (in particular, the optimal transmission probabilities will still be identical region-wise), but with higher complexity and more cumbersome notation [30]. We will later address this point by considering numerical evaluations for the case where the nodes adopt different transmission probabilities, showing that they are considerably suboptimal.

The essence of a game theoretic perspective is that, instead of imposing the optimal value t^* to all nodes, an individual player, say terminal 1, is put under the spotlight, and the minimization of its AoI is done over t_1 only, leaving the other values $t_2, \ldots t_N$ unchanged. Because of symmetry, this once again results in an equilibrium where all t_j's are equal, but usually to a different value than before. As a result, we are led to another operating point, in general more aggressive than the optimal one since nodes are driven by their selfish objective - a very well established game theoretic principle known as *the tragedy of the commons* [31].

In a game theoretic analysis, it is common to introduce a cost term that the nodes pay to access the channel [8], [9], [29]. This can be connected with some practical motivation, such as energy expenditure within the terminals, or simply to control their access. As a result, the transmission probabilities of the individual nodes can be controlled, to the point that a better NE arises when the cost is above a given threshold, which can be computed analytically [11].

Due to the inefficiency of slotted ALOHA, it is generally required to introduce a high cost to control the NE when the number of terminals is large. A more realistic characterization of the medium access will possibly lead to a better equilibrium without introducing too high transmission costs. For this reason, in the present analysis we resort to our previous characterization of the capture effect in random medium access presented in [14]. That paper proposes several closed-form derivations of how to represent multi-packet reception capabilities of the terminals. We just take some sample approaches to address this point, but the analysis is general and can be extended to any scenario presented in the paper.

Starting from such existing work by the authors, the present paper evolves the analysis in a novel manner achieving new, original results. Our contribution can be summarized as follows. First, we give an analytical derivation of the AoI when the capture effect is present, which, to the best of our knowledge, is not available in the present literature. Moreover, we apply game theory as detailed above, focusing on the transmission probability of a specific terminal, chosen so as to optimize its *individual* objective (a linear combination of its AoI and paid cost). The resulting NE is then discussed and quantified, highlighting the role played by key parameters.

3 Analysis of AoI and Capture Effect

We consider a network of N terminals that are synchronized on a discrete (slotted) time reference. The terminals share a common transmission channel which is used to send packets of identical size towards a single receiver (sink). The time slot is hence assumed to be equal to the packet transmission time, and transmissions can only occur according to the slot pattern [4]. We further assume that, during each time slot, terminal i is actively transmitting with probability t_i, independently of all the other nodes, and the packet transmitted always contains up-to-date information. Thus, the AoI of the data sent by a specific terminal is separately counted at the sink, and whenever a packet transmission is successful, that AoI value is set to 0, otherwise it is increased by 1 in each time slot [32].

We denote the average AoI and the probability of successful transmission of the ith terminal as Δ_i and ρ_i, respectively. These quantities can be put in relation with one another. For a discrete time, we can write [11, Eq. (1)]

$$\Delta_i = \rho_i^{-1} - 1 \,. \tag{1}$$

In turn, a successful transmission for i, which serves to compute the value of ρ_i, depends on the simultaneous occurrence of two events: (i) node i is transmitting, and (ii) its transmission is successful. In classic slotted ALOHA analysis, condition (ii) would require that all other nodes are not engaging in simultaneous transmissions, hence $\rho_i = t_i \prod_{k \neq i}(1 - t_k)$. If capture effect is considered, then i's transmission can be successful even in the presence of other nodes transmitting at the same time. Now, denote with $q_i(j)$ the probability that a packet sent by a transmitting node i is captured in the presence of j competing transmitters, including node i itself. Notably, $q_i(1) = 1$, when i is the only transmitter, while, for a regular slotted ALOHA system without capture, $q_i(j) = 0$ if $j > 1$. This allows us to compute ρ_i by considering the probability that j nodes out of N are active, and i's transmission is captured, and averaging over j.

Without loss of generality, we focus on terminal 1, whose transmission probability is t_1. This node must create a *belief* about the strategic choices of the other nodes, i.e., their selected transmission probabilities $t_2, t_3, \ldots, t_N$, where only

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t_1 is decided upon by terminal 1. The probability ρ_1 of successful transmission of terminal 1 can be written in general terms as

$$\rho_1 = t_1 \sum_{\mathbf{x}\in\mathbb{Z}_2^{N-1}} \prod_{h=2}^{N} t_h^{x_{h-1}} (1-t_h)^{1-x_{h-1}}\, q_1(w_{\mathbf{x}}+1)\ , \qquad (2)$$

where $\mathbb{Z}_2 = \{0,1\}$ is the binary set and $w_{\mathbf{x}} = \sum_{h=1}^{N-1} x_h$ is the Hamming weight of binary vector $\mathbf{x} = (x_1, \dots x_{N-1})$.

The aforementioned discussion about the symmetric choice of the terminals turns out to be also true in a game theoretic setup, where the nodes seek for an individually selfish best choice of t, rather than a global optimum; still, they will transmit with the same probability, i.e., $t_1{=}t_2{=}\dots{=}t_N$. In fact, at the NE, each terminal must believe that all other nodes play a *best response* as their t_j, which gives to each of them the highest possible utility. If the utility is AoI-based, according to (2), the value of the best response will be unique. This implies that if some node chooses a different t_j than the others, it is not playing a best response.

According to the reasoning above, when looking for the optimal choices of the transmission probabilities (from either a global perspective or the individual choices of the nodes), we drop the subscripts. In particular, when considering node 1, we denote all the transmission probabilities of other nodes as t, i.e., $t_2 = t_3 = \dots = t_N = t$. One can remark that, further pushing this reasoning, t_1 must also be equal to t. Yet, we keep the notations separate in order to distinguish between terminal 1, on which we focus, and all other terminals. This distinction will become useful when dealing with the computation of the NE.

The probability of successful transmission of terminal 1 can be written as

$$\rho_1 = t_1 \sum_{j=1}^{N} \binom{N-1}{j-1} t^{j-1}(1-t)^{N-j}\, q_1(j)\ . \qquad (3)$$

To compute $q_1(j)$, we can follow existing analytical frameworks available in the literature, e.g., [14], where we analyzed the capture effect under the assumption that received powers by different nodes are independent and identically distributed (i.i.d.). The other underlying hypothesis is that a transmission from node 1 is captured if j nodes are in set $\mathcal{T}_j$ of active transmitters (with $j{\geq}1$, and $\mathcal{T}_j$ containing node 1) as long as node 1's received power is greater than b times the sum of all other received powers, i.e.,

$$q_1(j) = \Pr\left[P_1 > b \sum_{k\in\mathcal{T}_j\setminus\{1\}} P_k\right]. \qquad (4)$$

The key parameter b is the *capture threshold*. The reader can refer to [14] for a more in-depth discussion on the model and the value of b. Here, we treat it like an adjustable parameter whose role in determining the AoI will be explored next.

If the received powers of active terminals (which are i.i.d.) have pdf $f_P(x)$ with x being within $[P_{\min}, P_{\max}]$, we write the probability of capturing node 1 as

$$q_1(j) = \int_{P_{\min}}^{P_{\max}} \int_{P_{\min}}^{x/b} f_P(x) f_P^{(j-1)}(y)\,\mathrm{d}y\,\mathrm{d}x \qquad (5)$$

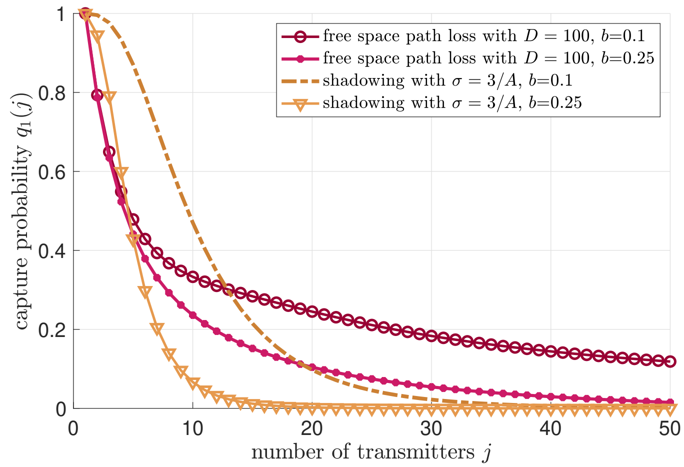

Fig. 2. Capture probability of a transmitter in the presence of j competing transmitters for shadowing and path loss models.

where $f_P^{(j-1)}(x) = f_P(x) \otimes f_P(x) \otimes \cdots \otimes f_P(x)$ is the $(j-1)$-fold convolution of $f_P(x)$. Note that this expression does not depend on t, which will be useful later.

Several meaningful choices are possible for $f_P(x)$. For example, in [15] a path loss model was considered, i.e., a scenario where transmitters are uniformly distributed within a circle of radius D whose center is occupied by the common receiver, i.e., an access point or base station. The distances of all users from the center are i.i.d. and their pdf is $f_d(a) = 2a/D^2$ for $0 \leq a \leq D$ and 0 otherwise. It is assumed that radio propagation just obeys a deterministic path loss, without any fading, so that the received power at distance d is $P(d) = (1+d)^{-\eta}$ and in particular we consider $\eta = 2$ (i.e., free space conditions) even though the result can be extended to other values. The standard power law of the path loss is slightly modified to avoid infinitely large powers when d goes to 0, but in the end this change is negligible for large distances. In the specific case of $\eta = 2$, the received power P (written without subscripts as the same statistical characterization holds for all terminals) is a random variable between $P_{\min} = (D+1)^{-2}$ and $P_{\max} = 1$, whose pdf is

$$f_P(x) = \left(x^{-2} - x^{-1.5}\right)/D^2\,. \qquad (6)$$

Another possible choice is a lognormal model, which describes a scenario where nodes have identical path loss as when being all placed at the same distance from the receiver, but each also has an independent shadowing term, in which case it is common to assume that received power P has a normal distribution if written in dB, that is,

$$f_P(x) = \frac{\exp(-(\log x - \mu)^2/(2\sigma^2))}{\sqrt{2\pi}\sigma x}\,, \qquad (7)$$

where μ and σ are parameters related to the mean and standard deviation of the associated normal distribution.

The resulting values of the capture probability from the perspective of player 1 for these two models are reported in Fig. 2. Here, we considered a path loss model of terminals distributed over a circular area with radius $D = 100$ and path loss exponent $\eta = 2$, and a shadowing model with $\mu = 0$ and $\sigma = 3/A_{10}$, where $A_{10} = 10/\log(10)$. Also, we remark that the capture threshold b is chosen as equal to either 0.1

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or 0.25. Since these values are below 1, multiple packets can be captured in the same slot, but this is consistent with the model of [14] as well as practical systems such as [33].

The plots highlight that the capture effect in the path loss scenario is relatively strong, in that it persists even when the number of contending transmitters increases considerably. Conversely, the shadowing model allows for a very high capture probability $q_1(j)$ in the presence of few overlapping transmitters, which rapidly decreases with increasing j, especially if the capture threshold is high. These points, combined with the theoretical results about the AoI derived in the next section, will help understand the performance of these systems from a game theoretic perspective.

Finally, we can consider a case where the received powers are i.i.d. and follow an exponential distribution with parameter λ, i.e., the probability density function (pdf) of all received powers is $f_P(x) = \lambda e^{-\lambda x}\mathbb{1}(x)$, with $\mathbb{1}(x)$ is the unit step function. This case corresponds to all transmitters being placed at the same distance and having identical shadowing, or alternatively using a power control mechanism compensating the attenuation over a medium-term timescale. However, signals are affected by multipath fading, which results in their received powers being exponentially distributed.

In this specific scenario, (5) admits a neat derivation, since it follows that, for $j>1$, $q_1(j) = \Pr[P_1 > b\, Q_{j-1}]$, where $Q_k \sim \mathrm{Erlang}(k, \lambda)$ is a random variable with Erlang distribution of index k and parameter λ. Without loss of generality, we will put $\lambda = 1$ in the following. From known properties of the Erlang distribution, we get

$$\Pr[P_1 > b\, Q_{j-1}] = \mathbb{E}_{Q_{j-1}}\left[e^{-bQ_{j-1}}\right] = \frac{1}{(1+b)^{j-1}}\,, \quad (8)$$

where we used the well-known moment generating function of the Erlang distribution, i.e., $\mathbb{E}_{Q_k}[e^{aQ_k}] = (1-a)^{-k}$.

Hence, putting $k = j-1$ in (3), we can write ρ_1 as

$$\rho_1 = t_1 \sum_{k=0}^{N-1} \binom{N-1}{k} \left(\frac{t}{1+b}\right)^k (1-t)^{N-1-k} \quad (9)$$

which reduces to

$$\rho_1 = t_1 \left(\frac{1+b(1-t)}{1+b}\right)^{N-1} . \quad (10)$$

4 Game Theoretic Formulation

A standard application of any derivation of the performance of a random access scheme is the evaluation of the best operating condition from a centralized point of view, which would quantify the throughput-optimal transmission probability of slotted ALOHA among N nodes as $1/N$. In other words, if we are interested in a symmetric centralized optimal solution for a given performance indicator, we derive it as a function of t, where we assume that all nodes, including node 1, follow the same transmission pattern due to symmetry, hence we also set $t_1 = t$. Then, we find the specific value of t maximizing the performance indicator, for example setting the first-order derivative in t of the performance indicator to 0 while also checking the boundary value at $t = 1$ (value $t = 0$ is always inefficient).

We can also think of broadening existing game theoretic frameworks discussing slotted ALOHA for a throughput objective so as to include an analysis of AoI as well. Unfortunately, for a distributed scenario where nodes transmit independently of one another, investigating the global optimal-AoI transmission probability turns out to be relatively uninteresting since, according to (1), it would just correspond to maximizing the success probability ρ_1, and the analysis follows the same steps as a throughput maximization.

However, it is interesting to take a game theoretic stance, where each node chooses its own transmission probability without following the others, driven by a selfish goal of minimizing its own AoI. Symmetry considerations will lead the terminals to converge to the same value of t for everyone, yet, the perspective is different since it implies that they follow different objectives. Such an analysis is made relatively simple by the separation found in (10) between t_1 (the value of choice for the terminal of interest) and t, i.e., the transmission probability of every other terminal. In a game theoretic analysis, we can consider the N terminals as the players of a static game of complete information [8], where they set their action as their transmission probability, chosen independently and unbeknownst to each other.

If we set the objective of the players as minimizing their AoI values, the minimization of the AoI from a selfish perspective will lead to a trivial NE where $t = 1$ for all the terminals. In fact, whatever the choice of the other terminals, it is always convenient for the terminal of interest to aggressively transmit with probability 1, which is a dominant strategy [6]. Indeed, choosing to always transmit is strictly better than never transmitting, and any intermediate strategy with transmission probability strictly between 0 and 1 obtains an outcome in between these two. Symmetry considerations imply that all terminals do the same and we get an NE where everyone transmits in every slot, with resulting AoI $\Delta_i = [q_1(N)]^{-1}$ for all terminals.

We remark that this can be considered as a *catastrophic* NE, a term that is inherited from standard slotted ALOHA networks. However, such an NE is in reality not that inefficient if the capture effect is in place. For a pure slotted ALOHA network, the catastrophic NE obtains perennial collisions and thus zero throughput, which is a particularly bad instance of the tragedy of the commons. If colliding transmissions can be captured, however, the situation can be slightly better. Nevertheless, if $q_1(j)$ tends to 0 for sufficiently high j, as is meaningful to assume, throughput is significantly lower than what an efficient management could achieve. Moreover, when nodes transmit so aggressively, the capture effect is not properly exploited, as it would be more effective if the set of concurrent transmitter $\mathcal{T}_j$ were relatively small.

The common solution to this catastrophic behavior in game theoretic approaches is the introduction of a cost incurred by each individual node i, proportional to its transmission probability t_i through a constant c [8], [22]. Such a cost term can be either related to actual physical phenomena, such as the energy consumption of the terminal when transmitting, or just introduced for the sake of limiting persistent access by the terminals. This will actually prompt a further discussion in the following.

For the purpose of a game theoretic analysis, we define the *utility* of the ith player as the value that terminal i seeks

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to maximize, defined as

$$u_i(\mathbf{t}) = -\Delta_i - c\,t_i = -\frac{1}{\rho_i} + 1 - c\,t_i \qquad (11)$$

where the negative sign indicates that all terminals actually seek to *minimize* AoI and/or transmission cost, and the individual utilities are defined to be functions of the *entire* array of transmission probabilities, $\mathbf{t} = (t_1, \ldots, t_N)$, which happens through Δ_i and thus through ρ_i.

The utilities defined in (11) can be employed in two ways. On the one hand, symmetry reasons lead to assuming that all t_i's are equal, i.e., $\mathbf{t} = (t, t, \ldots, t)$ and under this condition an optimal t can be found, such that the utilities are maximized. Note that, because of the introduction of the cost term, this optimization is no longer the same as maximizing the success probability alone, and therefore the throughput. At any rate, the numerical derivation of such a maximum is immediate by setting the first-order derivative $\mathrm{d}u_i/\mathrm{d}t = 0$. This corresponds to a fully coordinated working point, which is often deemed to be impractical in a game theoretic spirit, as individual selfish players may have an incentive to deviate.

On the other hand, we can also explore a unilateral maximization of the utility of a player of interest, say, terminal 1, while the moves of the others are kept unchanged. This requires to separate the transmission probability t_1 of such a terminal, while all the others can be assumed to use transmission probability t. Note that in the end, also t_1 will be set equal to this very value, but only after taking the first order derivative, which is now $\mathrm{d}u_1/\mathrm{d}t_1$, and setting it equal to 0. In game theoretic terms, this is a NE, since each player chooses a *best response* to the moves of the others.

To this end, we observe that, using (3), the utility function u_1 in (11) can be written as

$$u_1(t_1, t) = -\frac{1}{t_1 K_1(t)} + 1 - ct_1 \qquad (12)$$

where we set

$$K_1(t) = \sum_{j=0}^{N-1} \binom{N-1}{j} t^j (1-t)^{N-1-j} q_1(j+1)\,. \qquad (13)$$

We can express (13) as $K_1(t) = \mathrm{E}[q_1(\nu+1)]$, where the expectation is taken with respect to the distribution of ν, which is a binomial random variable with index $N-1$ and parameter t modeling the number of signals that overlap with node 1's transmission. Since $q_1(j) \in [0,1]$ is monotonically non-increasing in j, it is easy to realize that $K_1(t)$ takes values in the interval $[0,1]$, with $K_1(0) = 1$ and $K_1(1) = q_1(N) \leq 1$, and is monotonically non-increasing in $t \in [0,1]$.

Note that $K_1(t)$ can be seen as a measure of the strength of the capture effect: the higher the capture probability, the higher the value of $K_1(t)$ and the slower its decrease with t (i.e., the higher the derivative of $K_1(t)$ in t). Also, $K_1(t)$ does not depend on t_1, which is extremely convenient when taking derivatives as we will do in the following.

To determine the NE, we need to find the value of t_1 that maximizes u_1, for a given t, which we can denote by $t_{1^*}(t)$. This is equivalent to assuming that the other nodes do not change their strategy while node 1 optimizes its own. However, for symmetry reasons, we know that the equilibrium is actually reached when all nodes adopt the same strategy, i.e., for $t_1 = t$. Hence, we need to find the fixed point $t_{1^*}(t^*) = t^*$.

In turn, this requires to solve $\mathrm{d}u_1/\mathrm{d}t_1 = 0$, which gives

$$t_{1^*}(t) = \min\left\{\frac{1}{\sqrt{K_1(t)c}}, 1\right\}. \qquad (14)$$

The NE are then found by solving

$$t_{1^*}(t^*) = t^*\,, \qquad (15)$$

which, for $t^* < 1$, (15) yields

$$A(t^*) = t^{*2} K_1(t^*) = \frac{1}{c}\,. \qquad (16)$$

where, for ease of notation, we introduced the function $A(t) = t^2 K_1(t)$. We observe that $A(t)$ is a continuous function of t, and $A(t) \in [0,1]$ for $t \in [0,1]$. Let τ be the point where $A(t)$ takes its maximum in the interval $[0,1]$. Consider that t^2 is monotonically increasing for positive t, whereas $K_1(t)$ is monotonically non-increasing for $t \in [0,1]$, and $A(0) = 0$ and $A(1) = q_1(N)$. Thus, $A(t)$ can either be monotonically increasing in $[0,1]$, thus reaching its maximum in the interval for $\tau = 1$, or have a maximum in an intermediate point $\tau < 1$, with $A(\tau) > q_1(N)$. The distinction between these two cases depends on the shape of $K_1(\cdot)$, i.e., on the strength of the capture effect: if $K_1(t)$ decreases less than quadratically with t (i.e., the capture effect is strong), then $A(t)$ reaches its maximum value $q_1(N)$ for $\tau = 1$, otherwise it will reach a maximum value higher than $q_1(N)$ in a point $\tau < 1$.

We can now analyze (15) under these observations. If $c < 1/A(\tau)$, (16) does not admit any solution, so there is a single fixed point in $t^* = 1$. Otherwise, other NEs exist. We discuss these results in the following theorems.

Theorem 1. For a cost parameter c lower than $[q_1(N)]^{-1}$, the system admits a catastrophic NE, consisting of all transmitters choosing $t = 1$.

Proof: This theorem follows from the very definition of NE as a working point where the joint decision by all the agents does not leave any incentive for unilateral deviation of any of them. In the specific case, we can check what happens if node 1 must make a decision on t_1 under the belief that all other nodes are choosing $t = 1$. The best response is to choose $t_1 = 1$ as well if u_1 is maximized. Since $u_1 = \rho_1^{-1} - 1 + ct_1$ and this is a continuous function of t_1, where in particular the term ρ_1 goes to $t_1 q_1(N)$ for $t_1 = 1$ since all nodes transmit, we just check whether $\mathrm{d}u_1/\mathrm{d}t_1 = 0$, which implies $cq_1(N) = 1$. This means that if $c \leq [q_1(N)]^{-1}$ either this condition is achieved precisely at $t_1 = 1$ or the maximizing value is achieved for $t_1 > 1$, which is not feasible, in which case node 1 should transmit with the highest probability possible (i.e., $t_1 = 1$) as the best response. □

The theorem implies that the catastrophic NE can be avoided only if the cost is high enough (notably, it must be $c > 1$ for sure). Also, it is evident that the condition is never verified for a standard slotted ALOHA scheme, for which the catastrophic NE is always in place [11].

It is convenient to seek for other better NEs, to which the network can be driven through a free evolution of

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the system just by selfish choices of the terminals. This is guaranteed if the cost is higher than $[q_1(N)]^{-1}$, because whenever the catastrophic NE disappears, another one must exist. But it can also happen that multiple NEs coexist. The condition for a further non-catastrophic NE to exist is given by the following theorem.

Theorem 2. A threshold γ exists, with $1 \leq \gamma \leq [q_1(N)]^{-1}$, such that when $c > \gamma$ there is a non-catastrophic NE with values of t strictly between 0 and 1.

Proof: This is a direct consequence of solving condition $A(t) = 1/c$. Since ρ_1 is a continuous function of t (as it consists of polynomial terms multiplying $q_1(j)$ that in turn does not depend on t), the function $A(t)$ admits a maximum M for $t = \tau \in [0, 1]$. This maximum M must be less than or equal to 1 and is also equal to $q_1(N)$ if $\tau = 1$, whereas it is greater than that if $\tau < 1$. The proof follows from setting $\gamma = 1/M$ and remarking that if $c = \gamma$ then the NE is for $t = \tau$, otherwise condition $A(t) = 1/c$ admits two solutions in $[0, 1]$ but only the leftmost one corresponds to an NE, the other being a minimum of the utility and therefore not a best response. □

The two theorems combined state that the value of c influences the resulting operating point of the system as the NE, in a way that can be summarized as

$$\begin{cases} \text{catastrophic NE} & \text{for } c \leq \gamma \\ \text{two NEs} & \text{for } \gamma < c \leq [q_1(N)]^{-1} \\ \text{non-catastrophic NE} & \text{for } c > [q_1(N)]^{-1} \end{cases} . \quad (17)$$

This implies that the catastrophic NE exists for low values of the cost, while the non-catastrophic one exists for sufficiently high cost. Note that there is always at least one NE in the system, consistently with [34]. Also, if $A(t)$ is monotonically increasing and therefore its maximum in $[0, 1]$ is reached in $\tau = 1$, then we obtain $\gamma = [q_1(N)]^{-1}$ and the inner interval with two NEs disappears. In this case, the two NEs are alternative to each other, whereas if $\gamma < [q_1(N)]^{-1}$ then the two equilibria coexist for a certain range of cost values.

At any rate, an efficient distributed management can be obtained by increasing the cost beyond $[q_1(N)]^{-1}$, so that there is only one non-catastrophic NE. However, for scenarios where $q_1(N) = 0$, as happens for the regular slotted ALOHA without capture, the third interval in (17) is instead empty, which means that the catastrophic NE never disappears and the only effect achieved by increasing c is the coexistence of both NEs, a result that is coherent with the findings of [11].

This leads to an interesting classification of the capture effect from the perspective of distributed system management. We can denote the systems where $\tau = 1$ as possessing a *strong capture effect*. In this case, the two NEs never coexist. Remember that the nodes are strategic, i.e., selfish, players. Thus, when the capture probability $q_1(t)$ is high, increasing the cost above the threshold not only creates a non-catastrophic NE, but also eliminates the catastrophic one, as the nodes do not need to behave aggressively. Conversely, if $q_1(t)$ is low, the catastrophic NE still persists because the nodes may still find it convenient to transmit more aggressively, in the hope of trumping over one another.

The condition of strong capture effect requires the values of either b or N, or both, to be low enough. More precisely, since $A(t)$ contains a quadratic term, it corresponds to requiring that ρ_1/t_1 decreases at least quadratically in t, so that $A'(1^-) > 0$ and the maximum of $A(t)$ is obtained at $\tau = 1$. Conversely, if ρ_1/t_1 decreases quadratically or more in t, the value of $q_1(N)$ is lower than M (which justifies calling the capture effect as weak) and therefore the two equilibria coexist for $c \in \left[\gamma, [q_1(N)]^{-1}\right]$.

The existence of a non-catastrophic NE would in principle allow for a network management without aggressive terminals. However, if the two NEs coexist, the system is ambiguously characterized, and the behavior of the nodes is difficult to characterize. If we allow for a practical interpretation of the results of this static game, we can apply a reasoning akin to the standard considerations for the instability of ALOHA-like systems: if one terminal chooses a value of t higher than the one of the non-catastrophic NE, a dynamic evolution of the system might lead to the instability of the catastrophic NE [22].

These results also trigger a discussion on the impact of cost. Indeed, a simple modification in the utilities related to a transmission cost may allow for an efficient NE. In particular, a sufficiently high cost is required for this NE but it is to be remarked that too high a cost would simply make the NE efficient by having no transmission whatsoever, due to the very high costs of transmitting, so this is not an ideal situation from a practical point of view, as the AoI values would be extremely high. However, if the cost is increased beyond the threshold γ, not only does the efficient equilibrium appear, but also the catastrophic NE disappears, if the capture effect is strong.

This also highlights the two-fold role of parameter c. On the one hand, it can be simply treated as a consequence of some inherent tolls paid by the transmitters, such as power consumption, monetary price, or energy harvesting cost [32]. On the other hand, the term c can reflect some sort of virtual cost, artificially introduced to make the management more efficient [9]. While our analysis is transparent to the nature of this parameter, the aforementioned criteria requiring c to be above a certain threshold can be considered in the spirit of finetuning some system parameters to allow for an efficient distributed management.

Finally, it is also reasonable to expect that the distributed management obtained by the non-catastrophic NE is close to the optimal performance. While this is not guaranteed to happen at low costs (in particular, if the capture effect is strong, the solution for $c = \gamma$ is still $t = 1$, which is not very near-optimal), the global optimum and the NE must asymptotically converge for very high values of c, even though this also represents an impractical management where transmissions are very rare. However, from an engineering standpoint, there is a tradeoff between these two conditions where an NE corresponds to a near-optimal working point without t being extremely low. This is easier to achieve for a strong capture effect because the NE asymptotically converges to the network optimum for high costs, but this happens early if the threshold $[q_1(N)]^{-1}$ is lower.

To give a quantitative understanding of a comparison between strong and weak capture effect, we can consider the case of exponentially distributed received powers, where

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the requirement $A(t) = 1/c$ combined with (10) leads to an $(N+1)$th degree equation

$$t^2 \left(\frac{1 + b(1-t)}{1+b} \right)^{N-1} = \frac{1}{c} \,. \tag{18}$$

It easy to see that the equation admits at most two solutions in t that fall within $[0, 1]$ and are therefore valid as probability values. These solutions only exist if c is sufficiently high, in which case the non-catastrophic NE is found as t being equal to the leftmost solution for all the nodes. More precisely, the number of solutions in $[0, 1]$, whenever they exist, depends on the maximizing point τ of $A(t)$; if $\tau = 1$ there is only one solution, which leads to the NE. Otherwise, there are two solutions and only the lower one gives the NE.

We can observe that if

$$\frac{\mathrm{d}A(t)}{\mathrm{d}t}(1^-) > 0 \quad \text{i.e., } b(N-1) < 2 \,, \tag{19}$$

then $A(t)$ is always increasing in $[0, 1]$ and its maximum is $[q_1(N)]_{t=1} = (1+b)^{1-N}$. Hence, the capture effect is strong, and γ coincides with $[q_1(N)]^{-1} = (1+b)^{N-1}$. In other words, if $b(N-1) < 2$, i.e., b and/or N are low, we get rid of the catastrophic NE and obtain another better equilibrium by simply increasing c beyond γ, without the two equilibria even coexisting. Conversely, if (19) is not verified, the capture effect is weak, and the more efficient NE appears when $c > \gamma$ but without the catastrophic NE disappearing; this is not even guaranteed to happen as it requires $c > [q_1(N)]^{-1}$, which may be impossible in cases where no capture happens for high N (or, the capture threshold may be too big).

5 Numerical Results

We show some practical evaluations of the equations derived in the previous analysis to draw some quantitative conclusions. We consider N terminals as players in a simultaneous-move game following individual utilities that are a linear combination of their negative AoI and their negative transmission cost, with coefficient $c = \widetilde{c}\,N$, and whose strategic choices are their individual transmission probability values. Their received power values are i.i.d., and medium access is slotted ALOHA with capture threshold b. A successfully captured transmission sends the AoI of a terminal back to 0.

To allow a comparison among different values of N on the same plot, results are always shown as a function of the normalized transmission cost $\widetilde{c}$. We vary the number of users N and the capture threshold b, and consider different pdfs of the received powers. All numerical settings are consistent with the choices usually made in the literature [8], [14], [15], [29].

We consider symmetric solutions where the transmission probability is the same for all terminals, but under two different approaches: global maximization of the utilities after choosing the same t for all nodes, or NE where one terminal sets the individual value of t as the best response to the others, and this is ultimately set equal for everyone due to symmetry. In this case, the term "NE" in the plots always refer to the non-catastrophic one, if present, as the catastrophic NE where $t = 1$ is just pointless to plot. The value of the transmission probability is the main one derived from the analysis, but actually other metrics of interest can be derived from it, such as the AoI and the utility function; in the following, we will show some samples. Even for the scenarios where they are not shown, these computations are possible, they just would be redundant, so we focus on the most interesting cases.

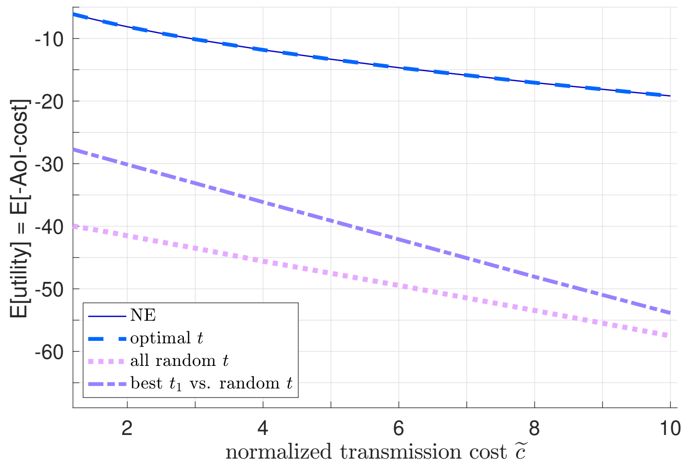

Fig. 3. Exponential powers: AoI-based utility as a function of the normalized transmission cost $\widetilde{c}$, for $N{=}10$ terminals and capture threshold $b = 0.02$. Transmission probabilities chosen as: NE; optimal; random (uniform distribution in $[0.15, 0.25]$); optimal t_1 with random t_j, $j{\neq}1$.

We apply the results to different expressions of the pdf of the received powers, by considering a path loss scenario as per (6), a lognormal received power as per (7), and finally the case of exponentially received powers with $f_P(x) = e^{-x}$ that admits a closed-form mathematical expression for the AoI. Only because of this reason, the latter is evaluated in more detail, to avoid redundant plots since all the derived metrics are based on the transmission probability t and therefore can be obtained for any choice of $f_P(x)$. However, the case of exponentially received power is more immediate to evaluate in terms of its structure of the NEs and the strength of the capture effect.

We start with Fig. 3 that shows the individual utility of the players, taken as the opposite of the sum of the average AoI and the transmission cost paid, in a scenario with exponentially distributed powers, for $N{=}10$ users and capture threshold $b{=}0.02$. The figure compares four possible cases: (i) the non-catastrophic NE; (ii) the globally optimal choice of the transmission probabilities; (iii) a case where each node chooses its transmission probability uniformly in $[0.15, 0.25]$; and (iv) a case where nodes 2 to 10 still select the transmission probability as in (iii) but player 1 chooses it so as to maximize its own utility. In this last case (iv), the expected utility plotted is that of player 1. Note that for cases (iii) and (iv), we used (2) since the transmission probabilities of the nodes are different, whereas (i) and (ii) are symmetric solutions where every node has the same t and therefore we can compute the results through (10).

The figure serves first of all to highlight that a choice like (iii) of non-optimized (and even possibly different) transmission probabilities is clearly suboptimal. A one-sided optimization of only one node, as in (iv), slightly improves its utility but still leaves a significant gap with

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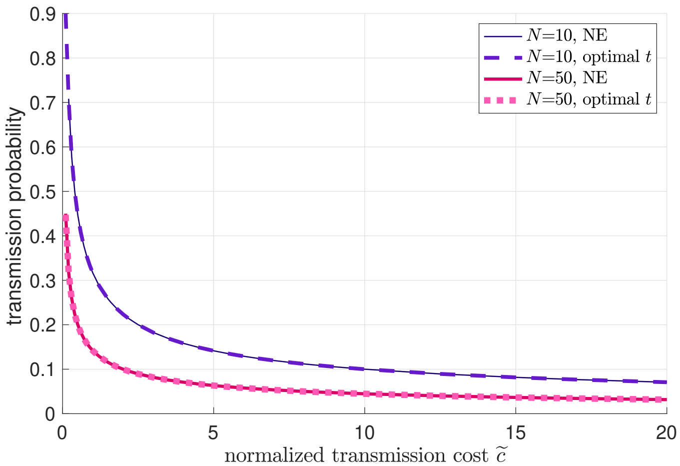

Fig. 4. Path loss model, $D = 100$, $\eta = 2$: transmission probability t as a function of the normalized transmission cost $\widetilde{c}$, at the NE or with an optimal setup, for $N \in \{10, 50\}$ terminals and capture threshold $b = 0.5$.

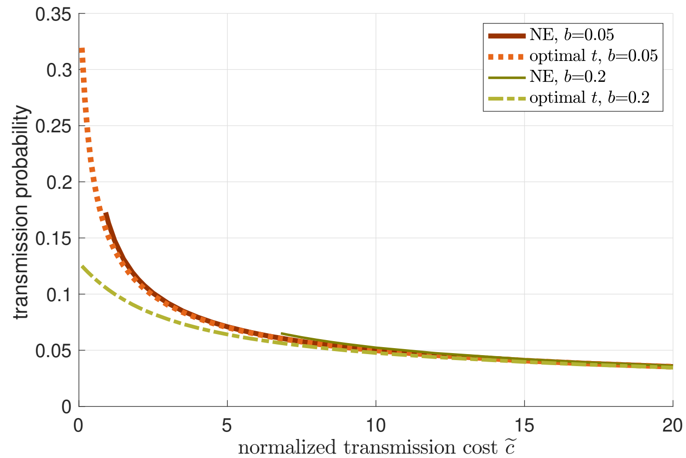

Fig. 5. Shadowing model with $\mu = 0$, $\sigma = 3/A_{10}$, $A_{10} = 10/\log(10)$: transmission probability t as a function of the normalized transmission cost $\widetilde{c}$, at the NE or with an optimal setup, for $N = 40$ terminals and capture threshold $b \in \{0.05, 0.2\}$.

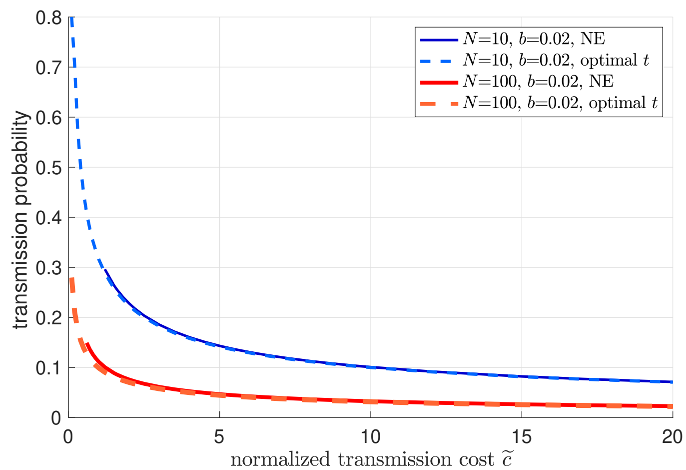

Fig. 6. Exponential powers: Transmission probability t as a function of the normalized transmission cost $\widetilde{c}$, at the NE or with an optimal setup, for capture threshold $b = 0.02$.

the optimum. Conversely, the NE and the global optimal choice of transmission probabilities are close. The inherent efficiency of the NE changes according to the statistics of the propagation scenario, the number of terminals, and the strength of the capture effect, as will be explored next. Still, a distributed choice of the transmission probability, without any preliminary signaling, is shown to be effective as long as it can leverage the common knowledge of the other users being strategic.

Now, we explore the point above for a detailed array of scenarios, starting from the one where the received powers are only determined by free space path loss. In Fig. 4 we report the values of the transmission probability at the NE or with an optimal setup, for a path loss based model with $D = 100$, $\eta = 2$, and a number of users that can be either $N = 10$ or $N = 50$. In accordance with the results of [14], and also as previously shown in Fig. 2, the capture effect is expected to be very strong in this case and the probability of successful transmission even in the presence of multiple contenders is indeed high. For a capture threshold of $b = 0.5$, we observe that the value of transmission probability t at the NE basically coincides with the optimal choice. This happens because, as per the previously found theoretical results, the catastrophic NE disappears for low values of c and is replaced by a more efficient one. It is evident that in this scenario the capture probability $q_1(j)$ does not decrease very rapidly with increasing j and therefore $K_1(t)$ has a sublinear decrease that causes the maximum for $A(t)$ to be in $t = 1$ and the actual value of t is selected by the cost parameter, resulting in a very efficient allocation even through distributed selfish choices.

In Fig. 5 we consider instead a case of received powers following a lognormal statistical model, i.e., a shadowing scenario. For low values of the number of terminals N, we would essentially face the same situation as in the previous result, since the capture effect is also extremely strong. For this reason, we explore a relatively high value of N where capture is more difficult, according to what shown in Fig. 2. In particular, we consider, analogous to the previous setup, a lognormal distribution with $\mu = 0$, $\sigma = 0.3\log(10)$, and $N = 40$ users. We also consider two values of the capture threshold, $b = 0.05$ and $b = 0.2$. The result is that, for the lower value of b, the capture effect is still strong enough for the NE equilibrium to be close to optimal. Conversely, if b increases and capture becomes less likely, we now see a separation and the NE does not coincide with an optimal allocation, even though they tend to overlap for high cost values. Also, the non-catastrophic NE appears only for relatively high values of $\widetilde{c}$, which is consistent with a weak capture effect as per the previous theoretical results.

To explore these findings in more depth, also considering other metrics, we focus on the case of exponentially received powers, since the closed form (10) allows for more efficient computations in a wider range of parameters, and (19) gives a precise condition on the required values of the parameters for the capture effect to be strong. Thus, we evaluate a scenario with exponential powers, for the number of users $N \in \{10, 100\}$, and considering two different capture thresholds, i.e., $b = 0.02$ and $b = 0.2$. With this choice of parameters, the capture effect is always strong if $b = 0.02$

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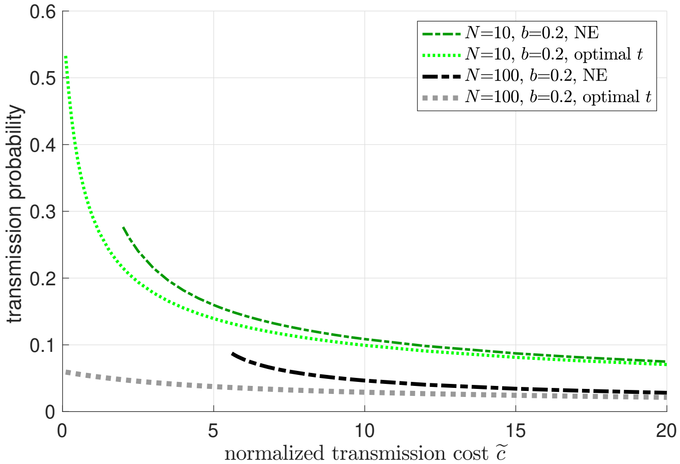

Fig. 7. Exponential powers: Transmission probability t as a function of the normalized transmission cost $\tilde{c}$, at the NE or with an optimal setup, for capture threshold $b = 0.2$.

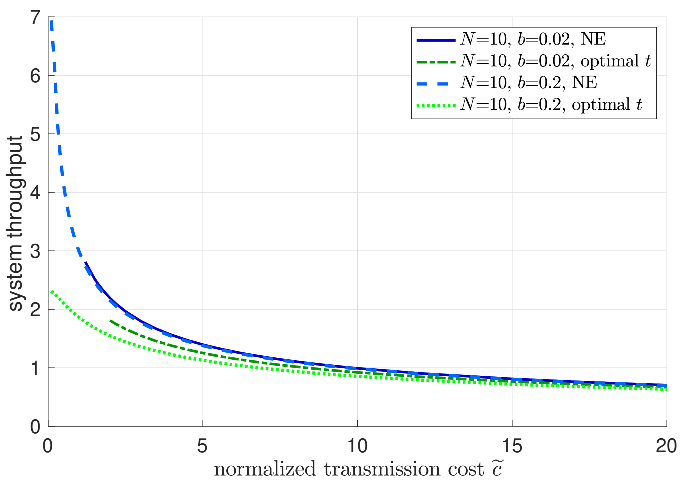

Fig. 8. Exponential powers: System throughput, as a function of the normalized transmission cost $\tilde{c}$, at the NE or with an optimal setup, for $N = 10$ terminals.

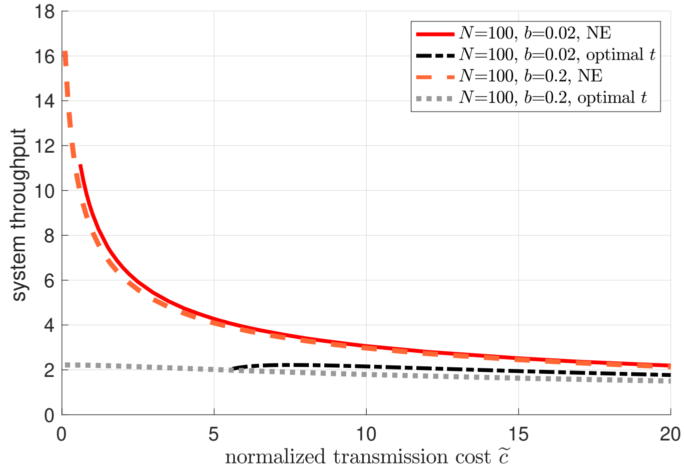

Fig. 9. Exponential powers: System throughput, as a function of the normalized transmission cost $\tilde{c}$, at the NE or with an optimal setup, for $N = 100$ terminals.

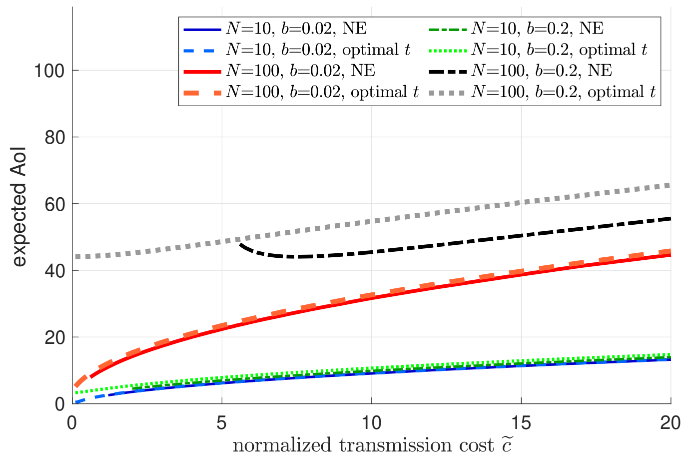

Fig. 10. Exponential powers: Expected AoI, as a function of the normalized transmission cost $\tilde{c}$, at the NE or with an optimal setup.

as condition (19) is always met even for $N = 100$, while when $b = 0.2$, the case $N = 10$ still corresponds to a strong capture effect, whereas $N = 100$ does not.

In Figs. 6–7, we report the resulting transmission probabilities of the terminals computed either at the NE (when available) or for the optimal centralized case. We remark that t at the NE is meaningful only if $c > \gamma$ and overall an increasing cost significantly lowers the transmission probability of both cases. For $b = 0.02$ the NE and optimal curves are very close (Fig. 6), while for $b = 0.2$ there is still some gap (Fig. 7). This implies that a strong capture effect allows even a decentralized system to work at near-optimal NEs.

However, this also suggests that the value of c deserves a technical discussion on its physical nature. Due to the need for a sufficiently high c, the near-optimality of the NE is achieved only if the persistence of selfish terminals is somehow limited. At the same time, it is convenient that c is not too high, since this would result in a very low transmission probability (and, as will be shown next, high AoI). This suggests that if c goes beyond being just a technical parameter, like the energy expenditure, and can be set with some slack, a proper regulation is key to obtain an efficient management.

In Figs. 8–9, we report the resulting values of the system throughput, for $N = 10$ and $N = 100$ terminals, respectively. These plots are straightforward extensions of the previous results on the transmission probabilities, yet they show that the system throughput at NE is even closer to the optimal case, given that we have a sort of compensation between a slightly higher t, resulting in more collisions but also a better chance of being captured. Also, the throughput rapidly decreases in $\tilde{c}$, except when $b = 0.2$, $N = 100$, for which it is already low anyway, as these parameters correspond to a situation where capture is difficult (many users and high capture threshold).

Fig. 10 shows the resulting expected AoI. Notably, this is *not* the objective of the players, since they actually try to minimize a utility combining AoI and transmission cost, which is instead shown in Fig. 11. At any rate, we notice that in all the resulting plots the optimal management and the NE are almost indistinguishable, with the only exception of the case $b = 0.2$ and $N = 100$, which is when (19) is violated. Overall, we conjecture that the efficiency of the NE

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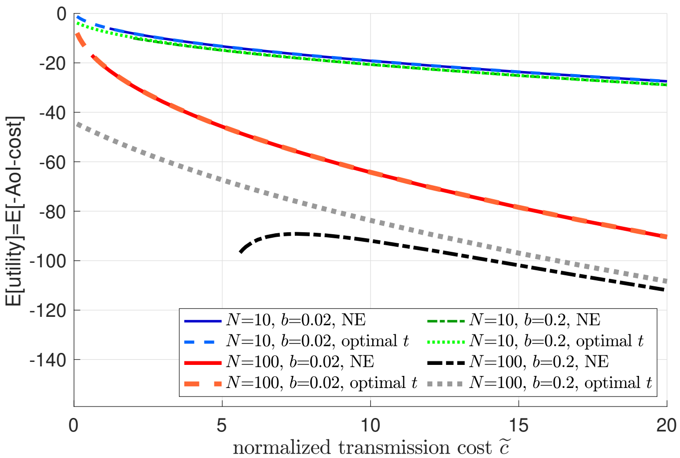

Fig. 11. Exponential powers: AoI-based utility of each terminal, as a function of the normalized transmission cost $\tilde{c}$, at the NE or with an optimal setup.

is related to its threshold structure and the actual value of the threshold, which is an issue that certainly deserves to be explored in future work. From a quantitative standpoint, we can say that the system-wide optimization can be replaced by a distributed setup through an NE, which is especially true when N and/or b are low enough, i.e., the capture effect is stronger.

6 Conclusions

We presented a game theoretic analysis of a large number of nodes sharing access following a slotted ALOHA protocol with capture effect, with their individual objectives being related to minimizing their AoI and also comprising a transmission cost. Based on previous analytical formulations of the AoI for a random access system, we showed that our framework is able to set an AoI-efficient working point, doing so in a distributed fashion where nodes act without coordination and driven by selfish objectives. This translates the system-wide optimization to a more practical approach based on individual actions of each nodes [7].

Future work may consider an expanded game theoretic formulation where the strategic choices of the nodes are more complex than just setting their transmission probability, possibly considering some sort of feedback from the receiver and an overall planning ahead over multiple update epochs [16], [35]. Even for these scenarios, game theory can be the proper tool to set a self-enforcing distributed management of nodes with minimal supervision from the network manager, which appears to be a desirable choice for future IoT implementations.

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