Three-Dimensional Field Intensity Shaping: The Scalar Case

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***Abstract*—Due to the plethora of possible applicative fallouts, in the recent years several efforts have been made to develop three-dimensional (3-D) field intensity shaping methods. While many approaches able to focus a wave-field into a target point are available, few strategies have been instead developed to generate a field having a given intensity spatial distribution. This is probably due to the higher challenging nature of such a problem. In this letter, with reference to scalar fields, we present an innovative 3-D shaping procedure, which casts the problem as the solution of a finite number of convex programming problems. An example of the performance of the proposed procedure in a 3-D inhomogeneous scenario is given to show the capability of ensuring uniform field intensity within a target area while keeping it arbitrarily bounded elsewhere.**

***Index Terms*—Antenna array, convex optimization, field synthesis, spatial field shaping.**

I. Introduction

GENERATING a field with a given intensity behavior into an inhomogeneous medium is a fundamental problem of wave physics. From the applicative point of view, the capability of arbitrarily shaping the field intensity is relevant in instances spanning from near-field focusing for RFID [1], through-the-wall imaging [2], to several biomedical applications including lithotripsy [3], microwave hyperthermia (MWH) [4], and many others.

In a very general fashion and in order to deal with flexible setups, the problem can be cast as the optimiziation of the complex excitations of a (fixed geometry) array. For such a problem, many strategies aimed at focusing a wave-field into a target point have been presented, such as time reversal (TR) [5], the optimization of the fraction of power delivered to the target [6], and the focusing via optimal costrained power (FOCO) [4], to name a few.

On the other hand, in many cases a shaped field (rather than a focused one) is of interest. As an example, in MWH treatment planning, one may have to deal with arbitrary extended target areas, such as late-stage tumors. In such a case, an accurate control of the spatial field intensity poses the major therapeutic challenge. Hence, one needs to develop a shaping strategy that is able to avoid the occurrence of undesired effects (say hot-spots) outside of the target region, while ensuring some uniformity of the field intensity into it.

Nonetheless, only a few shaping approaches can be found in the literature, and none of them, as far as we know, is able to fulfill both the above-mentioned requirements.

For example, the wave-field synthesis method proposed in [7] and [8], as well as the alternate projection-based shaping approach, proposed in [9], just aim to reproduce a given field distribution, rather than searching for the optimal field intensity distribution for the scenario at hand.

In [10], a shaping procedure has been proposed relying on the maximization of the ratio between two quadratic forms, representing the integral of the field intensity in the target area and everywhere, respectively. Such an approach, meant to address MWH problems, tackles the hotspots issue by exploiting a frequency multiplexing strategy. The idea is that by changing frequency during the treatment, undesired side field peaks may change location in time, counteracting undesired hotspots. However, such a strategy requires multifrequency applicators, and it is not able to guarantee the avoidance of hotspots. Last, but not least, it does not address the field intensity uniformity issue.

A different approach is to combine the results gathered by solving, through TR, several focusing problems concerning a number of target points placed into the focal area [11]. While this technique allows some control of the field intensity through a proper choice of the distances among the target points, the method cannot be proved to be optimal. Even more relevant is that the method is unable to keep under control the field intensity outside of the target region.[1]

By taking advantage from both [11] and [12], we propose in the following a new approach to field intensity shaping that combines the adoption of several "target" points (say control points) with the FOCO theory. In such a way, one can exploit the inherent advantages of FOCO as compared to TR (see [12]) and address both issues related to the occurrence of undesired

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[1]Note just the real part of the field intensity is shown in the figures of [11], while field intensity is instead of actual interest to appraise the shaping.

side field peaks and to the field intensity uniformity in the target region.

As a step toward the optimal solution of the vector problem, we deal in the following with the shaping of the intensity of a scalar field,[2] while shaping the intensity of a vector field will require to merge the approach that follows with the one in [13]. On the other hand, the presented approach, which is very general, can be directly applied to ultrasound fields in biological media and to any case where one knows *a priori* that one of the field components is dominant with respect to the other ones.

The letter is organized as follows. In Section II, after a brief recall of the standard FOCO, the formulation of the proposed shaping strategy is given. The numerical validation assessing the performances of the proposed technique is reported in Section III. Conclusions and remarks follow in Section IV.

II. Proposed Shaping Approach

A. FOCO—Focusing via Optimal Costrained Power

Let us consider N elementary monochromatic electric sources surrounding the 3-D region of interest, Ω. Assuming that $\Phi_n(\underline{r})$ is the total scalar field induced by the unitary excited nth antenna in Ω when all the other antennas are off, the overall field, say $U(\underline{r})$, in a generic point in the domain of interest, when the array is excited by a set of complex excitation coefficients, $I_n\,(n=1,2,\ldots,N)$, can be expressed as

$$U(\underline{r}) = \sum_{n=1}^{N} I_n \Phi_n(\underline{r}). \quad (1)$$

If we denote by $\underline{r}_t \in \Omega$ the target point, the constrained focusing problem can be stated as follows.

Determine the set of the array's complex excitations coefficients, I_n $(n=1,\ldots,N)$, such that to maximize the squared amplitude of the field in the target point, i.e., $|\,U(\underline{r}_t)\,|^2$, while enforcing arbitrary upper bounds in the rest of the domain of interest.

Such a cost functional would be a nonnegative quadratic polynomial with respect to the unknowns. Thus, the overall maximization problem would be nonlinear and belonging to the class of NP-hard problems [14].

For the case at hand, the FOCO theory circumvents the above difficulty by assuming that the field in the target point is real. Notably, such an assumption does not reduce the available degrees of freedom, as it simply corresponds to a change of the overall phase reference.

By doing so, the problem can be stated as follows.

Determine the real and the imaginary parts of the complex excitations I_n $(n=1,\ldots,N)$ such that

$$\max\{\Re\{U(\underline{r}_t)\}\} \quad (2.a)$$

subject to

$$\Im\{U(\underline{r}_t)\} = 0 \quad (2.b)$$

$$|U(\underline{r})|^2 \le UB(\underline{r}) \qquad \underline{r} \in \Omega \setminus \Pi(\underline{r}_t). \quad (2.c)$$

As $\Im\{U(\underline{r}_t)\}$ is a linear function of the unknown excitations, and $|\,U(\underline{r}_t)\,|^2$ a positive semidefinite quadratic form, the intersection of the convex constraints (2.b), (2.c) define a convex set in the space of the unknowns [15]. Moreover, the cost function (2.a) is a linear function of the unknowns. Then, the overall constrained focusing problem is conveniently cast as a convex programming (CP) problem. As a consequence, the globally optimal solution can be efficiently determined via local optimization procedures [15].

[2]Note such a scalar field can also be a component of a vector field.

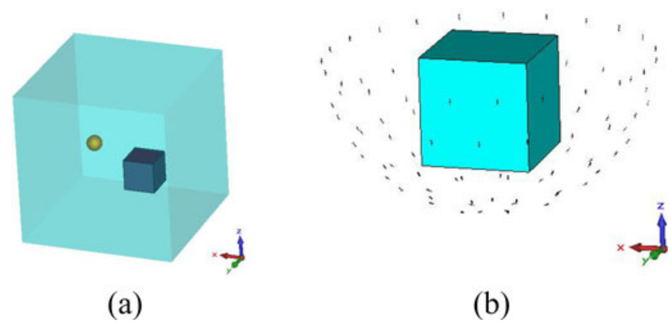

Fig. 1. (a) Scenario used for validation. (b) Antenna array configuration.

Note $UB(\underline{r})$ is a nonnegative arbitrary function (a *"mask"* function) that enforces the upper bound constraint on the power deposition outside the chosen focal area, $\Pi(\underline{r}_t) \in \Omega$. $UB(\underline{r})$ can be *ad hoc* chosen depending on the specific application's requirements.

B. Optimal Shaping via Multi-Target FOCO

Taking advantages from the above, a multitarget shaping approach (indicated as mt-FOCO) can be developed as follows.

Introducing $\underline{r}_{t\,i}$ $(i=1,\ldots,L)$ as a set of *"control points"* located into the chosen target area and, for a better understanding, considering initially the case of $L=2$, namely only $\underline{r}_{t\,1}$ and $\underline{r}_{t\,2}$, the problem of shaping the field intensity in a given target region can be formulated as follows.

Find the set of array's complex excitations coefficients I_n $(n=1,\ldots,N)$ such that

$$\max\left\{\sum_{i=1}^{2} \Re\{U(\underline{r}_{t\,i})\}^2 + \Im\{U(\underline{r}_{t\,i})\}^2\right\} \quad (3.a)$$

subject to

$$|U(\underline{r}_{t\,1})| = |U(\underline{r}_{t\,i})|\Big|_{i=2} \quad (3.b)$$

$$|U(\underline{r})|^2 \le UB(\underline{r}) \qquad \underline{r} \in \Omega \setminus \Pi(\underline{r}_t). \quad (3.c)$$

Formulation (3) is able to guarantee both the uniformity of the field at the target points as well as avoid the occurrence of unwanted hotspot, by means of constrains (3.b) and (3.c), respectively. However, the problem is again nonlinear and belonging to the class of the NP-hard problems. Also, no simple trick as the one used for the focusing problem can be exploited. Luckily, as we are going to show, the problem can be recast in terms of several (different) CP problems.

Assuming the field in $\underline{r}_{t\,1}$ to be purely real, similarly to problem (2), and exploiting the redundancy between (3.a) and (3.b), problem (3) can be formulated as follows:

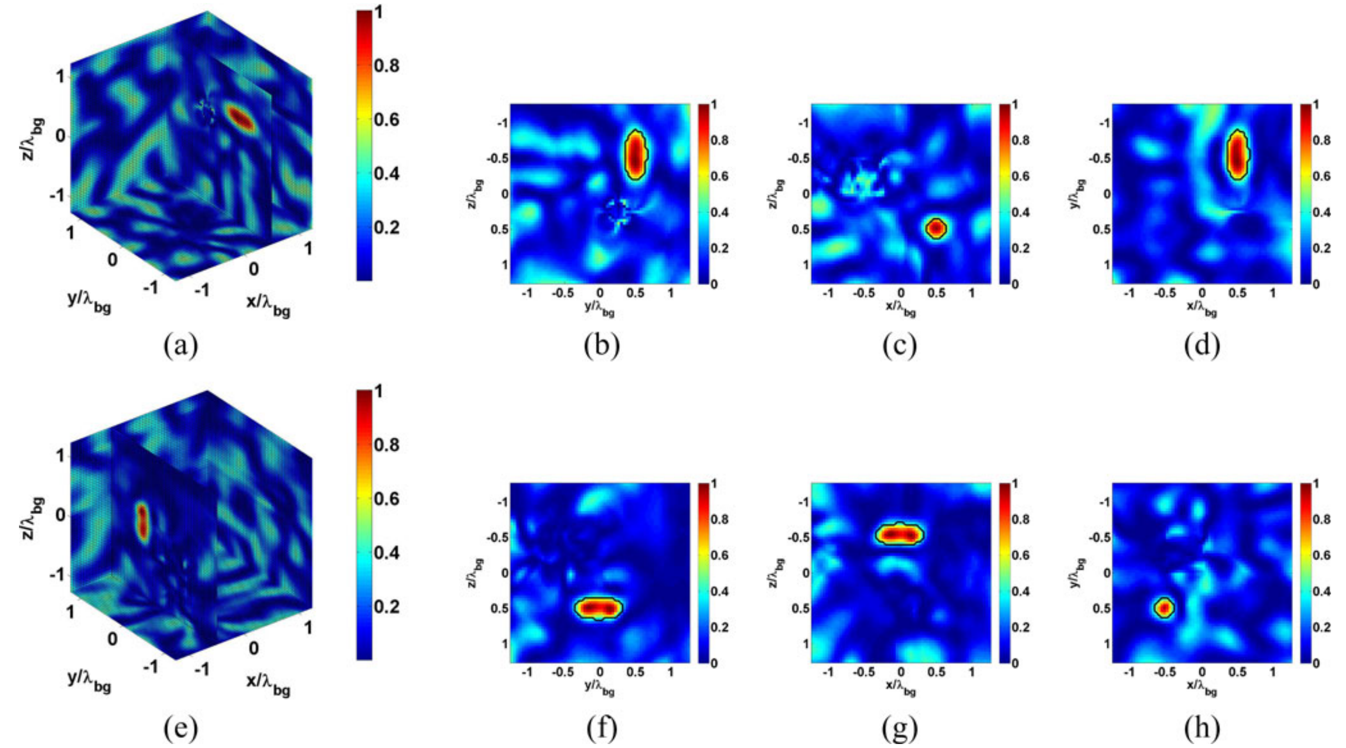

Fig. 2. Shaped field intensity by means of mt-FOCO for the configuration **I** [**II**]. (a) [(e)] squared amplitude, $|U(\underline{r})|^2$, of the shaped field. (b)–(d) [(f)–(h)] are respectively the *x*-, *y*-, and *z*-cut view of the target area.

For a sufficiently dense grid of points ϕ *in the interval* $[-\pi, \pi]$*, determine* I_n $(n = 1, \ldots, N)$ *such that*

$$\max \left\{ \Re\{U(\underline{r}_{t1})\} \right\} \tag{4.a}$$

subject to

$$\Im\{U(\underline{r}_{t1})\} = 0 \tag{4.b}$$

$$\Re\{U(\underline{r}_{t2})\} = \Re\{U(\underline{r}_{t1})\} \cos(\phi) \qquad \phi \in [-\pi, \pi] \tag{4.c}$$

$$\Im\{U(\underline{r}_{t2})\} = \Re\{U(\underline{r}_{t1})\} \sin(\phi) \qquad \phi \in [-\pi, \pi] \tag{4.d}$$

$$|U(\underline{r})|^2 \leq UB(\underline{r}) \qquad \underline{r} \in \Omega \setminus \Pi(\underline{r}_t) \tag{4.e}$$

where taking advantage from an additional parameter, represented by the phase shift, ϕ, between the fields in the two target points, constraints (3.b) have been conveniently turned into (4.c) and (4.d), which are convex.

In fact, for any fixed value of ϕ, problem (4) recasts the shaping problem as the maximization of a linear function in a convex set, which corresponds to a CP problem. Then, the globally optimal solution of overall optimization problem can be directly determined by looking into the values of the cost function achieved in the different CP problems. By doing so, the set of excitations would be chosen as the one that maximizes the function (3.a), e.g., notably, as all the different possibilities for ϕ are explored (in a sampled fashion), one is able to guarantee the global optimality of the final excitations.

The procedure can be extended in a conceptually easy fashion to the case of more control points. Of course, a price is paid in terms of computational complexity. In fact, one will need $L-1$ (phase shifts) auxiliary variables, so that if M is the number of sampling points in each auxiliary variable, the overall number of CP problems rises as M^{L-1}. On the other side, advantages can be taken from parallel computing, as well as from non-enumerative optimization procedures for the identification of the auxiliary variables' optimal values.

As a final remark, let us note that while a TR-based technique would require sensors and applicators entirely surrounding the region of interest, the proposed approach will also work (in a provable globally optimal fashion[3]) with a restricted set of antennas (possibly covering just a part of the possible impinging directions).

III. 3-D Numerical Validation

A. *Numerical Test Bed*

The 3-D scenario used for the validation is depicted in Fig. 1(a) and consists of two dielectric objects (a cube and a sphere) hosted in free space into the cubic region of interest, i.e., Ω, of side $l \approx 2\lambda_{bg}$, with λ_{bg} being the wavelength in the background medium. The cube has a side of $l_c = \lambda_{bg}/2$ and $\epsilon_c = 2$, whereas the sphere has a radius of $r_s = \lambda_{bg}/4$ and $\epsilon_s = 3$. The antenna array is an hemispherical array of radius $r \approx 4\lambda_{bg}$ made up by 92 very small unitary-excited dipoles (i.e., approximating point-sources), arranged, along the *z-axis*, evenly distributed over 6 equispaced circles (23, 21, 19, 15, 9, and 5 dipoles, respectively), as shown in Fig. 1(b).

Full-wave numerical simulations have been performed with CST Microwave Studio with a working frequency of 1.5 GHz. In order to deal with a scalar field, only the *z-component* of

[3]We mean globally optimal for the given set of antennas.

the field has been considered. However, note that enforcing constraints (4.e) on the overall vector field is straightforward. The extension to the case where the shaping is required on $|\hat{U}_x(\underline{r})|^2 + |\hat{U}_y(\underline{r})|^2 + |\hat{U}_z(\underline{r})|^2$ is instead more cumbersome (see [13]).

To test the reliability of the proposed approach, in this study an extended elliptical shaped target area, of axes $\approx 3\lambda_{bg}/4 \times \lambda_{bg}/4$, has been placed in two different positions in Ω (configurations **I** and **II** in the following).

B. Results: Analysis and Discussion

The results, reported in Fig. 2, have been obtained by means of the proposed mt-FOCO exploiting 2 control points evenly spaced with respect to the center of the target area and 20 values for the auxiliary variable ϕ. The globally optimal solution has been selected as the highest energy point, i.e., the largest value of the original cost functional (3.a).

Figs. 2(a) and (e) report the squared amplitude of the globally optimal solution field intensity for configurations **I** and **II**, respectively.

Note that, defining the coverage factor (CF) as the fraction of the target area in which $|U(\underline{r})|^2$ is higher than 50 % of its maximum value, the mt-FOCO allows us to have CFs approximately equal to 80 % and 95 %, respectively, for configurations **I** and **II**, while keeping the maximum sidelobe level amplitude outside the target region, respectively, as low as 48 % and 42 %. In order to appraise the aforementioned performances of the proposed mt-FOCO, Fig. 2(b)–(d) and Fig. 2(f)–(h) report, respectively, for the two configurations, three cut views along the target area.

The reported examples have been run on a workstation equipped with two Intel Xeon E5-2687W processors and 256 GB RAM with a calculation time of approximately 2 h each. Finally, note that no significant shaping capability of the field intensity has been achieved in both configurations (as well as in many others) by using the approach in [11].

IV. Conclusion

A method has been proposed for shaping the intensity of a scalar field in such a way as to achieve a maximum and nearly uniform behavior in a given region of space, and a sufficiently small field elsewhere.

By using a set of control points and auxiliary variables related to possible phase shifts among the corresponding fields, the approach is able to guarantee the global optimal solution through the solution of a number of CP problems.

It is worth of noting that, while the very satisfactory shaping performances herein obtained would obviously be affected by an uncertain knowledge of the scenario, an extensive analysis proving the robustness of the underlying CP approach has already been given in [4], so that we expect similar performances for the mt-FOCO.

While increasing the number of control points implies a price in terms of computational complexity, it has to be noted that this is somehow unavoidable because of its NP-hard characteristic [14], and that advantage can possibly be taken from parallel processing and non-enumerative optimization strategies.

Moreover, note that the strict equalities (4.c) and (4.d) could be possibly relaxed into inequalities and/or equalities enforcing given ratios among the different intensities, thus resulting in even more powerful methods.

Finally, it is worth noting that the proposed mt-FOCO shaping procedure can be applied also to canonical far-field problems. In this framework, mt-FOCO can be seen as a generalized and optimized version of the classical Woodward–Lawson method [16], in which the phase shifts between the field in the control points are optimally determined rather than fixed, so enabling optimal performances. In addition, the presented mt-FOCO can be also applied to those cases (e.g., planar or conformal arrays and shaped far field) where the classical procedure [17] (and the related [18]) cannot be adopted.

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