Encoder-Less Acquisition System for Rotating-Coil Measurements

Piotr Rogacki, Lucio Fiscarelli, and Stephan Russenschuck

***Abstract*—The established approach to rotating-coil measurements involves the integration of the induced voltage signal, triggered by an angular encoder, to re-parameterize the signal from time to the traveled arc length. The encoder is thus deemed necessary to ensure the rejection of speed variation errors. However, the encoder introduces additional constraints on the measurement system design and has limited performance in certain applications, such as field measurements at cryogenic temperatures. This paper presents an alternative method for rotating-coil measurement that does not require an angular encoder. The induction-coil signals themselves are used to reconstruct the rotation speed and the angular position. This is possible thanks to the field quality of particle accelerator magnets, with errors in the range of 10^{-4} relative to the main field. The approach also leverages the compensation capabilities of induction-coil magnetometers. A feasibility study and a metrological characterization of a prototype system are presented, which includes phase-shifted coils and measurements of the absolute field orientation.**

***Index Terms*—Accelerators, induction coils, magnetic measurements, magnetometers, magnets.**

I. Introduction

ROTATING-COIL magnetometers are one of the most widely used instruments for the characterization of accelerator magnets, in particular, the ones with a straight circular aperture. The strength of rotating-coil systems lies in an elegant and robust mathematical principle, and high precision of multipole field measurements [1]. Since its introduction in the 1950 s, this method has consistently relied on external tracking of the rotation angle, such as mechanical references and encoders.

However, besides introducing additional design constraints, the proper use of an encoder that does not affect the measurement precision can prove challenging in the measurements of long superconducting magnets at cryogenic temperatures. There, the high fields, of the order of 10 T, and low temperatures may prevent the installation of the encoder sufficiently close to the induction coils for reliable operation.

In the case of horizontal test benches, this can be alleviated by using extremely rigid materials, such as ceramics, to transfer the rotation angle from the motor and encoder unit placed outside of the magnet to the local coordinate system of the induction coil [2]. Unfortunately, this significantly increases the complexity of the system in its construction and operation, and is prohibitively hard to apply in a vertical configuration.

This paper proposes an alternative method of tracking the magnetometer's angular position. It is based solely on the voltage signals generated by the induction coils, and leverages the possibility of tuning the sensitivity of those coils, as well as the quality of the field (with multipole errors in the range of 10^{-4} relative to the main field component) typically achieved in accelerator magnets.

The method is validated experimentally and compared with a system equipped with an angular encoder. The results show negligible loss of precision in measurements at ambient temperature.

II. Description of the Method

A. Coil Sensitivity

The sensitivity of a coil to a field component depends on its geometric shape, as well as its position with regard to the rotation axis. A coil or a coil array can be designed to be sensitive purely to a single multipole [3], such as the main field component. While rotating the coil in a magnetic field, a voltage signal is induced that is a nearly perfect sine wave, modulated only by the speed variation.

However, using a coil design sensitive to a single multipole introduces a strong constraint on the order of the magnet to be characterized - a coil designed for a dipole cannot be used in the quadrupole. A more general solution based on coil arrays exists for most accelerator magnets, where the main component of the field is 3–4 orders of magnitude larger than the higher-order multipoles. This solution requires only minor modifications compared to the standard rotating-coil magnetometer layouts, and has been chosen as the baseline for the proposed method.

A rotating-coil transducer is usually equipped with an array of multiple coils on a single support shaft, used mainly to compensate for the fundamental field component as described in [4]. One of the typical arrangements of coils in rotating-coil magnetometers is shown in Fig. 1, coils A-E. In a simplified case, the sensitivity of a coil to the multipole of the n-th order is proportional to the rotation radius r to the power of $n-1$. For example, dipole sensitivity is independent of the radius, while quadrupole and sextupole components have linear and quadratic dependencies, respectively. By combining the coils in series, either with the same or opposite polarity, the ensemble sensitivity to certain field multipole orders can be tuned.

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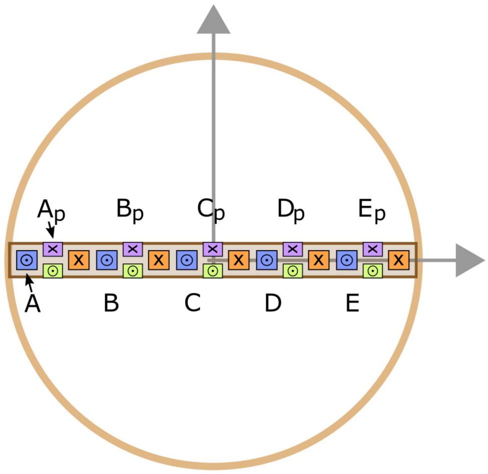

Fig. 1. Example optimal arrangement with nested phase-shifted windings. Each main coil has the phase-shifted winding inside, rotated by 90° around the coil center.

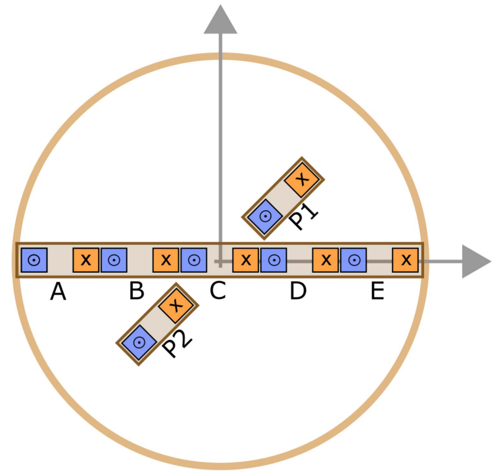

Fig. 2. Alternative coil arrangement, optimal for quadrupole measurements. P1 and P2 are the additional, phase-shifted coils obtained by rotating coils B and D by 45° around the shaft axis.

Conventionally, compensation aims at minimizing the sensitivity to the fundamental field component without significantly affecting the sensitivity to higher-order components. This is instrumental in increasing the accuracy of the measurement of high-order field multipoles, by reducing random and systematic errors. This is known as the bucking scheme.

Here we take the opposite approach: the sensitivities of the individual coils are combined in a way that compensates the higher-order multipoles, while preserving the sensitivity to the main component. For example, in a quadrupole magnet measurement, using the B and D coils (see Fig. 1) connected with an opposite polarity yields a voltage signal that is significantly less sensitive to high-order multipoles, compared to the quadrupole component. By virtue of symmetry, the sensitivity to odd multipole terms is nearly zero. Furthermore, even with a generous assumption of identical sensitivity to all multipoles for coils A and E, the sensitivity of coils B and D to the first allowed harmonic (the dodecapole - 6th order) is 32 times lower than that of the quadrupole.

Since higher-order multipole components are significantly smaller than the fundamental component n, rotating coils in this arrangement generate an almost perfect sinusoidal voltage signal, modulated only by the rotation speed variation:

$$U(t) = n\dot{\varphi}(t) S_n B_n \sin\left(n\varphi(t)\right), \tag{1}$$

where φ is the angular position, $\dot{\varphi}$ is the rotation speed, S_n is the coil sensitivity, and B_n is the normal, main-field component. The skew field component A_n can be assumed to be zero without loss of generality.

The induced voltage signal can therefore be used to calculate the rotation speed and angular position at all acquisition points. An issue with this basic implementation is the ill-conditioning of the problem around the zero-crossings of the induced voltage, where the acquisition noise can become larger than the effect of the speed variation. To address this problem, we propose additional coils that provide a phase-shifted signal.

B. Analytical Speed Reconstruction

For the *phase-shifted* coils, the induced voltage is:

$$U^{\mathrm{p}}(t) = n\dot{\varphi}(t) S_n^{\mathrm{p}} B_n \sin\left(n\varphi(t) + \varphi_{\mathrm{p}}\right), \tag{2}$$

with the superscript p denoting the phase-shifted coils, and φ_{p} corresponding to the signal phase difference between the phase-shifted and main coils. Ideally, the phase-shifted coils should generate a signal that is shifted by a quarter-period to the main coils, which corresponds to $\varphi_{\mathrm{p}} = \frac{\pi}{2}$. In such a case, combining (1) and (2) yields

$$\left(\frac{U(t)}{S_n}\right)^2 + \left(\frac{U^{\mathrm{p}}(t)}{S_n^{\mathrm{p}}}\right)^2 = \left(n\dot{\varphi}(t) B_n\right)^2, \tag{3}$$

which leads to the expression for the angular speed:

$$\dot{\varphi}(t) = \omega(t) = \frac{\sqrt{\left(\frac{U(t)}{S_n}\right)^2 + \left(\frac{U^{\mathrm{p}}(t)}{S_n^{\mathrm{p}}}\right)^2}}{n B_n}. \tag{4}$$

The most general way to create the optimal phase-shifted coils is to introduce *nested* windings rotated by 90° inside each main coil (see Fig. 1, coils $\mathrm{A_p}$-$\mathrm{E_p}$). In this case, the main coils are sensitive to the angular field component, while the phase-shifted ones are sensitive to the radial field component. Since the components are perpendicular, this arrangement provides a quarter-period phase shift for all magnet orders and, in most cases, can be readily created using PCB (Printed Circuit Board) technology [5].

In the case of the test system presented here, the available PCBs were not equipped with the nested coils, and therefore, a different arrangement (optimized only for quadrupole; see Fig. 2) was used for the validation of the proposed method. It involves the installation of two additional coils on an existing measurement head. Even though the windings are not perfectly aligned (3° deviation from the expected 45°), the results are promising; see Fig. 5.

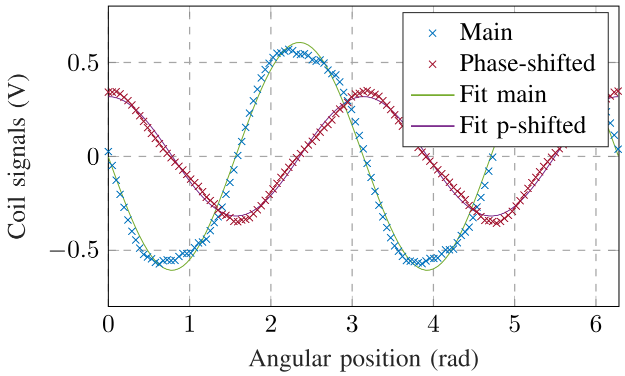

Fig. 3. Signals generated by the main and phase-shifted coil arrangements together with their respective fitted sines.

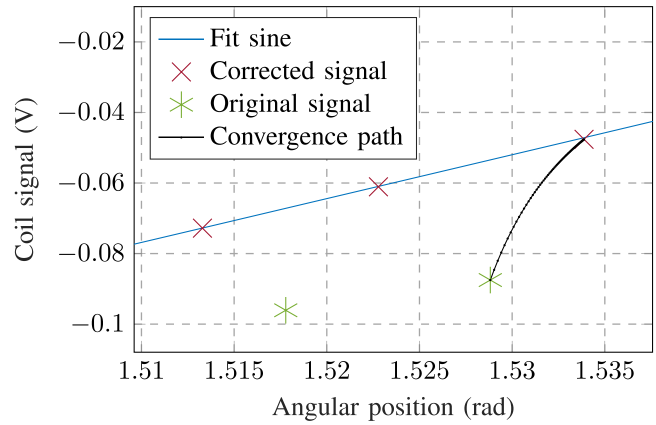

Fig. 4. Reconstruction convergence plot of an example point.

C. Numerical Speed Reconstruction

An alternative numerical speed reconstruction can be applied if creating the nested coils with sufficient precision for the analytical reconstruction is not possible due to mechanical or sensitivity constraints. In this case, the precision of the coil positioning can be relaxed, and the coil arrangement shown in Fig. 2 can also be used in dipoles and sextupoles, albeit with less accuracy of the speed reconstruction. If the voltage signals are not shifted by a half-period, the reconstruction is less robust in the areas where the sum of the two squared signals is smaller (as opposed to the almost constant sum of squares for a half-period shift).

The proposed algorithm for the reconstruction of the speed can be summarized in the following steps:

1) Fit a sine to the coil signal (see Fig. 3) over several rotations to account for non-periodic variations of rotation speed and reduce the sensitivity to noise. This implies that the analysis is done in a post-processing step.
2) Calculate the average angular velocity from the angular frequency of the fitted sine, accounting for the magnet order.
3) Convert the samples from the time domain to the angular domain, with the initial assumption of constant speed.
4) Select a starting point: the choice is arbitrary, but a zero-crossing is convenient.
5) Search for the instantaneous angular speed value that would result in the observed deviation of the voltage sample from the fitted sine. Alternatively, find the correction necessary to move the point onto the fitted sine as in Fig. 4. For example, in the case of the shown point:
   - The absolute value of the sample is larger than expected from the fit. This means that the instantaneous speed is higher than average, therefore, the point should be shifted further in the angular coordinate.
   - Since the speed is higher, the expected voltage should be adjusted. Equivalently, an inverse correction can be applied to the sample (for clarity of display, as is the case here).
   - A simple line search algorithm [6] can be used to iteratively find the angular speed that would correspond to the observed modulation of the perfect sine.

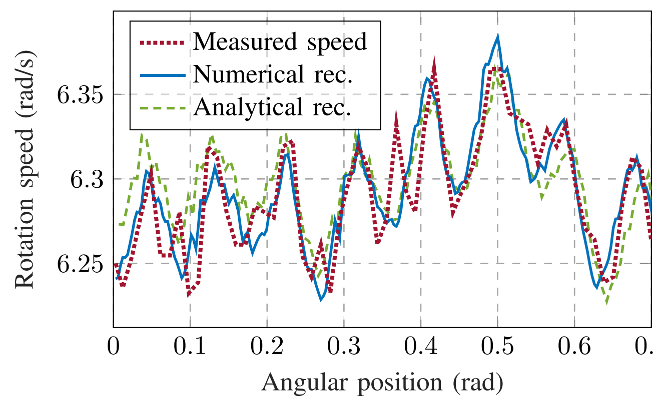

Fig. 5. Comparison between the angular speed as measured by the encoder and the reconstructed ones.

6) Move to the next sample, taking into account the new angular position stemming from the last and all the previous speed estimations.
7) Repeat steps 4 and 5 until the last sample is reached.

The results of the rotation-speed estimation for both the main and the phase-shifted coils are combined. The simplest approach is to weigh each rotation-speed estimate by the corresponding signal level. The results of a reconstruction are shown in Fig. 5. Thanks to the phase-shifted coils, the reconstruction is well-conditioned over the full rotation. This approach has been tested with the rotation speed variation of up to 20% peak-to-peak.

D. Field Orientation Measurements

An external angular reference is necessary to measure the field orientation. Usually, a precise inclinometer is used to provide the reference to gravity, which in turn can be related to the magnet orientation that is given, for example, by a fiducial surface [2].

Due to the absence of an encoder, which normally provides a reference point (index) in the rotation, an alternative alignment procedure is necessary. The proposed solution involves the acquisition of the signals starting from the leveled position given by an electrolytic inclinometer. According to Faraday's induction law, integrating the voltage signal yields the magnetic flux intercepted by the coils:

$$U(t) = -\frac{\mathrm{d}\Phi(t)}{\mathrm{d}t} \Rightarrow \Phi(t) = \int U(t)\mathrm{d}t, \tag{5}$$

where Φ denotes the magnetic flux. If the coil, or an array of coils, is sensitive almost exclusively to one multipole (of order n), the magnetic flux obtained by integration of acquired voltages has a purely sinusoidal dependence on the angle:

$$\tilde{\Phi}(\varphi) = \Phi_A \sin\left(n\left(\varphi + \varphi_0\right)\right) - C, \tag{6}$$

where Φ_A is the amplitude of the flux signal, and the integration constant C depends on the initial condition and represents the flux intercepted by the coil in the leveled position. In this case, the integration begins at zero, yielding:

$$\tilde{\Phi}(0) = 0 = \Phi_A \sin\left(n\left(\varphi + \varphi_0\right)\right) - C \tag{7}$$

$$C = \Phi_A \sin\left(n\left(\varphi_0\right)\right) \tag{8}$$

$$\varphi_0 = \frac{1}{n}\arcsin\left(\frac{C}{\Phi_A}\right). \tag{9}$$

However, due to the large time constant of electrolytic inclinometers, this method requires the coil to be stationary at the beginning of the integration. Because of that, the shape of the integrated signal as a function of time significantly deviates from a sinusoid in the initial acceleration phase. As there is no angular reference, this deviation is unknown and therefore, the phase of the signal cannot be established.

The proposed solution is to rely solely on the flux peaks, which are independent from the phase. Considering that $\tilde{\Phi} = \Phi_A - C = \tilde{\Phi}_{\text{max}}$ for $n(\varphi + \varphi_0) = \frac{\pi}{2} + k \cdot 2\pi$, where $k = 0, 1, 2\ldots$, C can be computed from flux maxima (or minima) as follows:

$$C = \Phi_A - \tilde{\Phi}_{\text{max}} = -\tilde{\Phi}_{\text{min}} - \Phi_A. \tag{10}$$

Combining (10) with (9) gives the final expression for the initial phase angle:

$$\varphi_0 = \frac{1}{n}\arcsin\left(-\frac{\tilde{\Phi}_{\text{max}} + \tilde{\Phi}_{\text{min}}}{2\Phi_A}\right). \tag{11}$$

The additional benefit of this approach is that it provides a stable reference for the polarity of the measured field.

III. Experimental Validation

The method has been validated in a quadrupole magnet, and compared with a calibrated reference system equipped with an angular encoder [7]. The magnet, with an aperture diameter of 124 mm and a length of 1.5 m, creates a field gradient of approximately 7 T/m in its center at 400 A. The higher-order multipoles produced by the magnet at the reference radius of 42 mm are within $5 \cdot 10^{-4}$ relative to the main field. The acquisitions were taken at 16667 Hz over 30 full rotations at varying speeds.

Table I and Fig. 6 show the measurement precision achieved with the proposed approach. The precision of the main field gradient and multipoles is provided relative to the main field gradient, in units of 10^{-4}. The values represent the maximum

TABLE I
Summary of Achieved Results

Parameter	Unit	Precision	Diff. to ref.
Main field gradient	10^{-4}	0.35	10
Field orientation	mrad	0.2	0.3
Field axis	mm	0.01	0.1
Field multipoles	10^{-4}	0.01	0.02

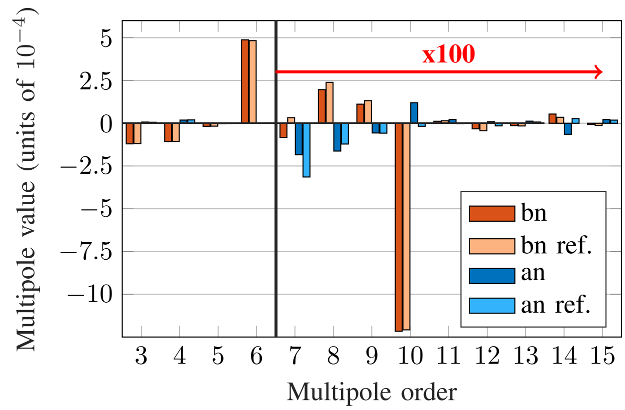

Fig. 6. Comparison between the normal and skew multipoles measured without an encoder, and the reference system with an encoder. In both cases, the values are obtained by averaging the results of 30 turns. Darker bars represent results obtained with the proposed method, while the lighter ones (ref.) correspond to the reference measurement. Multipoles of order seven and above are multiplied by 100 for display clarity.

variation between the measurements. The multipoles of all orders up to 15 agree with the reference within 0.02 units and, thanks to the field angle measurement, have consistent polarity. The precision of the proposed method is close to the precision of a conventional setup with an encoder.

IV. Conclusion

The presented method allows precise rotating-coil measurements without an encoder. The most significant improvements can be expected in measurements of long superconducting magnets, such as those used in LHC (Large Hadron Collider) [8], HL-LHC (High Luminosity Large Hadron Collider) [9] or possibly in FCC (Future Circular Collider) [10], at cryogenic temperatures. There, strong fields and more restrictive mechanical constraints often limit the achievable precision and functionality. For example, the precision, robustness, and simplicity of scanning magnetometers, such as the one presented in [11], can benefit from the encoder-less operation.

In addition, the reduction of the design constraints and the resulting system simplification contribute towards increasing the application range of the rotating-coil magnetometers. This synergizes well with the development of systems based on commercially available components, as presented in [12].

Possible further research involves confirming the feasibility of embedding the phase-shifted coils on the same PCB and studying the alternatives to the precision inclinometers for field orientation measurements, such as an onboard accelerometer.

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