Resource Efficiency: A New Paradigm on Energy Efficiency and Spectral Efficiency Tradeoff

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Abstract—Spectral efficiency (SE) and energy efficiency (EE) are the main metrics for designing wireless networks. Rather than focusing on either SE or EE separately, recent works have focused on the relationship between EE and SE and provided good insight into the joint EE–SE tradeoff. However, such works have assumed that the bandwidth was fully occupied regardless of the transmission requirements and therefore are only valid for this type of scenario. In this paper, we propose a new paradigm for EE–SE tradeoff, namely the resource efficiency (RE) for orthogonal frequency division multiple access (OFDMA) cellular network in which we take into consideration different transmission-bandwidth requirements. We analyse the properties of the proposed RE and prove that it is capable of exploiting the tradeoff between EE and SE by balancing consumption power and occupied bandwidth; hence simultaneously optimizing both EE and SE. We then formulate the generalized RE optimization problem with guaranteed quality of service (QoS) and provide a gradient based optimal power adaptation scheme to solve it. We also provide an upper bound near optimal method to jointly solve the optimization problem. Furthermore, a low-complexity suboptimal algorithm based on a uniform power allocation scheme is proposed to reduce the complexity. Numerical results confirm the analytical findings and demonstrate the effectiveness of the proposed resource allocation schemes for efficient resource usage.

Index Terms—Green radio (GR), orthogonal frequency division multiple access (OFDMA), resource efficiency (RE), energy efficiency (EE), spectral efficiency (SE).

I. Introduction

ONE of the major challenges in wireless communications is the increased energy consumption as a result of the rising number of emerging high data rate wireless applications such as multimedia and interactive services. Excessive power usage in such networks is a critical issue for the mobile operators in particular. To meet these challenges, the green evolution has become an urgent need for wireless networks today. Green radio, a research direction for the evolution of future wireless architectures and techniques, has thereby been pursued as an important trend for future energy-efficient wireless communications.

Up to now, the spectral efficiency (SE) metric has been the main performance indicator for designing and optimizing wireless communication networks. It is an important measure for quantitatively evaluating the effectiveness of cellular systems and has been extensively studied for various scenarios [1]–[3]. Although SE measures how efficiently a limited frequency spectrum is utilized, it fails to account for how efficiently power is consumed. Due to high network power consumption, research on energy efficiency (EE) has attracted much interests recently, including single link optimization [4], single cell scenario [5], multi-cell deployment [6], cognitive radio network [7] and cooperative relaying network [8].

Orthogonal frequency division multiple access (OFDMA) has been recognized as one of the best candidates for next generation broadband wireless standards because of its natural ability to utilize different portions of the spectrum. In OFDMA networks, multiple users share the total bandwidth to exploit multiuser diversity and improve the system capacity with dynamic resource allocation strategies. OFDMA has been extensively studied from the SE perspective. Recently, more attention has been paid to energy-efficient design in OFDMA networks. Energy-efficient OFDMA is first addressed in [9] which shows at least a 20% reduction in power consumption when performing EE optimization. Authors in [10] studied resource allocation for energy-efficient communication in multi-cell orthogonal OFDMA downlink networks with cooperative base stations, and extended the work to the case of a large number of transmit antennas in [11]. Energy-efficient design for multi-user OFDMA has also been studied. Energy-efficient radio resource scheduling with quality of service (QoS) guarantees in a multi-user OFDMA system has been studied in [12]. An optimization problem is formulated to maximize energy efficiency subject to users QoS requirements, and addressed using Lagrangian duality. Authors in [13] developed a low complexity energy-efficient scheduling scheme for uplink OFDMA scenario. The problem of resource allocation for the uplink of a multicell wireless network has been studied in [14]. Furthermore, downlink scenario has also been investigated. Energy-efficient design for downlink OFDMA with delay-sensitive traffic has been studied in [15]. In [16], an energy-efficient resource allocation scheme in both downlink and uplink OFDMA networks has been studied.

Unfortunately, optimizing EE and SE do not always coincide and may even conflict sometimes [17], [18]. Hence, how to balance EE and SE is a problem well worth studying. Conventional design of wireless networks mainly focuses on either SE or EE. Recently, the relationship between EE and SE has become an important research topic. In [19], a framework to

integrate the connections between EE and SE tradeoff has been proposed. Authors in [20] investigates the tradeoff between EE and SE for interference-limited wireless networks. In [21], EE–SE tradeoff with consideration of circuit power has been studied for energy-constrained wireless multihop networks with a single source-destination pair. In [22], a generic closed-form approximation of the EE–SE tradeoff has been derived for the uplink of coordinated multi-point (CoMP) system. Authors in [23] investigated the EE–SE relationship in a single cell down-link OFDMA network, which is important for designing energy efficient networks that require a better balance between EE and SE. They proved that the EE–SE relationship is a quasiconcave function. Based on that, a tight lower bound and a tight upper bound on the EE–SE curve are provided by Lagrange dual decomposition (LDD) and continuous relaxation, respectively. However, all such works about EE–SE tradeoff assumed that bandwidth are fully occupied regardless of the transmission requirements. As a result, this may lead to inefficient use of network resources.

A. Main Contributions

In this paper, we propose a new paradigm for EE–SE trade-off, namely the resource efficiency (RE) for OFDMA cellular network which accounts for all scenarios of bandwidth loading. We analyze the properties of the proposed RE and prove that it can exploit the tradeoff between EE and SE. We then formulate the generalized RE optimization problem for the downlink transmission with guaranteed QoS as a multiobjective optimization problem. This optimization problem is in general NP-hard for the optimal solution. To obtain insight into the problem, we first investigate the properties of the case with a given subcarrier assignment and obtained an analytical solution. By using water-filling algorithm for power allocation, we developed a gradient based optimal power adaptation scheme to solve the resource allocation problem. Since the optimal solution needs to check all possible number of active subcarriers, and for each case apply the water-filling algorithm and gradient updated scheme to find the optimal power allocation, the complexity is comparatively high however. We also provide an upper bound near optimal method to jointly solve the optimization problem based on interior point method. Although convex programming is numerically stable, its computational complexity depends on the number of optimizing variables, which can be large if the number of subcarriers and/or the number of UEs is still large. Therefore, we further explore the property of the optimization problem using the uniform power allocation scheme and propose a novel low-complexity suboptimal algorithm to solve the original problem. Simulation results confirm the theoretical findings and reveal that by allowing a slight increase in energy consumption (or bandwidth usage), the proposed approach can significantly reduce the amount of bandwidth used (or energy consumed). Thus the proposed RE can efficiently optimize the use of available resources in the network.

B. Organization and Notation

The rest of this paper is organized as follows. The system model and the limitation for energy-efficient or spectral-

efficient only design are described in Section II. In Section III, we introduce the proposed metric description and formulate the optimization problem. In Section IV, we first develop a gradient based optimal power adaptation scheme to solve the resource allocation problem. We then provide an upper bound near optimal method to jointly solve the optimization problem. To reduce complexity, a novel low-complexity suboptimal algorithm is also proposed to solve the original problem. Simulation results are provided in Section V and conclusions are drawn in Section VI.

The following notations are used in the paper. Bold upper and lower case letters denote matrices and vectors, respectively; $[x]^+$ denotes $\max(x, 0)$ and $[\cdot]_{K \times N}$ denotes an $K \times N$ matrix.

II. SYSTEM MODEL

In this section, we first introduce the system model and then discuss the limitation for energy-efficient or spectral-efficient only design.

A. System Description

Consider a single cell downlink OFDMA network with K active user equipments (UE). The total bandwidth, W_{tot}, is divided into N subcarriers, each with a bandwidth of $W_C = (W_{tot}/N)$. We assume that subcarriers are allocated centrally and each subcarrier cannot be assigned to more than one user to avoid interference among different UEs. The transmit power and the channel frequency response of the kth UE on the nth subcarrier are denoted as $p_{k,n}$ and $H_{k,n}$, respectively. Assuming perfect channel state information in both transmitter and receiver, the maximum achievable data rate of the kth UE on the nth subcarrier is accordingly

$$r_{k,n} = W_C \log_2 \left(1 + \frac{p_{k,n}|H_{k,n}|^2}{N_0 q W_C} \right) \qquad (1)$$

where N_0 is the single-sided noise spectral density. Hence, the aggregate rate for the kth UE and the overall throughput are shown, respectively as follows:

$$R_k = \sum_{n \in \mathcal{N}} \rho_{k,n} r_{k,n} \qquad (2)$$

$$R = \sum_{k \in \mathcal{K}} R_k = \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}} \rho_{k,n} r_{k,n} \qquad (3)$$

where $\rho_{k,n} \in \{1, 0\}$ indicates whether the nth subcarrier is assigned to the kth UE, $\mathcal{N} = \{1, 2, \ldots, N\}$ and $\mathcal{K} = \{1, 2, \ldots, K\}$ denote the sets of all subcarriers and all UEs, respectively. Obviously, a feasible subcarrier assignment indicator matrix, $\boldsymbol{\rho} = [\rho_{k,n}]_{K \times N}$, should satisfy

$$\sum_{k \in \mathcal{K}} \rho_{k,n} \leq 1, \ \forall n \in \mathcal{N} \qquad (4)$$

$$\rho_{k,n} \in \{0, 1\}, \ \forall k \in \mathcal{K}, \ \forall n \in \mathcal{N} \qquad (5)$$

$$\sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}} \rho_{k,n} = Q \qquad (6)$$

where Q denotes the number of used subcarriers, and hence $Q \leq N$. $\mathcal{Q}$ is defined as the sets of used subcarriers, $\mathcal{Q} \subseteq \mathcal{N}$. The subcarrier assignment constraint in (4)–(6) can be equivalently viewed from the following perspective:

$$\bigcup_{k=1}^{K} \mathcal{S}_k \subseteq \mathcal{Q} \text{ and } \mathcal{S}_k \bigcap \mathcal{S}_{k'} = \emptyset, \ \forall k \neq k' \quad (7)$$

where $\mathcal{S}_k$ is the set of subcarriers assigned to the kth UE. Since the total transmit power of either base station or UE is nonnegative and also limited, any possible power allocation matrix, $\mathbf{P_T} = [p_{k,n}]_{K \times N}$, should satisfy the following conditions:

$$p_{k,n} \geq 0, \ \forall k \in \mathcal{K}, \ \forall n \in \mathcal{N} \quad (8)$$

$$\sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}} p_{k,n} \leq P_{\max} \quad (9)$$

where $P_{\max}$ represent the maximum total transmit power at the base station for downlink transmission.

The overall transmit power for the kth UE and the total transmit power are shown as follows:

$$P_k = \sum_{n \in \mathcal{N}} p_{k,n} \quad (10)$$

$$P_T = \sum_{k \in \mathcal{K}} P_k = \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}} p_{k,n}. \quad (11)$$

In addition to transmit power, other active circuit blocks also consume energy [24]. For the downlink transmission, the overall power consumption model at the base station is given by [24]

$$P = \zeta P_T + P_C \quad (12)$$

where ζ is the reciprocal of drain efficiency of power amplifier and P_C represents the circuit power. Similarly, the overall power budget at the base station is modeled as

$$P_{tot} = \zeta P_{\max} + P_C. \quad (13)$$

B. Energy Efficiency

Conventional EE for downlink transmission is defined as the total delivered bits per unit energy, where energy consumption includes transmission energy consumption and circuit energy consumption in active mode, i.e.,

$$\lambda_{EE} \triangleq \frac{R}{P} = \frac{\sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}} \rho_{k,n} r_{k,n}}{\zeta P_T + P_C}. \quad (14)$$

Since EE represents the delivered bits per unit energy, optimizing EE has its limitation. As shown in Fig. 1(a), system will occupy as much bandwidth as possible to maximize the energy efficiency. Hence, it reduces the efficient use of resource in terms of bandwidth usage. Therefore, EE has its limitation in terms of optimally using the existing resources.
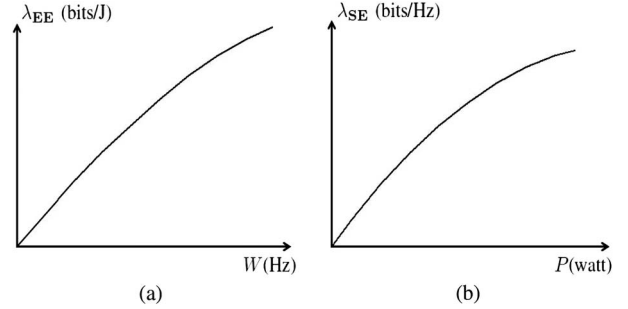

Fig. 1. EE and SE versus bandwidth $\mathbf{W}$ and power $\mathbf{P}$.

C. Spectral Efficiency

On the other hand, SE is defined as the total (average) number of delivered bits per unit bandwidth

$$\lambda_{SE} \triangleq \frac{R}{W} = \frac{\sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}} \rho_{k,n} r_{k,n}}{W} \quad (15)$$

where W represent the occupied bandwidth. In OFDMA network, the occupied bandwidth relates to the number of active subcarriers,

$$W = Q \times W_C = Q \times \frac{W_{tot}}{N}. \quad (16)$$

Similar to EE, optimizing SE also has its limitation. It will use as much power as possible to maximize the spectral efficiency which is shown in Fig. 1(b), and hence reduces the resource utilization in terms of power usage.

III. RESOURCE EFFICIENCY AND PROBLEM FORMULATION

As the conventional system design using EE or SE has limitation in terms of inefficient use of resources, we therefore introduce multi-objective optimization to optimize both EE and SE. In general, a constrained multi-objective optimization problem is defined as follows [25]:

$$\min_{\mathbf{x}} \quad F(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_k(\mathbf{x})) \quad (17)$$

$$\text{s.t.} \quad g_i(\mathbf{x}) \leq 0, i = 1, 2, \dots, m, \quad (18)$$

$$\qquad h_j(\mathbf{x}) = 0, j = 1, 2, \dots, n, \quad (19)$$

where $F(\mathbf{x})$ is the set of objective functions, $g_i(\mathbf{x})$ is the set of inequality constraints and $h_j(\mathbf{x})$ is the set of equality constraints. A constrained multi-objective optimization problem minimizes k objective functions simultaneously, where the objective functions represent (usually) competing or conflicting objectives. A multi-objective problem is often solved by combining its multiple objectives into a single-objective scalar function. This approach is in general known as the weighted-sum or scalarization method. In particular, the weighted-sum method minimizes a positively weighted convex sum of the objectives, that is

$$\min_{\gamma_l, \mathbf{x}} \quad \sum_{l=1}^{k} \gamma_l f_l(\mathbf{x}) \quad (20)$$

$$\text{s.t.} \quad \sum_{l=1}^{k} \gamma_l = 1, \ \gamma_l > 0, l = 1, 2, \dots, k, \quad (21)$$

$$\qquad g_i(\mathbf{x}) \leq 0, i = 1, 2, \dots, m, \quad (22)$$

that represents a new optimization problem with a unique objective function. It can be proved that the minimizer of this single-objective function is an efficient solution for the original multi-objective problem [25], i.e., its image belongs to the Pareto curve.

In this paper, since both EE and SE are our research target, we consider both of them as our objective functions and transformed the constrained multi-objective optimization problem to single-objective function as follows:

$$\max \quad \gamma_1 \lambda_{EE} + \gamma_2 \lambda_{SE} \tag{23}$$
$$\text{s.t.} \quad \gamma_1 + \gamma_2 = 1, \tag{24}$$
$$\gamma_1 > 0, \gamma_2 > 0. \tag{25}$$

However, the unit for EE is bits/Joule while the unit for SE is bits/s/Hz, and hence it is inappropriate to directly add EE and SE due to inconsistence of metric unit. Moreover, since the bandwidth W is numerically larger then the transmission power P_T, a simple summation of EE and SE will tend to focus the optimization problem on EE. This will defeat the purpose of balancing between EE and SE. As a result, we now introduce a new system metric called resource efficiency defined as

$$\lambda_{RE} \triangleq \frac{R}{P}\left(1 + \beta\frac{\eta_P}{\eta_W}\right) \tag{26}$$

where η_P and η_W represent power utilization and bandwidth utilization, respectively, given by

$$\eta_P \triangleq \frac{P}{P_{tot}}, \quad \eta_W \triangleq \frac{W}{W_{tot}}. \tag{27}$$

These variables are to denote the ratio of power and bandwidth used out of those that are available. β is a weighted factor to control the balance of EE and SE. In the following, we show that this new system metric is capable of exploiting the tradeoff between EE and SE by balancing consumption power and occupied bandwidth.

Property I: Resource efficiency is capable of exploiting the tradeoff between EE and SE, with the weighting between them controlled by $\beta(W_{tot}/P_{tot})$.

Proof: By substituting (27) into (26), we have

$$\begin{aligned}\lambda_{RE} &= \frac{R}{P}\left(1 + \beta\frac{\frac{P}{P_{tot}}}{\frac{W}{W_{tot}}}\right)\\ &= \frac{R}{P} + \beta\frac{R}{P}\frac{P}{P_{tot}}\frac{W_{tot}}{W}\\ &= \frac{R}{P} + \beta\frac{R}{W}\frac{W_{tot}}{P_{tot}}\\ &= \lambda_{EE} + \bar{\beta}\lambda_{SE}\end{aligned} \tag{28}$$

where $\bar{\beta} = \beta(W_{tot}/P_{tot})$. Hence, RE can be rewritten as a combination of EE and SE, W_{tot}/P_{tot} acts as unit normalizer for SE and EE while β acts as the weighted factor to control the balance of EE and SE. Furthermore, $\bar{\beta}$ in the proposed RE is equal to γ_2/γ_1 in the scalarization method in (23)–(25). ∎

Interestingly, due to the normalization factor W_{tot}/P_{tot}, the unit of the proposed RE is the same as EE which is still bits/Joule, which is a good measure for green communications. Furthermore, considering a fixed of β, when the total bandwidth W_{tot} is larger than the total power budget P_{tot}, RE emphasis

more on SE. On the other hand when the total bandwidth W_{tot} is smaller compared to the total power budget P_{tot}, RE will put more weight on EE. Practically, it reveals that RE will emphasis more on SE to save occupied bandwidth when the system has sufficient bandwidth to support the transmission requirement. For this case, if RE emphasize more on EE, the system will occupy as much bandwidth as possible to maximize the EE and hence reduces the efficient use of resource in terms of bandwidth usage. On the other hand, RE will put more weight on EE to save transmission power when the system has ample power to support the transmission requirement. For this case, if RE emphasize more on SE, the system will use as much power as possible to maximize SE and hence reduce the green resource usage in terms of power usage. Consequently, this proposed system metric is capable of exploiting the tradeoff between EE and SE and adapt its optimization based on the available resources.

On the other hand, β is the weight between EE and SE such that RE optimize EE when $\beta = 0$ but optimize SE when $\beta = \infty$. There is no a priori correspondence between a weight vector and a solution vector and hence it is up to the decision maker to choose appropriate weights. Consequently, without loss of generality, we consider β as a constant in our RE optimization problem. Since providing different service priorities and guaranteeing QoS for each UE is important in communication system design, we consider the generalized RE under a series of traffic-related minimum rate requirements, γ_k, and the peak transmit power, $P_{\max}$. Hence, the generalized RE optimization problem for the downlink transmission can be mathematically formulated as follows:

$$\max_{\boldsymbol{\rho},\mathbf{P_T}} \frac{\sum_{k\in\mathcal{K}}\sum_{n\in\mathcal{N}}\rho_{k,n}r_{k,n}}{\zeta P_T + P_C}\left(1 + \beta\frac{\eta_P}{\eta_W}\right) \tag{29}$$

subject to

$$\sum_{n\in\mathcal{N}}\rho_{k,n}r_{k,n} \geq \gamma_k, \; \forall k\in\mathcal{K} \tag{30}$$
$$\sum_{k\in\mathcal{K}}\sum_{n\in\mathcal{N}}p_{k,n} \leq P_{\max} \tag{31}$$
$$\sum_{k\in\mathcal{K}}\sum_{n\in\mathcal{N}}\rho_{k,n} \leq N. \tag{32}$$

IV. PROPOSED OPTIMAL AND NEAR-OPTIMAL RESOURCE ALLOCATION SCHEMES

In this section, we will develop a gradient based optimal power adaptation scheme and a near optimal upper bound approach based on interior point method for the resource-efficient resource allocation in the downlink transmission.

A. Optimal Resource Allocation Scheme

Problem (29)–(32) is in general NP-hard for the optimal solution. To obtain an insight into the problem, we first investigate the properties of the case with a given subcarrier assignment, which are summarized in the following theorem.

Theorem I: For any fixed subcarrier assignment indicator matrix $\boldsymbol{\rho}$ and its corresponding subcarrier assignment sets

$\mathcal{S}_k (\forall k \in \mathcal{K})$, the maximum achievable RE at a certain total transmit power, P_T, namely

$$\bar{\lambda}_{RE}(P_T) \triangleq \max_{p_{k,n} \geq 0} \frac{\sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{S}_k} r_{k,n}}{\zeta P_T + P_C} \left(1 + \beta \frac{\eta_P}{\eta_W} \right) \quad (33)$$

subject to

$$\sum_{n \in \mathcal{S}_k} r_{k,n} \geq \gamma_k, \quad \forall k \in \mathcal{K} \quad (34)$$

$$\sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{S}_k} p_{k,n} = P_T \quad (35)$$

$$P_T \leq P_{\max} \quad (36)$$

has the following properties:

  (i) $\bar{\lambda}_{RE}(P_T)$ is continuously differentiable and quasiconcave in P_T,
 (ii) The derivative of RE satisfies

$$\frac{d\bar{\lambda}_{RE}(P_T)}{dP_T} = \frac{\left(\beta \frac{\eta_P}{\eta_W} + 1 \right) \frac{d\bar{R}(P_T)}{P_T} - \zeta \bar{\lambda}_{EE}(P_T)}{\zeta P_T + P_C} \quad (37)$$

where

$$\bar{\lambda}_{EE}(P_T) = \frac{\bar{R}(P_T)}{\zeta P_T + P_C} \quad (38)$$

$$\bar{R}(P_T) \triangleq \max_{p_{k,n} \geq 0} R(P_T) = \max_{p_{k,n} \geq 0} \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{S}_k} r_{k,n} \quad (39)$$

is the maximum sum rate under constraints (34)–(36), and its derivative satisfies

$$\frac{d\bar{R}(P_T)}{P_T} = \max_{k \in \mathcal{K}, n \in \mathcal{S}_k} \frac{W_C g_{k,n} \log_2 e}{1 + \bar{p}_{k,n} g_{k,n}} \quad (40)$$

where $g_{k,n} \triangleq (|H_{k,n}|^2 / N_0 W_C)$ is the channel gain to noise ratio of the kth UE on the nth subcarrier and $\bar{p}_{k,n} (n \in \mathcal{S}_k)$ is the optimal power on the nth subcarrier for achieving $\bar{R}(P_T)$.

Proof: See Appendix A.

Since transmission power P_T and subcarrier assignment indicator matrix ρ are fixed in this case, RE $\bar{\lambda}_{RE}$ can be rewritten as follows:

$$\bar{\lambda}_{RE}(P_T) = \max_{p_{k,n} \geq 0} R \left(\frac{1}{P} + \beta \frac{W_{tot}}{W P_{tot}} \right) = \omega \bar{R}(P_T) \quad (41)$$

where $\omega \triangleq (1/P) + \beta (W_{tot}/W P_{tot})$. Therefore, our next step is to find out the optimal power allocation scheme to maximize (41). Similar to [16], the optimal power can be calculated using the following water-filling scheme:

$$\tilde{p}_{k,n} = \left(\mu_k - \frac{1}{g_{k,n}} \right)^+, \quad \forall n \in \mathcal{S}_k, \quad (42)$$

$$\sum_{n \in \mathcal{S}_k, \tilde{p}_{k,n} > 0} W_C \log_2(\mu_k g_{k,n}) = \gamma_k, \quad (43)$$

$$\bar{p}_{k,n} = \tilde{p}_{k,n} + \left(\mu - \frac{1}{g_{k,n}} - \tilde{p}_{k,n} \right)^+, \quad (44)$$

TABLE I
GRADIENT-BASED OPTIMAL POWER ADAPTATION SCHEME

1) **FOR** each UE $k \in K$
2)   Do single-user water-filling using (42)-(43) to get $\tilde{p}_{k,n}$ and μ_k, calculate the power consumption P_S;
3) **END FOR**
4) **IF** $P_S > P_{max}$
5)   infeasible;
6) **ELSE**
7)   Initial Power $P_T(1) \in [P_S, P_{max}]$;
8)   **REPEAT**
9)   For the remaining power, do the multilevel water-filling in (44)-(45);
10)   Update transmission power using gradient of RE as in (46);
11)   **STOP** when $|P_T(n) - P_T(n-1)| \leq \varepsilon$;
12) **END IF**

$$\sum_{k \in \mathcal{K}} \sum_{n \in \{ \mathcal{S}_k | \bar{p}_{k,n} > \tilde{p}_{k,n} \}} \left(\mu - \frac{1}{g_{k,n}} - \tilde{p}_{k,n} \right)$$
$$= P_T - \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{S}_k} \tilde{p}_{k,n}, \quad (45)$$

where μ_k and μ are intermediate variables. The basic idea of the power allocation process contains two steps. The first step is to allocate power to make each UE merely satisfy its rate requirement. The power used in this step is $P_S = \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{S}_k} \tilde{p}_{k,n}$. The second step is to allocate the remaining power to the subcarriers that can further maximize the sum rate. This scheme can be straightforwardly realized by the method of Lagrange multiplier and the derivations of (42)–(45).

There exists a unique global maximum for any quasiconcave function. As a result, Property (i) in *Theorem I* guarantees the existence and uniqueness of the global maximum and reveals the differentiability of $\bar{\lambda}_{RE}(P_T)$. Furthermore, $\bar{\lambda}_{RE}(P_T)$ either strictly decreases or first increases and then strictly decreases with P_T starting from $\sum_{k \in \mathcal{K}} R_k^{-1}(\mathcal{S}_k, \gamma_k)$ which is the minimum transmit power required for realizing aggregate rate γ_k over subcarrier set for all the UEs. Property (ii) further indicates that the maximum point is always achieved at a finite transmit power. It connects the sign of the first derivative with the relative size of RE and the scaled reciprocal of the water-filling level. For the fixed subcarrier assignment, *Theorem I* indicates that with an initial power value $P_T(1)$, the optimal power allocation scheme for (33)–(36) can be easily obtained by a derivative-assisted gradient scheme which is based on the single-UE water-filling in (42) and (43), and the multilevel water-filling in (44) and (45). The algorithm is called gradient-based power adaptation. The power is updated using the gradient of RE as follows:

$$P_T(n) = P_T(n-1) + t \times \frac{d\bar{\lambda}_{RE}(P_T)}{dP_T} \quad (46)$$

where t is the step size. Since $\bar{\lambda}_{RE}(P_T)$ either strictly decreases or first increases and then strictly decreases with P_T, the proposed approach will end with either convergence or $P_0 = \sum_{k \in \mathcal{K}} R_k^{-1}(\mathcal{S}_k, \gamma_k)$ if $\bar{\lambda}_{RE}(P_T)$ is monotonically decreasing in $[P_0, P_{\max}]$ and $P_{\max}$ if $\bar{\lambda}_{RE}(P_T)$ is monotonically increasing in $[P_0, P_{\max}]$. The algorithm is detailed in Table I.

The optimal solution to (29)–(32) can be obtained by applying the gradient-based power adaptation algorithm to every feasible subcarrier assignment and then choose the one with the maximum RE. However, the complexity is extremely high and makes it prohibitive for practical scenarios.

B. Upper Bound Near Optimal Resource Allocation Scheme

To facilitate practical implementation of the optimal resource efficient design, we will first exploit and prove the quasiconcave relation between an upper bound $\max_{\rho, p_{k,n}} \tilde{\lambda}_{RE}(P_T)$ and total transmit power, P_T. Different from previous definition where $\rho_{k,n} \in \{1, 0\}$ indicates whether the nth subcarrier is assigned to the kth UE, we need to relax $\rho_{k,n}$ as follows:

$$0 \leq \rho_{k,n} \leq 1 \text{ and } Q = \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}} \rho_{k,n}. \tag{47}$$

The fractional $\rho_{k,n}$ can be either interpreted as time domain sharing of subcarriers [26], [27]. Also, the maximum achievable data rate of the kth UE on the nth subcarrier is accordingly

$$\tilde{r}_{k,n} = W_C \log_2 \left(1 + \frac{p_{k,n} |H_{k,n}|^2}{\rho_{k,n} N_0 W_C} \right). \tag{48}$$

Clearly, by relaxing $\rho_{k,n}$ as (47) and reformulating the maximum achievable data rate $\tilde{r}_{k,n}$ in (48), the RE optimization problem always yields an upper bound on the RE of (29)–(32), i.e., $\tilde{\lambda}_{RE}^{UB} \geq \bar{\lambda}_{RE}$, although it does not necessarily guarantee a solution where $\rho_{k,n}$ is either 0 or 1.

Theorem II: The upper bound on the maximum achievable RE, $\max_{\rho, p_{k,n}} \tilde{\lambda}_{RE}(P_T)$, at a given number of subcarriers Q and a certain total transmit power, P_T, namely

$$\tilde{\lambda}_{RE}^{UB}(P_T) \triangleq \max_{\rho, p_{k,n} \geq 0} \frac{\sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{S}_k} \rho_{k,n} \tilde{r}_{k,n}}{\zeta P_T + P_C} \left(1 + \beta \frac{\eta_P}{\eta_W} \right) \tag{49}$$

subject to

$$\sum_{n \in \mathcal{N}} \rho_{k,n} \tilde{r}_{k,n} \geq \gamma_k, \ \forall k \in \mathcal{K} \tag{50}$$

$$\sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{S}_k} p_{k,n} = P_T \tag{51}$$

$$\sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}} \rho_{k,n} = Q \tag{52}$$

has the following properties:

(i) $\tilde{\lambda}_{RE}^{UB}(P_T)$ is continuously differentiable and quasiconcave in P_T,

(ii) The derivative of upper bound RE satisfies

$$\frac{d\tilde{\lambda}_{RE}^{UB}(P_T)}{dP_T} = \frac{(\beta \frac{\eta_P}{\eta_W} + 1)\frac{d\tilde{R}(P_T)}{P_T} - \zeta \tilde{\lambda}_{EE}(P_T)}{\zeta P_T + P_C} \tag{53}$$

where

$$\tilde{\lambda}_{EE}(P_T) = \frac{\tilde{R}(P_T)}{\zeta P_T + P_C} \tag{54}$$

$$\tilde{R}(P_T) \triangleq \max_{\rho, p_{k,n} \geq 0} R(P_T) = \max_{\rho, p_{k,n} \geq 0} \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}} \rho_{k,n} \tilde{r}_{k,n} \tag{55}$$

is the maximum sum rate under constraints (34)–(36), and its derivative satisfies

$$\frac{d\tilde{R}(P_T)}{P_T} = \max_{k \in \mathcal{K}, n \in \mathcal{N}} \frac{W_C \tilde{\rho}_{k,n} g_{k,n} \log_2 e}{1 + \tilde{\rho}_{k,n} \tilde{p}_{k,n} g_{k,n}} \tag{56}$$

where $\tilde{\rho}_{k,n}$ and $\tilde{p}_{k,n}$ is the optimal subcarrier and power allocation for achieving $\tilde{R}(P_T)$.

Proof: The proof is similar to that of *Theorem I* in Appendix A but with the inclusion of time sharing factor ρ. Therefore, the proof is ignore here.

More importantly, as a result of the quasiconcavity, problem (49)–(52) can be decomposed into two layers and solved iteratively by the joint inner- and outer-layer optimization as follows:

(i) Inner-layer: For a given transmit power, $P_T \leq P_{\max}$, finds the maximum RE $\tilde{\lambda}_{RE}^{UB}(P_T)$, and the gradient of RE $d\tilde{R}(P_T)/P_T$.

(ii) Outer-layer: Finds the optimal RE, λ_{RE}^{opt}, via a gradient-based power adaption algorithm in accordance with *Theorem II*.

The key lies in the inner-layer algorithm that finds $\tilde{\lambda}_{RE}^{UB}(P_T)$ and the gradient of RE $d\tilde{R}(P_T)/P_T$. For a given total transmit power P_T, the inner-layer subproblem to find $\tilde{\lambda}_{RE}^{UB}(P_T)$ is equivalent to maximizing the constrained sum rate, which is proved to be strictly and jointly concave in $\rho_{k,n}$ and $p_{k,n}$ [28]. The constrained sum rate maximization is in the standard form of a convex programming problem that can be solved by standard numerical methods such as the interior-point method [29]. When the optimal subcarrier assignment $\rho_{k,n}$, and power allocation $p_{k,n}$, for the constrained sum rate maximization problem are obtained, the gradient of RE $d\tilde{R}(P_T)/P_T$ can be readily determined following Property (ii) in *Theorem II*. Hence, problem (49)–(52) can be successfully solved by the aforementioned joint inner- and outer-layer optimization.

When $\tilde{\lambda}_{RE}^{UB}$ EE is found, the corresponding optimal $\rho_{k,n}$ are not ensured to be either 0 or 1. To get a feasible solution to the original downlink RE maximization problem, we need to round the possibly fractional $\rho_{k,n}$ to 0 or 1 and then perform the gradient-based algorithm to get the maximum RE for the round-off $\rho_{k,n}$. Such manipulations may not result in the optimal solution to (49)–(52). Nevertheless, this is rarely a problem when the number of subcarriers is large comparing to the number of UEs and in this case $\tilde{\lambda}_{RE}^{UB}$ RE is quite close to $\bar{\lambda}_{RE}$. In fact, the optimal $\rho_{k,n}$ for the constrained sum rate maximization problem mostly tend to be either 0 or 1 when $K \ll N$ [28]. On the other hand, such fine tightness of the RE upper bound and the fact that the optimal $\rho_{k,n}$ are almost either 0 or 1 implicitly enable the use of the original sum rate, instead of the constrained sum rate for maximization in the inner-layer optimization with an expectation of good performance. This enables us to precisely solve the original problem (49)–(52) by the joint inner- and outer-layer optimization framework with the constrained sum rate maximization as the inner-layer subproblem.

The solution to (29)–(32) can be obtained by applying the interior point method to all different number of subcarriers, i.e., from $Q = K$ to $Q = N$, and then choose the one with the maximum RE.

V. SUBOPTIMAL LOW COMPLEXITY SCHEME

Although convex programming is numerically stable, its computational complexity depends on the number of optimizing variables, which can be large if the number of subcarriers or the number of UEs is large. As for the optimal solution, we need to check all possible number of active subcarriers and apply water-filling algorithm and gradient updated scheme for each case to find the optimal power allocation. The complexity of this scheme is comparably high. Here, we will explore the property of proposed optimization problem in (29)–(32) using uniform power allocation scheme and propose a novel low-complexity suboptimal algorithm which is based on the following theorems.

Theorem III: For a fixed transmission power P_T, with uniform power allocation scheme, the maximum achievable RE at a certain bandwidth, W, namely

$$\check{\lambda}_{RE}(W) \triangleq \max_{\boldsymbol{\rho}} \frac{\sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}} \rho_{k,n} r_{k,n}}{\zeta P_T + P_C} \left(1 + \beta \frac{\eta_P}{\eta_W}\right) \quad (57)$$

subject to

$$\sum_{n \in \mathcal{N}} \rho_{k,n} r_{k,n} \geq \gamma_k, \ \forall k \in \mathcal{K} \quad (58)$$

$$p_{k,n} = \rho_{k,n} \frac{P_T}{Q}, \ \forall k \in \mathcal{K}, \ \forall n \in \mathcal{N} \quad (59)$$

$$\sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}} \rho_{k,n} W_C = W \quad (60)$$

either strictly decreases or first strictly increases and then strictly decreases with Q.

Proof: See proof in Appendix B.

Theorem IV: For any fixed subcarrier assignment indicator matrix $\boldsymbol{\rho}$ and its corresponding subcarrier assignment sets $\mathcal{S}_k (\forall k \in \mathcal{K})$, with uniform power allocation scheme, the maximum achievable RE at a certain total transmit power, P_T, namely

$$\hat{\lambda}_{RE}(P_T) \triangleq \max_{p_{k,n} \geq 0} \frac{\sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{S}_k} r_{k,n}}{\zeta P_T + P_C} \left(1 + \beta \frac{\eta_P}{\eta_W}\right) \quad (61)$$

subject to

$$\sum_{n \in \mathcal{S}_k} \rho_{k,n} r_{k,n} \geq \gamma_k, \ \forall k \in \mathcal{K} \quad (62)$$

$$p_{k,n} = \frac{P_T}{Q}, \ \forall k \in \mathcal{K}, \ \forall n \in \mathcal{S}_k \quad (63)$$

is continuously differentiable and quasiconcave in P_T, and has a derivative that satisfies $d\hat{\lambda}_{RE}(P_T)/dP_T = ((\beta(\eta_P/\eta_W) + 1) \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{S}_k} (W_C \log_2 e/(Q/g_{k,n}) + P_T) - \zeta \hat{\lambda}_{EE}(P_T)/ \zeta P_T + P_C)$, where $\hat{\lambda}_{EE}(P_T) = (\sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{S}_k} W_C \log_2(1 + (P_T/Q)g_{k,n})/\zeta P_T + P_C)$.

Proof: See proof in Appendix C.

The basic idea of the proposed suboptimal scheme is solving the problem in (57)–(60) using water-filling scheme similar to (42)–(45) to find out how much bandwidth is suitable for the network with initial power setting P_T. Since *Theorem III* reveals that $\check{\lambda}_{RE}(W)$ either strictly decreases or first strictly increases and then strictly decreases with Q, we can determine the optimal number of subcarriers by comparing two consecutive RE values (i.e., $\check{\lambda}_{RE}(Q \times W_C)$ and $\check{\lambda}_{RE}((Q + 1) \times W_C)$). If $\check{\lambda}_{RE}(Q \times W_C) < \check{\lambda}_{RE}((Q + 1) \times W_C)$, we treat the number of subcarriers used in the previous iteration as the target subcarriers. Otherwise, we keep increasing the number of active subcarriers. With the defined bandwidth, i.e., number of subcarriers, and the gradient of RE we find in *Theorem IV* $(d\hat{\lambda}_{RE}(P_T)/dP_T)$, we solve the problem in (61)–(63) to find out the optimal transmission power. We then update the transmission power P_T to solve the problem in (57)–(60) again. This procedure is repeated until convergence (both subcarriers and transmission power). Details of the proposed suboptimal scheme is discussed in the following.

We first initiate a transmission power $P_T(1)$ and an initial number of subcarriers $Q = K$. The reason for choosing the initial number of subcarriers as K is that there exists K users in the network, we should have at least K subcarriers to satisfy each user's minimum data rate requirements. Using uniform power allocation, we store the current RE value $\check{\lambda}_{RE}^{(l)}$ in the buffer. Then we increment the number of subcarriers as $Q = Q + 1$ and calculate the RE value $\check{\lambda}_{RE}^{(l+1)}$. If the current RE is larger than the previous one, we further increase the number of subcarriers as $Q = Q + 1$. Otherwise, we treat the number of subcarriers used in the previous iteration as the target subcarriers, we store this result in the buffer as $L(n) = Q - 1$. This procedure is referred to as subcarrier selection. Then we need to find out the suitable transmission power P_T based on the chosen subcarriers. We can update the transmission power P_T using the gradient $d\hat{\lambda}_{RE}(P_T)/dP_T$ from *Theorem IV* as follows:

$$P_T(n) = P_T(n - 1) + t \times \frac{d\hat{\lambda}_{RE}(P_T)}{dP_T} \quad (64)$$

where t is the step size. This power updating procedure is repeated until its convergence (i.e., $|P_T(n) - P_T(n - 1)| \leq \varepsilon$). Once we have the updated transmission power P_T, we use it as the updated initial transmission power for the next iteration, and run the subcarrier defining method again. The overall procedure is repeated until its convergence (both subcarriers and transmission power are fixed). Finally, based on the chosen subcarriers and transmission power, we apply water-filling algorithm in (42)–(45) to find out the optimal power allocation. The algorithm is presented in Table II.

In Table III, the complexity of the aforementioned optimal, near-optimal upper bound, and low-complexity alternative is listed for comparison. We calculate the complexity based on the number of floating point.

VI. SIMULATION RESULTS

In this section, we present simulation results to verify the benefit of the proposed RE and the proposed allocation algorithm. In our simulation, the circuit power is normalized to 1 W while the drain efficiency of the power amplifier is 38% for our simulation [16]. These parameters are chosen to demonstrate the concept of RE, and can easily be modified

TABLE II
LOW COMPLEXITY SUBOPTIMAL SCHEME

1) Initial transmission power $P_T(1)$, $\varepsilon = 0.01$, $n = 1$;
2) **REPEAT**
3)   Initial number of subcarrier $Q = K$, $l = 1$, exit = 0;
4)   **WHILE** exit = 0
5)   Solve the problem in (57)-(60) using uniform power allocation scheme and store the value of RE $\check{\lambda}_{RE}^{(l)}$;
6)   **IF** $\check{\lambda}_{RE}^{(l)} > \check{\lambda}_{RE}^{(l-1)}$
7) $Q = Q + 1$, $l = l + 1$;
8)   **ELSE**
9)   exit = 1; $L(n) = Q - 1$;
10)   **END IF**
11) **END WHILE**
12) Update $P_T(n)$ using (64); $n = n + 1$;
13) **STOP** when $L(n) = L(n-1)$ and $|P_T(n) - P(n-1)| \le \varepsilon$;
14) Initial transmission power $P_T(n)$ and number of subcarrier $Q = L(n)$;
15) Do single-user water-filling using (42)-(43);
16) For the remaining power, do the multilevel water-filling in (44)-(45).

TABLE III
COMPLEXITY COMPARISON FOR THE PROPOSED ALGORITHMS

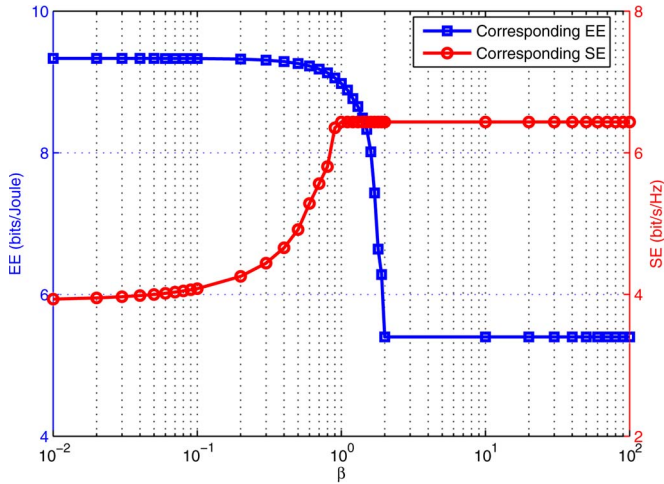| Algorithm	Complexity
Optimal scheme	$\mathcal{O}(LK^K \frac{1-K^{(N-K+1)}}{1-K})$
Upper bound scheme	$\mathcal{O}(\frac{1}{\delta^2}LK(N-K+1)\frac{K+N}{2})$
Suboptimal scheme	$\mathcal{O}(L\frac{N-K+1}{2})$

Fig. 2. Impact of weighted factor β to the corresponding EE and SE ($P_{\max} = 2$, $P_C = 1$, $K = 4$, $N = 16$, $W_C = 1$ Hz, and $\gamma_k = 4$ bit/s/Hz).

to any other values for different scenarios. The channels are frequency selective Rayleigh faded with zero mean and unit variance. To provide different service priorities and guaranteed QoS for each UE, we consider the generalized RE under the minimum rate requirements γ_k of 4 bit/s/Hz.

In the first simulation, we investigate the impact of weighted factor β to the corresponding EE and SE. Fig. 2 plots the weighted factor β versus the corresponding EE and SE. As can be seen from the figure, the corresponding EE decreases with increasing β while the corresponding SE increases with increasing β. This is because increasing β leads to more weight
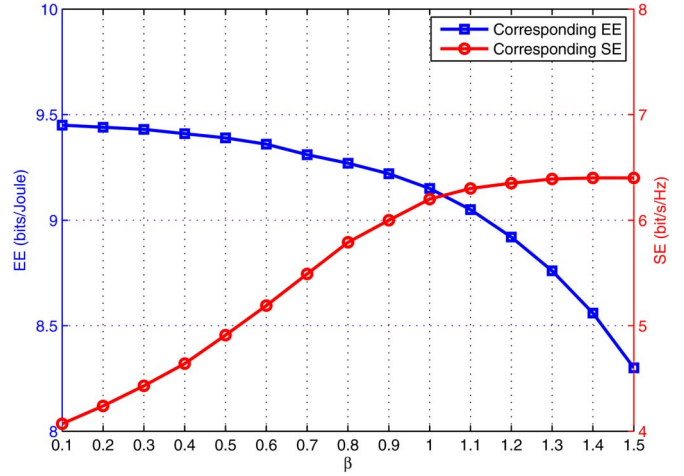
Fig. 3. Impact of weighted factor β to the corresponding EE and SE ($P_{\max} = 2$, $P_C = 1$, $K = 4$, $N = 16$, $W_C = 1$ Hz, and $\gamma_k = 4$ bit/s/Hz).
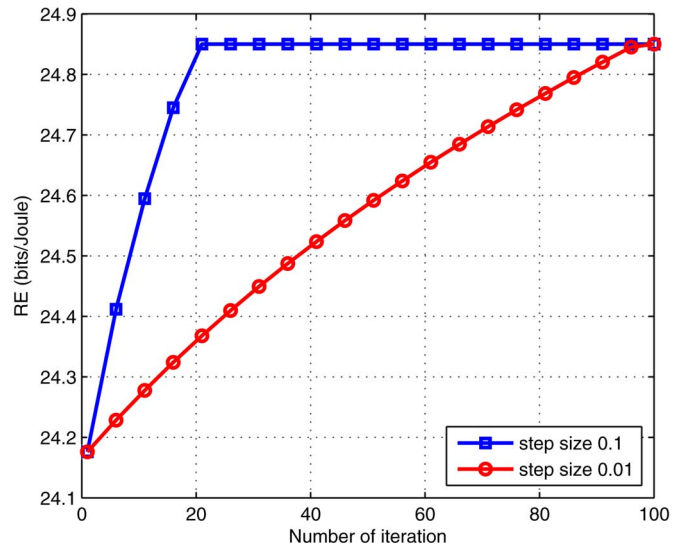

Fig. 4. Convergence behavior of the proposed gradient-based optimal power adaptation scheme ($P_{\max} = 2$, $P_C = 1$, $K = 4$, $N = 16$, $W_C = 1$ Hz, and $\gamma_k = 4$ bit/s/Hz).

putting on SE and hence more resources are allocated for maximizing SE. Furthermore, the corresponding EE and SE remains the same when weighted factor β is either small or large. As explained before, when β is small, RE will focus on optimizing EE, but will optimize SE when β is large. As shown in Fig. 3, when β is in the range from 0.1 to 1, the corresponding EE decreases gradually with increasing β while the corresponding SE increases gradually with increasing β. In particular, when $\beta = 1$, both corresponding EE and SE are close to the maximum EE (when $\beta = 0$) and maximum SE (when $\beta = \infty$). The amount of degradation is only 4% on EE and 2% on SE. This result justify the proposed use of W_{tot}/P_{tot} as the normalization factor in the RE definition. As a result, we set β to unity in the following simulation sections.

We then evaluate the performance of all the proposed schemes. We first show a figure for the proposed gradient-based optimal power adaptation scheme which illustrates how the system performance improves with the number of iterations. Fig. 4 plots resource efficiency versus the number of iterations of
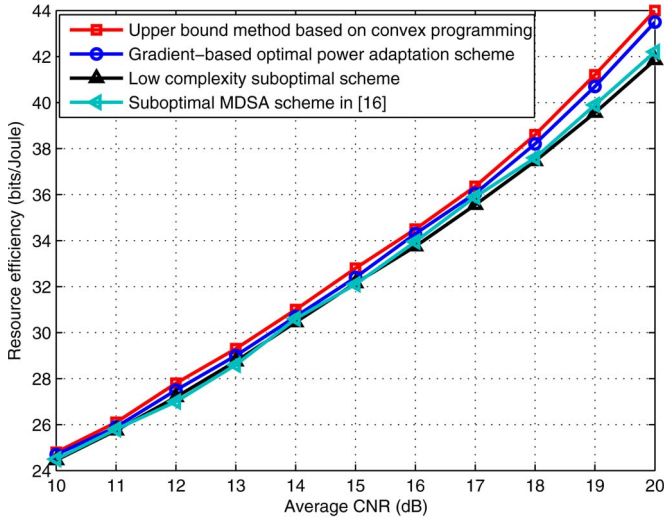
Fig. 5. Comparison of suboptimal low complexity scheme with gradient-based optimal power adaptation scheme and near optimal upper bound scheme. ($P_{\max} = 2$, $P_C = 1$, $K = 4$, $N = 16$, $W_C = 1$ Hz, and $\gamma_k = 4$ bit/s/Hz).
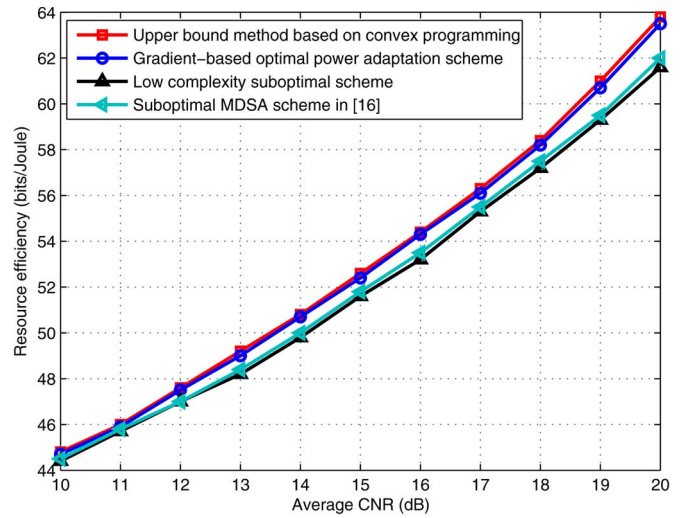

Fig. 6. Comparison of suboptimal low complexity scheme with gradient-based optimal power adaptation scheme and near optimal upper bound scheme. ($P_{\max} = 30$, $P_C = 15$, $K = 50$, $N = 256$, $W_C = 1$ Hz, and $\gamma_k = 4$ bit/s/Hz).

the proposed gradient-based optimal power adaptation scheme for step sizes $t = 0.1$ and $t = 0.01$. As can be seen from the figure, the step size affects the accuracy and convergence speed of the algorithm. Nevertheless, both step size converge to the same optimal point. We then compare the proposed suboptimal low complexity scheme with the gradient-based optimal power adaptation scheme, near optimal upper bound approach and the suboptimal resource allocation scheme proposed in [16]. The average channel gain to noise ratio is varied from 10 dB to 20 dB. The total number of subcarriers N and UEs K in our network is first set to 16 and 4, respectively. Fig. 5 shows that the RE achieved by the near optimal upper bound method is close to that of the gradient-based optimal power adaptation scheme. Specifically, the proposed suboptimal scheme results in an RE that is at least 95% of the optimal RE. Furthermore, the proposed suboptimal scheme is very close to the suboptimal resource allocation scheme proposed in [16] but with much lower complexity. Hence, we can employ the proposed suboptimal scheme to lower the system's complexity while achieving near optimal performance. We then increase the total number of subcarriers N and UEs K to 256 and 50, respectively. As can be seen from Fig. 6, the trend is similar with large number of subcarriers, whereby the RE achieved by the near optimal upper bound method is almost the same as that of the gradient-based optimal power adaptation scheme. The proposed low complexity suboptimal scheme is also very close to suboptimal resource allocation scheme proposed in [16].

In the next simulation, we analyse the impact of different number of subcarriers N to the maximum achievable RE. Fig. 7 shows the comparison of maximum achievable RE obtained by the proposed gradient-based optimal power adaptation scheme for different number of subcarriers from 16 to 24. As expected, the maximum achievable RE increases as the number of subcarriers increases. This is because when the number of subcarriers increases, the network has more freedom in terms of choosing subcarriers subset, and hence can have better resource utilization. Thus this increases the maximum achievable RE.
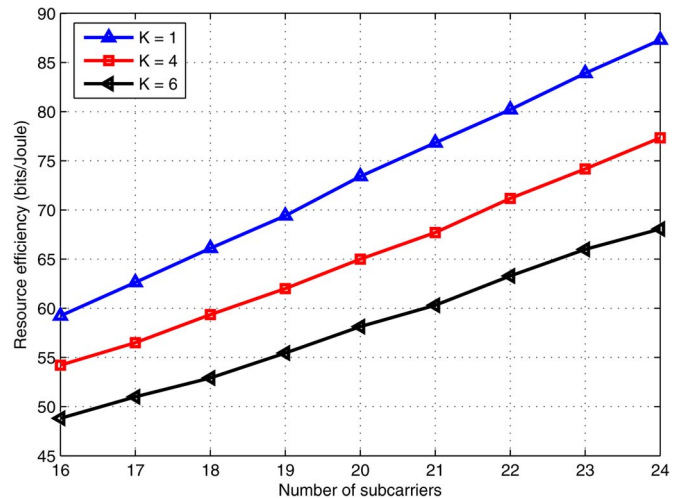

Fig. 7. Comparison of the maximum achievable RE obtained by gradient-based optimal power adaptation scheme for different number of subcarriers. ($P_{\max} = 1$, $P_C = 1$, $K = 4$, $W_C = 1$ Hz, and $\gamma_k = 4$ bit/s/Hz).

Furthermore, the trends of RE are similar for different number of UEs (K). Interestingly, Fig. 7 reveals that a higher number of users leads to lower achievable RE. Although more users in the network provide user diversity, satisfying each user's minimum rate requirements (γ_k of 4 bit/s/Hz in the simulation) leads to inefficient use of resources hence lowering the achievable RE.

We analyse the impact of different transmission power budget $P_{\max}$ to the maximum achievable RE. The number of subcarriers is fixed to 16. Fig. 8 shows the comparison of the maximum achievable RE obtained by the proposed gradient-based optimal power adaptation scheme for different transmission power budget from 0.2 W to 2 W. As can be seen from Fig. 8, the maximum achievable RE first increases then decreases with a turning point around 0.4 W for $K = 1$ and 0.8 W for $K = 6$. This is because RE is defined as $(R/P)(1 + (\eta_P/\eta_W))$, where $\eta_P \triangleq (P/(\zeta P_{\max} + P_C))$. When $P_{\max}$ is comparably low (i.e., $P_{\max} < 0.8W$ for $K = 6$), the maximum
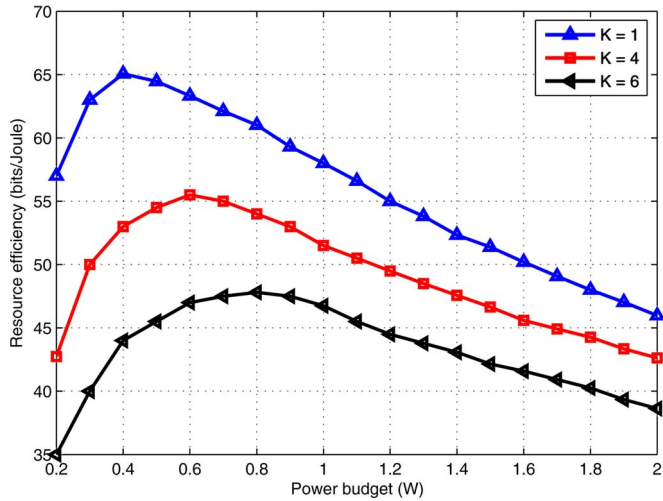
Fig. 8. Comparison of the maximum achievable RE obtained by gradient-based optimal power adaptation scheme for different transmission power budget. ($P_C = 1$, $K = 4$, $N = 16$, $W_C = 1$ Hz, and $\gamma_k = 4$ bit/s/Hz).
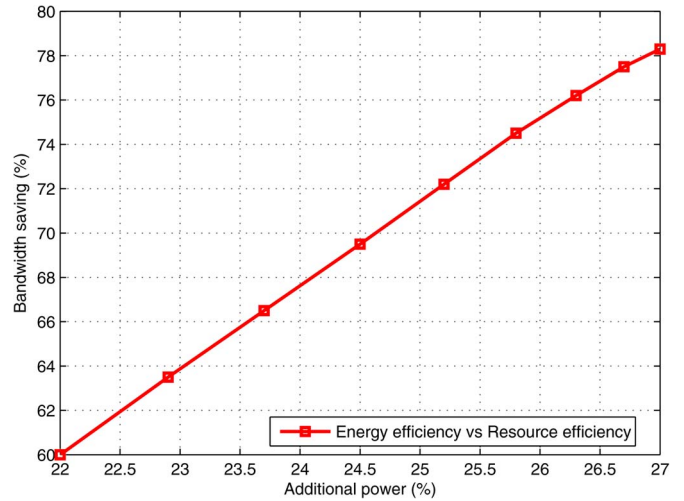

Fig. 10. Energy efficiency versus resource efficiency for tradeoff between extra power and bandwidth saving. ($P_{\max} = 1$, $P_C = 1$, $K = 50$, $N = 256$, and $W_C = 1$ Hz).
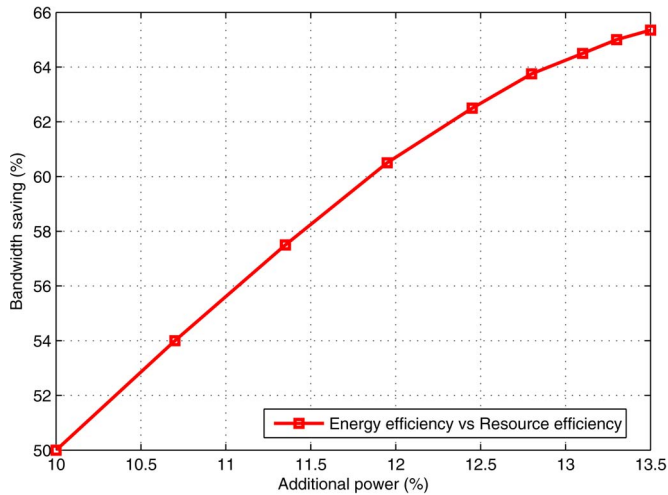

Fig. 9. Energy efficiency versus resource efficiency for tradeoff between extra power and bandwidth saving. ($P_{\max} = 1$, $P_C = 1$, $K = 4$, $N = 16$, and $W_C = 1$ Hz).
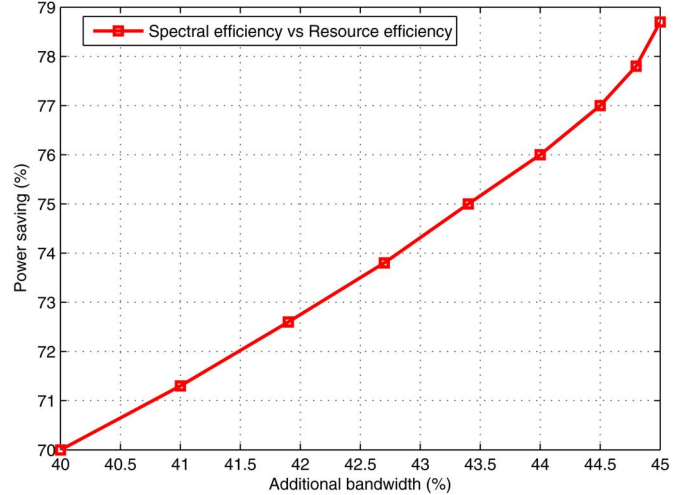

Fig. 11. Spectral efficiency versus resource efficiency for tradeoff between extra bandwidth and power saving. ($P_{\max} = 1$, $P_C = 1$, $K = 4$, $N = 16$, and $W_C = 1$ Hz).

Fig. 12. Spectral efficiency versus resource efficiency for tradeoff between extra bandwidth and power saving. ($P_{\max} = 1$, $P_C = 1$, $K = 50$, $N = 256$, and $W_C = 1$ Hz).

achievable RE increases when the transmission power budget $P_{\max}$ increases. However, when $P_{\max}$ is comparably high, the maximum achievable RE decreases when the transmission power budget $P_{\max}$ increases. This is very important for green cellular networks as we can save much power by lowering the transmission power budget.

We finally show the performance of the RE based approach compared to conventional EE and SE ones in terms of resource saving by varying the minimum rate requirements from 1 bit/s/Hz to 5 bit/s/Hz (at 0.5 interval). Figs. 9 and 10 show EE versus RE for tradeoff between extra power and bandwidth saving. With 16 subcarriers and 4 UEs in the network, Fig. 9 reveals that by using 10%–13.5% extra power compared to optimal energy efficient approach, our proposed scheme is capable of saving 50%–66% bandwidth. With 256 subcarriers and 50 UEs in the network, Fig. 10 reveals that by using 22%–27% extra power compared to optimal energy efficient approach, our proposed scheme is capable of saving 60%–80% bandwidth. Furthermore, Figs. 11 and 12 show SE versus RE for tradeoff
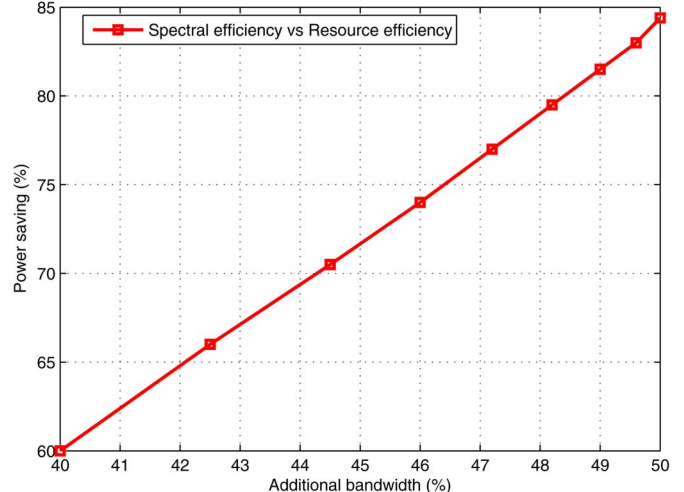
between extra bandwidth and power saving. Fig. 11 reveals that by using 40%–45% extra bandwidth compared to SE based approach, the proposed RE approach is capable of saving 70%–79% power. Fig. 12 reveals that with the same parameters as in Fig. 10 by using 40%–50% extra bandwidth, the proposed RE approach is capable of saving 60%–85% power. Therefore, a significant amount of power can be reduced by expanding the bandwidth usage. Thus the proposed RE can optimize the use of available resources in the network. It should also be noted that the amount of power or bandwidth saving can be varied by choosing a different β. Thus the desirable operation region can be adapted according to the network requirement.

VII. CONCLUSION

In this paper, we propose resource efficiency as a new system metric for EE–SE tradeoff in OFDMA based cellular network. We analysis the properties of the proposed RE and provide the optimal and solution. We also provide an upper bound near optimal method to jointly solve the optimization problem. We further develop a suboptimal but low-complexity approach by exploring the inherent structure and property of the resource efficiency approach. Numerical results confirm the theoretical findings and demonstrate the effectiveness of the proposed resource allocation scheme for achieving optimal RE. A major finding is that by using the proposed RE, significant amount of bandwidth can be saved with a slight increase in energy consumption. A similar conclusion can also be drawn on energy saving by bandwidth expansion. This shows that the proposed approach can improve the efficient use of available network resources.

APPENDIX A
PROOF OF THEOREM I

Proof: Since subcarrier assignment indicator matrix ρ and its corresponding subcarrier assignment sets $\mathcal{S}_k (\forall k \in \mathcal{K})$ are fixed in this case, we can rewrite $\bar{\lambda}_{RE}(P_T)$ as $(\bar{R}(P_T)/P) + \alpha \bar{R}(P_T)$, where $\alpha = \beta(W_{tot}/WP_{tot})$. We first prove that $\bar{R}(P_T)$ under the constraints (34)–(36) is strictly concave and continuously differentiable in P_T. With the nature of water-filling, it is easy to prove that the transmit power on each subcarrier is nondecreasing with the total transmit power. Then we consider the limit under the constraint $\sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{S}_k} \Delta p_{k,n} = \Delta P_T$. The existence of the limit indicates that $\bar{R}(P_T)$ is continuously differentiable in P_T and

$$
\frac{d\bar{R}(P_T)}{dP_T} = \frac{d\bar{R}(P_T)}{dp_{k,n}}
$$
$$
= \max_{k \in \mathcal{K}, n \in \mathcal{S}_k} \frac{W_C g_{k,n} \log_2 e}{1 + g_{k,n} \bar{p}_{k,n}}. \tag{65}
$$

Moreover, $W_C g_{k,n} \log_2 e/(1 + g_{k,n} \bar{p}_{k,n})$ is nonincreasing with P_T for $k \in \mathcal{K}$ and $n \in \mathcal{S}_k$ while $\max_{k \in \mathcal{K}, n \in \mathcal{S}_k}(W_C g_{k,n} \log_2 e/(1 + g_{k,n} \bar{p}_{k,n}))$ is strictly monotonically decreasing with P_T. Thus, $(d^2 \bar{R}(P_T)/dP_T^2) < 0$ and $\bar{R}(P_T)$ is strictly concave in P_T.

To prove the quasiconcavity of $\bar{\lambda}_{RE}(P_T)$, we first introduce the definition of quasiconcave function. According to [30], a function $f : \mathbf{R}^n - \mathbf{R}$ is called quasiconvex if its domain and all its sublevel sets

$$
\mathcal{S}_\theta = \{x \in \mathbf{dom}\, f | f(x) \le \theta\} \tag{66}
$$

for $\theta \in \mathbf{R}$, are convex. A function is quasiconcave if $-f$ is quasiconvex, i.e., every superlevel set $x|f(x) \ge \theta$ is convex. According to [16] and [30], $\bar{R}(P_T)/P_T$ is strictly quasiconcave in P_T if $\mathcal{S}_\theta$ is strictly convex for any real number θ. When $\theta < 0$, no points exist on the counter $\bar{R}(P_T)/P_T = \theta$. When $\theta \ge 0$, $\mathcal{S}_\theta$ is equivalent to $\mathcal{S}_\theta = \{P_T \ge \sum_{k \in \mathcal{K}} R_k^{-1}(\mathcal{S}_k, \gamma_k) | \theta \zeta P_T + \theta P_C - \bar{R}(P_T)\} \le 0$, where $R_k^{-1}(\mathcal{S}_k, \gamma_k)$ is the minimum transmit power required for realizing aggregate rate γ_k over subcarrier set for the kth UE. Since $\bar{R}(P_T)$ is strictly concave in P_T, $\mathcal{S}_\theta$ is strictly convex in P_T. Therefore, $\bar{R}(P_T)/P_T$ is continuously differentiable and quasiconcave in P_T. Since $\bar{R}(P_T)$ is strictly concave in P_T and $\bar{R}(P_T)/P_T$ is quasiconcave in P_T, $\bar{\lambda}_{RE}(P_T)$ is continuously differentiable and quasiconcave in P_T and this completes the proof of Property (i).

Furthermore, since $\bar{\lambda}_{RE}(P_T)$ can be rewritten as $(\bar{R}(P_T)/P) + \alpha \bar{R}(P_T)$, the derivative of RE $d\bar{\lambda}_{RE}(P_T)/dP_T$ satisfies $d\bar{\lambda}_{RE}(P_T)/dP_T = (d(\bar{R}(P_T)/P)dP_T + \alpha(d\bar{R}(P_T)/dP_T)$. We analysis $(d(\bar{R}(P_T)/P)dP_T$ as follows:

$$
\frac{d\frac{\bar{R}(P_T)}{P}}{dP_T} = \lim_{\triangle P_T \to 0} \frac{\frac{\bar{R}(P_T + \triangle P_T)}{\zeta(P_T + \triangle P_T) + P_C} - \frac{\bar{R}P_T}{\zeta P_T + P_C}}{\triangle P_T}
$$
$$
= \lim_{\triangle P_T \to 0} \frac{\frac{\bar{R}(P_T + \triangle P_T) - \bar{R}P_T}{\triangle P_T} - \zeta \bar{\lambda}_{EE(P_T)}}{\zeta(P_T + \triangle P_T) + P_C}
$$
$$
= \frac{\frac{d\bar{R}(P_T)}{dP_T} - \zeta \bar{\lambda}_{EE}(P_T)}{\zeta P_T + P_C}. \tag{67}
$$

Therefore, we have

$$
\frac{d\bar{\lambda}_{RE}(P_T)}{dP_T} = \frac{\left(\beta \frac{\eta_P}{\eta_W} + 1\right) \frac{d\bar{R}(P_T)}{P_T} - \zeta \bar{\lambda}_{EE}(P_T)}{\zeta P_T + P_C} \tag{68}
$$

where $d\bar{R}(P_T)/dP_T = \max_{k \in \mathcal{K}, n \in \mathcal{S}_k}(W_C g_{k,n} \log_2 e/(1 + g_{k,n} \bar{p}_{k,n}))$, $\bar{\lambda}_{EE}(P_T) = (\bar{R}(P_T)/\zeta P_T + P_C)$. This completes the proof of Property (ii) of *Theorem I*. ∎

APPENDIX B
PROOF OF THEOREM III

Proof: For fixed transmission power P_T, i.e., $P_T \in [P_1, P_{\max}]$ where $P_1 = \sum_{k \in \mathcal{K}} R_k^{-1}(\mathcal{S}_k, \gamma_k)$ and $R_k^{-1}(\mathcal{S}_k, \gamma_k)$ is the minimum transmit power required for realizing aggregate rate γ_k over subcarrier set for the kth UE by using uniform power allocation, we have the following proof.

Applying the result form *Property I*, we have $\check{\lambda}_{RE}(W) = \max_{\rho}(\check{R}(W)/P_T) + \bar{\beta}(\check{R}(W)/W)$, where $\check{R}(W) \overset{\Delta}{=} \max_{\rho_{k,n}} R(W) = \max_{\rho_{k,n}} \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}} \rho_{k,n} r_{k,n}$, under the constraint of (58)–(60), and hence

$$
\frac{d\check{\lambda}_{RE}(W)}{dW} = \frac{1}{P_T} \frac{d\check{R}(W)}{dW} + \frac{\bar{\beta}}{W} \frac{d\check{R}(W)}{dW} - \frac{\bar{\beta}\check{R}(W)}{W^2}. \tag{69}
$$

Therefore, we have

$$
\begin{aligned}
\frac{d^2 \check{\lambda}_{RE}(W)}{dW^2} &= \frac{1}{P_T} \frac{d^2 \check{R}(W)}{dW^2} + \frac{\bar{\beta}}{W} \frac{d^2 \check{R}(W)}{dW^2} - \frac{\bar{\beta}}{W^2} \frac{d\check{R}(W)}{dW} \\
&\quad + \frac{2\bar{\beta} \check{R}(W)}{W^3} - \frac{\bar{\beta}}{W^2} \frac{d\check{R}(W)}{dW} \\
&= \left(\frac{1}{P_T} + \frac{\bar{\beta}}{W} \right) \frac{d^2 \check{R}(W)}{dW^2} \\
&\quad + \frac{2\bar{\beta}}{W^2} \left(\frac{\check{R}(W)}{W} - \frac{d\check{R}(W)}{dW} \right).
\end{aligned}
\tag{70}
$$

Since $W = Q \times W_C = \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}} \rho_{k,n} W_C$, we have

$$
dW = W_C d\rho_{k,n} \text{ and } d\rho_{k,n} = \frac{dW}{W_C}.
\tag{71}
$$

Furthermore, $\rho_{k,n} \in \{1, 0\}$ indicates whether the nth subcarrier is assigned to the kth UE. Hence, W is not continuous and we need to relax $\rho_{k,n}$ as follows:

$$
0 \le \rho_{k,n} \le 1 \text{ and } Q = \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}} \rho_{k,n}.
\tag{72}
$$

Thus, we now analyze the term $d\check{R}(W)/dW$ as follows:

$$
\begin{aligned}
\frac{d\check{R}(W)}{dW} &= \frac{d \max_{\boldsymbol{\rho}} \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}} \rho_{k,n} W_C \log_2(1 + p_{k,n} g_{k,n})}{W_C d\rho_{k,n}} \\
&= \max_{\rho_{k,n}} \log_2(1 + p_{k,n} g_{k,n}) \\
&\quad + \max_{\rho_{k,n}} \frac{\rho_{k,n} g_{k,n} \frac{dp_{k,n}}{d\rho_{k,n}} \log_2 e}{1 + p_{k,n} g_{k,n}}.
\end{aligned}
\tag{73}
$$

With uniform power allocation scheme and subcarrier maximum assignment scheme, $p_{k,n} = \rho_{k,n}(P_T/Q) = \rho_{k,n}(P_T W_C/W)$, (73) can be rewritten as follows:

$$
\begin{aligned}
\frac{d\check{R}(W)}{dW} &= \log_2 \left(1 + \frac{P_T W_C}{W} \check{g}_{k,n} \right) + \frac{P_T(W - W_C) \log_2 e}{P_T W + \frac{W^2}{\check{g}_{k,n} W_C}} \\
&= \log_2 \left(1 + \frac{P_T}{W} f_{k,n} \right) + \frac{P_T(W - W_C) \log_2 e}{P_T W + \frac{W^2}{f_{k,n}}}
\end{aligned}
\tag{74}
$$

where $\check{g}_{k,n} = \max_{k \in \mathcal{K}, n \in \mathcal{N}} g_{k,n}$ and $f_{k,n} = \check{g}_{k,n} W_C$. Based on this result, we now have $d^2 \check{R}(W)/dW^2$ as follows:

$$
\frac{d\check{R}^2(W)}{dW^2} = \left(-\frac{1}{W} + \frac{W_C}{W^2} + \frac{W_C - W}{W^2 + P_T W f_{k,n}} \right) \frac{\log_2 e}{1 + \frac{W}{P_T f_{k,n}}}.
\tag{75}
$$

We then analysis $(\check{R}(W)/W) - (d\check{R}(W)/dW)$ as (76), shown at the bottom of the page. Since $\Upsilon \le 0$, we have

$$
\frac{\check{R}(W)}{W} - \frac{d\check{R}(W)}{dW} \le \frac{P_T(W_C - W) \log_2 e}{P_T W + \frac{W^2}{f_{k,n}}}.
\tag{77}
$$

Substituting (75) and (77) in (70), we have

$$
\begin{aligned}
&\frac{d^2 \check{\lambda}_{RE}(W)}{dW^2} \\
&= \left(\frac{1}{P_T} + \frac{\bar{\beta}}{W} \right) \frac{d^2 \check{R}(W)}{dW^2} + \frac{2\bar{\beta}}{W^2} \left(\frac{\check{R}(W)}{W} - \frac{d\check{R}(W)}{dW} \right) \\
&< \frac{\bar{\beta}}{W} \left(-\frac{1}{W} + \frac{W_C}{W^2} + \frac{W_C - W}{W^2 + P_T W f_{k,n}} \right) \frac{\log_2 e}{1 + \frac{W}{P_T f_{k,n}}} \\
&\quad + \frac{2\bar{\beta}}{W^2} \frac{P_T(W_C - W) \log_2 e}{P_T W + \frac{W^2}{f_{k,n}}} \\
&= \frac{\bar{\beta}}{W} \left(-\frac{1}{W} + \frac{W_C}{W^2} + \frac{W_C - W}{W^2 + P_T W f_{k,n}} \right) \frac{\log_2 e}{1 + \frac{W}{P_T f_{k,n}}} \\
&\quad + \frac{2\bar{\beta}(W_C - W)}{W^3} \frac{\log_2 e}{1 + \frac{W}{P_T f_{k,n}}} \\
&= \frac{\log_2 e}{1 + \frac{W}{P_T f_{k,n}}} \left[\frac{2\bar{\beta}(W_C - W)}{W^3} \right. \\
&\quad \left. + \frac{\bar{\beta}}{W} \left(-\frac{1}{W} + \frac{W_C}{W^2} + \frac{W_C - W}{W^2 + P_T W f_{k,n}} \right) \right] \\
&\le \frac{\log_2 e}{1 + \frac{W}{P_T f_{k,n}}} \left[\frac{2\bar{\beta}(W_C - W)}{W^3} + \frac{\bar{\beta}}{W} \left(\frac{W_C}{W^2} - \frac{1}{W} \right) \right] \\
&= \frac{3\bar{\beta} \log_2 e}{W^2 \left(1 + \frac{W}{P_T f_{k,n}} \right)} \left(\frac{W_C}{W} - 1 \right).
\end{aligned}
\tag{78}
$$

Since $(3\bar{\beta} \log_2 e/W^2(1 + (W/P_T f_{k,n})))$ is always positive, the sign$(d^2 \check{\lambda}_{RE}(W)/dW^2)$ depends on

$$
\text{sign} \left(\frac{d^2 \check{\lambda}_{RE}(W)}{dW^2} \right) = \frac{W_C}{W} - 1 = \frac{1}{Q} - 1.
\tag{79}
$$

Since $Q \ge 1$, sign$(d^2 \check{\lambda}_{RE}(W)/dW^2) < 0$. Therefore, $\check{\lambda}_{RE}(W)$ either strictly decreases or first strictly increases and then strictly decreases with Q, this completes the proof of *Theorem III*. ∎

APPENDIX C
PROOF OF THEOREM IV

Proof: Since subcarrier assignment indicator matrix $\boldsymbol{\rho}$ and its corresponding subcarrier assignment sets $\mathcal{S}_k(\forall k \in \mathcal{K})$ are fixed in this case, we rewrite $\hat{\lambda}_{RE}(P_T)$ as $(\hat{R}(P_T)/P) + \alpha \hat{R}(P_T)$, where $\hat{R}(P_T) \overset{\Delta}{=} \max_{p_{k,n} \ge 0} \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{S}_k} r_{k,n}$, under the

---

$$
\frac{\check{R}(W)}{W} - \frac{d\check{R}(W)}{dW} = \underbrace{\frac{\max_{\boldsymbol{\rho}} \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}} \rho_{k,n} W_C \log_2(1 + p_{k,n} g_{k,n})}{\sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}} \rho_{k,n} W_C} - \max_{\rho_{k,n}} \log_2(1 + p_{k,n} g_{k,n})}_{\Upsilon} + \frac{P_T(W_C - W) \log_2 e}{P_T W + \frac{W^2}{f_{k,n}}}
\tag{76}
$$

constraint of (62) and (63). We first prove that $\hat{R}(P_T)$ is strictly concave and continuously differentiable in P_T. We consider the limit under the constraint $\sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{S}_k} \triangleq p_{k,n} \triangleq P_T$, the existence of the limit indicates that $\hat{R}(P_T)$ is continuously differentiable in P_T. Then we analyze $d\hat{R}(P_T)/dP_T$ as follows:

$$\frac{d\hat{R}(P_T)}{dP_T} = \frac{d \max_{p_{k,n} \geq 0} \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{S}_k} W_C \log_2(1 + p_{k,n} g_{k,n})}{dP_T}. \tag{80}$$

Substitute (63) into (80), we have

$$\frac{d\hat{R}(P_T)}{dP_T} = \frac{d \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{S}_k} W_C \log_2 \left(1 + \frac{P_T}{Q} g_{k,n}\right)}{dP_T}$$
$$= \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{S}_k} \frac{W_C \log_2 e}{\frac{Q}{g_{k,n}} + P_T}. \tag{81}$$

Since $W_C \log_2 e/((Q/g_{k,n}) + P_T)$ is decreasing with P_T for $k \in \mathcal{K}$ and $n \in \mathcal{S}_k$, $\sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{S}_k}(W_C \log_2 e/((Q/g_{k,n}) + P_T))$ is strictly monotonically decreasing with P_T. Thus, $(d^2\hat{R}(P_T)/dP_T^2) < 0$ and $\hat{R}(P_T)$ is strictly concave in P_T.

Similar as the proof of *Theorem I*, $\hat{R}(P_T)/P_T$ is strictly quasiconcave in P_T if $\mathcal{S}_\sigma$ is strictly convex for any real number σ. When $\sigma < 0$, no points exist on the counter $\hat{R}(P_T)/P_T = \sigma$. When $\sigma \geq 0$, $\mathcal{S}_\sigma$ is equivalent to $\mathcal{S}_\sigma = \{P_T \geq \sum_{k \in \mathcal{K}} R_k^{-1}(\mathcal{S}_k, \gamma_k) | \sigma \zeta P_T + \sigma P_C - \hat{R}(P_T)\} \leq 0$, where $R_k^{-1}(\mathcal{S}_k, \gamma_k)$ is the minimum transmit power required for realizing aggregate rate γ_k over subcarrier set for the kth UE by using uniform power allocation. Since $\hat{R}(P_T)$ is strictly concave in P_T, $\hat{R}(P_T)/P_T$ is continuously differentiable and quasiconcave in P_T. Therefore, $\hat{\lambda}_{RE}(P_T)$ is continuously differentiable and quasiconcave in P_T.

Hence, the derivative of RE $d\hat{\lambda}_{RE}(P_T)/dP_T$ satisfies

$$\frac{d\hat{\lambda}_{RE}(P_T)}{dP_T} = \frac{d\frac{\hat{R}(P_T)}{P}}{dP_T} + \beta \frac{W_{tot}}{W P_{tot}} \frac{d\hat{R}(P_T)}{dP_T}. \tag{82}$$

Substituting (67) and (81) into (82), we have

$$\frac{d\hat{\lambda}_{RE}(P_T)}{dP_T} = \frac{(\beta \frac{\eta_P}{\eta_W} + 1) \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{S}_k} \frac{W_C \log_2 e}{\frac{Q}{g_{k,n}} + P_T} - \zeta \hat{\lambda}_{EE}(P_T)}{\zeta P_T + P_C} \tag{83}$$

where $\hat{\lambda}_{EE}(P_T) = \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{S}_k} W_C \log_2(1 + (P_T/Q)g_{k,n})/(\zeta P_T + P_C)$ and this completes the proof of *Theorem IV*. ∎

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