Optimal Beamforming Schemes and its Capacity Behavior for Downlink Distributed Antenna Systems

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***Abstract*—In this paper, we investigate the outage and ergodic capacity of downlink distributed antenna systems (DAS) where each distributed antenna unit (DAU) has multiple antennas with per-DAU power constraint. We first derive the optimal beamforming vector in a closed form by applying a matrix minor condition to relax the positive semi-definite constraint. We observe that our derived solution has a form of maximum ratio transmission per each DAU with full power. Based on the derived optimal beamforming, the outage and ergodic capacity under Rayleigh fading channels are analyzed. To this end, we show that a distribution of the received signal-to-noise ratio is characterized as a Gamma distribution by approximating a sum of non-identical independent Nakagami-m random variables as a single Nakagami-m random variable based on the moment matching method. Then, we present an accurate formula of the outage and ergodic capacity in a closed form which matches well with the simulation results. Furthermore, we derive an upper bound of an achievable average rate of DAS with limited feedback. We then propose a new feedback bit allocation algorithm to maximize the derived metric. Simulation results confirm the accuracy of the derived outage and ergodic capacity expressions and the efficiency of the proposed bit allocation method.**

***Index Terms*—Distributed antenna systems, beamforming, limited feedback.**

I. Introduction

LATELY, distributed antenna systems (DAS) have been studied with a lot of interest as a promising technique to satisfy growing demands for future wireless communication networks [1]. Saleh et al. originally introduced the DAS to simply cover dead spots in indoor wireless systems [2]. Unlike conventional systems with co-located antennas at the cell center, distributed antenna units (DAU) in DAS are geometrically separated at different physical locations, and are connected by fiber links. Thus, the DAS can reduce the access distance, the transmit power and co-channel interference, which result in power savings and the enhanced system capacity [3]–[8].

Manuscript received November 9, 2011; revised August 31, 2012; accepted February 27, 2013. The associate editor coordinating the review of this paper and approving it for publication was M. Win.

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This work was supported in part by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MEST) (No. 2010-0017909). The material in this paper was presented in part at the IEEE International Conference on Communications (ICC), Ottawa, Canada, June 2012.

Digital Object Identifier 10.1109/TWC.2013.041613.111987

Recently, many efforts have been devoted to analyze the system performance of the DAS including a design of antenna locations [9] [10]. The asymptotic sum-rate capacity in the multi-user uplink scenario was investigated with total power constraint [11] and per-user power constraint [12]. For single user multicell DAS, a downlink capacity gain obtained by reducing other-cell interference was examined from an information theoretic point of view under per-DAU power constraint when DAU has a single antenna [5]. The problem of power allocation among distributed antennas in terms of the downlink capacity maximization was solved by adopting random matrix theory with sum power constraint in [13].

One fundamental limit which should be considered in a downlink DAS design is power constraint. Since DAUs in DAS are spaced apart by large distance, they cannot share power with each other in practical situations. Thus understanding the performance under the per-DAU power constraint in the downlink is important. However, there are few papers which consider the downlink DAS under the per-DAU power constraint. The paper [5] showed that phase steering with full power transmission is the capacity maximizing transmission strategy for DAS where each DAU has a single antenna. Recently, in point-to-point multiple-input multiple-output (MIMO) systems under per-antenna power constraint, the same result was also derived with more rigorous proof by using the relaxation method of the positive semi-definite constraint [14].

Another inevitable issue in real-world system designs is finite-rate feedback where each mobile feeds back a finite number of bits regarding instantaneous channel realizations. The limited feedback system was well studied for point-to-point MIMO and MIMO broadcast channels in [15]–[18]. Several feedback bit allocation schemes were proposed for conventional antenna systems (CAS) with limited feedback in multi-cell environments [19] [20]. In [19] and [20], bit allocation methods were introduced which minimize a mean rate loss due to quantization error for multiple-input single-output interference channels. Unlike CAS, the authors of [21] addressed various issues of DAS with limited feedback focusing on the challenges in codebook designs and proposed a suboptimal method suitable for DAS.

In this paper, we investigate the downlink DAS where each DAU has multiple antennas with per-DAU power constraint, assuming channel knowledge at both transmitter and receiver. We first derive the optimal beamforming which maximizes an achievable transmission rate. In order to obtain the optimal solution, we solve a relaxed problem by replacing the positive semi-definite constraint with a 2×2 matrix minor condition as in [14] without sacrificing optimality. Our derived solution takes a form of maximum ratio transmission for each DAU. It can also be regarded as a generalized version of equal gain transmission [22] in systems with single antenna DAU.

Next, we analyze the outage and ergodic capacity of the DAS with the optimal beamforming under Rayleigh fading channels. In order to compute the capacity, we first study a distribution of received signal-to-noise ratio (SNR). By approximating a sum of non-identical independent Nakagami-m random variables (RVs) as a single Nakagami-m RV based on the moment matching method [23], the SNR distribution is characterized by a Gamma distribution, and then the outage and ergodic capacity are derived in a closed form. From numerical simulations, we verify the validity of our analysis.

Furthermore, we develop a new limited feedback strategy suitable for DAS with the derived optimal beamforming. We quantify an upper bound of an achievable average rate by applying random vector quantization (RVQ) for channel direction information (CDI) and uniform quantization for phase difference information. Formulating the problem which maximizes the derived upper bound with total bit constraint, we propose a feedback bit allocation technique which utilizes the available feedback resources efficiently. Simulation results confirm that the system with the proposed algorithm outperforms conventional equal bit allocation methods.

The rest of this paper is organized as follows: Section II describes a system model and expresses an achievable transmission rate under the per-DAU power constraint. In Section III, we derive the optimal beamforming vector in terms of maximizing the achievable rate subject to per-DAU power constraint. The capacity of DAS with the optimal beamforming under Rayleigh fading channels is presented in Section IV. Section V proposes a new limited feedback bit allocation scheme. We provide simulation results in Section VI. Finally, Section VII concludes this paper.

Throughout the paper, we employ uppercase boldface letters for matrices and lowercase boldface for vectors. The superscripts $(\cdot)^T$, $(\cdot)^H$, $(\cdot)^*$ and $\|\cdot\|$ stand for transpose, conjugate transpose, element-wise conjugate and 2-norm, respectively. For a matrix $\mathbf{A}$, tr$(\mathbf{X})$ denotes trace and $\mathbf{A} \succcurlyeq 0$ indicates positive semi-definiteness. The expectation of a RV is given by $\mathbb{E}(\cdot)$.

II. System Model

We consider a downlink DAS where N DAUs with M_i antennas $(i = 1, \ldots, N)$ communicate to a single user with a single antenna, as shown in Figure 1. A user is randomly deployed within the coverage area. Since DAUs are largely separated in DAS, we consider the channel model which includes not only small scale fadings but also distance-dependent pathloss [24]. Note that our channel model accounts for arbitrary placement of DAU and user locations and different numbers of transmit antennas per DAU. We assume that optical fibers are employed to transfer information and signaling between the central processor and DAUs.

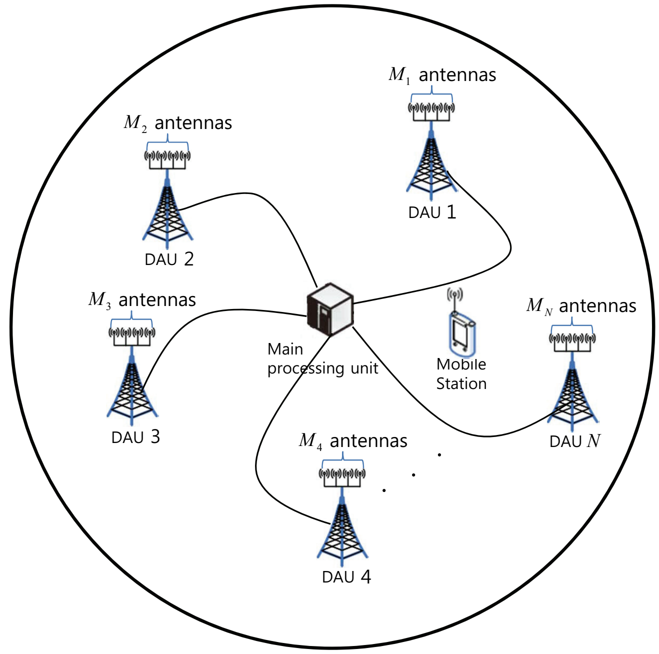

Fig. 1. Cell structure of a distributed antenna system.

The received signal at the user can be expressed as

$$y = \sum_{i=1}^{N} \sqrt{\alpha_i} \mathbf{h}_i^H \mathbf{x}_i + n \tag{1}$$

where α_i and $\mathbf{h}_i$ represent the distance-dependent pathloss and the $M_i \times 1$ channel vector between DAU i and a user, respectively, $\mathbf{x}_i \in \mathbb{C}^{M_i}$ is the transmitted signal vector from the i-th DAU, and n denotes the additive white Gaussian noise (AWGN) with zero mean and unit variance. The elements of $\mathbf{h}_i$ are independently and identically distributed with $\mathcal{CN}(0, 1)$. We assume that both the DAUs and the user know channel state information (CSI) perfectly. Unlike conventional systems where all antennas are co-located, the DAUs have no ability to share or allocate power among themselves and hence per-DAU power constraint holds. Accordingly, it is assumed that the input vector $\mathbf{x}_i$ satisfies the per-DAU power constraint tr $\left(\mathbb{E}[\mathbf{x}_i \mathbf{x}_i^H]\right) \leq P_i$, where P_i is the total power available at the i-th DAU.

We form $\overline{\mathbf{h}} \in \mathbb{C}^M$ and $\mathbf{x} \in \mathbb{C}^M$ by $\overline{\mathbf{h}} = \left[\overline{\mathbf{h}}_1^H \ldots \overline{\mathbf{h}}_N^H\right]^H$ and $\mathbf{x} = \left[\mathbf{x}_1^H, \ldots, \mathbf{x}_N^H\right]^H$, respectively, where $\overline{\mathbf{h}}_i$ is given as $\overline{\mathbf{h}}_i = \sqrt{\alpha_i} \mathbf{h}_i$ and $M = \sum_{i=1}^{N} M_i$. Denoting $\mathbf{w} \in \mathbb{C}^M$ as a transmit beamforming vector, the precoded input signal $\mathbf{x}$ is obtained by

$$\mathbf{x} = \mathbf{w} s \tag{2}$$

where s indicates the transmitted data symbol with zero mean and unit variance. Then letting $\mathbf{Q} = \mathbb{E}[\mathbf{x}\mathbf{x}^H] = \mathbf{w}\mathbf{w}^H$ be the covariance of the Gaussian input, the achievable transmission rate is written as

$$R = \log\left(1 + \overline{\mathbf{h}}^H \mathbf{Q} \overline{\mathbf{h}}\right). \tag{3}$$

In order to impose the per-DAU power constraint, we denote $\mathbf{\Phi}_i$ as an $M \times M$ diagonal matrix with all zeros, except for M_i

consecutive ones, corresponding to positions from $M_{1:i-1}+1$ to $M_{1:i}$ on the main diagonal where $M_{1:k}$ is defined as

$$M_{1:k} = \sum_{j=1}^{k} M_j \qquad \text{and} \qquad M_{1:0} = 0.$$

Then, the per-DA power constraint is expressed in terms of the partial trace of the transmitted signal covariance matrix as

$$\text{tr}\left(\mathbf{\Phi}_i \mathbf{Q}\right) \le P_i \qquad \text{for } i = 1, \ldots, N.$$

III. Optimal Beamforming under Per-DAU Power Constraint

In this section, we establish the optimal $\mathbf{Q}$ that maximizes the transmission rate (3) according to the per-DAU power constraint. To this end, we formulate the optimization problem and then solve the problem by using relaxations. Finally, we show that a solution of the relaxed problem is also optimal in the original problem. The problem which computes the optimum input covariance matrix for maximizing the achievable rate can be expressed as

$$\max_{\mathbf{Q}} \quad R \tag{4}$$

$$\text{subject to} \quad \text{tr}\left(\mathbf{\Phi}_i \mathbf{Q}\right) \le P_i \text{ for } i = 1, \ldots, N \tag{5}$$

$$\mathbf{Q} \succcurlyeq 0 \tag{6}$$

$$\text{Rank}(\mathbf{Q}) = 1.$$

It is clear that due to a rank-one constraint for $\mathbf{Q}$, this problem is non-convex in general. Note that the problem (4) without a rank-one constraint becomes a convex problem. However, even in this case, there is generally no closed-form solution available because of positive semi-definite constraints (6). The paper [14] recently derived a closed-form solution by applying relaxation in (6) for the special case of $M_i = 1$ for all i where only one diagonal element of $\mathbf{\Phi}_i$ equals 1 and the rest elements are all zeros.

To solve the problem (4), we first relax semi-definite constraints[1] (6) to 2×2 principal minors of $\mathbf{Q}$ which is given as

$$\mathbf{M}_{ij} = \begin{bmatrix} q_{ii} & q_{ij}^* \\ q_{ij} & q_{jj} \end{bmatrix}$$

where q_{ij} denotes the (i,j)-th element of $\mathbf{Q}$. Here, a principal minor is obtained by removing $M-2$ columns (except columns i and j) and the corresponding rows of $\mathbf{Q}$.

By relaxing the rank-one and positive semi-definite constraint and using the fact that the log function is a monotonic increasing function with respect to $\overline{\mathbf{h}}^H \mathbf{Q} \overline{\mathbf{h}}$, we then form the following relaxed problem

$$\max_{\mathbf{Q}} \quad \overline{\mathbf{h}}^H \mathbf{Q} \overline{\mathbf{h}} \tag{7}$$

$$\text{subject to} \quad \text{tr}\left(\mathbf{\Phi}_i \mathbf{Q}\right) \le P_i \quad \text{for } i = 1, \ldots, N$$

$$\mathbf{M}_{mn} \succcurlyeq 0 \qquad \text{for } m, n = 1, \ldots, M \ (n > m).$$

Note that since this problem is a relaxed version of (4), if the optimal $\mathbf{Q}^\star$ is a positive semi-definite and rank-one matrix, then it is also an optimal solution of (4).

[1]Note that the positive semi-definite constraint is equivalent to the case where all principal minors of $\mathbf{Q}$ are positive semi-definite [25].

Based on the fact that the constraint $\mathbf{M}_{mn} \succcurlyeq 0$ is identical to $|q_{mn}|^2 \le q_{mm} q_{nn}$, the Lagrangian for (7) is given by

$$\begin{aligned} \mathcal{L}(\mathbf{Q}, \boldsymbol{\lambda}, \boldsymbol{\mu}) &= \overline{\mathbf{h}}^H \mathbf{Q} \overline{\mathbf{h}} - \sum_{i=1}^{N} \mu_i \left(\text{tr}\left(\mathbf{\Phi}_i \mathbf{Q}\right) - P_i\right) \\ &\quad - \sum_{m=1}^{M} \sum_{n=m+1}^{M} \lambda_{mn} \left(|q_{mn}|^2 - q_{mm} q_{nn}\right) \end{aligned}$$

where μ_i and λ_{mn} are the Lagrange multipliers. Let us denote $\mathcal{K}_k = \{M_{1:k-1}+1, \ldots, M_{1:k}\}$ as the set of indices belonging to DAU k. Then, the stationarity conditions of the KKT conditions are written by

$$\frac{\partial \mathcal{L}}{\partial q_{mn}^*} = \overline{h}_m^* \overline{h}_n - \lambda_{mn} q_{mn} = 0 \qquad \text{for } m, n = 1, \ldots, M, \ n > m \tag{8}$$

$$\frac{\partial \mathcal{L}}{\partial q_{mm}} = |\overline{h}_m|^2 + \sum_{j=1}^{m-1} \lambda_{jm} q_{jj} + \sum_{j=m+1}^{M} \lambda_{mj} q_{jj} - \mu_k = 0 \qquad \text{for } m \in \mathcal{K}_k \tag{9}$$

where $\overline{h}_i$ represents the i-th element of $\overline{\mathbf{h}}$. The remaining KKT conditions corresponding to the complementary slackness conditions are given by

$$\mu_i \left(\text{tr}\left(\mathbf{\Phi}_i \mathbf{Q}\right) - P_i\right) = 0 \qquad \text{for } i = 1, \ldots, N \tag{10}$$

$$\lambda_{mn} \left(|q_{mn}|^2 - q_{mm} q_{nn}\right) = 0 \qquad \text{for } m, n = 1, \ldots, M, \ n > m. \tag{11}$$

We can see that from (8), λ_{mn} has a nonzero value if $\overline{h}_m$ and $\overline{h}_n$ are not zero. Even if $\overline{h}_m$ equals zero, $\mathbf{Q}$ and $\overline{\mathbf{h}}$ in (7) can be replaced with new variables whose components corresponding to m are removed from $\mathbf{Q}$ and $\overline{\mathbf{h}}$. Therefore, without loss of generality, we can assume that $\overline{h}_m$ has a nonzero value. Thus, we have

$$q_{mn} = \frac{\overline{h}_m^* \overline{h}_n}{\lambda_{mn}} \qquad \text{for } m, n = 1, \ldots, M, \ n > m. \tag{12}$$

Also, since λ_{mn} is nonzero, $|q_{mn}|^2$ should be equivalent to $q_{mm} q_{nn}$ for satisfying (11). Combining (12) and $|q_{mn}|^2 = q_{mm} q_{nn}$, it follows

$$\lambda_{mn} = \frac{|\overline{h}_m^* \overline{h}_n|}{\sqrt{q_{mm} q_{nn}}}. \tag{13}$$

Next, plugging (13) into (9) leads to

$$|\overline{h}_m|^2 + \sum_{j=1}^{m-1} \frac{|\overline{h}_j^* \overline{h}_m|}{\sqrt{q_{mm}}} \sqrt{q_{jj}} + \sum_{j=m+1}^{M} \frac{|\overline{h}_m^* \overline{h}_j|}{\sqrt{q_{mm}}} \sqrt{q_{jj}} - \mu_k = 0. \tag{14}$$

By multiplying the above equation by $\sqrt{q_{mm}}$ and using $|\overline{h}_j^* \overline{h}_m| = |\overline{h}_j \overline{h}_m^*|$, we have

$$\sum_{j=1}^{M} |\overline{h}_j^* \overline{h}_m| \sqrt{q_{jj}} - \mu_k \sqrt{q_{mm}} = 0 \quad \text{for } m \in \mathcal{K}_k. \tag{15}$$

Since μ_i is nonzero, $\text{tr}\left(\mathbf{\Phi}_i \mathbf{Q}\right)$ should be P_i in order to satisfy (10). Thus, it follows

$$\sum_{m \in \mathcal{K}_k} q_{mm} = P_k \qquad \text{for } k = 1, \ldots, N. \tag{16}$$

From now on, we compute q_{mm} and q_{mn} from (12), (15) and (16). Let us consider m_1 and m_2 which belong to $\mathcal{K}_k$. From (15), two equations for m_1 and m_2 are given by

$$\sum_{j=1}^{M} |\overline{h}_j^* \overline{h}_{m_1}| \sqrt{q_{jj}} - \mu_k \sqrt{q_{m_1 m_1}} = 0 \tag{17}$$

$$\sum_{j=1}^{M} |\overline{h}_j^* \overline{h}_{m_2}| \sqrt{q_{jj}} - \mu_k \sqrt{q_{m_2 m_2}} = 0. \tag{18}$$

Multiplying (17) and (18) by $|\overline{h}_{m_2}|$ and $|\overline{h}_{m_1}|$, respectively, and subtracting them yields

$$\mu_k (|\overline{h}_{m_2}| \sqrt{q_{m_1 m_1}} - |\overline{h}_{m_1}| \sqrt{q_{m_2 m_2}}) = 0. \tag{19}$$

Since μ_k is nonzero, we obtain

$$q_{m_2 m_2} = \frac{|\overline{h}_{m_2}|^2}{|\overline{h}_{m_1}|^2} q_{m_1 m_1} \text{ for } m_1, m_2 \in \mathcal{K}_k. \tag{20}$$

Applying (20) into (16), q_{mm} is computed as

$$q_{mm} = \frac{|\overline{h}_m|^2}{\sum_{j \in \mathcal{K}_k} |\overline{h}_j|^2} P_k = \frac{|\overline{h}_m|^2}{\|\overline{\mathbf{h}}_k\|^2} P_k \qquad \text{for} \quad m \in \mathcal{K}_k. \tag{21}$$

Inserting (21) into (13) leads to

$$\lambda_{mn} = \frac{\|\overline{\mathbf{h}}_{k_1}\| \|\overline{\mathbf{h}}_{k_2}\|}{\sqrt{P_{k_1} P_{k_2}}} \qquad \text{for} \quad m \in \mathcal{K}_{k_1} \text{ and } n \in \mathcal{K}_{k_2}. \tag{22}$$

Lastly, combining (12) and (22), the off-diagonal elements q_{mn} of $\mathbf{Q}$ can be obtained as

$$q_{mn} = \frac{\overline{h}_m \overline{h}_n^*}{\|\overline{\mathbf{h}}_{k_1}\| \|\overline{\mathbf{h}}_{k_2}\|} \sqrt{P_{k_1} P_{k_2}} \qquad \text{for} \quad m \in \mathcal{K}_{k_1} \text{ and } n \in \mathcal{K}_{k_2}.$$

Since λ_{ij} and μ_k are positive, the covariance matrix $\mathbf{Q}^\star$ composed of q_{mm} and q_{mn} is the optimal solution of the relaxed problem (7).

Next, we exploit a structure of the optimal beamforming for (7) based on (21) and (23). Denoting $\mathbf{z}$ as

$$\mathbf{z} = \left[\frac{\sqrt{P_1} \overline{h}_1}{\|\overline{\mathbf{h}}_1\|} \ \frac{\sqrt{P_1} \overline{h}_2}{\|\overline{\mathbf{h}}_1\|} \cdots \frac{\sqrt{P_1} \overline{h}_{M_1}}{\|\overline{\mathbf{h}}_1\|} \frac{\sqrt{P_2} \overline{h}_{M_1+1}}{\|\overline{\mathbf{h}}_2\|} \ \cdots \right.$$
$$\left. \frac{\sqrt{P_N} \overline{h}_{M_{1:N-1}+1}}{\|\overline{\mathbf{h}}_N\|} \cdots \frac{\sqrt{P_N} \overline{h}_M}{\|\overline{\mathbf{h}}_N\|} \right]^T,$$

we can easily observe that $\mathbf{z}\mathbf{z}^H$ is equivalent to $\mathbf{Q}^\star$ which consists of q_{mm} and q_{mn}. Therefore, by applying eigenvalue decomposition $\mathbf{Q}^\star = \mathbf{W} \mathbf{\Lambda} \mathbf{W}^H$ where $\mathbf{W}$ is a unitary matrix and $\mathbf{\Lambda}$ equals a real diagonal matrix, $\mathbf{Q}^\star$ has only one nonzero eigenvalue $P = \sum P_i$ and its corresponding eigenvector is given as

$$\left[\frac{\sqrt{P_1}}{\sqrt{P}} \frac{\overline{\mathbf{h}}_1^H}{\|\overline{\mathbf{h}}_1\|} \quad \cdots \quad \frac{\sqrt{P_N}}{\sqrt{P}} \frac{\overline{\mathbf{h}}_N^H}{\|\overline{\mathbf{h}}_N\|} \right]^H.$$

Thus $\mathbf{Q}^\star$ has rank one. From the above result and $\overline{\mathbf{h}}_i = \sqrt{\alpha_i} \mathbf{h}_i$, the optimal beam vector can be expressed as

$$\mathbf{w}^\star = \left[\sqrt{P_1} \frac{\mathbf{h}_1^H}{\|\mathbf{h}_1\|} \quad \cdots \quad \sqrt{P_N} \frac{\mathbf{h}_N^H}{\|\mathbf{h}_N\|} \right]^H. \tag{23}$$

We can observe that since $\mathbf{Q}^\star$ satisfies the positive semi-definite condition and has rank one, $\mathbf{Q}^\star$ is also the optimal solution of the original problem (4).

Now, we discuss the meaning of this solution. The beamforming vector $\mathbf{w}^\star$ accounts for maximum ratio transmission per each DAU, and the transmission power at each DAU is fixed as full power according to its power constraint. As a special case, this solution reduces to the case with sum power constraint by setting the number of DAUs to one ($N = 1$) in our model. Also, the per-antenna power constraint case can be obtained by considering a single antenna for each DAU ($M_i = 1 \ \forall i$).

IV. Ergodic and Outage Capacity with the Optimal Beamforming

In this section, we analyze the outage and ergodic capacity of DAS with the optimal beamforming derived in the previous section under Rayleigh fading channels. The ergodic capacity is expressed as

$$C_{erg} = \mathbb{E}[\log_2(1+\gamma)]$$

where γ represents SNR. Also the outage capacity C_ϵ which is the largest rate of reliable communication at a certain outage probability ϵ can be computed by solving $Pr\{\log_2(1+\gamma) \leq C_\epsilon\} = \epsilon$. Clearly, our goal is to find a distribution of $\log_2(1+\gamma)$. For this matter, we first investigate the distribution of SNR γ.

Before starting, we list several definitions and useful properties of RVs as follows:

- Definition 1: A chi distribution with a positive integer parameter k, denoted as $\chi(k)$, has the probability density function (pdf) given as

  $$f(x;k) = \frac{2^{1-\frac{k}{2}} x^{k-1} e^{-\frac{x^2}{2}}}{\Gamma(\frac{k}{2})} \qquad \text{for} \quad x > 0,$$

  where $\Gamma(\cdot)$ is the Gamma function.
- Definition 2: A Nakagami-m distribution with parameters $m \geq 0.5$ and $\Omega > 0$, denoted as $\eta(m, \Omega)$, has the pdf

  $$f(x; m, \Omega) = \frac{2m^m}{\Gamma(m)\Omega^m} x^{2m-1} \exp\left(-\frac{m}{\Omega} x^2\right) \text{ for } x > 0$$

  where m is the fading parameter which ranges from $\frac{1}{2}$ (half-Gaussian model) to ∞ (AWGN channel) and Ω equals the average fading power [26].
- Definition 3: A Gamma distribution with a finite shape $k > 0$ and a finite scale $\theta > 0$, defined as $\psi(k, \theta)$, has the pdf

  $$f(x; k, \theta) = x^{k-1} \frac{e^{-x/\theta}}{\theta^k \, \Gamma(k)} \qquad \text{for} \quad x \geq 0.$$

- Property 1: If X_i are K independent, circularly symmetric complex Gaussian RVs with $\mathcal{CN}(0,1)$ (variance $\frac{1}{2}$ per dimension), then $\sqrt{2} \sum_{i=1}^K |X_i|$ has a chi distribution $\chi(2K)$.
- Property 2: A RV with a chi distribution $\chi(2K)$ equals a RV with a Nakagami-m distribution $\eta(K, 2K)$.
- Property 3: When X follows a Nakagami-m distribution $\eta(m, \Omega)$, the pdf of αX is $\eta(m, \alpha^2 \Omega)$ with a scalar $\alpha > 0$, and X^2 has a Gamma distribution $\psi(m, \Omega/m)$.

From now on, we find a distribution of γ by using these properties. Applying the optimal beamforming $\mathbf{w}^\star$ into (1), γ is given by

$$\gamma = \overline{\mathbf{h}}^H \mathbf{w}^\star \mathbf{w}^{\star H} \overline{\mathbf{h}} = \left(\sum_{i=1}^{N} \sqrt{\alpha_i P_i} \|\mathbf{h}_i\| \right)^2 . \tag{24}$$

From $\|\mathbf{h}_i\| = \sqrt{\sum_{i=1}^{M_i} |h_i|^2}$ where $h_i \sim \mathcal{CN}(0,1)$, $\sqrt{2}\|\mathbf{h}_i\|$ follows $\chi(2M_i)$ according to Property 1. From Property 2, the distribution of $\sqrt{2}\|\mathbf{h}_i\|$ can be expressed as a Nakagami-m distribution $\eta(M_i, 2M_i)$. Then, using Property 3, we can see that $\sqrt{\alpha_i P_i}\|\mathbf{h}_i\|$ has a distribution $\eta(M_i, M_i \alpha_i P_i)$. Therefore, $\sqrt{\gamma} = \sum_{i=1}^{N} \sqrt{\alpha_i P_i}\|\mathbf{h}_i\|$ in (24) is equal to a sum of Nakagami-m RVs with the parameter $(M_i, M_i \alpha_i P_i)$.

Now, we obtain a distribution of SNR based on the above results. It is generally difficult to compute an exact distribution of a sum of non-identical Nakagami-m RVs, if not impossible. To overcome this problem, we adopt an approximation of the sum of N independent and non-identically distributed (i.n.d.) Nakagami-m RVs with arbitrary m and Ω by using the moment matching method [23]. It was shown in [23] that for K i.n.d. Nakagami-m RVs Y_ks with parameter (m_{Y_k}, Ω_{Y_k}), the pdf of $\sum_{k=1}^{K} Y_k$ is approximated by a Nakagami-m distribution $\eta(m_Y, \Omega_Y)$ where the parameters m_Y and Ω_Y follow the relation

$$m_Y = \frac{\Omega_Y^2}{\mathbb{E}[(\sum_{k=1}^{K} Y_k)^4] - \Omega_Y^2}, \quad \Omega_Y = \mathbb{E}[(\sum_{k=1}^{K} Y_k)^2].$$

Here, $\mathbb{E}[(\sum_{k=1}^{K} Y_k)^2]$ and $\mathbb{E}[(\sum_{k=1}^{K} Y_k)^4]$ can be calculated using

$$\mathbb{E}[Y_k^n] = \frac{\Gamma(m_{Y_k} + n/2)}{\Gamma(m_{Y_k})} \left(\frac{\Omega_{Y_k}}{m_{Y_k}} \right)^{n/2}.$$

It will be shown in the simulation section that this approximation is quite accurate.

By appropriately applying this approach to our case and denoting $\sqrt{\alpha_i P_i}\|\mathbf{h}_i\|$ as X_i, the pdf of $\sqrt{\gamma}$ is given by $\eta(m_s, \Omega_s)$ with

$$m_s = \frac{\Omega_s^2}{\mathbb{E}[(\sum_{i=1}^{N} X_i)^4] - \Omega_s^2}, \quad \Omega_s = \mathbb{E}[(\sum_{i=1}^{N} X_i)^2], \tag{25}$$

where m_s and Ω_s can be obtained from

$$\mathbb{E}[X_i^n] = \frac{\Gamma(M_i + n/2)}{\Gamma(M_i)} (\alpha_i P_i)^{n/2}.$$

Finally, from Property 3, the distribution of γ in (24) can be expressed as

$$\gamma \sim \Gamma(k_s, \theta_s) \tag{26}$$

where we have the parameter mapping relation $k_s = m_s$ and $\theta_s = \frac{\Omega_s}{m_s}$.

Next, based on the above results, we derive the ergodic and outage capacity in a closed form. The ergodic capacity can be expressed as

$$C_{erg} = \mathbb{E}[R] = \frac{1}{\Gamma(k_s)\theta^{k_s}} \int_0^\infty \log_2(1+x) x^{k_s - 1} e^{-x/\theta_s} \mathrm{d}x. \tag{27}$$

According to [27], $\ln(1+x)$ can be expressed in terms of the Meijer's G-function as

$$\ln(1+x) = G_{2,2}^{1,2}\left(x \,\middle|\, \begin{matrix} 1,1 \\ 1,0 \end{matrix} \right) \tag{28}$$

where $G_{p,q}^{m,n}(\cdot)$ is the Meijer's G-function [28, eq. (9.301)]. Applying the above equation into (27) and using the integration formula [28, eq. (7.813.1)]

$$\begin{aligned} &\int_0^\infty x^{-\rho} e^{-\beta x} G_{p,q}^{m,n}\left(\alpha x \,\middle|\, \begin{matrix} a_1, \ldots, a_p \\ b_1, \ldots, b_q \end{matrix} \right) dx \\ &\quad = \beta^{\rho - 1} G_{p+1,q}^{m,n+1}\left(\frac{\alpha}{\beta} \,\middle|\, \begin{matrix} \rho, a_1, \ldots, a_p \\ b_1, \ldots, b_q \end{matrix} \right), \end{aligned} \tag{29}$$

the ergodic capacity can be evaluated as

$$\begin{aligned} C_{erg} &= \frac{1}{\Gamma(k_s)\theta^{k_s} \ln 2} \int_0^\infty x^{k_s - 1} e^{-x/\theta_s} G_{2,2}^{1,2}\left(x \,\middle|\, \begin{matrix} 1,1 \\ 1,0 \end{matrix} \right) \mathrm{d}x \\ &= \frac{1}{\Gamma(k_s)\ln 2} G_{3,2}^{1,3}\left(\theta_s \,\middle|\, \begin{matrix} 1-k_s, 1, 1 \\ 1, 0 \end{matrix} \right). \end{aligned} \tag{30}$$

In particular, for a positive integer k_s, C_{erg} reduces to [29]

$$C_{erg} = \frac{1}{\ln 2} \, e^{1/\theta_s} \sum_{i=0}^{k_s - 1} \left(\frac{1}{\theta_s} \right)^i \Gamma\left(-i, \frac{1}{\theta_s} \right) \tag{31}$$

where $\Gamma(s,x)$ stands for the complementary incomplete gamma function defined as $\Gamma(s,x) = \int_x^\infty t^{s-1} \, e^{-t} \, \mathrm{d}t$.

Also, the outage probability at a certain target rate C_ϵ can be computed as

$$P_{out} = Pr\{\gamma \leq 2^{C_\epsilon} - 1\} = F(2^{C_\epsilon} - 1; k_s, \theta_s) \tag{32}$$

where $F(x; k, \theta)$ represents the cumulative distribution function of a Gamma distribution, which is defined as $\frac{\gamma(k, x/\theta)}{\Gamma(k)}$ and $\gamma(s,x)$ indicates the incomplete Gamma function given as $\int_0^x t^{s-1} e^{-t} \mathrm{d}t$. Then, solving $P_{out} = \epsilon$ yields [30]

$$C_\epsilon = \log_2(1 + F^{-1}(\epsilon)) \text{ bits/s/Hz}.$$

By using relations among RVs and approximations, we have characterized SNR as one single Gamma RV, and derived a closed-form formula of the outage and ergodic capacity. The accuracy of our derived analysis will be confirmed in the simulation section.

V. Limited Feedback Designs

So far, we have assumed the perfect CSI is available at transmitter. In this section, we analyze an average rate of DAS with limited feedback by employing RVQ for CDI of each DAU and uniform quantization for phase information among DAUs. Then, we propose a new feedback bit allocation algorithm to maximize the overall performance of systems with limited feedback. In our work, it is assumed that a user perfectly feeds back channel quality information for all channel links.

A. Quantization of CDI and Phase

Under the assumption that a user has perfect knowledge of CSI for all channel links $\mathbf{h}_i$ by exploiting orthogonal reference signals, the user send back CDI for each link to the DAUs. Before feeding back, the user quantizes the direction of all received channel vector $\tilde{\mathbf{h}}_i = \mathbf{h}_i/\|\mathbf{h}_i\|$ for all i by adopting a distinct quantization codebook $\mathcal{C}_i = \{\mathbf{c}_i^1, \ldots, \mathbf{c}_i^{N_i}\}$ which consists of M_i dimensional unit norm vectors of size $N_i = 2^{B_i}$, where B_i is the number of feedback bits for DAU i. By using the minimum chordal distance metric, the index n_i for $\tilde{\mathbf{h}}_i$ and its corresponding quantized CDI vector $\hat{\mathbf{h}}_i$ are obtained as

$$n_i = \arg \max_{1\le m\le 2^{B_i}} |\mathbf{c}_i^{mH}\tilde{\mathbf{h}}_i|, \qquad \hat{\mathbf{h}}_i = \mathbf{c}_i^{n_i}. \tag{33}$$

For simple analysis, we apply RVQ [17] where each codeword is independent and isotropically distributed on a complex M_i dimensional hypersphere.

When the optimal beamforming $\mathbf{w}^\star$ in (23) is applied for the case of perfect CSI, the received signal in (1) can be rewritten as

$$y = \sum_{i=1}^{N} \sqrt{\alpha_i P_i}\mathbf{h}_i^H\tilde{\mathbf{h}}_i s + n = \sum_{i=1}^{N}\sqrt{\alpha_i P_i}\|\mathbf{h}_i\| s + n. \tag{34}$$

In contrast, employing $\hat{\mathbf{h}}_i$ in (33) instead of $\tilde{\mathbf{h}}_i$, the received signal in the limited feedback system is expressed as

$$y = \sum_{i=1}^{N} \sqrt{\alpha_i P_i}\mathbf{h}_i^H\hat{\mathbf{h}}_i s + n = \sum_{i=1}^{N}\sqrt{\alpha_i P_i}\|\mathbf{h}_i\|\tilde{\mathbf{h}}_i^H\hat{\mathbf{h}}_i s + n, \tag{35}$$

and thus the received SNR γ_{LF} is given by

$$\gamma_{LF} = \left|\sum_{i=1}^{N}\sqrt{\alpha_i P_i}\|\mathbf{h}_i\|\tilde{\mathbf{h}}_i^H\hat{\mathbf{h}}_i\right|^2. \tag{36}$$

Let us denote ξ_i and θ_i as the magnitude and the phase of the inner product between quantized CDI and actual CDI, $\tilde{\mathbf{h}}_i^H\hat{\mathbf{h}}_i$, respectively. Then, γ_{LF} is rewritten as

$$\gamma_{LF} = \left|\sum_{i=1}^{N}\sqrt{\alpha_i P_i}\|\mathbf{h}_i\|\xi_i e^{j\theta_i}\right|^2. \tag{37}$$

As shown in (37), the difference among θ_i's should be considered for coherent transmission in limited feedback systems unlike the perfect CSI case, since the phase mismatch causes a degradation of the received SNR. In other words, γ_{LF} is maximized when θ_i's are aligned along the same angle. Thus, we can improve the received SNR γ_{LF} by properly rotating the beamforming vector $\hat{\mathbf{h}}_i$ for all i. It is also noted that since RVQ is applied for the quantized CDI vector, ξ_i^2 becomes the maximum of 2^{B_i} independent beta random variables with parameter 1 and $M_i - 1$ as in [17], whereas θ_i is independent of B_i and uniformly distributed as in [31].

From now on, we develop a phase compensation method in order to enhance the received SNR. First, we address the phase difference between θ_i and θ_j as $(\theta_i - \theta_j)_{2\pi}$ where $(\cdot)_{2\pi}$ indicates the modulo operation with 2π. Without loss of generality, we set phase θ_1 as the reference phase. Then, defining the phase difference between DAU 1 and DAU i as

$$\Delta\theta_i = (\theta_1 - \theta_i)_{2\pi} \qquad \text{for } i = 2, 3, \ldots, N, \tag{38}$$

γ_{LF} in (37) can be rewritten as

$$\gamma_{LF} = \left|\sqrt{\alpha_1 P_1}\|\mathbf{h}_1\|\xi_1 + \sum_{i=2}^{N}\sqrt{\alpha_i P_i}\|\mathbf{h}_i\|\xi_i e^{-j\Delta\theta_i}\right|^2. \tag{39}$$

Since θ_i and θ_1 are independent and uniformly distributed, one can show that $\Delta\theta_i = (\theta_1 - \theta_i)_{2\pi}$ follows a uniform distribution in $[0, 2\pi]$. Therefore, a user utilizes uniform quantization on $\Delta\theta_i$ to compensate the phase difference. Then, the quantized phase difference for $\Delta\theta_i$ can be modeled as

$$\hat{\theta}_i = \Delta\theta_i + \delta_i U_i \tag{40}$$

where $\delta_i U_i$ represents the quantization error for the phase compensation, we have $\delta_i = 2\pi/(2\cdot 2^{B_i^\theta})$ and U_i is a RV distributed uniformly in $[-1, 1]$.

For example, when a user allocates B_i^θ feedback bits for $\Delta\theta_i$, $2^{B_i^\theta}$ candidates are considered. Then we compare candidates to the actual phase difference and map the closest candidate to the quantized phase difference $\hat{\theta}_i$. Note that as B_i^θ increases, the phase quantization error decreases, as the number of candidates grows exponentially. After quantization, the user informs all indices of $\hat{\mathbf{h}}_i$ for $i = \{1, 2, \ldots, N\}$ and $\hat{\theta}_i$ for $i = \{2, \ldots, N\}$ through error free feedback channels.

Based on the quantized channel for all DAUs and (23) after the phase compensation, the overall beamforming vector in limited feedback systems can be rewritten as

$$\mathbf{w}_{LF}^\star = \left[\sqrt{P_1}\hat{\mathbf{h}}_1^H \;\; \sqrt{P_2}\hat{\mathbf{h}}_2^H e^{j\hat{\theta}_2} \quad \ldots \quad \sqrt{P_N}\hat{\mathbf{h}}_N^H e^{j\hat{\theta}_N}\right]^H. \tag{41}$$

As a result, the received SNR γ_{LF} is given by

$$\gamma_{LF} = \left|\sqrt{\alpha_1 P_1}\|\mathbf{h}_1\|\xi_1 + \sum_{i=2}^{N}\sqrt{\alpha_i P_i}\|\mathbf{h}_i\|\xi_i e^{-j(\Delta\theta_i - \hat{\theta}_i)}\right|^2. \tag{42}$$

B. Upper bound of the average rate and bit allocation strategy

From (42) and (40), the average rate of DAS with limited feedback is expressed by

$$C_{erg}^{LF} = \mathbb{E}\left[\log_2\left(1 + \left|\sqrt{\alpha_1 P_1}\|\mathbf{h}_1\|\xi_1 + \sum_{i=2}^{N}\sqrt{\alpha_i P_i}\|\mathbf{h}_i\|\xi_i e^{j\delta_i U_i}\right|^2\right)\right].$$

Applying Jensen's inequality, after some manipulations, C_{erg}^{LF} is upper-bounded as

$$\begin{aligned} C_{erg}^{LF} \;\le\; \log_2\Bigg(1 &+ \sum_{i=1}^{N}\alpha_i P_i \mathbb{E}[\|\mathbf{h}_i\|^2]\mathbb{E}[\xi_i^2] \\ &+ 2\sum_{i=1,\ i<j}^{N}\sqrt{\alpha_i P_i}\sqrt{\alpha_j P_j}\mathbb{E}[\|\mathbf{h}_i\|]\mathbb{E}[\|\mathbf{h}_j\|]\mathbb{E}[\xi_i]\mathbb{E}[\xi_j]\varphi(i,j)\Bigg) \end{aligned} \tag{43}$$

where

$$\varphi(i,j) = \begin{cases} \mathbb{E}[\cos(\delta_j U_j)] & \text{if } i = 1 \text{ and } j = 2, \ldots, N \\ \mathbb{E}[\cos(\delta_i U_i - \delta_j U_j)] & \text{if } i, j = 2, \ldots, N. \end{cases} \tag{44}$$

Since each element of $\mathbf{h}_i$ is a complex Gaussian RV, we can obtain $\mathbb{E}[\|\mathbf{h}_i\|^2]$ and $\mathbb{E}[\|\mathbf{h}_i\|]$ as

$$\mathbb{E}[\|\mathbf{h}_i\|^2] = M_i, \qquad \mathbb{E}[\|\mathbf{h}_i\|] = \frac{\Gamma(M_i + \frac{1}{2})}{\Gamma(M_i)}. \tag{45}$$

Also, from [17], $\mathbb{E}[\xi_i^2]$ can be expressed as

$$\mathbb{E}[\xi_i^2] \approx 1 - 2^{-\frac{B_i}{M_i-1}}. \quad (46)$$

In addition, $\varphi(i,j)$ is calculated as

$$\varphi(i,j) = \begin{cases} \frac{2\sin\delta_j}{\delta_j} = \frac{2^{B_j^\theta+1}}{\pi}\sin(\pi 2^{-B_j^\theta}) \\ \qquad \text{if } i=1 \text{ and } j=2,\ldots,N \\ \frac{4\sin\delta_i\sin\delta_j}{\delta_i\delta_j} = 4\frac{2^{B_i^\theta}2^{B_j^\theta}}{\pi^2}\sin(\pi 2^{-B_i^\theta})\sin(\pi 2^{-B_j^\theta}) \\ \qquad \text{if } i,j=2,\ldots,N. \end{cases}$$

Finally, C_{erg}^{LF} is represented as

$$\begin{aligned} C_{erg}^{LF} \leq\ & \log_2\left(1+\sum_{i=1}^{N}\alpha_i P_i M_i\left(1-2^{-\frac{B_i}{M_i-1}}\right)\right. \\ &+ 2\sum_{i=1,i<j}^{N}\sqrt{\alpha_i P_i}\sqrt{\alpha_j P_j}\frac{\Gamma(M_i+\frac{1}{2})}{\Gamma(M_i)}\frac{\Gamma(M_j+\frac{1}{2})}{\Gamma(M_j)} \\ &\left.\times\sqrt{1-2^{-\frac{B_i}{M_i-1}}}\sqrt{1-2^{-\frac{B_j}{M_j-1}}}\varphi(i,j)\right) \\ \triangleq\ & \Psi(B_i, B_i^\theta). \end{aligned}$$

Our fundamental question is that if the total number of feedback bits $B_t = \sum_{i=1}^{N} B_i + \sum_{i=2}^{N} B_i^\theta$ is fixed, how many feedback bits should be allocated to channel quantization and phase quantization for DAUs in order to maximize an upper bound of the average rate. To answer this question, we formulate our problem as a function of B_i and B_i^θ

$$\max_{B_i, B_i^\theta} \quad \Psi(B_i, B_i^\theta) \quad (47)$$
$$\text{s.t.} \quad \sum_{i=1}^{N} B_i + \sum_{i=2}^{N} B_i^\theta = B_t.$$

In (47), the optimal B_i and B_i^θ which maximize the upper bound of C_{erg}^{LF} can be determined by searching all possible bit combinations. Then, a user utilizes RVQ for the CDI $\hat{\mathbf{h}}_1,\ldots,\hat{\mathbf{h}}_N$ and adopts uniform quantization for the phase compensation $\hat{\theta}_2,\ldots,\hat{\theta}_N$ according to the allocated bits. Finally, the user feeds back the codebook indices which correspond to $\hat{\mathbf{h}}_1,\ldots,\hat{\mathbf{h}}_N$ and $\hat{\theta}_2,\ldots,\hat{\theta}_N$ towards each DAU.

Note that in order to obtain the optimum bit allocation, we do not need to consider the case allocating any bits for $\hat{\theta}_i$ when the number of allocated bits for $\hat{\mathbf{h}}_i$ is zero. Therefore, the search size can be reduced compared to the original search in (47). Furthermore, it should be emphasized that the bit allocation algorithm is computed only once for a fixed user position, since the objective function in (47) does not depend on instantaneous channel realizations.

VI. Numerical Results

In this section, we verify the validity of our analysis through Monte Carlo simulations. We generate 100,000 independent channel realizations for all simulations. We denote the overall transmit antenna configuration of N DAUs as $M = [M_1\ M_2\ \ldots\ M_N]$ and the overall pathloss as $\alpha = [\alpha_1\ \alpha_2\ \ldots\ \alpha_N]$. First, in Figure 2, we compare a distribution of SNR obtained from our analysis (26) with that of actual values from simulations in DAS with $M = [2\ 3\ 4\ 5]$ and

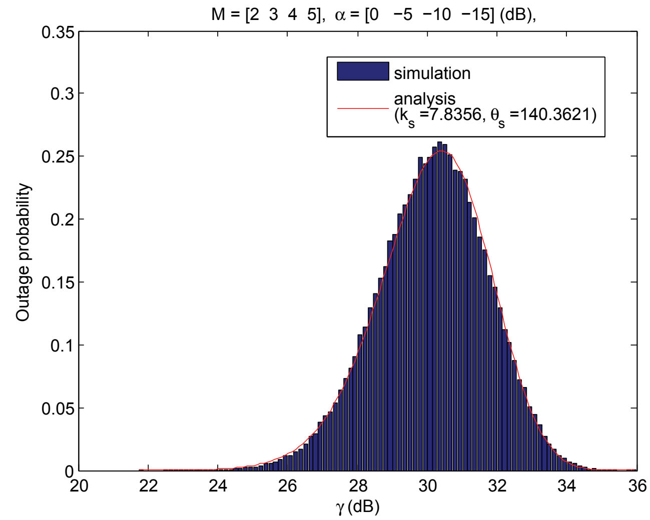

Fig. 2. Probability density function of SNR γ with $N = 4$.

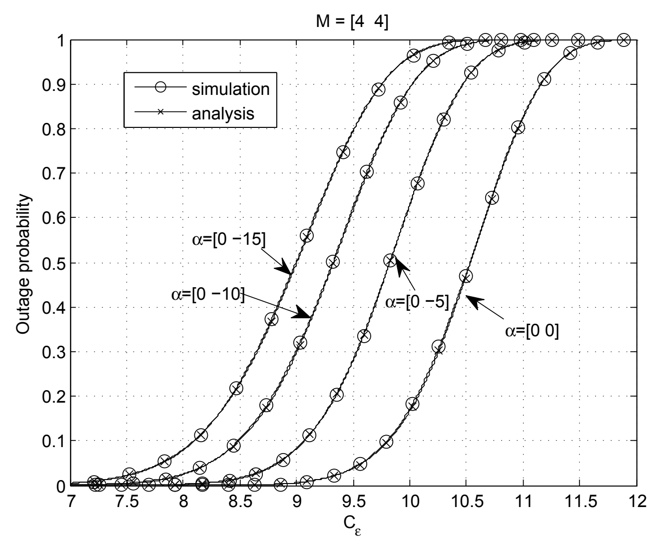

Fig. 3. Outage probability of capacity for various pathloss values.

$\alpha = [0\ -5\ -10\ -15]$ dB at $P_i = 20$ dB. Here, we set k_s and θ_s as 7.83 and 140.36, computed from (25). From this plot, we can see that our analysis of a distribution of SNR provides very accurate results.

In Figure 3, we present the comparison of the outage probability with respect to the target rate C_ϵ for different pathloss values in dB with $M = [4\ 4]$ at $P_i = 20$ dB. It can be seen that the simulation results match well with the theoretical results presented in (32). We observe that as α_2 increases from -15 to 0, about a 1.5 bps/Hz transmission rate gain in the supported SNR can be achieved at the outage probability of 10%.

Next, in Figures 4 and 5, we evaluate the capacity performance under various configurations. Figure 4 compares the 10% outage capacity of DAS with different antenna settings with $\alpha = [0\ -10]$ dB. Note that the system with DAUs equipped with equal numbers of antennas outperforms that with unequal numbers of antennas. Figure 5 shows the ergodic capacity of DAS with $M = [4\ 4]$ and $\alpha_1 = 0$ with respect to

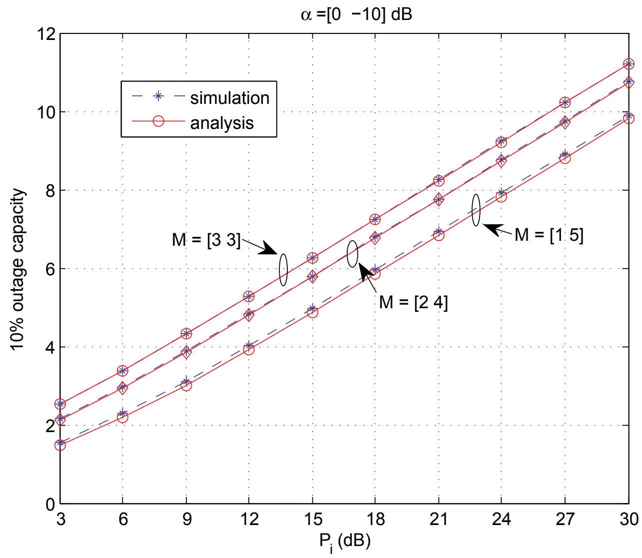

Fig. 4. 10% outage capacity for different antenna configurations.

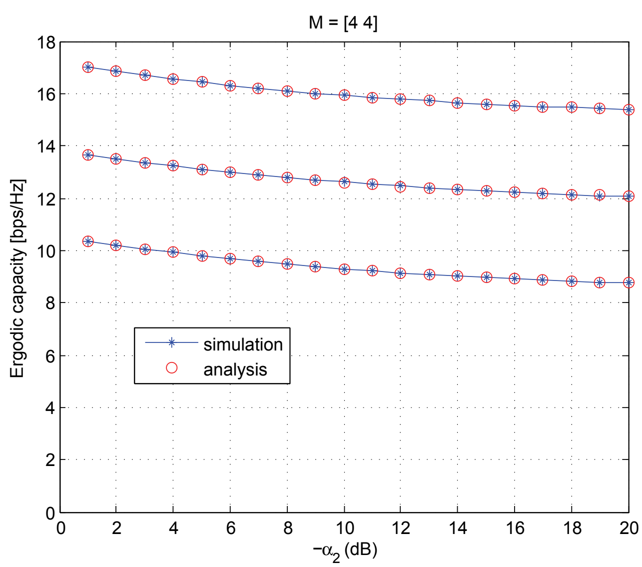

Fig. 5. Ergodic capacity with respect to $-\alpha_2$ for various P_i.

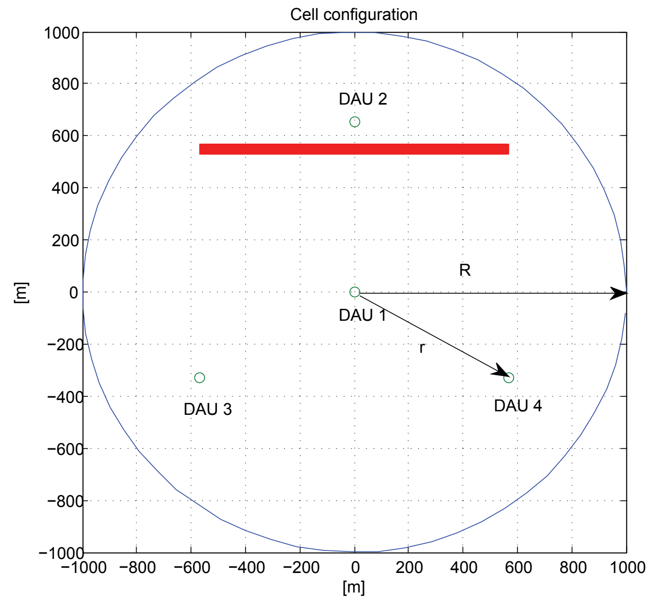

Fig. 6. DAS configuration with 4 DAUs.

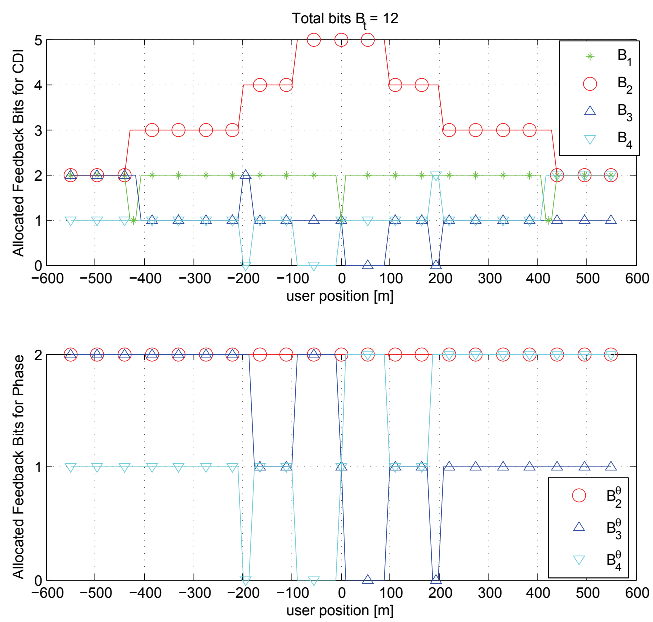

Fig. 7. Allocated feedback bits depending on the user position.

$-\alpha_2$ for $P_i = 20, 30$ and 40 dB. In this plot, we can see that the analysis results calculated from (30) are very close to the actual value.

In what follows, we provide the cell average throughput of the proposed limited feedback bit allocation scheme in practical environments. We consider DAS having a circular antenna layout with the cell radius R, where $N-1$ DAUs are deployed circularly at $\left(r\cos\left(\frac{2\pi(j-1)}{N-1}+\frac{\pi}{2}\right), r\sin\left(\frac{2\pi(j-1)}{N-1}+\frac{\pi}{2}\right)\right)$ for $j = 1, \cdots, N-1$ with $r = \sqrt{\frac{3}{7}}R$ as in [5], and one DA port is also located at the cell center $(0, 0)$. For example, Figure 6 describes DAS with 4 DAUs and $R = 1$ km. Also, in order to evaluate the effect of pathloss values, we use $\alpha_i = (d_0/d_i)^\eta$, where d_0 indicates the reference distance for the antenna far field, d_i denotes the distance between a user and DAU i, and η is the pathloss exponent. Based on this pathloss model, we define the average received signal power from DAU i as

$$P_i^r = P_i\left(\frac{d_0}{d_i}\right)^\eta.$$

Throughout simulations, we consider the case with parameters $\eta = 3.7$ and $d_0 = 1$ m.

In order to illustrate the proposed bit allocation strategy, we assume that a user travels from $(-550, 550)$ to $(550, 550)$ as indicated by a straight bold line in Figure 6. Figure 7 shows the result of the proposed bit allocation algorithm according to the user location with $M = [4\ 4\ 4\ 4]$ and $B_t = 12$. Here, it is assumed that P_i is set to satisfy $P_i^r = 10$ dB with $d_i = \frac{1}{2}r$. When the user approaches DAU 2, the corresponding allocated bits grow up to 5 bits, since the terms related with DAU 2 in the ergodic capacity formula become dominant due to pathloss. In contrast, the allocated bits for others

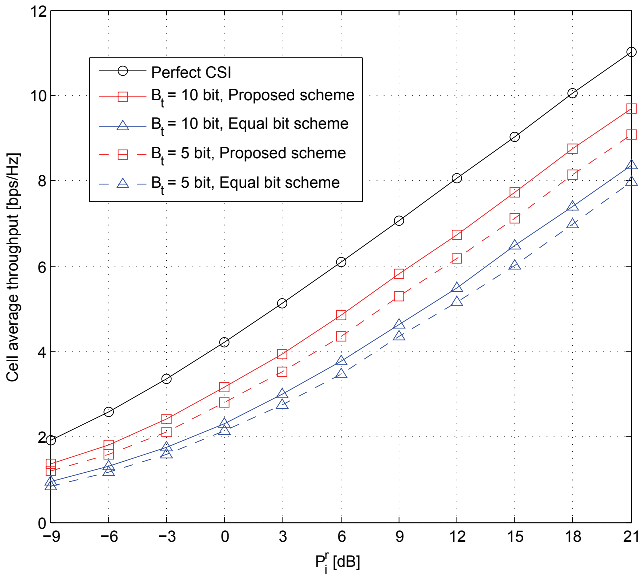

Fig. 8. Cell average throughput performance for various B_t.

are upper-bounded by 2 bits, which results from relatively larger pathloss values compared to DAU 2. Bit allocation for phase compensation also exhibits a similar tendency. We can easily observe that both allocated bits for CDI and phase are symmetric according to user positions.

Finally, we present the cell average throughput for a system with 5 DAUs where a user is uniformly distributed within a cell. In Figure 8, the performance is evaluated as a function of the average received SNR P_i^r with $d_i = \frac{1}{2}r$. As shown in the figure, the proposed scheme shows a good performance gain over conventional equal bit allocation methods with $B_i = B_t/5$. The cell average throughput performance of the proposed bit allocation scheme provides 19% and 22% gains at $P_i^r = 12$ dB compared with that of the equal bit allocation method with $B_t = 5$ and 10, respectively. Thus, compared to the equal bit allocation, the proposed scheme substantially reduces performance degradations caused by the limited feedback.

VII. Conclusions

In this paper, we have investigated the outage and ergodic capacity of the downlink DAS with optimal beamforming under per-DAU power constraint. We have first derived the optimal beamforming in a closed form which is applicable to DAS with arbitrary numbers of antennas and different power levels per each DAU. It can be seen that our derived solution has a form of maximum ratio transmission per each DAU with full power. Next, we have characterized a distribution of the received SNR of the DAS with the optimal beamforming in Rayleigh fading channels. Then, the formula of the outage and ergodic capacity is expressed in a closed form by utilizing the obtained SNR distribution. Simulation results confirm the accuracy of our analysis for the outage and ergodic capacity. Furthermore, we have proposed a new bit allocation method which maximizes an upper bound of the average rate of DAS with limited feedback. We have compared the proposed scheme with the equal bit allocation method and have shown that improved performance can be attained.

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