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Some Reflections on Arnold Sommerfeld—Theoretical Physicist, Teacher, Influencer

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Arnold Sommerfeld is one of the most influential theoretical physicists of all times. His contributions to electromagnetics comprise mathematical theories of diffraction and fundamental work on wave propagation above planar ground, leading to the concept of Sommerfeld integral representations. Probably even more recognized are his contributions to the theory of atoms and to the development of quantum physics. In his role as educator and teacher, Sommerfeld's performance is unprecedented—four of his doctoral graduates and three of his postdocs are recipients of a Nobel Prize in physics or chemistry; one even received an additional Nobel Peace Prize. In his

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EDITOR'S NOTE

In this month's column, we are treated to a fascinating summary of the contributions of one of the foremost scholars in electromagnetics and quantum theory during the late 19th and early 20th centuries. This is Arnold Sommerfeld. He was a great theoretician and formidable mathematician. Besides contributions to electromagnetics, he worked in other fields of physics and in mathematics. For example, an integral identity, now called the *Sommerfeld Identity*, and Sommerfeld integrals are still the most common way to solve problems related to a Hertzian dipole radiating over a conducting Earth. As a reminder, the Sommerfeld Identity [S1] in its most generic form is

$$\frac{\exp(-\mathrm{j}kr)}{r} = -\mathrm{j}\int_0^\infty d\xi \frac{\xi}{k_z} \mathrm{J}_0(\xi\rho)\exp[(-\mathrm{j}k_z|z|)] \qquad \text{(E1)}$$

where $k_z = \sqrt{|k^2 - \xi^2|}$ or $-j\sqrt{|\xi^2 - k^2|}$ if $\xi > k$; r is the distance from the origin; ρ is the distance in a cylindrical coordinate system (ρ, ϕ, z); and J_0 is the ordinary Bessel function. A physical interpretation is that a spherical wave can be represented as a summation of cylindrical waves. A special form of (E1) occurs in some antenna mutual coupling calculations when the source is in the $z = 0$ plane. This leads to integrals of Bessel functions, which can be expressed in a closed form [S2]. An identity in Bessel function series also bears his name and is known as the *Kneser–Sommerfeld expansion* [S3].

In electromagnetics, Sommerfeld is best known for his first rigorous solution to diffraction by a perfectly conducting half-plane. This was published in 1896 [S4]. His method involved seeking a solution to the wave equation that had a periodicity of 4π. Sommerfeld's result is famous partly because he obtained the exact solution in terms of the well-known Fresnel integral. The article was completed in the summer of 1895 and published the following year.

Sommerfeld is primarily known for his contributions to the initial theory of the atom and his supervision of doctoral students. Relating to the latter, by the early 1920s, he had supervised about half the physics professors in Germany in their doctoral theses, four of whom ultimately won Nobel prizes. Sommerfeld's style as a professor and institute director was one of collaboration with staff and students. At all times, he worked closely with them, and their ideas also modified his approach to his problems. Sommerfeld invited them to his home and met with them in cafés before and after seminars. He owned an alpine ski hut where students were often invited for discussions on physics as well as going on walking excursions in the summer or skiing in winter.

I recommend the article by Thomas Eibert to readers of this magazine as it captures the important work of Sommerfeld and shows why he was one of the most important scientists of the 20th century.

References

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role as an influencer, Sommerfeld was instrumental for the development of theoretical physics in the first half of the 20th century, where his way of doing physics influenced scientists all over the world, and dozens of his students and postdocs came into leadership positions in universities, companies, and other institutions. The goal of this article is to highlight some cornerstones and developments in the life and career of Arnold Sommerfeld and to give some insight into his way of doing science and education, where the focus is on electromagnetics and wave propagation.

INTRODUCTION

In the year 2000, the IEEE Antennas and Propagation Society International Symposium took place in Salt Lake City, UT. In a millennium session, organized by Robert E. Collin and Yahya Rahmat-Samii, an impressive collection of presentations reviewed important research progress in the fields of antennas and propagation. At least four of these presentations highlighted the contributions of Arnold Sommerfeld [1], [2], [3], [4], which are, in particular, Sommerfeld's mathematically exact solution of the half-plane diffraction problem [5], [6, Vol. 4] as well as his work on wave propagation above a possibly lossy dielectric half-space [6, Vol. 6], [7]. The diffraction solution of the half-plane problem can be considered as the starting point of mathematically exact solutions of diffraction problems. The wave propagation problem of a Hertzian dipole above a realistic ground half-space was, in a similar manner, a fundamental contribution toward the mathematical treatment of wave propagation above ground and also in multilayer planar material configurations. It opened up a long-lasting discussion about the role of guided waves such as the Zenneck wave [8], [9], and the calculation of the related Sommerfeld integrals is still a topic of intensive research [10], [11], [12].

Other important contributions of Sommerfeld to the field of electromagnetics are his works on wave propagation along metallic wires [13] and on the theory of electrons, e.g., [14] and [15]. Related to the theory of atoms, in particular, his extensions of Bohr's atomic model [16] by elliptic electron paths with relativistic treatment of the electron movement [17] can be considered as a key contribution to the development of quantum theory. His book *Atomic Structure and Spectral Lines* [18] made him famous throughout the physics and chemistry communities all over the world. With his scientific success and with his exceptional skills as an academic teacher, he attracted a growing number of excellent students and postdocs, not only in Europe, but many young researchers even came from the United States to study atomic physics in Munich.

In the "Physics and Engineering at the Beginning of the 20th Century" section, we will start with a short look at physics and engineering education and research, in particular in Germany, at the beginning of the 20th century and its development during the active time of Sommerfeld. The "Childhood and Education" section will shed some light on the childhood and education of Sommerfeld in Königsberg, Prussia (nowadays Kaliningrad, Russia). The first stations of his professional career in Göttingen, Clausthal, and Aachen will be discussed in the "Early Career in Göttingen, Clausthal, and Aachen" section, where we also look in particular into some aspects of his mathematical theory of diffraction. Sommerfeld's role as a professor of theoretical physics at Ludwig-Maximilians-University (LMU) Munich is the topic of the "Teaching and Research at LMU in Munich" section, and then "The Influencer" section will try to explain his role as an influencer. An attempt of drawing a final conclusion is found in the "Conclusion" section.

It is clear that most of the information conveyed in this article comes from the given references, where, in particular, the biographic books and articles directly related to the person of Arnold Sommerfeld and his work (i.e., [19], [20], [21], [22]) need to be mentioned. Quite a bit of information was obtained from Wikipedia [23], and of course, also from the articles of Sommerfeld himself. Considerable credit goes to Deutsches Museum, Munich [24], where quite a lot of material out of the estate of Sommerfeld is archived and where historical studies about Arnold Sommerfeld have been performed, in particular by Michael Eckert and his colleagues [20], [21], [22]. Moreover, it should be mentioned that there are many honorary articles and notes around that were written and published in appreciation of the work and life of Arnold Sommerfeld; see, for example, the articles by M. Planck [25], W. Pauli [26], W. Heisenberg [27], P. Ewald [28], M. Born [29], and P. Debye [30].

PHYSICS AND ENGINEERING AT THE BEGINNING OF THE 20TH CENTURY

At the beginning of the 20th century, physics was mostly an experimental subject, and theoretical physics was not really well accepted as a scientific discipline with a great future. Lectures on theoretical physics were often given by assistant or associate professors at the universities. Universities with a chair professor of theoretical physics in Germany were, e.g., in Berlin, Königsberg, and Göttingen. The first chair professor of theoretical physics in Germany was Gustav Kirchhoff, who got his appointment at the University of Berlin in 1875. His successor was Max Planck in 1889. In 1890, Ludwig Boltzmann became the first chair professor of theoretical physics at LMU Munich, but he left after four years, and his successor was Arnold Sommerfeld in 1906.

Engineering education in Germany was performed at technical schools called *Technische Hochschulen*, which were heavily fighting for improved academic rights at this time. For example, the Technical University of Munich was founded as Polytechnic School in 1868 and named *Technische Hochschule* (*TH*) in 1877, and it received the right to award doctoral degrees in 1901. In 1970, it was renamed into *Technische Universität München* (*Technical University of Munich*).

As a strong mathematician, who was very interested in technical and physical problems, Sommerfeld found a wide field of specific problems that he could treat and solve with sophisticated mathematical techniques. Actually, he was a master in doing this, and he had a great influence on the establishment and growth of the field of theoretical physics in the first half of the 20th century. During his time in Göttingen, his teacher Felix Klein supported his affinity for solving specific physical and technical problems. At the Mining School ("Bergakademie") in Clausthal, Sommerfeld gave mathematics lectures for technicians; as a professor of mechanics at TH Aachen, he educated engineers in physics, and he solved a wide variety of technical problems. As a professor of theoretical physics at LMU Munich, he looked into many problems that were of practical relevance at this time, and a first summit of his research at LMU was related to experiments on the understanding of X-rays, where Wilhelm Conrad Röntgen, the discoverer of X-rays, was a professor of experimental physics at LMU at the same time.

An important ignition point for the development of theoretical physics was the model of atoms introduced by Niels Bohr in Copenhagen in 1913. Sommerfeld advanced this model systematically, and he was able to establish himself as one of the leading scientists in the theory of atoms. In particular, his book on the structure of atoms and spectral lines [18], where he systematically collected and discussed the relevant research results of the entire community, found attention throughout the world and made him famous. His institute at LMU became one of the centers of theoretical physics in the world, together with Copenhagen, where Bohr was active, and Göttingen, where Max Born was active. Max Planck and Albert Einstein were, of course, giants in Berlin, and eventually, important centers of theoretical physics were established in Zurich with Wolfgang Pauli and Erwin Schrödinger and in Leipzig with Werner Heisenberg. Among all of these great physicists, Sommerfeld was quite obviously the one who was most successful in educating and promoting students and young researchers. Toward the end of Sommerfeld's academic career, at least 12 chair professorships in Germany and quite a few in other countries were headed by his students.

CHILDHOOD AND EDUCATION

Arnold Johannes Wilhelm Sommerfeld (see Figure 1 for some photographs of him) was born in Königsberg, Prussia, on 5 December 1868 as son of the practicing medical doctor Franz Sommerfeld and his wife Cäcilie. Together with his siblings, he grew up in a well-situated environment. His school education at the "Altstädische Gymnasium" in Königsberg, one of the oldest high schools in Germany, gave him a wide foundation of knowledge in natural sciences and mathematics but also in literature, history, and music. At this time, the "Altstädische Gymnasium," and afterward, the University in Königsberg were also attended by the future mathematician Hermann Minkowski and the future physicists Max and Willy Wien (Nobel Prize in Physics in 1911). With all of them, Sommerfeld had many interactions throughout his professional career. In terms of music, Sommerfeld became a very good pianist, and he played the piano at all kinds of social occasions throughout his life, which, of course, often helped him to be at the center of attention.

After finishing high school, Sommerfeld decided to study mathematics and natural sciences at the Albertus-University in Königsberg. The Albertus-University had a very good reputation at this time. It was the university of the philosopher Immanuel Kant, and in the field of mathematics and physics, professors such as Friedrich Wilhelm Bessel, Carl Gustav Jacobi, and Franz Ernst Neumann had been teaching there in the past. At the time of Sommerfeld's studies, the chair professor of mathematics was Ferdinand Lindemann, who succeeded in proving that the circle number π is a transcendental number. Adolf Hurwitz was associate professor of mathematics and David Hilbert was lecturer (Privatdozent). Sommerfeld started with some general studies on various topics, but later on, he was in particular excited by the lectures in mathematics, where a lecture by David Hilbert on ideal theory quite obviously had a deep impression on him.

Toward the end of his studies, Sommerfeld conceived and wrote a doctoral dissertation titled "Arbitrary Functions

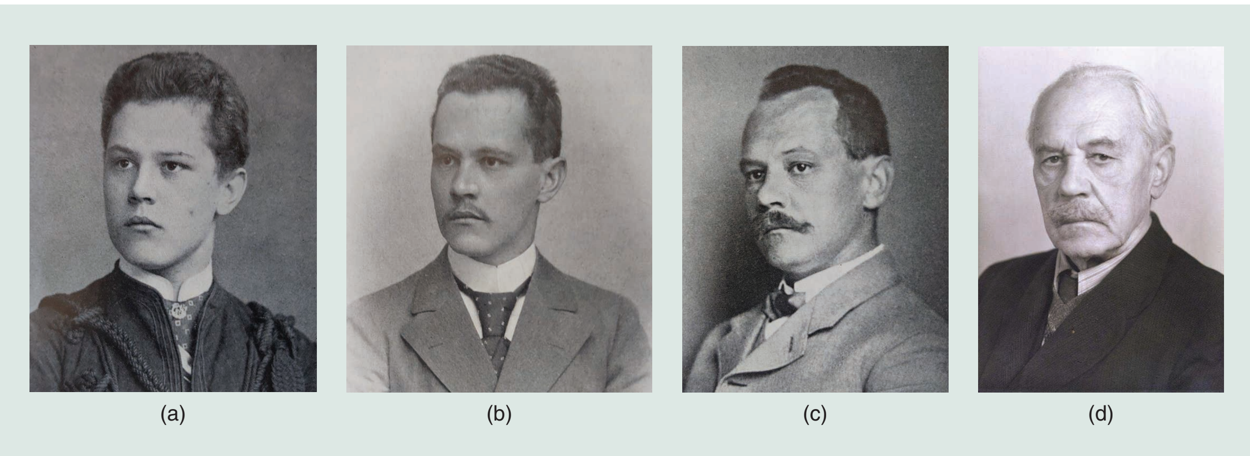

FIGURE 1. Photographs of Arnold Sommerfeld. (a) As a student in Königsberg in 1888. (b) As a lecturer in Göttingen in 1896. (c) As a professor in Munich in 1906. (d) As a professor emeritus in Munich in 1944. (Source: [19] and Deutsches Museum, Munich.)

in Mathematical Physics" [31], where he looked into Fourier integral representations as well as into function representations utilizing Bessel and spherical functions, which are related to the corresponding 2D and 3D Fourier integral representations, respectively. In this thesis, he already studied integration contours in the complex plane and their influence on the convergence of the integrals. The supervisor and examiner of his thesis was Prof. Lindemann, who did, however, not care much and gave Sommerfeld the minimum grade of "rite." Sommerfeld himself got the impression that nobody in Königsberg had even read his doctoral thesis. Still, this doctoral thesis was actually the starting point of Sommerfeld's extremely successful work in the field of partial differential equations throughout his entire career.

Very important for the professional development of Sommerfeld toward the end of his doctoral studies was a prize competition sponsored by the Association of Physics and Economics of Königsberg, where ground temperature recordings in the Botanic Garden in Königsberg had to be evaluated toward a better understanding of temperature variations in the Earth. Sommerfeld decided to participate in this competition, and his goal was to utilize Fourier series representations of the temperature data to solve the heat conduction into Earth, a procedure that was, of course, strongly related to his doctoral thesis. Since the Fourier series analysis of the temperature data was not a trivial task in an age without computers, he decided to construct a mechanical Fourier analyzer (harmonic analyzer), which was even realized in the mechanical workshop of the Institute of Physics at the Albertus-University in collaboration with Emil Wiechert, the scientific assistant at this time [32]. Sommerfeld submitted his solution for the prize problem; however, due to an error in a boundary condition of his theoretical work, he finally withdrew it.

After this excursion into heat conduction, Sommerfeld got attracted to electrodynamics. His attention was caught by a publication of Heinrich Hertz [33], where Maxwell's equations were assumed as the starting axioms of electrodynamics, without an attempt of a mechanical explanation. Nowadays, this is accepted knowledge, but at this time, Sommerfeld got inspired to work on the mechanical explanation of electrodynamics. By assuming the ether to be a liquid with friction, he was able to arrive at Hertz's equations for conductors, which was in contrast to a solid ether, as used to describe isolators. With these insights, Sommerfeld published an article in the *Annalen der Physik* [34], which attracted the attention of Ludwig Boltzmann, who wrote him a letter of appreciation. Boltzmann did, of course, also criticize his work, but as a young researcher, Sommerfeld was extremely happy about Boltzmann's attention.

As with many other young scientists, Sommerfeld decided to take the examination for qualification as a high school teacher, and he got certified in 1892. Next, he intended to do his military service, but first, he went on a longer trip to southern Germany, where he was very impressed by the Alps and Munich. While he was on his trip, a letter from Adolf Hurwitz arrived in Königsberg, who was meanwhile a professor at the Eidgenössische Technische Hochschule (ETH) in Zurich. Hurwitz offered him an assistant position in Zurich, but his mother replied to Hurwitz that he was currently not there and that he was going to start his military service after his return. When Sommerfeld came back, he was very excited about the offer and wanted to accept it, but the position was already gone. With this experience, he was, however, convinced that he wanted to pursue an academic career and work for a habilitation degree as the next step.

EARLY CAREER IN GÖTTINGEN, CLAUSTHAL, AND AACHEN

In 1893, Sommerfeld became the scientific assistant of the mineralogy professor Theodor Liebisch at the University of Göttingen. He was not really convinced about this position, but he was actually much more attracted by the University of Göttingen as one of the leading centers of mathematics at this time. It was the university of Carl Friedrich Gauss, Gustav Lejeune Dirichlet, and Bernhard Riemann. In 1893, the leading mathematician in Göttingen was Felix Klein (called *the Great Felix*), who had a great reputation throughout Germany and was also well connected to the government.

Sommerfeld recognized rather quickly that mineralogy and the work with Liebisch were not the right choice for him, and he took any effort to attract the attention of Felix Klein and possibly become an assistant with him. Liebisch gave him the permission to spend some time on studying mathematics, and Klein invited him for a one-hour discussion on his mathematical studies every week. Klein liked Sommerfeld's doctoral dissertation, and also, Sommerfeld recognized many similarities of his work with the research of Klein at this time. Klein was in favor of application-oriented mathematics, and he was, for instance, teaching a course on partial differential equations in physics, which was one of the favorite topics of Sommerfeld throughout his career ([6, Vol. 6]).

In October 1894, Sommerfeld became the assistant responsible for the reading room at Klein's institute. The reading room was kind of a library of the institute, where, in particular, all the lectures of the professor were also archived in written form. Sommerfeld intensely studied mathematical methods for physical problems, and he got a lot of support by Klein into this direction. He studied heat conduction problems based on the image principle and the method of principal solutions (Green's functions in homogeneous space), where he was, in particular, excited by the utilization of the image principle on Riemann surfaces and the construction of the corresponding branched solutions [35]. Eventually, he came up with the idea of solving the half-plane diffraction problem with these techniques, and this work would become his famous "Mathematical Theory of Diffraction" [5], which is well known as the first mathematically rigorous and exact solution of the half-plane diffraction problem.

For Sommerfeld's method to work, he actually needed two key ingredients. First, to apply the image principle, he needed a geometrical arrangement with the corresponding symmetry. If we assume a half-plane in the xz-plane of a coordinate system with the edge located along the z-axis, then the solution space is described by the polar angle φ ranging from 0 to 2π, and the primary excitation might be an incident plane wave or a line source in the direction $\varphi = \alpha$, as depicted in Figure 2. If we look at the half-plane as a wedge with wedge angle $\varphi_W = 0 = (2 - n)\pi$, i.e., $n = 2$, then the 0-sheet of the half-plane is found for $\varphi_0 = 0$, and the n-sheet is found for $\varphi_n = 2\pi - \varphi_W = 2\pi$. If we now map the angle φ onto an angle $\xi = (\varphi - \alpha)/2$, then the 0-sheet is found at $\xi_0 = -\alpha/2$, and the n-sheet is found at $\xi_n = \pi - \alpha/2$, so that we now have a full plane in the space described by the angle ξ, for which the image principle is well applicable.

Mapping the complete space described by ξ ranging from 0 to 2π back onto the original space described by φ, we obtain obviously two complete spaces, i.e., φ ranges over a period from 0 to 4π, which correspond to the two sheets of a Riemann double surface with branch points at the origin (at the edge) and at infinity, together with a branch cut advantageously chosen along the direction $2\xi = \varphi - \alpha = \pm\pi$, which corresponds to the shadow boundaries of the diffraction problem. The image sources located in the direction $\xi = -\alpha$ need, of course, to be found on the Riemann surface sheet, which is invisible in the original space. For more general 3D excitation configurations, the concept may be extended by considering a Riemann double space instead of a Riemann surface, and the procedure can, of course, also be extended to Riemann surfaces with several sheets.

Now, where we have a configuration with the right symmetry for the utilization of the image principle, we need as a second ingredient the solution functions of Maxwell's equations (only the time-harmonic case is considered), which can be taken as the incident field and its image. Sommerfeld constructed these solution functions by integral representations, where he connected his considerations strongly to potential theory and complex analysis. In modern terminology and for the case of plane-wave incidence, it is probably most intuitive to say that Sommerfeld started with a propagating plane-wave representation of the incident field in the form of a complete k-space integral over the Ewald sphere (according to Paul Ewald), which is actually only an Ewald circle in the 2D case of an incident plane wave. (Paul Peter Ewald earned a doctoral degree/habilitation with Sommerfeld. He became an associate professor/chair professor of theoretical physics at TH Stuttgart, and he was well known for the Ewald sphere in k-space and the Ewald transformation for the acceleration of infinite series evaluations [36].) As the spectral solution function, Sommerfeld chose an analytic function with a simple pole, which reproduces the incident plane wave according to the residue theorem.

Due to the regularity of the spectral solution function, the integration contour could then easily be deformed in the complex β-plane, where β is the integration angle along the Ewald circle. By choosing an integration contour that starts and ends at appropriate locations at infinity with a vanishing spectral integrand, the generation of the 4π-periodic solution function on the spatial Riemann double surface is now achieved by replacing the spectral angle β in the spectral solution function by $\beta/2$, and the pole angle α is similarly replaced by $\alpha/2$ to keep it in the correct position. Since the plane-wave propagator is not modified in this process, the resulting spatial function is still a solution of Maxwell's equations. It fulfills the necessary conditions at infinity and near the edge, and it has the right symmetry property to fulfill the boundary condition on the half-plane via the image principle.

By the performed conformal angular mapping, the original incident wave field is kind of deformed in a way that the correct solution of the diffraction problem is obtained. With this, the problem can formally be considered to be solved, and the resulting integral representation can be evaluated by the methods of complex integration. Moreover, it must be decided which sheet of the spatial Riemann surface is visible and, thus, contributing in a certain region of the actual solution space, where the different regions are separated by the corresponding incident and reflection field shadow boundaries, corresponding to the branch cut of the Riemann surface.

In his original work [5], which became his habilitation thesis in 1895, Sommerfeld considered not only incident plane waves but also localized sources, and he did careful convergence analyses. Actually, he rigorously constructed the solutions of the Helmholtz equation from potential solutions by utilizing spherical functions and performing an infinite number of derivatives [5]. A more compact and simplified approach with direct integral representation was presented by him in [37] for constructing Green's functions of the Poisson equation, where

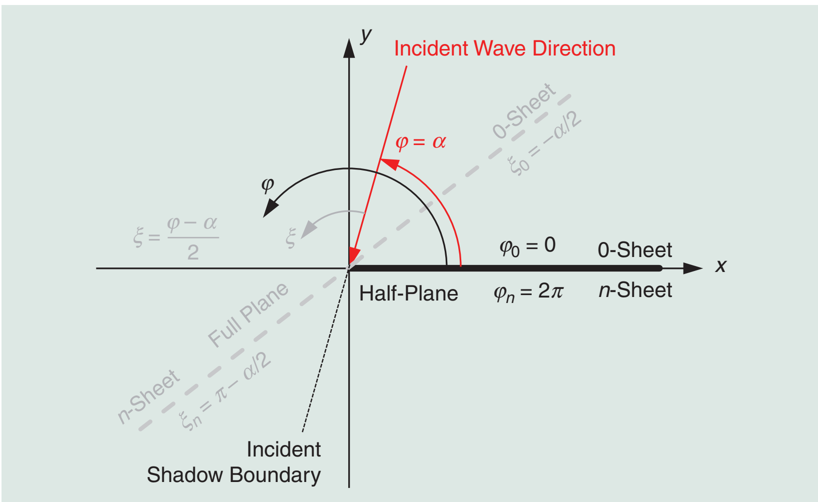

FIGURE 2. Geometrical arrangement of the half-plane problem in the original space with polar angle φ and the mapped space with polar angle ξ.

the possible extension for the Helmholtz equation and for the heat conduction equation were just outlined. The solution for the Helmholtz equation with plane-wave incidence was then published by H. Carslaw in [38], who was a student with Sommerfeld in 1897. In the direct integral representation for plane-wave incidence, the solution function is derived from a closed integration contour in the complex β-plane around the pole in the spectral solution function representation as mentioned before.

In his later publication [14], where he treats the diffraction of temporal pulses to gain insight on the generation of X-rays, Sommerfeld himself called this approach *heuristic*, which is maybe the reason why some people do not consider his treatment of the half-plane diffraction problem as mathematically rigorous. A summary of the state of the art of the rigorous treatment of diffraction problems was given by the Sommerfeld student Paul Epstein in the *Encyclopedia of Mathematical Sciences* [39] in 1915. (Paul Sophus Epstein earned a doctoral degree with Sommerfeld; he also became a professor of mathematical physics at the California Institute of Technology, Pasadena, CA.) Epstein also follows the concept of the more compact integral representation, and he discusses the extension to wedges with commensurate and incommensurate wedge angles. Moreover, he outlines a solution for the slot problem with two half-planes, as published by Karl Schwarzschild in 1902. A textbook representation of Sommerfeld's branched solution of the half-plane diffraction problem for incident plane waves is finally found in Volume 4 (*Optics*) of his *Lecture Notes on Theoretical Physics* [6, Vol. 4].

FIGURE 3. Prof. Sommerfeld while cutting a branch cut into a Riemann surface—a drawing by one of his students. (Source: [19].)

With his diffraction work, Sommerfeld demonstrated impressively that he was a master in complex integration techniques, as, e.g., expressed by Philip Morse, who earned a Rockefeller scholarship in 1930/1931 and was with Sommerfeld as a postdoc at LMU Munich. Morse later became a professor and the director of the Operations Research Center at the Massachusetts Institute of Technology (MIT). In his memories about his research stay with Sommerfeld in Munich in 1930/1931 [40], he said:

> I sat in on as many lectures as I could (…) Sommerfeld was a master of the application of mathematical analysis to the classical theory of fields. It was what I needed; I drank it in.

During his entire career, Sommerfeld built on these capabilities again and again, for instance, in his work on wave propagation above ground; in his atomic model with elliptical electron paths; and for many other problems, which he tackled by using his excellent mathematical skills in integration and differential equations. Later, when he was a professor, Sommerfeld's ingenuity in complex integration techniques was also well known among his students as, e.g., seen in Figure 3, where Prof. Sommerfeld is shown while working hard on a Riemann surface.

The next major project of Sommerfeld, together with his teacher Felix Klein, was related to the theory of the gyroscope. Klein put a lot of attention on this topic and also treated it in his lectures. Sommerfeld was heavily involved in this work, and it finally became a topic for many years, resulting into a series of publications [41]. Klein was obviously an impressive personality and a great teacher, who gave excellent lectures, always well prepared and with great didactic skills. Sommerfeld was very impressed by Klein and his lecturing style, and with his responsibility for the reading room, he had all the opportunities to learn from his teacher, but for himself, he eventually drew the conclusion that his own teaching should be a little less formal, similar to what he had also experienced, e.g., with David Hilbert. In honor of the one hundredth birthday of his teacher Felix Klein, Sommerfeld wrote in 1949 [42]:

> What lectures they were! Perfectly prepared, most hauntingly presented, every hour a small, also stylistically well-rounded masterpiece; a summary dictation in concise form every 10 minutes. When I wanted to imitate this method of dictation during my first professorship in Clausthal, I aroused the displeasure of the students and had to stop it. Klein captivated his audience by using his personality. No proofs by deductio ad absurdum, but an organic, natural construction of the train of thought, from which what was to be proved seemed to emerge without any effort.

Overall, Klein can be seen as the most important teacher and promoter of Sommerfeld. He gave him the opportunity to dig into mathematical problems, but he always encouraged him to also find the right connection to physical problems. In this style of mathematics (or physics), mathematical methods were used heavily, but there was not much focus on mathematical proofs. The goal was rather to arrive at a description consistent with physics.

Klein was so happy with Sommerfeld that he offered him the editorship for *Volume 5: Physics* of the *Encyclopedia of Mathematical Sciences* [43]. The *Encyclopedia of Mathematical Sciences* was an ambitious project, which Klein had conceived together with Walther Dyck and Franz Meyer and which had the goal to present the state of the art of mathematical sciences at this time, and in particular, also its relation to and influence on neighboring disciplines such as physics. Sommerfeld first wanted to avoid this duty because he felt that he was too busy with the gyroscope, and he tried to convince Willy Wien to take it over. However, Willy Wien rejected the offer since he did not see much future in theoretical physics at that time, and Sommerfeld accepted the job, which would become a great promoter for his own scientific development. As an editor, it was his task to motivate leading experts in the various disciplines to write the different sections (summary articles), and this helped him, of course, to get to know many of the top physicists at that time.

Together with Felix Klein, he made several trips, e.g., to The Netherlands and to the U.K., and he built up relations with scientists such as Hendrik Antoon Lorentz (Nobel Prize in Physics in 1902), who would become a very close friend of the Sommerfeld family. The work on the *Encyclopedia of Mathematical Sciences* was very demanding and would last for many years. Sommerfeld himself wrote an article titled "Boundary Value Problems in the Theory of Partial Differential Equations," which appeared in Volume 2 in 1904, and he very intensively discussed the other articles with the corresponding authors and partially even rewrote certain parts.

In 1897, Sommerfeld got an appointment as a professor of mathematics at the Mining School in Clausthal, which was more or less a teaching appointment with quite a few lecturing duties, but he remained, of course, very active in his scientific work. He continued to work on the gyroscope and on the *Encyclopedia of Mathematical Sciences*. Also, he started research work on the propagation of electrodynamic waves along metallic wires [13] and on the theoretical explanation of X-rays [14]. Overall, Sommerfeld was not that satisfied with the teaching position in Clausthal, and he was very happy about the appointment to the position of a chair professor of mechanics at the TH Aachen as the successor of Willy Wien.

In Aachen, Sommerfeld worked very intensely on technical problems, again with great success. He approached his colleagues to identify relevant problems, which he could solve with his excellent mathematical skills, and he worked, for example, on the brakes of trains; on the electrical impedance of coils; and on the oscillations of electrical machines. A lot of attention in the field of mechanics was given to his work on the hydrodynamic theory of the friction of lubricants [44], where the introduced "Sommerfeld number" is still well known.

In 1904, the Mining Academy in Berlin offered Sommerfeld a chair professorship of mathematics and mechanics, which he declined, and in 1905, he was a top candidate for the position of a professor of applied mathematics at the TH Hannover, which was, however, not supported by the responsible ministry in Berlin. Meanwhile, his position in Aachen was so strong that he was assigned an assistant position, which he gave to Peter Debye, who had just finished his engineering studies. (Peter Josephus Wilhelmus Debye earned a doctoral degree/habilitation with Sommerfeld, and in 1936, he was awarded the Nobel Prize in Chemistry.) In 1906, the Technical University in Delft offered Sommerfeld a chair professorship of applied mathematics, which he declined with the argument that he was happy in Aachen.

TEACHING AND RESEARCH AT LMU IN MUNICH

After Ludwig Boltzmann had left Munich in 1894, the chair of theoretical physics was vacant for several years. When the LMU got ready to reactivate the chair at around 1904, the number one candidate of Wilhelm Conrad Röntgen, the chair professor of experimental physics at this time, for the position was Hendrik Antoon Lorentz from The Netherlands, who did, however, decline eventually. In the next attempt to fill in the position, the search committee agreed on the appointment list "Primo loco and ex aequo Emil Cohn and Emil Wiechert, secundo loco Arnold Sommerfeld" [19]. The LMU Munich search committee, in its statement from 20 July 1905 for considering Sommerfeld as the number three candidate for the professorship of theoretical physics at LMU, stated [19]:

> We were made aware of Sommerfeld by very well-known theoretical physicists such as Boltzmann, Lorentz, and Wien, and the expectation was expressed that he would still achieve a lot with his great talent and his great diligence…. Sommerfeld is described as a pleasant colleague and an excellent teacher.

After Emil Cohn and Emil Wiechert, with whom Sommerfeld had worked on his harmonic analyzer in Königsberg, had declined the offer, Sommerfeld accepted the appointment (together with Peter Debye) and came to Munich in the autumn of 1906. After his arrival in Munich, Sommerfeld spent quite a lot of effort in setting up his teaching, where he could also rely on Peter Debye. Peter Debye said about the early teaching of Sommerfeld in Munich [19]:

> We studied together in order that he was able to give his lectures…. There were four main lectures: Theoretical Mechanics, Thermodynamics, Electrodynamics, and Optics… He did it as in Aachen before. He talked and I had to sit there and give comments.

Together with Debye, Sommerfeld was successful in educating a group of very promising doctoral researchers during the first years in Munich, where, in particular, his excellent teaching skills were helpful. The impression of Sommerfeld's lectures on Paul Ewald as a young student in Munich is for example expressed in the quotation [45]:

> It was Hondros who, at the beginning of the summer semester of 1908, dragged me almost with brute force into a two-hour lecture by Sommerfeld on hydrodynamics — so unwilling was I to let my circles of pure Pringsheim-style mathematics be disturbed. The result was that I, who had hitherto neither be in touch with mechanics nor vector calculus, was so fascinated starting from the very first lecture that I knew from then on that my love was not for abstract mathematics, but for this wonderful harmony of descriptive mathematical thinking and physical phenomena in theoretical physics. Nothing is more characteristic of the uncoerced artistry of Sommerfeld's lecture than that of introducing vector algebra and

analysis, the basic hydrodynamic equations in Lagrangian and Eulerian forms, complex mapping methods and their flow interpretation, wave propagation in deep and shallow water and in canals, ship waves and then the problem of the Reynolds number and the onset of turbulence in a comprehensible manner. From then on I was completely addicted to theoretical physics. I am always grateful to Hondros, who later became a highly respected Professor of Physics at the University of Athens, for putting me on this initially reluctant path.

Another impression about the lecturing style of Sommerfeld by Helmut Hönl (who earned a doctoral degree with Sommerfeld and became the chair professor of theoretical physics at the University of Freiburg) is given in the statement [19]:

> In the years 1923-25, Arnold Sommerfeld used to give a two-hour special college throughout several semesters in addition to his already famous main lecture with the promising title: "Spectroscopic Problems". The subject matter of this lecture was not fixed in advance. Sommerfeld presented what he was currently doing (only in the introductions he used to work with known material). This fact alone exerted an incomparable attraction on the listener: Even the young student had to get the impression of being right at the front of research and was carried away by its development. Similarly inspiring were the great presentation skills of the teacher. The lecture, which was clearly structured in terms of the subject matter, did not seem to present any great difficulties for its understanding. The slow diction, occasionally halting, gave the impression of an immediate, often ingeniously formulated idea. More mature listeners have occasionally complained that Sommerfeld's presentation is so artful and clear that it often hides difficulties. This might be the case, but outside of the lectures Sommerfeld was always ready to discuss in detail and completely undogmatically the difficulties and obscure points that the situation at that time necessarily entailed.

Even Albert Einstein got to know about the lecturing skills of Sommerfeld. Albert Einstein, in a letter to Sommerfeld on 14 January 1908 [22], wrote, "…if I were in Munich and had the time, I would attend your lectures to complete my mathematical-physical knowledge."

Since many of the young students were often not able to easily follow the lectures of Sommerfeld, Ewald approached the older student Demetrios Hondros (who earned a doctoral degree with Sommerfeld and went on to become a professor of physics at the University of Athens), and he eventually approached Debye to initiate a colloquium among the doctoral students only to deepen the understanding and give room for discussion. Sommerfeld approved this kind of colloquium, and he donated a box of cigars to get it started and "for sharpening the thinking" [45].

Sommerfeld was very open minded and accessible for his students, and thus, a remarkable culture of discussion developed during the first years in Munich. Many of these discussions were continued in the "Hofgarten Café," where over time students from other professors, such as Prof. Röntgen and from TH Munich, also participated. Beyond this, the culture of discussion was even extended into the Bavarian Alps. Sommerfeld and his students went on hiking and skiing tours on a regular basis, and many of the students enjoyed this kind of science a lot. Together with his mechanic, Karl Selmayr, Sommerfeld maintained a skiing hut in the "Sudelfeld" near Bayerischzell on a permanent basis, with the scientifically very efficient coincidence that the neighboring hut was owned by Jonathan Zenneck [46], who was a professor at TH Munich. Another center of networking for the "Munich Physics Community" was Mittenwald (near Garmisch-Partenkirchen), where Willy Wien maintained a country house and where regular meeting events took place [22].

In his early research in Munich, Sommerfeld continued to work on the theory of electron radiation related to the explanation of X-rays and on wave propagation along wires. Eventually, telegraphy also got into focus. Jonathan Zenneck got inspired by Sommerfeld's work on electromagnetic waves along metallic wires [13] when he looked into wave propagation above Earth, resulting in his well-known article on the Zenneck wave [8], and it was, of course, a very exciting prospect that the Zenneck wave was important for wave propagation over large distances. Eventually, Sommerfeld was obviously not too critical to see that the significance of the Zenneck wave was an outcome of his own investigations, which are documented in his famous 1909 article [7]. In terms of scientific methods, Sommerfeld's propagation work bears a lot of similarities with his work on diffraction at the half-plane. He started by constructing a suitable integral representation of the incident field for which the image principle was applicable, in this case based on cylindrical waves. Next, he applied the image principle, and he evaluated the resulting integral representation by sophisticated methods of complex contour integration.

Sommerfeld's propagation work got a lot of attention in the scientific community and was, for example, criticized by Hermann Weyl, who published his famous article with the well-known "Weyl-identity" in the form of a 2D plane-wave integral in 1919 [47]. Sommerfeld responded to Weyl with a rebuttal [48] in 1920, and in 1926, he published another article with the same title as his 1909 article and a somewhat revised summary presentation of the topic [9]. Some further discussion related to wave propagation above planar ground related to Sommerfeld's work is, e.g., found in [11] and [12], where, in particular, Michalski and Mosig have been working in the field of Sommerfeld integrals for many years. From Sommerfeld's *Lecture Notes: Volume 6, Partial Differential Equations in Physics* [6, Vol. 6], it becomes clear that he was very well aware of the problems of his original 1909 solution, as, e.g., discussed by Collin in [11], but he classifies the whole discussion related to the Zenneck wave as immaterial. Highly recommended references for more general considerations on wave propagation in complex environments are, e.g., the books by Ishimaru [49] and Chew [50].

A key accomplishment at Sommerfeld's institute were the experiments of Max von Laue (a lecturer at LMU with Sommerfeld from 1909 to 1914), Walter Friedrich, and Paul Knipping related to the Bragg scattering of X-rays, for which Max von Laue received the Nobel Prize in Physics in 1914, together with William Henry Bragg and William Lorentz Bragg. There are many rumors around these experiments, and the statement of Peter Debye that quite a lot of randomness was involved in this research is certainly one of the more friendly descriptions. Laue did obviously follow some misconceptions in the design of the experiments, and he also did not interpret the results correctly. Nonetheless, the experiments opened up a wide range of applications related to X-rays, in particular also related to the investigation of crystal lattices. Sommerfeld recognized this immediately, and with his marketing and presentation skills, he became a great promoter of X-rays and also of his own institute and work.

The next cornerstone in the development of atomic physics was the model of atoms published by Niels Bohr in Copenhagen in 1913 [16], which assumed electrons moving on circular paths around the atom core. Sommerfeld recognized its importance rather quickly, and he started to work on its extension. He came up with elliptical electron paths, and he treated the electron movement with relativistic correction, which allowed him to describe some of the fine structures of the related spectral lines of emission. The mathematical skills of Sommerfeld, in particular in the evaluation of integrals and differential equations, found here the right challenges. He calculated the energy based on phase integrals, and his student Adalbert Rubinowicz had the idea to include the conservation of momentum into the considerations. (Adalbert Rubinowicz earned a doctoral degree with Sommerfeld; he later became chair professor of physics at the University in Ljubljana, and then, he became chair professor of theoretical physics at Polytechnic Hochschule, Lemberg in Lwów, Poland.)

With the publication of his "Quantum Theory of Spectral Lines" [17], Sommerfeld was probably at the summit of his scientific career, and he got the best possible appreciation from many sides, e.g., from Albert Einstein and from Hendrik Antoon Lorentz. For example, in a letter to Sommerfeld on 3 August 1916, Albert Einstein wrote [22]:

> Your spectral investigations are among my most beautiful physical experiences. It is only through them that Bohr's idea becomes completely convincing. If only I knew which little screws the Lord God is using here.

In a letter to Arnold Sommerfeld on 14 February 1917, Hendrik Antoon Lorentz wrote [22]:

> Your results are among the most beautiful that has ever been achieved in theoretical physics. Who would have thought just a few years ago that relativity mechanics would provide us with the key to unravel so many mysteries.

Over time, the resulting Bohr-Sommerfeld model of atoms became very sophisticated, and Sommerfeld started to write his book *Atomic Structure and Spectral Lines* [18], which would become one of the most significant resources collecting the relevant results on atomic research obtained over many years. The book was written for a rather broad audience, in particular also in chemistry, and it was relatively quickly translated into English. It was continuously updated with the latest results, and it was read by a large number of atomic researchers around the world.

In 1918 and 1920, respectively, two exceptional students started their studies with Sommerfeld: Wolfgang Pauli and Werner Heisenberg. Wolfgang Ernst Pauli earned a doctoral degree with Sommerfeld; became a professor in Hamburg; became a professor at ETH Zürich; and was awarded the Nobel Prize in Physics in 1945. Werner Heisenberg earned a doctoral degree with Sommerfeld; became a professor of theoretical physics at the University of Leipzig; and was awarded the Nobel Prize in Physics in 1932. In addition, Heisenberg became the director of the Kaiser-Wilhelm (Max-Planck) Institute in Berlin; attained a professorship at the University of Berlin; and became a professor at LMU.

It was Sommerfeld's habit to assign project tasks to new students relatively quickly, and both Pauli and Heisenberg performed exceptionally in this respect. Pauli proved to have, e.g., excellent knowledge on relativity, and Sommerfeld eventually assigned him the task of writing the section on relativity for the *Encyclopedia of Mathematical Sciences*, which he originally had in mind for Einstein. Heisenberg got an assignment related to the Zeemann effect and the corresponding spectral lines, where his conclusion was that half-quantum numbers are needed to solve the problem, which was unacceptable for Sommerfeld at first but was obviously the right way to go.

The fourth Nobel Prize Winner among Sommerfeld's doctoral graduates is Hans Bethe, who joined LMU in 1926 to start his impressive scientific career. Hans Albrecht Bethe earned a doctoral degree with Sommerfeld; became a professor at Cornell University in Ithaca, NY; and was the chief scientist on atom theory in the Manhattan Project to develop the first atomic bomb in Los Alamos, NM. In 1967, he was awarded the Nobel Prize in Physics.

With his scientific reputation and his quite obvious pedagogical skills, Sommerfeld attracted increasingly more excellent students during these years, causing quite a lot of admiration within the community, and his institute became one of the leading centers of theoretical physics in the world. In a letter to Sommerfeld on 14 January 1922, in particular also under the impression of the great performance of the young Wolfgang Pauli in the field of relativity, Albert Einstein wrote [23]:

> What I especially admire about you is that you have, as it were,

pounded out of the ground a large number of young talents. This is something very peculiar. You must have a gift for ennobling and activating the spirits of your students.

THE INFLUENCER

Already very early in his career, Sommerfeld strove to sell his achievements and promote himself. With his appointment to a "Royal Secret Counselor" (Königlicher Geheimer Hofrat) in 1916, he became in many respects an advisor to government authorities, and in academia, his advice was asked for, e.g., whenever suitable scientists were needed for vacant professorships. His book *Atomic Structure and Spectral Lines* was a masterpiece in his role as an influencer. He conceived it in a popular fashion for a very wide audience from the very beginning, and it became a classic throughout the world that was continuously updated to cover the relevant topics over many years.

In 1922, Sommerfeld was awarded a Carl Schurz Memorial Professorship to teach at the University of Wisconsin in Madison during the upcoming winter semester. Together with Albert Einstein, he was probably the most popular German physicist in the United States at this time, and the Carl Schurz Memorial Professorship, which was awarded for the first time after World War 1 in 1922, had quite a lot of significance in science and in politics. Sommerfeld's visit to the United States was, e.g., announced in *Science* and in *The New York Times*, where even the intended topics of his lectures were mentioned [22]. During his stay, Sommerfeld gave lectures on atomic theory and on the mathematical treatment of wave propagation, and he had, of course, lots of opportunities to discuss his research in colloquia. Also, he visited many nearby universities in short trips, such as the University of Illinois in Urbana Champaign or the University of Michigan in Ann Arbor.

After finishing his teaching duties in Wisconsin, he went on a large trip throughout the United States, with stays in California, Washington, and New York, where he gave numerous presentations on topics related to atomic research, which was not yet well known in the United States at this time. The popularity of Sommerfeld soared to new heights, and the result was that numerous students and postdocs came to Munich to study atomic physics with Sommerfeld. Well known among these visiting students and researchers are for example:

- Linus Pauling, who from 1926 to 1927 was a postdoc in Europe (Guggenheim Fellowship) with Sommerfeld, Schrödinger, and Bohr, and who was awarded the Nobel Prize in Chemistry in 1954 and the Nobel Peace Prize in 1962
- Isidor Rabi, who from 1927 to 1929 had a fellowship in Europe, working with Sommerfeld, Bohr, Pauli, and Heisenberg, and who was awarded the Nobel Prize in Physics in 1944
- Ernst Guillemin, who was an American research fellow at LMU with Sommerfeld in 1926; earned a doctoral degree with Sommerfeld; and became a professor at MIT
- Philip Morse, who (as mentioned earlier) earned a Rockefeller scholarship in 1930/1931 and who was with Sommerfeld as a postdoc at LMU Munich.

In 1927, Sommerfeld started to plan a large lecturing trip around the world, which lasted from summer 1928 until spring 1929. On this trip, he visited a large number of scientific centers in India, China, Japan, and the United States, and he gave numerous presentations on his latest research work in atomic theory, especially related to electron theory in metals. In the United States, Sommerfeld was amazed to see how dramatically research in atomic physics had progressed since his last visit.

Figure 4 gives impressions about his trip, showing him during his stays

(a) (b)

FIGURE 4. Photographs of Arnold Sommerfeld. (a) In Calcutta, India, together with K. S. Krishnan (left) and C. V. Raman (right), the discoverers of the "Raman effect," in 1928. During his stay in Calcutta, Sommerfeld had the chance to become a witness of the experiments related to the "Raman effect," and he kind of promised to nominate Raman for the Nobel Prize [22]. (b) On a nature trip in Hakone, Japan, together with Hantaro Nagaoka in 1928. (Source: Deutsches Museum, Munich.)

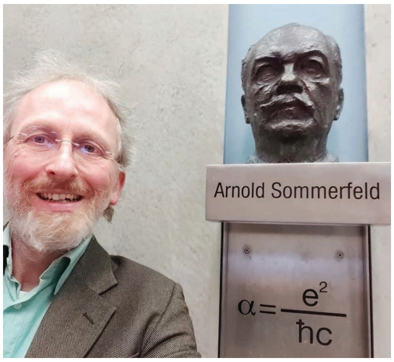

FIGURE 5. The author of this article together with the bust of Arnold Sommerfeld at LMU Munich. The constant α below the bust is the famous fine structure constant with its probably even more famous value of close to 1/137 found by Sommerfeld in his work on the Bohr-Sommerfeld model of atoms. The constant is here given in Gaussian centimeter–gram–second units. In International System of Units (SI) units, it would read as $\alpha = (1/4\pi\varepsilon_0)(e^2/\hbar c)$.

in India and in Japan. Probably the last major project of Sommerfeld as an influencer was the writing and publication of his *Lecture Notes on Theoretical Physics* [6], a book in six volumes covering the contents of the main lectures given by Sommerfeld over the years. Whoever has read Sommerfeld's lecture notes will even there immediately agree with the characterization of his lecturing style given by Helmut Hönl as found in the "Teaching and Research at LMU in Munich" section.

At LMU Munich, a bust, as seen in Figure 5, and a lecture hall commemorate Arnold Sommerfeld. Theoretical physics research and teaching are performed there in the "Arnold Sommerfeld Center for Theoretical Physics."

CONCLUSION

Arnold Sommerfeld was a great mathematician and a great physicist who succeeded in particular in the combination of both disciplines to something that enabled him to have this great success. Of course, he was also lucky in the way that the right problems came up at the right time, fitting to his capabilities. Important above all for his success was his remarkable teacher personality, helping him to reach, activate, motivate, and guide a large number of young talents in the fields of physics, mathematics, and engineering. First, his students came from Germany and other European countries, and later, he hosted visiting students and scientists from all over the world. Over time, he extended his lectures and presentations into the popular field, which also made him successful in neighboring disciplines, such as chemistry, and as a science and technology influencer in general. With his trips to the United States and around the world, he became a scientific and cultural ambassador, in particular for the new fields of atomic physics.

Despite his great success throughout his career, Sommerfeld did, of course, also suffer quite a number of setbacks, both on a personal and on a professional level. We did not address many of them in this article, but the remarkable and important lesson that we can learn out of such things is that Sommerfeld never lost his enthusiasm for his science and for his teaching. In particular, in this respect, he can be a role model for all of us. If one wonders whether Sommerfeld received a Nobel Prize himself, the answer is no, but he received a record number of 84 nominations over the years. From the archive of the Nobel Prize Committee, it can obviously be learned that his work was considered as too theoretical to qualify as fundamental physics contribution. He can be happy with this in his role as a mathematician.

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