The Color Top and the Distinction Between Additive and Subtractive Color Mixing

Giulio Peruzzi and Valentina Roberti

This article provides a brief overview of the history of the color top as a research instrument. Specific focus is given to James Clerk Maxwell's experiments, which allowed him to obtain the first colorimetric equations using this simple device, and to Hermann von Helmholtz's use of the top to exhibit, in a very simple and clear manner, the difference between additive and subtractive color mixture. As a matter of fact, among Maxwell's and Helmholtz's key contributions to the theory of color, it is worth mentioning that they clarified, for the first time and independently from one another, the distinction between additive and subtractive color mixing, eliminating definitively Newton's confusion between the optical and pigment mixture of colors.

EDITOR'S NOTE

This issue's "Historical Corner" column presents an article by Giulio Peruzzi and Valentina Roberti (the University of Padua) focused on the history of the color top as an instrument to comprehend the physiology of color vision. The authors analyze the development of the top by many scientists, from ancient Roman times up to the 21st century, but a large part of the article is devoted to James Clerk Maxwell, who was the first to devise quantitative rules in color mixing.

Giuseppe Pelosi

Maxwell began to study vision problems shortly after earning his diploma at the University of Cambridge, and, with this aim, he designed an instrument to examine the bottom of the eye, which he tested on his dog, Toby. Subsequently, to investigate the theory of color composition, he built a box to decompose white light by a prism, select colors, and recombine them. Based on these experiments and on those with the color top reported in this article, he developed a trichromy theory: he asserted that color photography was possible by taking a picture of the scene with three different filters to select different colors and subsequently combining the three separate images. He managed to do it technically, and, in 1861, he presented the world's first color photograph in a London lesson, obviously a picture of a Scottish kilt (see Figure S1).

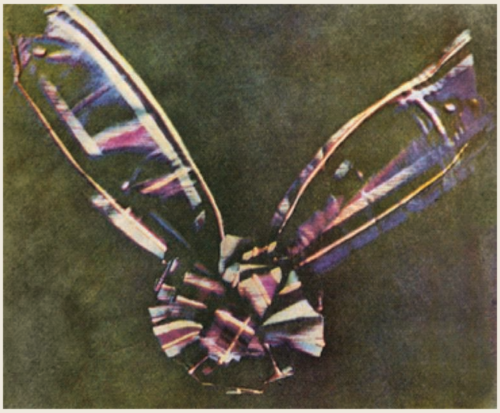

FIGURE S1. The first color photograph ever, obtained by James Clerk Maxwell in 1861 [S1].

Reference

[S1] G. Pelosi, "James Clerk Maxwell: His journey to Italy; and the first color photograph," *IEEE Antennas Propag. Mag.*, vol. 50, no. 1, pp. 240–243, Feb. 2008.

James Clerk Maxwell (1831–1879) was the first to use a color top to obtain colorimetric equations and, together with Hermann von Helmholtz (1821–1894), the first to exhibit the difference between additive and subtractive color mixture by means of this experimental device. Among the most debated topics in the field of light and color in the 18th and the first half of the 19th centuries was the relationship between the spectral colors (additive color mixture), which

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when mixed together furnish white light, and those colors obtained by painters mixing different pigments (subtractive color mixture).

Spinning disks are the simplest instruments for obtaining additive mixtures of colors. This may seem misleading because these disks use papers painted with different pigments. The reason we perceive an optical mixture of these colors, however, instead of a pigment mixture lies in our visual system, which requires only a brief exposure to a stimulus to determine its nature. When visual stimuli change very rapidly, they cannot be individually resolved, involving, therefore, the additive process instead of the subtractive one. Our intention in this article is to provide a brief sketch of the history of disk color mixture, from antiquity to the 19th century, when Maxwell's and Helmholtz's contributions in the field of color science led to the quantitative theory of color measure. We then describe Maxwell's color top and its key role in shedding light on the human vision system. Indeed, experiments carried out on subjects affected by color blindness provided valuables clues for Young's three-receptor theory. We focus particularly on the use of the color top to demonstrate the difference between additive and subtractive color mixtures, reporting Helmholtz's ideas and Rood's quantitative results to explain the two kinds of mixtures.

HISTORY OF ROTATING DISKS

This way of mixing colors has been known since antiquity. In the second century AD, the Greco-Roman natural philosopher Claudius Ptolemaeus (ca. 100–170 AD), who was most famous for the elaboration of the geocentric model of the universe that was named after him, gave the first written record of the disk mixture illusion in his second book of optics, describing the effect of a potter's wheel in motion painted with several colors. Another clear description of disk mixture is to be found in Alhazen's *Book II of Optics*. Abu Ali al-Hasan ibn al-Haytham (965–1039), Latinized as Alhazen, was an Arab mathematician, astronomer, and physicist who studied Ptolemaeus's *Book of Optics* and wrote in

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his own *Book of Optics* a description of a revolving top [1, p. 144]:

> A clear and visible proof that perception of the quiddity of colour must take place in time is furnished by what can be observed in a revolving top. If the top is painted in different colours forming lines that extend from the middle of its visible surface, close to its neck, to the limit of its circumference, then forcefully made to revolve, it will turn round with great speed.

We had to wait about 700 years before finding another written description of the color top, that is, as the Dutch natural philosopher Pieter van Musschenbroek (1692–1761) mentioned it in his book *Introductio ad philosophiam naturalem*, published posthumously in 1762. Before proceeding, it is worth pointing out that the celebrated physicist and mathematician Isaac Newton (1642–1727) presented in his monumental work *Opticks, or, a Treatise of the Reflections, Refractions, Inflections and Colours of Light*, first published in 1704 [2], a method for the composition of the colors of the spectrum, the famous Newton's color circle. He envisaged the chosen primary colors arranged around a circumference and proposed a method for determining the outcome of their combinations in analogy with the construction of the center of gravity, as we discuss in the section "Maxwell's Color Top." Although there is no historical source, as far as we know, attesting Newton's proposal to compound painted color disks by rotating a color top [3, p. 116], his color-mixing law and, more generally, his theory of light and color had been taken as a starting point for all subsequent investigations on the subject, such as those conducted by van Musschenbroek.

Helmholtz was aware of van Musschenbroek's experiments with the top, as testified by the following words from an English reprint of his *Handbuch der physiologischen Optik*, whose complete edition was first published in German in 1867 [4, p. 347], [5, pp. 215–216] (see Figure 1):

> The simplest contrivance of rotating disk, mentioned first by MUSSCHENBROEK, is the top. In most of his experiments the author uses a simple top turned out of brass …. It is set spinning by hand; and so it can be easily started at any time without preparation, and its velocity can be regulated at will, although the greatest velocity that can be imparted by hand is not more than about six

FIGURE 1. A color top with a chord, which allowed it to spin faster [4, p. 216].

> revolutions per second; which is enough to keep it going three or four minutes. … To make the top spin faster, a cord can be wound around the stem and pulled.

At the end of the 18th century and the beginning of the 19th century, not only scientists but also artists, writers, and philosophers made use of rotating disks for studies concerning perception, successive contrast, and persistence of visual stimuli in the human eye, animated by the same intent of deepening the connections between art and science. Between 1790 and 1830, in fact, a vast movement of ideas took off in Germany and then spread across Europe, under the name of *Naturphilosophie*, a term that indicates this rich intertwining of philosophical ideas and scientific perspectives, expressive of the common purpose of reaching an overall and unitary understanding of nature. Among the various debate topics, pivotal roles were played by the nature of colors and their perception, topics that increasingly fascinated not only natural philosophers but also painters, poets, and writers, such as Johann Wolfgang von Goethe (1749–1832), who published in 1810 his work *Zur Farbenlehre*, and the German painter Philipp Otto Runge (1777–1810). Runge made a number of experiments using the color top that allowed him to compare a given gray tint, resulting from the mixture of three colors, with a combination of white and black, as he wrote in a letter addressed to his friend Goethe and dated April 1808 [6].

In the 19th century, James David Forbes (1809–1868), a professor of natural philosophy at the University of Edinburgh and who was interested in the problem of color classification, gave an account of his experiments with a color top in his work, "Hints Toward a Classification of Colours" [7]. Forbes endeavored to form a neutral tint by the combination of three colors only, and he found, contrary to expectations, that when using as primary colors the yellow, blue, and red of painters, the resulting tint could not be rendered neutral by any combination of the three.

Under the mentoring guidance of Forbes, James Clerk Maxwell, studying

Not only scientists but also artists, writers, and philosophers made use of rotating disks for studies concerning perception, successive contrast, and persistence of visual stimuli.

in Edinburgh, began his research on colors in the spring of 1849. Maxwell and Forbes's first aim was to obtain quantitative expressions for the mixing of colors that were more satisfactory than those available at that time. To this purpose, Maxwell built a color top described in detail in his article of 1855, "Experiments on Colour, as Perceived by the Eye, With Remarks on Colour Blindness" [9], and presented his results at the Royal Society in Edinburgh in the same year. We give a brief description of the top in the following section. (For a more detailed description of Maxwell's color top, or "chromatic teetotum," see "Manuscript on the Comparison of Colours Using a Spinning Top," dated 27 February 1855 [8, pp. 284–286].)

MAXWELL'S COLOR TOP

The first set of colored paper disks for Maxwell and Forbes's experiments were furnished by David Ramsey Hay (1798–1866). Maxwell was introduced by his father, John Clerk Maxwell, and uncle, John Cay, to the Royal Scottish Society of Arts, of which they were prominent members, and on that occasion he came into contact with Hay, a decorative painter and color theorist living in Edinburgh at that time, who, inspired by the current *Naturphilosophie*, expressed his ideas about the possibility of defining laws of symmetry underlying the primary beauty of forms in his volume, *First Principles of Symmetrical Beauty* [10]. His treatise, *Nomenclature of Colours* [11], a color sample collection, was considered a landmark for painters. Maxwell's color top consists of three colored paper disks in ultramarine, vermilion, and emerald green, placed on the top of a wooden disk. It may be asked why some variety of yellow was not chosen in place of green, which is commonly placed among the secondary colors, while yellow ranks as a primary [9, p. 276]. Maxwell answered this question by remarking on the impossibility of obtaining a neutral tint using the painters' primaries, which disappointed Forbes's expectations.

The circumference of the plate was divided into 100 equal parts to quantify the amount of color that entered into a given mixture. Maxwell modified the first version of the color top realized by Forbes, inserting a central disk, of smaller dimensions, containing black and white paper disks. The top was then set in rapid rotation, and an observer could compare the color of the outer with that of the inner circle. The arrangement could then be changed to render the resultant tint of the outer and of the inner circles alike. The number of divisions occupied by the different colors would be read and registered in the form of an equation. It is worth reading Maxwell's original words used to describe his color-mixing experiment [9, pp. 275–276] (see Figure 2):

> When this system of discs is set in rapid rotation, the sectors of the different colours become indistinguishable, and the hole appears of one uniform tint. The resultant tints of two different combinations of colours may be compared by using a second set of discs of a smaller size, and placing these over the center of the first set, so as to leave the outer portion of the larger discs exposed. The resultant tint of the combination will then appear a ring round that of the second, and may be very carefully compared with it. … As an example of the method of experimenting, let us endeavour to form a neutral gray by the combination of vermilion, ultramarine and emerald green.
>
> The most perfect results are obtained by two persons acting in concert, when the operator arranges the colours and spins the

top, leaving the eye of the observer free from the distracting effect of the bright colours of the papers when at rest.

After placing discs of these three colours on the circular plate on the top, and smaller discs of white and black above them, the operator must spin the top and demand the opinion of the observer respecting the relation of the outer ring to the inner circle.

… The arrangement must then be changed, so as to render the resultant tint of the outer and inner circles more nearly alike.

Maxwell registered the obtained results in the form of colorimetric equations. Here, the typical form of his equation is reported, referring to an experiment conducted "on the sixth March 1855, in daylight without sun":

$$0.37\,V + 0.27\,U + 0.36\,EG = 0.28\,SW\ 0.72\,Bk.$$

The colors are labeled using letters *V* for vermillion, *U* for ultramarine, *EG* for emerald green, *SW* for snow white, and *Bk* for black, and the coefficients before each letter represent the number of divisions occupied by the different colors that could be read on the edge of the plate.

Helmholtz was aware of Maxwell's research in the field and reported in his *Handbuch* a clear description of the top, underlining also its pivotal role in verifying the correctness of Newton's law of color mixing in analogy with the construction of the center of gravity [5, p. 140]:

NEWTON's device of exhibiting the laws of colour mixture by the method used for constructing centers of gravity was intended originally simply as a kind of mathematical picture for expressing graphically a large mass of facts …. However, quite recently the method has been still further supported by careful quantitative measurements made by MAXWELL.

Newton's theory of color represents, in fact, a fundamental starting point for both Maxwell's and Helmholtz's research. Newton reported in his *Opticks* his famous color circle and laid down a clear method for predicting the compound color that results from a combination of seven principal colors [2, pp. 114–117]. Indeed, although he believed that the range of colors between violet and red was infinite, he divided the spectrum into seven principal colors, in line with his analogy between colors and musical harmony: violet, indigo, blue, green, yellow, orange, and red. He divided the circumference of his color circle into seven parts that were proportional to the musical intervals contained in an octave. The arches corresponded to the

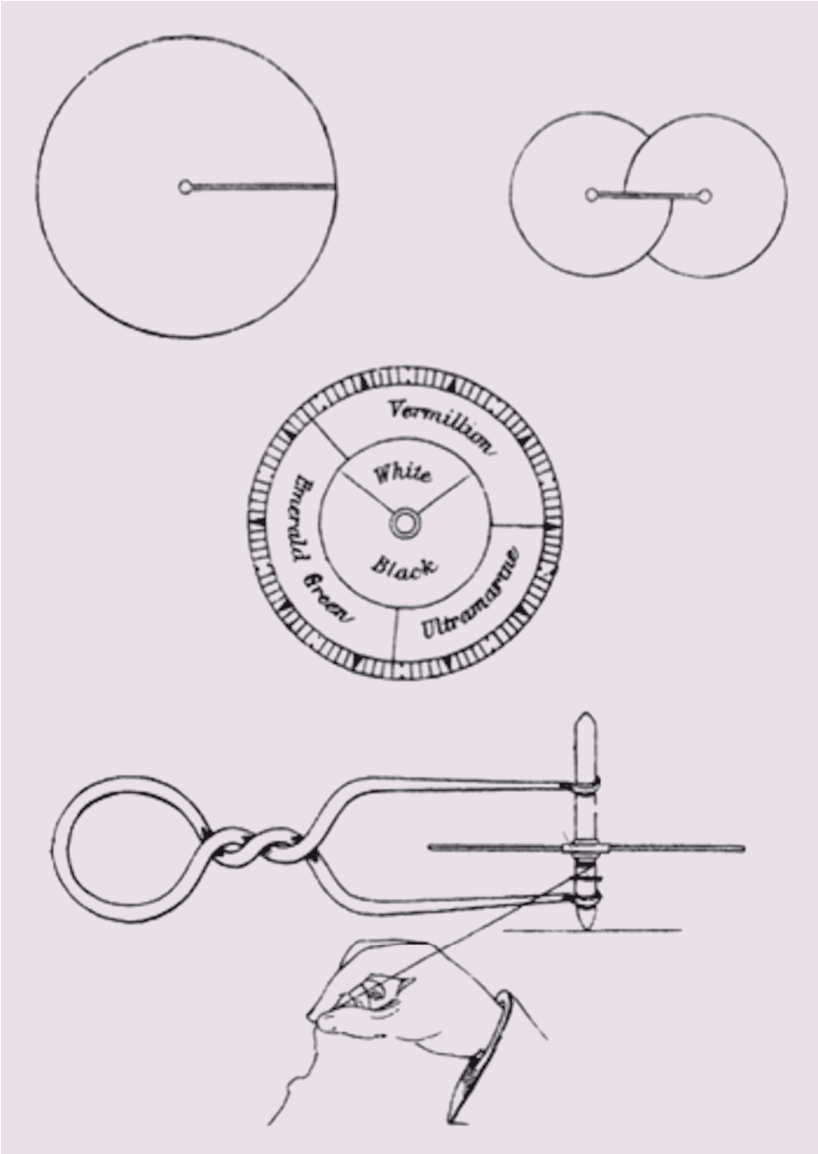

FIGURE 2. An illustration of the working principle of Maxwell's color top, indicating also the correct disposition of the five disks of paper [9].

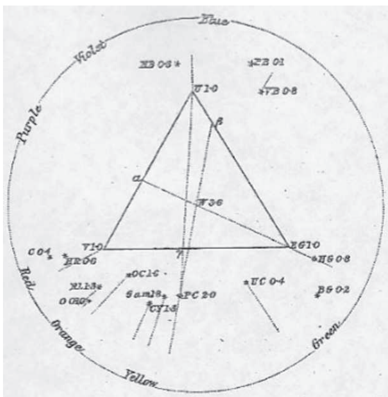

FIGURE 3. Maxwell's equilateral triangle of colors inside Newton's color circle. The number of principal colors of Newton's model appears redundant [9].

previously mentioned principal colors, from which he derived all the others. At the center of gravity of these sections, a small circle was to be found. As the area of each circle was proportional to the number of rays of that color that entered into a given mixture, the common center of gravity of these circles could be easily determined. (The implicit reference of Newton's construction of color mixture to the mixture of different quantities, or masses, of pigments is clear.) Using the color top, Maxwell obtained the first colorimetric equations, after the work of renormalization to take into account the total intensity of the white. From these equations he built his triangular color diagram, eliminating definitively Newton's arbitrary choice of the number of primary colors. To render it evident on Newton's color circle, Maxwell superimposed the color triangle, as depicted in Figure 3. [From Maxwell's color triangle has evolved the CIE (*Commission Internationale d'Eclairage*, or International Commission on Illumination) model, first published in 1931, which is the most popular system of color representation in use today and considered the basis of modern colorimetry.]

Maxwell came, thus, to the conclusion that the nature of a color was dependent on three things—the greenness, the redness, and the blueness—and was in general described by three variables. In the same work of 1855, Maxwell reported "in shade, hue, and tint, we have another mode of reducing the elements of colour to three" [9, p. 279], adapting terms from Hay's *Nomenclature of Colours* and from the German mathematician Hermann Grassmann (1809–1877), who used the words intensity (*Intensität der Farbe*), tint (*Farbenton*), and intensity of the intermixed white (*Intensität des beigemischten Weißs*) to denote the three color variables in his seminal article on color mixture "Zur Theorie der Farbenmischung," published in 1853 [12]. [Grassmann came to the study of color with the aim of finding connections between the laws of nature and the forms of geometrical analysis he had developed in his first major work of 1844, the *Ausdehnungslehre* ("theory of extension").] Maxwell showed that these methods of considering color could be deduced one from the other: it became, therefore, a matter of geometry.

Maxwell and Forbes's first aim was to obtain quantitative expressions for the mixing of colors that were more satisfactory than those available at that time.

COLOR TOP TO STUDY COLOR BLINDNESS

The first clear and detailed account of color blindness was provided by John Dalton (1766–1844), who read his paper before the Manchester Literary and Philosophical Society on 31 October 1794. This essay, "Extraordinary facts relating to the vision of colours: With observations" [13], was published in the memoirs of the society in 1798. Dalton described his own color-vision deficiency, which we now call *protanopia* or *red-blindness*. In the last section of his paper, Dalton put forward a hypothesis about the cause of his anomaly. His loss of sensitivity could be due to one of the humors of his eye that absorbed the red [13, p. 42]:

> I was led to conjecture that one of the humours of my eye must be a transparent, but coloured, medium, so constituted as to absorb red and green rays principally, because I obtain no proper ideas of these in the solar spectrum; and to transmit blue and other colours more perfectly.

Thomas Young (1773–1829) and John Herschel (1792–1871), his contemporaries, disagreed with this idea. Young found here a key argument in favor of his trichromatic theory. In November 1801, Young delivered a lecture, "On the Theory of Light and Colours," at the Royal Society of London [14], in which he presented his arguments in favor of a wave theory of light and proposed, in this connection, the idea of trichromatic color vision. He suggested that the existence of three kinds of particles placed in the human retina were responsible for color vision. Young, in fact, attributed Dalton's deficit to the absence or paralysis of one of the three receptors in the retina, as he remarked in the bibliography of *Lectures of Natural Philosophy*, listing Dalton's paper [15, p. 514]:

> He [Dalton] thinks it probable that the vitreous humor is of a deep blue tinge: but this has never been observed by anatomists, and it is much more simple to suppose the absence or paralysis of those fibers of the retina, which are calculated to perceive red.

George Wilson (1818–1859), professor of technology at the University of Edinburgh, gave an account of a wide variety of cases of color blindness—or *chromato-pseudopsis* (i.e., false vision of colors)—in *Researches on colour blindness* [16], becoming an authority on the subject. (*Daltonism* was the original name attributed to this kind of defect because Dalton was the first to provide a detailed account of it. Wilson explains in his volume the reasons why this name was then substituted: "The countrymen of Dalton have protested against the immortalising of his name, in connection with a personal defect; and the chemists of all countries might claim that the term Daltonian is needed by them to distinguish those who adopt the famous opinions of Dalton, concerning the atomic constitution of matter" [16, p. 5].) Among the main cases of color blindness, Wilson expressed some considerations on the most frequent variety, that is, the inability to distinguish between the primary colors, writing that "red is the primary colour most distracting to the subjects of colour-blindness: for some it has absolutely no existence" [16, p. 13]. Thus, Maxwell, who was familiar with Wilson's work, concentrated on color-blind subjects with eyes insensitive to red light. He used his color top to verify Young's hypothesis of the absence or inhibition of one receptor in the eye. Maxwell could conclude that one of the receptors should have a particularly high sensitivity to the red portion of the spectrum. It is worth emphasizing with Maxwell's own words the key role of color blindness in the investigation of the true nature of color: "the most valuable evidence which we have as to the true nature of colour vision is furnished by the colour blind" [17, p. 621].

Using his color top, Maxwell found a mathematical formulation of the difference between ordinary and color-blind eyes, expressed in term of equations, as he wrote in his paper of 1855 [9, p. 285]:

> The equations thus obtained do not require five colours including black, but four only. For instance, the mean of several observations gives $0.19\,G + 0.05\,B + 0.76\,Bk = 1.00\,R$.

Maxwell used his color top with four colored papers only instead of five, and the mean of his observations gave the expression reported previously, where letters G, B, Bk, and R denote respectively the green, blue, black, and red papers prepared by D.R. Hay. A mixture of green, blue, and black on the left side of the equation gave rise to a full red on the right side. Maxwell was then able to identify point D on his color diagram, which is invisible to the color blind, as we can see in Figure 4.

Maxwell's color diagram can be portioned in two sectors separated by the boundary line that passes through point D (invisible to the color-blind subject and located in the red region) and point W (white). Maxwell gave a full exposition of the twofold nature of color space for color-blind subjects: all colors for a color-blind person appear as if composed of blue and yellow, while for an ordinary eye, color is a function of three independent variables, which he identified with redness, greenness and blueness: "… all colours on the upper side of DW will be varieties of blue and all colours on the under side varieties of yellow" [9, p. 286]. In a letter addressed to George Wilson, he described the method used to obtain lines that appeared identical to a color-blind person inside his color triangle [18, pp. 123–124] (see Figure 5 and Figure 6):

> If we find two combinations of colours which appear identical to a Colour-Blind person, and mark their position on the triangle of colours, then the straight line passing through these points will pass through all points corresponding to other colours, which to such a person, appear identical with the first two. We may in the same way find other lines passing through the series of colours which appear alike to the Colour-Blind.

Maxwell's results using the top gave valuable clues to the three-receptor theory of human vision. He could conclude, furthermore, that one of the receptors should have a particularly high sensitivity to the red portion of the spectrum, confirming red as one of the primary colors.

COLOR TOP TO STUDY THE DIFFERENCE BETWEEN ADDITIVE AND SUBTRACTIVE COLOR MIXTURE

Among Maxwell's and Helmholtz's key contributions to the theory of color, it is worth mentioning that they eliminated definitively Newton's confusion between optical and pigment mixtures of colors (confusion still existing in Young's works). Maxwell reported the crucial result obtained by the Belgian physicist Joseph Plateau (1801–1883), which anticipated Forbes's observations [20, p. 12]:

> The first experiment on the subject is that of M. Plateau, who, before 1819, made a disk with alternate sectors of Prussian blue and gamboge [yellow] and observed that, when spinning, the resultant tint was not green but a neutral gray, inclining sometimes to yellow or blue, but never to green. Prof. J.D. Forbes made similar experiments in 1849, with the same result. Prof Helmholtz of Königsberg, to whom we owe the most complete investigation on visible colour, has given the true explanation of the phaenomenon.

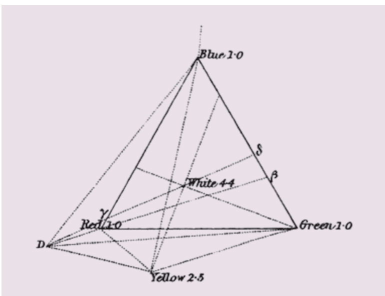

FIGURE 4. Maxwell's color diagram, with indication of point D, from which he could derive all his results on color blindness. Point D corresponds to a color that should be added to a color-blind eye to obtain ordinary vision. From this representation, furthermore, we can see that white lies outside the blue-red-yellow triangle, further evidence of the fact that white cannot be obtained from a mixture of the three primaries used by painters (blue, red, and yellow) [9].

Helmholtz indeed offered a clear and complete explanation of the difference between additive and subtractive color mixture in his paper, "Über die Theorie der zusammengesetzen Farben," published in 1852 [21]. On the basis of

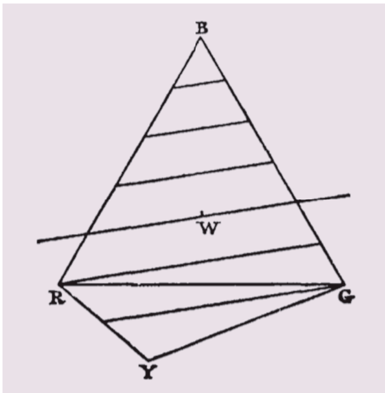

FIGURE 5. The Maxwell color triangle with the parallel lines indicating colors that appeared the same to a color-blind subject [18, p. 124].

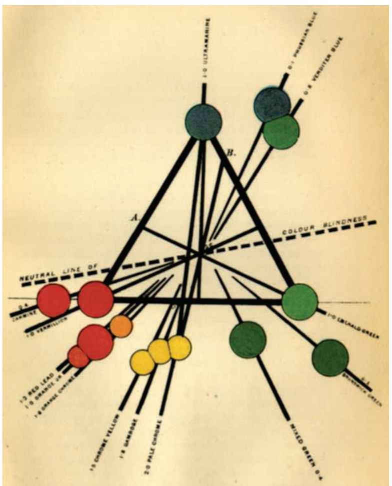

FIGURE 6. The Maxwell color triangle illustrating the chromatic relations of colored papers and showing the neutral line of color blindness (the dotted line) [19, p. 228].

these observations, he underlined here that the mixture of pigments (*Mischung der Farbstoffe*) and the composition of spectral colors (*Zusammensetzung des gefärbten Lichtes*) are two different phenomena. The mixture of pigments determines a unique color stimulus that reaches directly the eye and the retina. In the case of composition of lights, by contrast, different stimuli come separate and unchanged to the eye, and their fusion takes place in the visual system. They are, therefore, phenomena that obey completely different rules and belong to different domains: the mixture of pigments belongs to the physical domain, while the composition of lights belongs to the physiological domain.

Using an apparatus composed basically of prisms and lenses and designed to mix two spectral colors without other interference, Helmholtz experimentally verified that the model used for pigment mixtures could not be applied to composition of lights: a yellow light and a blue light, in fact, if combined in the right proportion, produce an achromatic color, while yellow and blue pigments, when mixed, give a green color. This, explained Helmholtz, is because pigments consist of series of layers of semitransparent particles that act as filters for the incoming light, which is reflected by the underlying layer. In a mixture of two pigments, the first absorbs part of the spectral radiation, and part of the remaining radiation is then absorbed by the second. Only the radiation not absorbed by either of the two pigments is reflected. He also described two methods of composing light coming from pigments that furnish the same results obtained by mixing prismatic colors. One of these methods regards the use of the color top. It is therefore worthwhile reporting the following extract taken from the English translation of the paper [22, p. 530]:

> The first of these methods is the union of the colours upon the rotating disk. It has been long noticed, that results thus obtained are different from those derived from the mixture of the pigments. I repeated the experiment with yellow and blue.

With these words, Helmholtz anticipated the fundamental proof to show the difference between additive and subtractive color mixing contained in his *Handbuch der physiologischen Optik*. He was the first to use a color top to exhibit this difference in a very simple and clear manner. We can find a clear description of the method he used in his *Handbuch* [5, pp. 124–125] (see Figure 7):

> A convenient way of exhibiting the distinction here is to paint the sectors a and b on the edge of a colour disk in two different colours, while the central portion c is painted with a mixture of the two pigments. Thus, if the sectors are cobalt-blue and chrome-yellow, the appearance is pale gray when the disk is rotated so as to get the impression of both colours at once; but the mixture of the two pigments in the center is a much darker green. Evidently, therefore, the result of mixing pigments cannot be used to

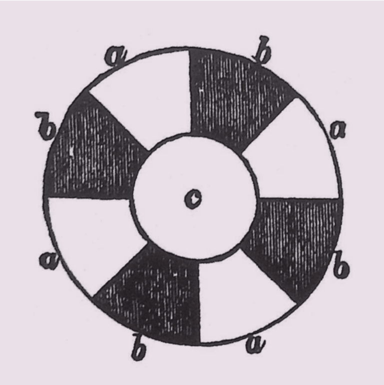

FIGURE 7. An illustration of the color top Helmholtz used to show the difference between additive and subtractive color mixing [4, p. 276].

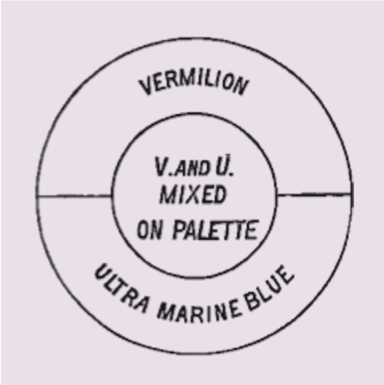

FIGURE 8. Rood's color top for exhibiting the differences between mixtures of colored lights and colored pigments [23, p. 146].

TABLE 1. RESULTS ROOD OBTAINED USING THE COLOR TOP TO COMPARE THE EFFECTS OF MIXING PIGMENTS BY ROTATION AND ON THE PALETTE [23, P. 148].

Pigments	By Rotation	On the Palette
Violet ("violet-carmine") Yellow-green (Hooker's green)	Yellowish-gray	Brown
Violet ("violet-carmine") Yellow (gamboge)	Pale yellowish-gray	Sepia-gray
Violet ("violet-carmine") Green (Prussian-blue and gamboge)	Greenish-gray	Gray
Violet ("violet-carmine") Prussian-blue	Blue-gray	Blue-gray
Violet ("violet-carmine") Carmine	Pink-purple	Dull red-purple
Gamboge Prussian-blue	Pale greenish-gray	Full blue-green
Carmine Hooker's green	Yellowish-orange (flesh-tint)	Brick-red
Carmine Green	Pale reddish (flesh-tint)	Dark-red

deduce conclusions as to the effect of combining different kinds of light. The statement that yellow and blue make green is perfectly correct in speaking of the mixture of pigments; but it is not true at all as applied to mixture of these lights.

Disk-mixture technology became very popular. Ogden Nicholas Rood (1831–1902), a Scottish-American physicist, described in his book, *Modern Chromatics, With Applications to Art and Industry*, published in 1879 [23], some experiments conducted using the color top. He was aware of Maxwell and Helmholtz's results in the field and made use of this device to obtain quantitative results to exhibit clearly the difference between additive and subtractive color mixture, inspired by Helmholtz's method.

Two watercolor pigments were prepared and used to paint the two outer sectors of the disk. Then the same numbers of drops of the two colors were mixed on the palette and used to paint the inner circle (see Figure 8). When the top was set in rapid rotation, the two colors placed at the outer circle underwent additive mixture, and the resultant tint could be directly compared with that furnished by the palette, placed at the inner circle. Rood repeated this process using eight pairs of colors and reported in a table the experimental results, reproduced here in Table 1.

For the eight cases under examination, Rood also obtained color equations, expressed in the form shown below:

$$50\,\text{violet} + 50\,\text{Hooker's green} = 21\,\text{violet} + 22.5\,\text{Hooker's green} + 52.5\,\text{black}.$$

The left side represents the mixture of two colors on the palette and the right side the mixture by rotation. Here, Rood gave indication of the quantities of black and of other colors necessary to be introduced on the right side to match the mixed pigments on the palette, offering a quantitative relationship between the additive and the subtractive mixture of colors. He found that the amount of black required to match the left side of the equation was a highly variable quantity, fluctuating from 4 to 52%. Rood explained that "It is for this reason that artists are so careful in their selection of pigments for the production of definite tones, particularly when they are to be luminous in quality" [23, p. 146].

CONCLUSIONS

Known since antiquity, the color top offered the possibility to establish for the first time color-matching equations, from which the forerunners of the modern chromaticity diagrams were created. Composed of disks of paper painted with different colors, rotating disks were used to obtain additive mixtures of colors. In the present article, we have described how Maxwell made use of his color top to prove the validity of the three-receptor theory of human vision proposed by Young, and we have reported Helmholtz's use of the top in exhibiting the difference between mixing pigments and mixing spectral lights. Although we have sketched only a few of the color top's applications of historical relevance, it is worth mentioning that it has also been adopted for luminosity comparison and to define complementary colors. This simple device has been used in the fields of physics, physiology, psychophysics, and art, offering an important link among different areas of knowledge.

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