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Early Pioneers of Electromagnetic Horns: From 1880 to 1960

Early electromagnetic (EM) horns were developed experimentally, but in the late 1930s, there was a fundamental shift in the approach. Barrow and Chu laid the mathematical foundations of EM horn antennas. An improved theory and basic practical information relating to horns were encapsulated by 1950 in Volume 12 of the Radiation Laboratory Series edited by Samuel Silver [1] and in the book by Schelkunoff and Friis [2]. The contributions of the early pioneers to the theory and design of EM horns dating from the 1880s until the 1960s, when computer methods became feasible, are reviewed in this article.

INTRODUCTION

The purpose of this article is to provide a historical overview of the development of EM horns from Hertz's time up to around 1960, during which time a landmark compendium of theory and results appeared in Volume 12 of the Radiation Laboratory Series that was edited by Samuel Silver [1]. With Chapter 12 of this book and the book by Schelkunoff and Friis [2], much of the practical information relating to horns and antenna feeds used by engineers in the 1950s and early 1960s was covered before computer methods changed the scene. A collection of articles and a brief history of horns were published in 1976 by Allan Love [3]. There were some articles on horns published up to 1960, but the standard had been set. It was not until computers came along did the methods of analysis improve, such as GTD and mode matching, allowing the more accurate design of horns.

Figure 1 shows the number of articles published by IEEE (previously the IRE) in the period from 1936 until 1960. According to IEEE *Xplore*, no articles on EM horns were published before Barrow's article on waveguides in 1936 [4]. However, the waveguide was not flared into a horn until slightly later. Therefore, 1936 is taken as the start date of the survey, although [4] is not counted in it. As an end date for this history, the introduction of computer methods in the early 1960s is an appropriate time to terminate this survey. In addition to IEEE, the *Bell Systems Technical Journal*, the *Journal of Applied Physics*, and the IEE in the United Kingdom published a similar number of articles over the same period. The author estimates that Figure 1 represents about 50% of the articles published on horns with a similar overall distribution.

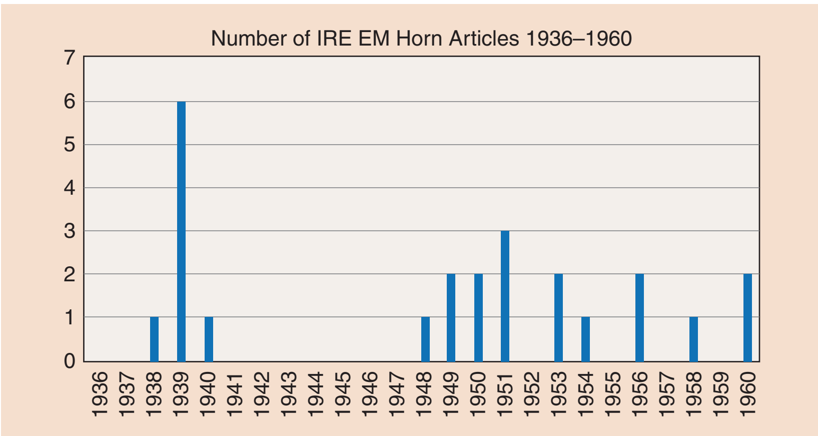

FIGURE 1. The number of articles published by IEEE (previously the IRE) on EM horns from 1936 to 1960.

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EARLY WAVE PROPAGATION

In the 1870s, Hermann von Helmholtz was attempting to understand Maxwell's theory, and in 1879, he called for experimental validation of it. This call resulted in a prize problem offered by the Prussian Academy of Science in Berlin, and hence, it is sometimes known as the *Berlin Prize*. At the time, Heinrich Hertz (1857–1894) was engaged in EM research at the Physical Institute in Berlin [5]. He completed his doctorate in the following year. He went to the University of Kiel as an instructor in theoretical physics. The Berlin prize became an incentive for his research. During his time at Kiel, he published an article on Maxwell's equations, which possibly gained him an appointment to Karlsruhe, where he later did most of his important work. Although Hertz did not use or contribute to EM horns, he had important correspondence with one who was a pioneer. Hertz was in contact with Oliver Lodge in London, who was attempting to prove wavelike phenomena of EM radiation.

Oliver Joseph Lodge (knighted 1900) FRS (1851–1940) was a British physicist and experimenter involved in the early development of wireless. He held several key patents connected with radio transmission. Lodge identified EM radiation independent of Hertz's proof, and at his 1894 Royal Institution lectures [6], he demonstrated an early radio wave detector called the *coherer*. In 1898, he was awarded the "syntonic" (or tuning) patent by the U.S. Patent Office. Lodge was the first principal of the University of Birmingham from 1900 to 1920. The author of his obituary in *The Times of London* wrote that he was "Always an impressive figure, tall and slender with a pleasing voice and charming manner…" [7].

In late Victorian times, people were extremely excited about new discoveries in science, and these were often communicated at evening lectures open to the public. Lodge became well known through his publications and evening lectures given at the Royal Society of London. In one of these, in 1874, he used his coherer to receive spark emissions with an open-ended waveguide. Later, in 1888, Lodge experimented with two 29-m-long wires that had a series of spark gaps. When the wires were excited by a Leyden jar, he noticed a large spark in the gap near the end of the wires. He thought that the large spark was due to interference with a wave reflected from the end of the wires. When the room was darkened, he could also see a glow at intervals along the wire close to half-wavelength spacing. He saw this as evidence of Maxwell's EM waves [8]. While vacationing in the Tyrolean Alps in July 1888, Lodge read an article by Hertz, who had conducted similar research. Hertz had published a series of articles proving the existence of EM waves and their propagation in free space (see [5]). Lodge presented his own paper on EM waves propagating along wires in September 1888 at the British Science Association meeting in Bath, U.K., adding a postscript acknowledging Hertz's work.

Although Hertz did not use or contribute to EM horns, he had important correspondence with one who was a pioneer.

Lodge's work and articles influenced William Strutt (the third Lord Rayleigh) to theoretically describe the radiation from the waveguide [9] and also the later research of Jagadish Chandra Bose.

THE FIRST HORNS

In 1897, Lord Rayleigh [10] solved the boundary-value problem of EM waves propagating through both conducting tubes and dielectric rods of arbitrary shape. He identified these mode types as the electric field (TE) or magnetic field (TM) that had cutoff frequencies, and above the cutoff, these modes could travel with little attenuation. He showed that these modes would radiate at an open end.

The first person who seemed to use EM horns was Jagadish C. Bose (knighted 1917) CSI CIE FRS (1858–1937). He was a pioneer in the investigation of radio microwave optics, and he also made significant contributions to botany. Bose became interested in radio following the 1894 publication of British physicist Oliver Lodge's demonstrations on how to transmit and detect radio waves [11]. He began his own research in the new field in November 1894, setting up his own equipment. Some of this work is described in his collected papers [12]. One of the first, in 1895, at the Asiatic Society of Bengal, involved a tube of 2.5-cm diameter because of "the efficiency of the radiating apparatus" [12, pp. 1–10], although from Bose's description, it sounds more like "reliability." Similarly, his justification in an evening lecture at the Royal Institution in January 1897 [12, p. 80] was that he wanted a "narrow pencil of electric radiation," and for this, he used very short waves so that the aperture was a reasonable size. He continued to use circular waveguides, and later, pyramidal horns for the measurement of the refraction of different materials, especially the relationship between the dielectric constant and refractive index, although he made no comment on the "electric source apparatus." In investigating polarization as well as double refraction at an evening lecture at the Royal Institution in London in January 1897, Bose used the apparatus pictured in Figure 2 [12, p. 91], which shows a pyramidal horn. This picture is evidence of the first use of rectangular horns.

MICROWAVE COMMUNICATION

After this promising start on EM horns and their use in demonstrating the properties of the so-called "electric" radiation, work seems to have stopped on horns for want of a direct application. There was much research to do at lower frequencies for the new long-distance Marconi radio, which, of course, had tremendous commercial and social value. World War (WW) I intervened, and by the 1920s, radio, wireless, and cable telegraphy made communications much easier. After the 1920s, some of the limitations of these systems were being noticed. Such limitations were signal delays, interference, and distortion. Although the cost of submarine cables, including laying, was expensive, the alternatives appeared even more expensive and less dependable. In 1931, an experimental 64-km microwave telecommunication link was established across the English Channel, followed by a commercial relay in 1935. This was not a commercial success due to the expense of the equipment compared with an undersea cable link. However, the higher carrier frequency did provide the possibility of more channels that potentially would allow more customers to use the service. In 1939, the British Army introduced the Wireless Set No. 10 for microwave relays to multiplex eight telephone channels over long distances. This set was used in 1945 to provide a link across the English Channel, allowing General Bernard Montgomery to remain in continual contact with his group headquarters in London. In addition, the development of microwave radar in WWII provided technology, such as the cavity magnetron, for the practical exploitation of microwave frequencies.

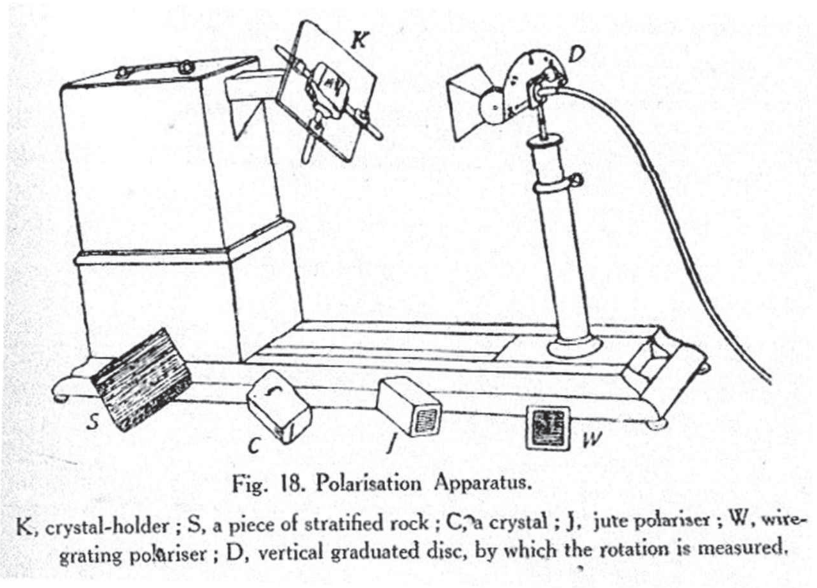

FIGURE 2. A figure of Bose's polarization apparatus. (Source: Bose collected papers [12, p. 91]).

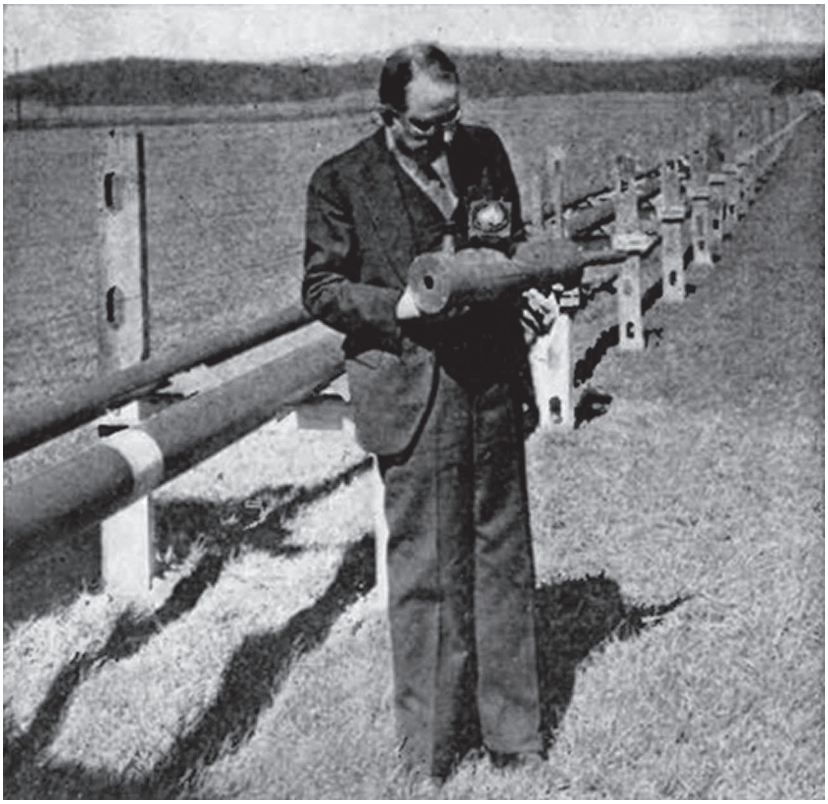

FIGURE 3. George Southworth beside the waveguide and holding test waveguide section in 1936.

WAVEGUIDES AND HORNS

The development and use of EM horns were intrinsically linked to the adoption of waveguides for long-distance communications. In the early 1930s, the limitations of the existing cable systems led to a search for alternatives. The work by Rayleigh and others on dielectric waveguides attracted attention, and two research groups commenced work on waveguides. One group was led by George Clark Southworth (1890–1972) at Bell Telephone Laboratories, Inc., and the other was William L. Barrow (1903–1975) at the Massachusetts Institute of Technology (MIT). Hollow pipes were attractive because they did not need a return conductor, although, instead, they had a cutoff frequency. This frequency was often higher than the available sources at the time. Some basic properties needed to be verified, too, such as the characteristic impedance, attenuation, and phase velocity as well as the selectivity and radiation.

In 1931, Southworth began to study wave propagation in dielectric rods, which have no cutoff frequency, but by early 1932, he had moved on to wave propagation in a water-filled copper pipe. By May 1933, he had transmitted waves through air-filled copper pipes up to 6 m in length. He had a 12.7-cm-diameter waveguide constructed at Bell Telephone Laboratories in Holmdel, NJ, USA, with a length of 26.7 m for testing trunk waveguide communications (see Figure 3). An early history of this is given by Southworth in an article published in *Proceedings of the IRE* in 1962 [13].

The cutoff frequency could be lowered by filling the pipe with a suitable low-cost dielectric that had a low loss. For instance, Southworth described work in [14] using frequencies from 100 to 400 MHz with circular pipes of 12.4-cm diameter and 25.4-cm diameter filled with water with a known dielectric constant of 81. The cutoff frequencies of the test pipes were 218 MHz and 131 MHz, respectively. He made measurements from 1.5 to 2 GHz as well. Southworth was one of the first to note that the TM modes have a descending attenuation with a frequency above the cutoff.

Regarding radiation, he commented that [14]

> "…radiation issues from the open end much the same as sound waves from a hollow tube. It has been possible to expand the ends of these pipes into horns, thereby obtaining effects very similar to those common in acoustics. Such an electrical horn not only possesses considerable directivity but may also provide a moderately good termination for the pipe."

It is realized through comments such as this that early work on EM horns was guided by methods from acoustics.

The first *Proceedings of the IRE* article on radiation from an open-ended waveguide came not from the Bell Laboratories group but from the MIT group [15]. William Barrow had built a group of students and technicians that included L.J. Chu, F.D. Lewis, and F.M. Greene. Barrow graduated in electrical engineering from Louisiana State University in 1927 and received a master's degree from MIT in 1929. This was followed by a Doctor of Science degree from the Technische Hochschule München in 1931. When in Munich, Barrow was a Redfield Proctor Fellow in physics and studied under Jonathan Zenneck and Arnold Sommerfeld. A talented professional, Barrow worked in electrical engineering at MIT from 1931 to 1943, becoming an associate professor in 1941, and was the director of MIT's radar school.

Like Southworth, Barrow commenced work on waveguides filled

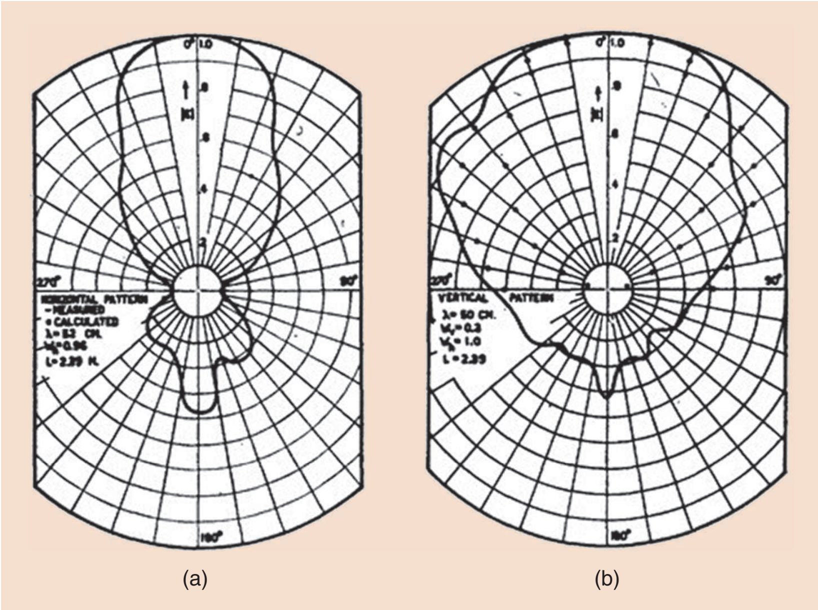

FIGURE 4. Measured radiation patterns (solid) and calculated (circles) in (a) the E-plane and (b) the H-plane by Barrow and Greene in 1938 [15], which are some of the first measured horn patterns. (Source: [15].)

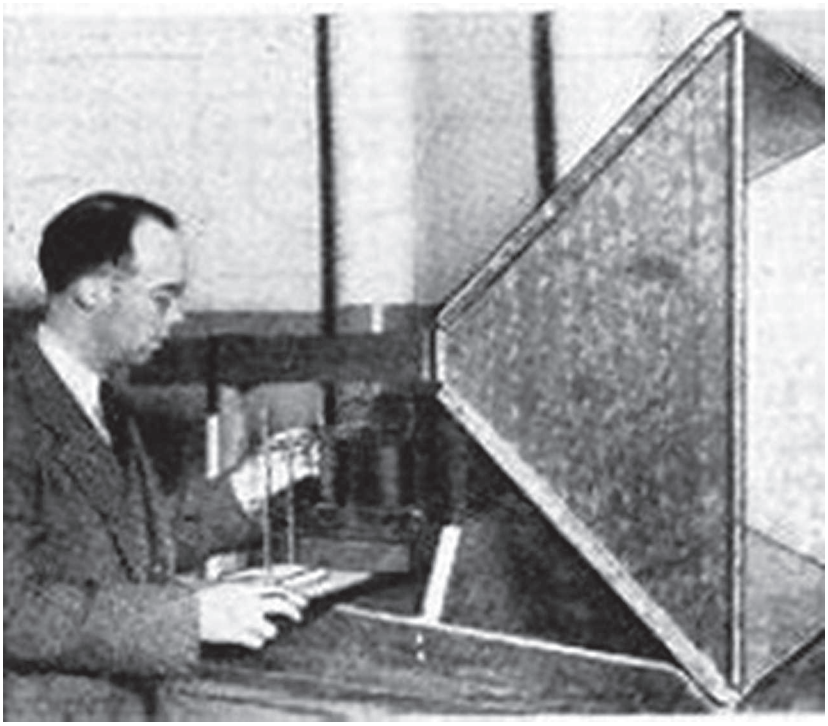

FIGURE 5. William Barrow with his test rectangular horn.

with water to reduce their size as he was using sources at UHF frequencies [4]. His first article with Greene on radiation from rectangular waveguides [15] was a logical extension of Barrow's waveguide work [4]. The waveguide was excited by a probe backed by an adjustable piston for tuning to maximize the signal going to the open end. A modal representation had been developed, and they assumed that $H_{0,m}$ modes were excited and that radiation was predominantly in the $H_{0,1}$ mode. The field distribution of this mode was assumed to represent the field at the mouth of the waveguide. Using the Hertzian vector, Huygen's principle was employed to calculate the Hertzian vector in the outside space at a large distance from the aperture. The classical field solution in terms of sinc functions was the result. These equations were verified through manual measurement with a mobile carriage (see Figure 4).

This work on rectangular waveguides led to an investigation of radiation from sectoral horns (see Figure 5), ultimately building a theory of radiation based on radial modes in a uniformly flared section. In 1939, two articles were published in *Proceedings of the IRE* on the sectoral horn by Barrow: one with Lewis [16] and the other on the theory of EM horns with his student L.J. Chu [17]. Another article followed on horn design [18] as well as another by L.J. Chu alone using a vector form of Kirchhoff's theorem, which is identical to the equivalence theorem [19].

Lan Jen Chu (1913–1973) is known today for a fundamental limitation for small antennas known as *Chu's limit*. He graduated with a science degree in electrical power in 1934 and received a Doctor of Science degree in electrical engineering from MIT in 1938. Chu was with the Radiation Laboratory at MIT from 1942 to 1946 and with the Department of Electrical Engineering from 1947 to 1973. During WWII, Chu supervised research at MIT on many types of antennas for use in radar and telecommunications.

Meanwhile, the Bell Telephone Laboratories team led by G.C. Southworth had also worked on horns, and in 1939, Southworth and A.P. King published an article on conical horns [20]. George Southworth graduated in 1914 with a physics degree, and after working with the National Bureau of Standards for five years, he joined Yale University and completed a doctorate in 1923 on the measurement of the dielectric constant of water at frequencies above 15 MHz.

Southworth left Yale for a position with a subsidiary of the American Telephone and Telegraph Company created in 1925 under the name Bell Telephone Laboratories, Inc. He first helped edit the *Bell System Technical Journal* and then commenced researching shortwave radio propagation. He spent the rest of his career at Bell before retiring in 1955.

The first analyses of EM horns were made through a radial mode representation. The present standard approach of modifying the equivalent currents due to the modes in the aperture with

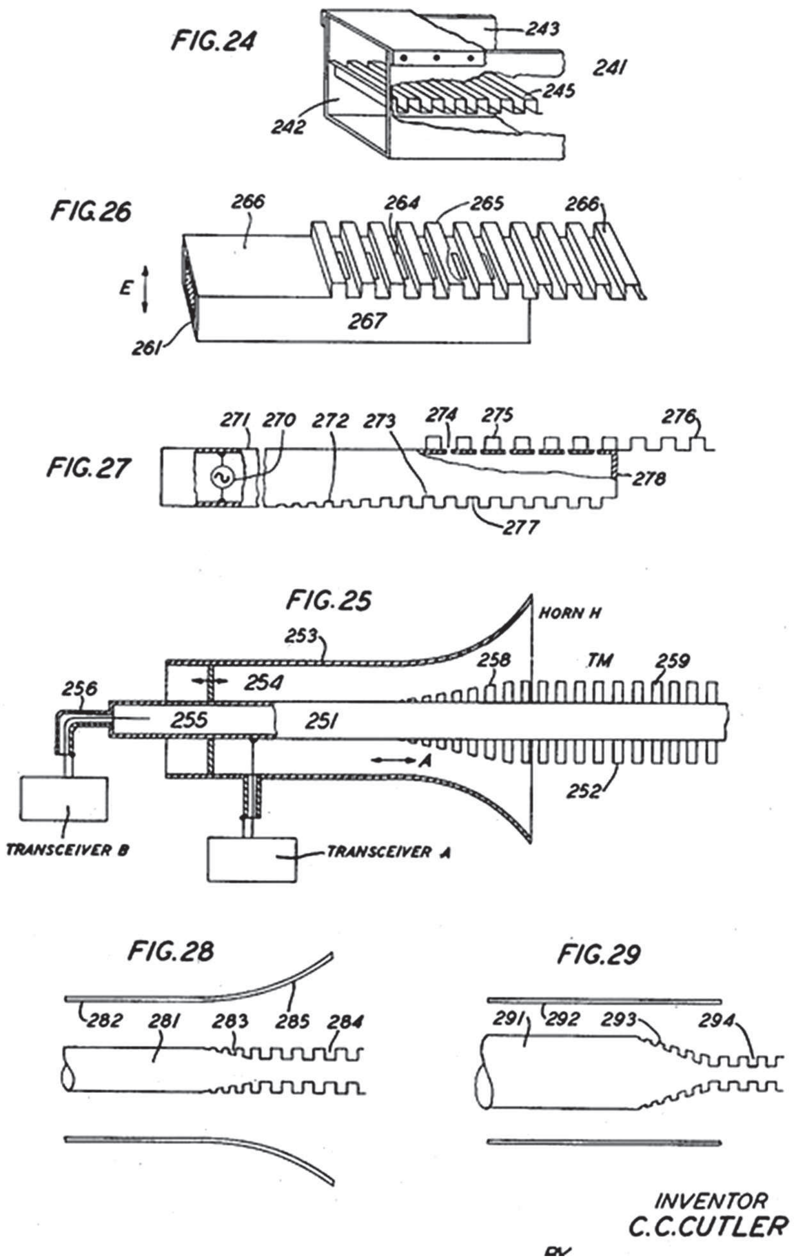

FIGURE 6. Corrugated horns proposed by Chapin Cutler in U.S. Patent #2912695 A [27] originating in 1948. (Source: [27].)

a spherical phase [21] factor came from the approach first described by Schelkunoff in 1939 [22]. A little later, in [2], Schelkunoff and Friis showed that the dominant phase of the wavefront over a planar aperture was quadratic in the aperture coordinates, which are weighted by the inverse of the radii of curvature in those directions. This showed, for example, that long horns, having a large radius of curvature of wavefront over a comparably smaller aperture, have an aperture phase that is almost uniform.

THE DECADE FOLLOWING WWII

At the end of WWII, the U.S. government continued to pay key people who had worked at the Radiation Laboratory for six months to enable them to write about their work in science and engineering as security allowed. As a result, basic material of great value became available and was published as the Radiation Laboratory Series in 28 volumes. Samuel Silver edited a collection of chapters in Volume 12 of the series on microwave antennas [1]. The material was meant to be generally accessible. As Silver wrote in the preface to the volume, "references to unpublished material of Radiation Laboratory notebooks have been assiduously avoided." Chapter 10 of Volume 12 on waveguide and horns feeds was written by J.R. Risser. This chapter is a wonderful collection of theories and experiments on EM horns that have proved valuable up to recent times. The following chapter, Chapter 11, is on dielectric and metal-plate lenses and is also similarly useful. Little is available on Risser in the open literature.

Similarly, there is little available in the open literature on A.P. King except that he was employed at the Bell Laboratories in Holmdel and that he contributed to several early articles in *Proceedings of the IRE* on horns with Southworth and C.C. Cutler, who is mentioned later. King had a single-author article published in 1950 [23] on conical horns that has become a classic. In this article, he coined the name *optimum horns* for conical horns of a given length that had maximum gain. There is a similar optimum geometry for sectoral and pyramidal horns [21]. Out of interest, King's measured results were compared with modern-day theory in [21]. The computed gain and E-plane pattern agree well, but the H-plane is narrower than predicted at wider angles. This is thought to be due to how the aperture of King's test horn was terminated or how it was mounted.

Burrow and Chu developed the mathematical foundation of horn antennas, which developed into an EM science for the analysis and design of horn antennas.

Due to the flaring of horns, it became apparent that the phase variation across the mouth of the horn resulted in a lower gain. An approach for correcting this error using metal partitions was discovered in the early 1940s by Rust and colleagues. This approach with results is described in a 1946 article [24].

Another early pioneer from Bell Laboratories who contributed to EM horn development up to 1950 was C. Chapin (Chuck) Cutler (1914–2002). Chuck became interested in electrical engineering through radio, and as a teen, enrolled at Worcester Polytechnic Institute, Worcester, MA, USA. He received a B.S. degree in 1937 and then took graduate courses at the Stevens Institute of Technology, Hoboken, NJ, USA. Cutler worked at Bell Laboratories, Inc., Deal Test

FIGURE 7. C. Chapin Cutler (left) with Allan W. Love at the 1994 APS Symposium in Seattle, WA, USA, during a session celebrating the 50th anniversary of soft and hard EM surfaces. (Source: Photo by T.S. Bird.)

Site, Murray Hill, NJ, USA, and Holmdel, NJ, USA, from 1937 until 1979. His research contributions extended over a broad area of electrical engineering, including shortwave radio technology for telecommunication, microwave radar antennas, microwave amplifiers, satellite communication, and digital signal coding. He was granted more than 70 patents, including fundamental patents on differential pulse-code modulation.

He invented the rear-radiating "Cutler feed" for parabolic antennas that was widely used during WWII [25]. Rear-radiating feeds [21] are used for front-fed reflectors as they have the inherent advantage of being self-supporting.

Cutler was the inventor in the 1940s of the corrugated surface now used in corrugated horns and corrugated waveguides as well as in a variety of multimode antenna feeds [26], [27], [28]. These patents were classified during the war, so Cutler did not receive recognition for these inventions until later years [28]. A figure from his 1948 patent is shown in Figure 6. At the 1994 Antennas and Propagation Symposium, there was a session on soft and hard EM surfaces on the 50th anniversary of Cutler's invention, which the author attended. Cutler presented a paper in this session on the background of his invention [29]. He is pictured in Figure 7 with Allan Love during this session [28].

In the postwar period, the development of microwave technology was rapid, which led to the construction of transcontinental microwave relay systems in North America, Europe, and Asia. During this time, Cutler continued to contribute strongly to new technologies, such as the theory of traveling wave tubes (TWTs), with Rudolph Kompfner, who invented the TWT, and John R. Pierce (e.g., [30]). With the arrival of the satellite era following the launch of Sputnik in 1957, Cutler worked on various satellite systems and projects, such as Echo (1969) and Telstar (1962). After he retired in 1979, he joined Stanford University as a professor.

The 1960s and thereafter saw an explosion of techniques as well as horn types that were stimulated by new applications, new analysis techniques, new numerical methods, and dramatic improvement in computing power for analysis and design. The developments after 1960 are summarized in a compendium of Allan Love [3], and more recently, in [31].

CONCLUSION

Through this article, the author has emphasized the contributions of the early pioneers to the theory and design of EM horns. It is based on information scattered among many sources and has been brought together for the first time. The author wanted to show that the EM horn we use today has come from many pioneering contributions and not just one inventor. In the beginning, the future EM horns were designed intuitively from the earlier knowledge of acoustics. However, between 1935 and 1940, there was a fundamental shift in the approach. Burrow and Chu developed the mathematical foundation of horn antennas, which developed into an EM science for the analysis and design of horn antennas.

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