Interview

An Interview with Professor Ingrid Daubechies

In July of 2022, the EiC of the *IEEE Circuits and Systems Magazine*-Yiran Chen and the Associate Editor-Fan Chen had a great chance to interview the legendary mathematician Professor Ingrid Daubechies, known as the Godmother of the digital image. In this interview, Prof. Daubechies shared with our CAS readers her insightful reflections and wisdoms on mathematical education and beyond, her illustrious career, and colorful life. A total of nine questions were compiled into this interview article. We hope that our CAS readers enjoy – Thank You!

Prof. Ingrid Daubechies's Short Bio

Ingrid Daubechies received the Ph.D. degree in theoretical physics. She has spent most of her career on bringing mathematical techniques and analysis to bear on applications not only from physics, but also in signal processing, brain imaging, geophysics, biological morphology, and art conservation and analysis, as well as on research in mathematics. She is currently a Professor at Duke University. During the isolation caused by the pandemic, she has spent part of her time on the realization, together with 23 other mathematicians and artists, of an art installation that seeks to communicate the wonder, the beauty, and the whimsy of mathematics. More details can be found at: mathemalchemy.org.

Question 1: Can you share with our readers about your earliest memories on mathematics? Did you receive any special early training in math? And what is your advice on encouraging children's interests in STEM?

Prof. Daubechies: At a very early age, I was interested in how things work, how things are put together, and why things are the way they are. I would ask many questions. My father was an engineer and he liked answering my questions. I did not receive any special tutoring on mathematics, but I have never dissociated my interest in mathematics from this curiosity into everything. It turned out that, for much of this, in order to delve into understanding you need a little bit of math, and on the other hand, once you know a little bit of math you'll see many more different things can be explained - that's basically what mathematics is about: find patterns in reasoning so you can use them in many different contexts because the same reasoning happens. This way of looking at mathematics came very early to me, and I also liked that it was so logical and put together so nicely. Later I studied physics because I was interested in the application of mathematics to the world. And then I drifted into what people call applied mathematics, which is working on problems that are connected to other branches by using mathematical tools. It all seemed a very natural progression to me.

I think the most important thing to encourage children is to just stimulate curiosity rather than a specific subject. The world needs a lot of different things and I believe that there is room for everybody and for all talents. In my case, both my son and my daughter love mathematics and majored in mathematics, which I hadn't expected because we didn't try to push mathematics, but we did push curiosity. We had an encyclopedia at home. Whenever a question came up at the dinner table, we would start looking it up and talk about it. I didn't do it consciously, but the children remember this. I have been very careful not to project a dream on my children. I just help them develop in the directions that they seek to build - my daughter is now a data scientist, and my son teaches high school mathematics in the south side of Chicago where I think he has a bigger possibility to impact young people than many college teachers, so I'm very proud of him, as well as of my daughter.

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Question 2: You received a B.S. degree in physics, and later a Ph.D. in quantum mechanics. Six years later after your Ph.D., you made your best-known discovery on the wavelet theory. Can you elaborate on how these diverse backgrounds connect and impact your research and career?

Prof. Daubechies: The work on quantum mechanics I had been done during my Ph.D. was very mathematical. I worked in a field called mathematical physics, which essentially means working on physical models, but with rigorous mathematics. Mathematical physics results can be useful elsewhere: for instance, the initial work on the Ising model is now widely used in machine learning. When you do signal analysis, you really are looking at the properties of functions locally in time and in frequency. This is very similar to what you do in quantum mechanics where you want to understand the wave function as it depends on space and momentum - the dual variable. In this sense, signal analysis and quantum mechanics are both related to the Fourier transform and their mathematical techniques are very similar. So it's not such a big jump to go from one to the other. My background in physics was very useful to me and I had learned some techniques different from what electrical engineers learn. Also, I had learned much more to look at time and frequency as two aspects of one reality - having a different point of view really helps.

Question 3: How did you come up with the idea of "Daubechies wavelets"? How to explain it to average people? How does the concept of wavelets compare to the currently widely used machine learning approaches in signal processing?

Prof. Daubechies: When I started working on wavelets, I built on research I had done in time and frequency. And then, when the first wavelet basis was constructed, it linked into harmonic analysis that I didn't know, and I started studying that. It was a beautiful theory, but for the practical application to signals analysis, it would have to be mutilated, and in this process lose some of its beautiful properties. Mathematicians think that is natural because when you work on practical applications rather than mathematical models you have to take aspects into account that don't fit into the model. I wanted to see if it was really true that I had to give up all the beautiful features in order to do the applications. To answer this question, you must think about it differently. You need to find out what the constraints are imposed by the applications, whether those constraints can be added to the ones of the model, and whether the whole construction can be rebuilt. All these questions inspired the construction of "Daubechies wavelets". This way of looking at the problem was particular to me and can be attributed to my mixed background. I solved the problem in a roundabout way, otherwise I don't think people would have solved it so easily.

In old-fashioned machine learning, before neural networks came along, algorithms were essentially trying to learn a representation in which the class of things that you were studying could be represented simply. A natural way of doing so is using orthogonal polynomials. Normally, the first few terms are going to be easy to compute and it becomes harder to find more terms. The situation is different when you can compute easily a large number, say a million of terms. In this case it may still be useful to retain only some small collection of these terms, but maybe not necessarily the first few terms, which becomes a completely different mathematical approach. So that's where wavelets come in - instead of looking at the original representation, you can make a basis transformation on the signal and characterize function spaces by only focusing on a few of the coefficients you are interested in. Furthermore, we can identify the types of functions that can provide sparse expansion for certain tasks, making it possible for compression. Neural networks, in a sense, also make it possible for learning compressed representations, though we don't well understand - at least not the level that I call understanding- the theory behind them. I think the next decade is going to see a lot of theory work that helps us to better understand neural networks. We will be able to get a better idea of how to construct neural nets, and their computation will be less wasteful than they are today.

Question 4: Can you talk about your recent works on applying mathematical skills to fine art? What projects have you been working on?

Prof. Daubechies: Art conservation often involves the use of very high-resolution images and has completely different tasks about these images from standard image analysis. Taking one of my recent projects as an example, which is X-ray image separation for artworks with concealed designs. X-ray images play a vital role in conservation and preservation of artworks, but many X-ray images of paintings have concealed sub-surface designs deriving from the reuse of the painting support or revision of a composition by the artist. In practice, it is very difficult to separate the contributions from both the surface painting and the concealed features. The best algorithm we have now is one that uses neural network, but there are still many open questions, which I believe in turn will lead to a lot of interesting research. For me and my students, working with these images (such as a fantastic famous painting) is much more fun than standard images. You have an opportunity to work with people from completely different domains, try to understand what their problem is, then abstract the problem and start thinking mathematically about the solution.

Question 5: What do you think is an exciting trend in signal processing, especially in the age of artificial intelligence? Can you share your vision on the possible innovations in your field?

Prof. Daubechies: I think we'll have a better understanding of how neural networks work and how they can do better. I do believe that mathematics and mathematical analysis, not necessarily mathematicians - there are a lot of people who do great mathematical analysis and who aren't mathematicians - will play a big role there and that will make these tools better, more targeted, more energy-efficient, and requires less data to learn from. I think there is still a lot to do. There will be people who say, "well, I don't want to wait for all the theory in order to work on applications" and that's absolutely fine and completely understandable and it would be terrible if that were not the case. So we should use them for applications, but we should at the same time, work on building the theories so that applications can become much better.

Question 6: Can you share the approach you take to advising your students in your research group? What do you think is the most important characteristics when recruiting PhD students?

Prof. Daubechies: I do want to see my students at least once a week, regardless of whether they have made progress or not. I ask my students to not avoid me if they have not made progress. All my graduate students have access into my calendar. I have an open door and I encourage them to come in and talk to me if they can find time on my schedule. I direct the reading and my students would also come to me with articles that they were interested in. We would discuss research papers, and I would also invite them to meetings I had with senior students and postdocs. I typically do not give my graduate students a dissertation problem. I tell my PhD students that what they really would be doing in the first couple of years is building an intellectual landscape where they would start seeing how different things that people are working on fit together and how they are all implicitly part of this landscape. And then, I tell them "You will start seeing that there are paths that are not completely dense and problems that have not been explored. If some of these problems interests you, then you will have found a thesis problem." This approach may not work well with some of my students, so I had to steer a bit more, but in most cases my students would indeed find research problems that they have a passion for and become independent researchers through this training process.

I think two things that are important, one is you really want to build understanding. For example, if you see a derivation, you may think can I do it better or in a simpler way rather than just try to learn it all. I mean you fight with it and in fighting with it, you learn it. This process gives you a much better understanding than just reading books and absorbing material cover to cover but haven't done much with it. The other thing is you are also willing to learn a lot. I encourage my students to take every pure mathematics course they have an appetite for even though we are working on applied mathematics.

Question 7: You are a legendary female researcher, just to name a few, you were the first woman to become a full professor of mathematics at Princeton University, the first woman to receive the National Academy of Sciences Award in Mathematics, and the first women to won the William Benter Prize in Applied Mathematics from City University of Hong Kong. Throughout your career, have you ever faced any specific challenges as a female researcher?

Prof. Daubechies: When I am asked how I feel about being the first female to win an award in mathematics, I said it wasn't my problem, it was theirs. I feel that there are perceptions about women that are not accurate, and I do make special efforts to encourage women and our young girls to stick with mathematics because they can do it just as well. I don't have big theories about it, and I think it's a cultural thing. Looking at the number of women in academia in mathematics in Europe, the percentage changes drastically from one country to the other. There may be a few genetic differences, but not that as much as in those wide variations in the presence of women in mathematics. It's a sad thing for everybody when you have a profession that could be done as well by women but there are very few women and underrepresentation in it. Encouraging the next generation, it's something even for the men in the profession today. That's why I work at these issues, and I can sometimes get angry when people tell me mathematics is not blind. Mathematics itself is blind and you cannot tell the difference between an article written by a woman or a man unless the style is so particular that you recognize that person, but it's not that you recognize it as a woman versus a man. But our profession is far from blind.

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(for HD video) or H.265 (for Ultra HF/4k video). When we capture video on our phones, watch TV, Netflix, Amazon video, or make a Zoom or Teams call, we use these formats, which are all DCT-based. Today, almost 80% of all data traffic on the Internet is on video streams in those formats. Ahmed et al.'s [2] original DCT work was also an inspiration for designing DCT variants and extensions, such as the modified discrete cosine transform (MDCT) and lapped transforms (LTs), with overlapping basis functions that improve compression and reconstruction quality for images and audio. Essentially all modern digital audio formats, such as MP3, WMA, and AAC, are based on these digital variants. The JPEG XR image format also uses one of those variants and is used extensively by hundreds of millions of people, as it is integrated with all Microsoft Office applications. Moreover, compressed domain image and video processing which leverages the DCT coefficients, their residues along with other information like block coding modes, motion vectors have offered a completely new paradigm for algorithm development courtesy the unique DCT properties.

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I do not agree when sometimes people say we have no prejudices, when you can cite example after example of things that wouldn't have happened if it wasn't for such a prejudice. So I hope this will change, and I think we have to work at it because unless we work at it, change will not come.

Question 8: You are on board of director of Enhancing Diversity in Graduate Education (EDGE). What role does this organization play?

Prof. Daubechies: I have worked with EDGE for many years. At first, I have been on the periphery, and now on the board. EDGE has helped young women, especially women from disadvantaged backgrounds who are interested in mathematics, to persist in mathematics. Even if their background may be less prepared than others, EDGE helps them with workshops, special activities, and a network that gives them both moral and mathematical support. The success rate of young people who came through EDGE and then went on to PhDs in mathematics is very high, which is very impressive. EDGE now also has a program that is not aimed only at women but at underrepresented mid-career fellows. It is a really remarkable program and I think it's the same idea again - but now for other groups in which we see underrepresentation in STEM, like people of color and African Americans, to an even more dramatic extent than for women. There are young people who have talent, and we should encourage them if they have the inclination to become STEM professionals. That's why I'm proud of my son, because at his school, 95% are black or Hispanic. Having a positive, fun math teacher can be life changing.

Question 9: What is your own definition of "success", or do you consider one of your past roles to be "success"?

Prof. Daubechies: Things that make me most proud is being a member of the community. If my students later say that they really appreciate my mentorship and they felt that I made a difference for them for the better, that is what I find most successful. I now have the next generation. My son has two small children and asked me to help. I am actually taking a week off next week to be a grandma, which also makes me feel very proud. I think the feeling that I have a meaningful role in people's lives gives me the greatest sense of accomplishment.