Outage Probability of One-Source-With-One-Helper Sensor Systems in Block Rayleigh Fading Multiple Access Channels

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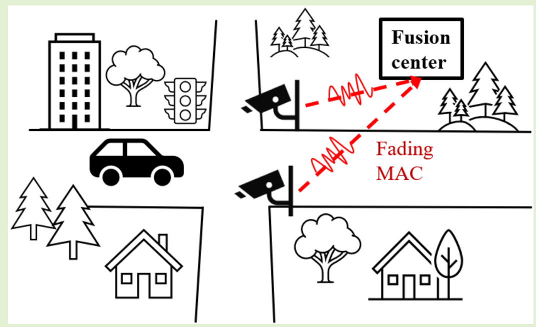
***Abstract*—Two correlated sources are transmitted over a block Rayleigh fading multiple access channel (MAC), where information recovery from only one of the sources is aimed at, and the other is used as a helper which helps recover the source sequences at the destination. In this article, the successful transmission is defined as in the case the transmission rates satisfy the intersection of both the MAC region and the Slepian-Wolf (SW) region with a helper, referred to as h-SW region, even though it is a sufficient condition. Then, the outage is defined as the average probability that the two regions do not have an intersection. The explicit expressions of the outage probability of the system are derived, in the form of multiple integrals with respect to the probability density function of instantaneous signal-to-noise ratios of each link. To have an in-depth insight regarding the contributions of some special cases to the successful transmission probability, the occurrence probabilities of these cases are also evaluated. The results indicate that outage probabilities decrease as the correlation between the source and helper information increases. The impacts of geometric gains of each link and the correlation of each link's fading variation are also considered. The most significant finding of this article is that h-SW over a MAC improves the throughput efficiency compared to the orthogonal transmission both in independent and correlated block Rayleigh fading channels without sacrificing the outage probability.**



***Index Terms*—Helper, multiple access channel, outage probability, Slepian-Wolf theorem.**

I. Introduction

IN WIRELESS sensor networks (WSNs), as exemplified in the figure in Abstract, multiple sensors track the states of targets, of which topics are, in general, categorized in statistical signal processing [1]. They then transmit the gathered *correlated* data to the fusion center, of which topics belong to network information theory. The issues of the location identification and tracking of the targets in WSNs have been investigated, e.g. [2], [3]. In this article, we investigate the outage probability of transmissions in the category of network information theory. The links between the sensors and the fusion center suffer from fading, occurring in the wireless propagation process. Since the information sequences obtained by the sensors are correlated, where the transmission rates for lossless recovery should satisfy the Slepian-Wolf (SW) region, they can help each other to retrieve the original information via joint decoding. Multiple access channel (MAC) is known to increase the up-link transmission efficiency, and especially, when used in WSN, it can significantly improve the throughput and reduce latency. In MAC systems, information sequences collected by the sensors are transmitted to the destination simultaneously. However, there arises a problem that if the information sequences are correlated, as in WSN, the SW and MAC regions, referred to as SW-MAC regions, have to intersect for the successful transmission. Even though having an intersection is a sufficient condition [4], it has been widely used in [5], [6] when analyzing the probability of successful transmissions. Reference [5] analyzes the outage probability of

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a two-correlated-source transmission system over independent block Rayleigh fading channels where both the two sources are aimed to be recovered. In [6], the author studies a two-hop multiple access relay cooperative communication system, where the admissible rate region of two correlated MAC sources with an exclusive-or (XOR) helper is investigated over independent and correlated block Rayleigh fading channels. Reference [5], [6] have motivated us to investigate the case where only one-hop MAC with one source to be detected and the other to work as a helper. The SW region with a helper, referred to as h-SW region, is determined by the source and helper entropy, which is fixed. On the contrary, MAC region is determined by the channel capacities, which vary according to the fading variations. Block Rayleigh fading is assumed in this article where the instantaneous signal-to-noise ratio (SNR) of each link does not change during one block, but changes block-by-block.

The main contributions of this article are summarized as follows:

1. The outage probability expression of one source with one helper over a fading MAC is derived in an explicit form. For the ease of the calculations, we use the approximated h-SW region which is known to be yet very accurate [7].

2. The mathematically derived outage probability is verified to be consistent to that obtained through the Monte-Carlo technique.

3. We investigate the outage probabilities over block independent fading channels with the information correlation between the source and the helper. The geometric gains of each link are also taken into account. It is shown that the 2^{nd} order diversity can be achieved with large information correlation. Also, lower outage probability can be achieved in the case the geometric gain of the source link is larger than that of the helper link.

4. The occurrence probabilities of three cases are calculated in order to evaluate how significant contributions those cases make to the successful transmission.

5. We make a comparison of the outage probabilities of orthogonal and MAC transmission systems with one source and one helper over correlated block Rayleigh fading channels.

This article is organized as follows. The system model of one source with one helper transmitting over a block Rayleigh fading MAC, investigated in this article, is provided in Section II. In Section III, we first investigate the scenarios that outage happens, of which the average outage probability is derived in an explicit form. Three successful cases (i)-(iii) are considered, and their average occurrence probabilities are derived also in the explicit forms. In Section IV, the performance curves are presented with the geometric gains of each link and the information correlation as parameters. In Section V, the mathematical expressions of the outage probability are further modified to include the correlation of the fading variations of the two links. Finally, Section VI concludes this article with some technical remarks.

II. System Model

Two correlated sources represented as u_1 and u_2 are encoded independently, and jointly decoded at the destination,

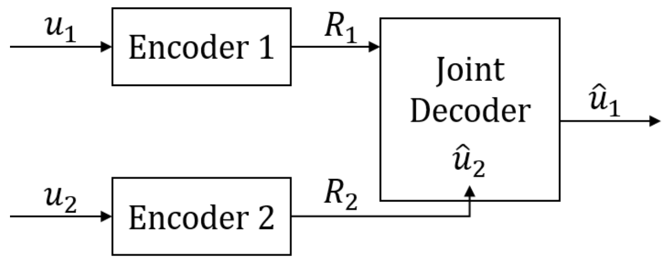

Fig. 1. Encoder and joint decoder of one correlated source with one helper.

as shown in Fig. 1. The sources are binary, and uniformly distributed. Hence, $\Pr\{u = 0\} = \Pr\{u = 1\} = 0.5$, and the entropy of the source is $H(u_1) = H(u_2) = 1$, where $H(p) = -p\log_2(p) - (1-p)\log_2(1-p)$ is the binary entropy function.

In this article, only source u_1 is aimed to be recovered, and u_2 serves as a helper. The sources u_1 and u_2 are correlated, which are represented by the bit-flipping model, as

$$u_1 = u_2 \oplus e, \tag{1}$$

with

$$e = \begin{cases} 1, & \text{with probability } P_e, \\ 0, & \text{with probability } 1 - P_e. \end{cases} \tag{2}$$

The probability that bit-flipping happens is $P_e \in [0, 0.5]$, where $P_e = 0$ and $P_e = 0.5$ correspond to the correlation being unity and zero, respectively [9].

Transmission links from the source and from the helper to the destination suffer from block Rayleigh fading. The received sequences can be expressed as

$$y_i = \sqrt{P_i} h_i u_i + n_i, \tag{3}$$

where $i = \{1, 2\}$ denotes specifically the source-destination and the helper-destination links, respectively. P_i represents the geometric gain, corresponding to the locations of sensors and the destination [8]. h_i represents the channel gain, which can be described as a complex Gaussian variable with $CN \sim (0, 1)$, n_i is a complex additive white Gaussian noise (AWGN) with $CN \sim (0, 1)$. Since the two-dimensional noise variance is normalized to unity, the average SNR is expressed as $\Gamma_i = P_i$, and the instantaneous SNR is given by $\gamma_i = P_i \cdot |h_i|^2$. For the simplicity, we ignore shadow fading here.

III. Outage Probabilities

A. SW and H-SW Rate Regions

Let the rates of the sources u_1 and u_2 be R_1 and R_2, respectively. According to Shannon's theorem, if the rate is larger than the entropy of the source, the source can be recovered with an arbitrarily small probability of error, when the length of block code $n \to \infty$ [10], as

$$\begin{cases} R_1 \geq H(u_1), \\ R_2 \geq H(u_2), \end{cases} \tag{4}$$

where u_1 and u_2 are independent sources. If the two sources are correlated, we can further compress the rates. According to SW theorem, u_2 can provide correlated information to help

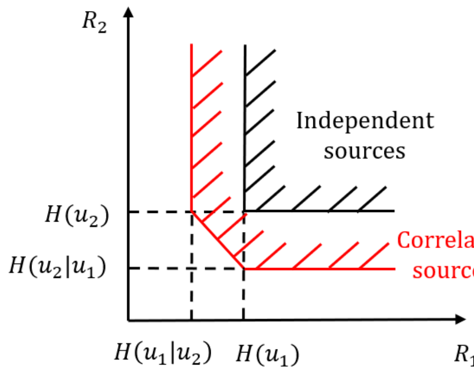

Fig. 2. Admissible rate region of two sources u_1 and u_2, where the black and red solid lines indicate the independent and correlated cases, respectively [11].

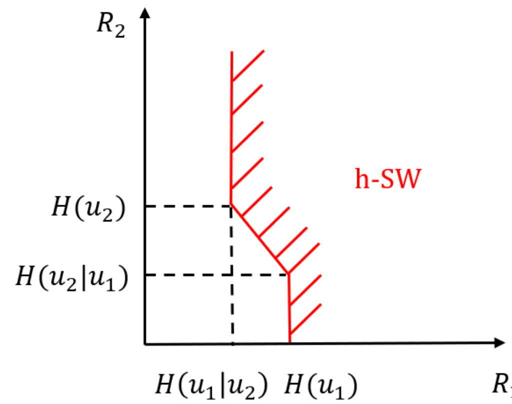

Fig. 3. Approximated admissible rate region of u_1, while u_2 is utilized as a helper [13].

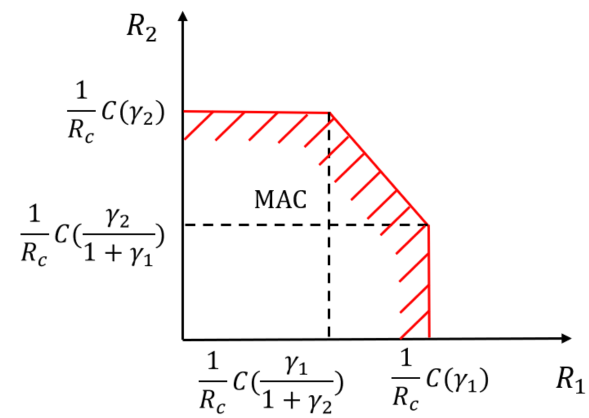

Fig. 4. Achievable rate region of MAC.

recover u_1, and hence R_1 only needs to be no less than the conditional entropy $H(u_1|u_2)$. Similarly, $R_2 \geq H(u_2|u_1)$. Moreover, if we consider two links as a whole, the sum rates have to be no less than the joint entropy of two sources for recovering both of them. R_1 and R_2 should satisfy the admissible SW rate region [12], shown in Fig. 2. It can be mathematically expressed as

$$\begin{cases} R_1 \geq H(u_1|u_2), \\ R_2 \geq H(u_2|u_1), \\ R_1 + R_2 \geq H(u_1, u_2), \end{cases} \tag{5}$$

where $H(u_1|u_2) = H(u_2|u_1) = H(P_e)$, and $H(u_1, u_2) = H(u_2) + H(u_1|u_2) = H(u_1) + H(u_2|u_1) = 1 + H(P_e)$ with bit-flipping model. Conversely, the correlation between u_1 and u_2 can be exploited to increase the information recovery probability.

However, since u_2 is utilized as a helper, the admissible rate should satisfy the following expressions [13]

$$R_1 \geq \begin{cases} H(u_1|u_2), & \text{for } R_2 \geq H(u_2), \\ H(u_1, u_2) - R_2, & \text{for } H(u_2|u_1) \leq R_2 \leq H(u_2), \\ H(u_1), & \text{for } 0 \leq R_2 \leq H(u_2|u_1), \end{cases} \tag{6}$$

which is an approximated, yet accurate, expression. The admissible rate region of h-SW is shown in Fig. 3.

B. MAC Rate Region and Source-Channel Separation

According to Shannon's lossless source-channel separation theorem [14], the transmission rate for point-to-point communications over discrete memoryless channels is given by

$$H(u)R_c \leq C(\gamma), \tag{7}$$

where R_c indicates the normalized spectrum efficiency including the channel coding rate and the modulation multiplicity. Throughout this article, $R_c = 0.5$ is assumed without loss of generality. $C(\gamma) = \log_2(1 + \gamma)$ denotes the channel capacity with the instantaneous SNR γ. The source index i is ignored in Eq. 7 for the simplicity.

In the same way as in [5], the separation theorem can be further extended to the h-SW and MAC case, as: if the h-SW and MAC regions have an intersection, source-channel separation holds. However, as stated before, this is a sufficient condition. The achievable MAC rate region is then shown in Fig. 4. The mathematical expressions of the MAC region with the assumption that the separation holds is given by [7]

$$\begin{cases} R_1 R_c \leq C(\gamma_1), \\ R_2 R_c \leq C(\gamma_2), \\ R_1 R_c + R_2 R_c \leq C(\gamma_1 + \gamma_2). \end{cases} \tag{8}$$

C. Expressions of Outage Probability

To achieve successful transmission of one source with one helper, the sufficient condition requires the MAC and h-SW regions to intersect. In the latter part of this article, this system is referred to as h-SW-MAC system to save the space. Conversely, if the intersection does not happen, as shown in Fig. 5 and Fig. 6, outage happens. Hence, the outage probability can then be calculated by

$$P_{out} = P_{out,1} + P_{out,2}. \tag{9}$$

In Fig. 5, since $\frac{1}{R_c}C(\gamma_1) < H(u_1 \mid u_2)$, the MAC and h-SW regions do not intersect. Thus, we have

$$\begin{aligned} P_{out,1} &= \Pr\left\{\frac{1}{R_c}C(\gamma_1) < H(u_1 \mid u_2)\right\} \\ &= \int_{\Phi(0)}^{\Phi[H(u_1|u_2)]} \int_{\Phi(0)}^{\Phi(+\infty)} p(\gamma_1)\, p(\gamma_2)\, d\gamma_2 d\gamma_1 \end{aligned}$$

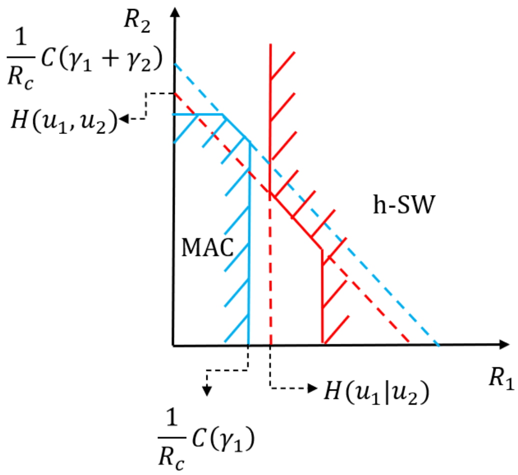

Fig. 5. Scenario 1 yielding $P_{out,1}$.

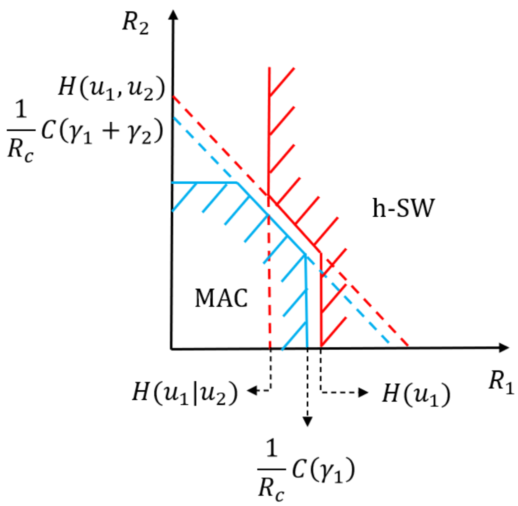

Fig. 6. Scenario 2 yielding $P_{out,2}$.

$$
\begin{aligned}
&= \int_{0}^{2^{R_c H(P_e)}-1} \int_{0}^{+\infty} p(\gamma_1)\, p(\gamma_2)\, d\gamma_2 d\gamma_1 \\
&= 1 - \exp\left\{-\frac{1}{\Gamma_1}\left[2^{R_c H(P_e)} - 1\right]\right\}. \qquad (10)
\end{aligned}
$$

In Fig. 6, with $H(u_1|u_2) \leq \frac{1}{R_c}C(\gamma_1) < H(u_1)$, the MAC and h-SW regions do not intersect when $\frac{1}{R_c}C(\gamma_1+\gamma_2) < H(u_1,u_2)$. Thus, we have

$$
\begin{aligned}
P_{out,2} &= \Pr\left\{\begin{array}{l} H(u_1|u_2) \leq \dfrac{1}{R_c}C(\gamma_1) < H(u_1); \\ \dfrac{1}{R_c}C(\gamma_1+\gamma_2) < H(u_1,u_2) \end{array}\right\} \\
&= \int_{\Phi[H(u_1|u_2)]}^{\Phi[H(u_1)]} \int_{\Phi(0)}^{\Phi[H(u_1,u_2)]-\gamma_1} p(\gamma_1)\, p(\gamma_2)\, d\gamma_2 d\gamma_1 \\
&= \int_{2^{R_c H(P_e)}-1}^{2^{R_c}-1} \int_{0}^{2^{R_c[1+H(P_e)]}-1-\gamma_1} p(\gamma_1)\, p(\gamma_2)\, d\gamma_2 d\gamma_1 \\
&= \exp\left\{-\frac{1}{\Gamma_1}\left[2^{R_c H(P_e)}-1\right]\right\} - \exp\left[-\frac{1}{\Gamma_1}\left(2^{R_c}-1\right)\right]
\end{aligned}
$$

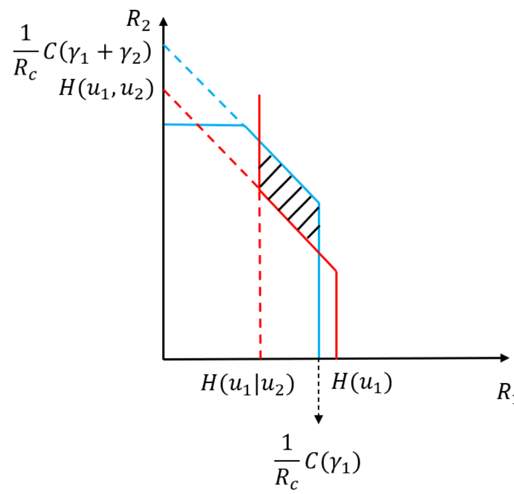

Fig. 7. Admissible h-SW-MAC region of Case (i).

$$
\begin{aligned}
&- \frac{\Gamma_2}{\Gamma_1-\Gamma_2}\exp\left[-\frac{1}{\Gamma_1}\left(2^{R_c}-1\right)\right] \\
&\cdot \exp\left\{-\frac{1}{\Gamma_2}\left[2^{R_c H(P_e)+R_c} - 2^{R_c}\right]\right\} \\
&+ \frac{\Gamma_2}{\Gamma_1-\Gamma_2}\exp\left\{-\frac{1}{\Gamma_1}\left[2^{R_c H(P_e)}-1\right]\right\} \\
&\cdot \exp\left\{-\frac{1}{\Gamma_2}\left[2^{R_c H(P_e)+R_c} - 2^{R_c H(P_e)}\right]\right\}. \qquad (11)
\end{aligned}
$$

The successful transmission probability is given by $P_{in} = 1 - P_{out}$.

The function $\phi(\cdot)$ represents the instantaneous SNR required to transmit rate R sequence with a Gaussian code book, i.e.

$$\gamma = \phi(R) = 2^{R_c R} - 1. \qquad (12)$$

$p(\gamma_i)$ is the probability density function of instantaneous SNR over Rayleigh fading channels, expressed as [15]

$$p(\gamma_i) = \frac{1}{\Gamma_i}\exp\left(-\frac{\gamma_i}{\Gamma_i}\right). \qquad (13)$$

D. Three Cases of Successful Transmissions

To have an in-depth insight of the contributions to the average successful probability P_{in}, three cases (i) - (iii) defined in the Introduction section are investigated in this subsection.

Case (i):

The admissible rate region in Case (i) is shown in Fig. 7, which corresponds to the situation where $H(u_1|u_2) \leqslant \frac{1}{R_c}C(\gamma_1) \leqslant H(u_1)$ and $H(u_1,u_2) \leq \frac{1}{R_c}C(\gamma_1+\gamma_2)$. When $\gamma_1 \ll \gamma_2$, Case (i) happens most likely. The average occurrence probability of the case can be derived mathematically in an explicit form, as

$$
\begin{aligned}
P_{in,case(i)} &= \Pr\left\{\begin{array}{l} H(u_1|u_2) \leqslant \dfrac{1}{R_c}C(\gamma_1) \leqslant H(u_1); \\ H(u_1,u_2) \leq \dfrac{1}{R_c}C(\gamma_1+\gamma_2) \end{array}\right\} \\
&= \int_{\Phi[H(u_1|u_2)]}^{\Phi[H(u_1)]} \int_{\Phi[H(u_1,u_2)]-\gamma_1}^{+\infty} p(\gamma_1)\, p(\gamma_2)\, d\gamma_2 d\gamma_1
\end{aligned}
$$

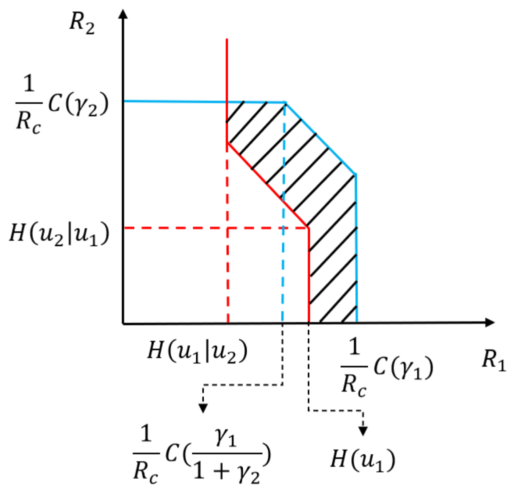

Fig. 8. Admissible h-SW-MAC region of Case (ii).

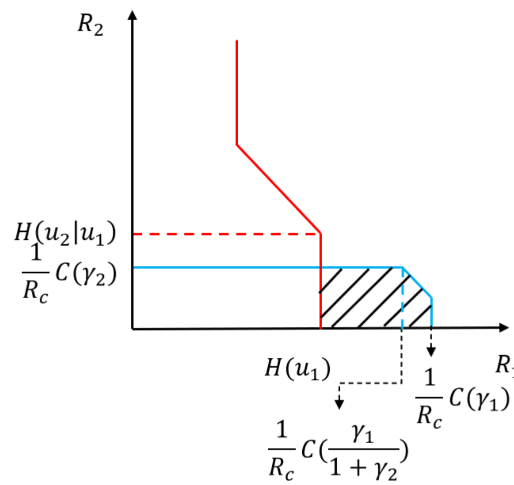

Fig. 9. Admissible h-SW-MAC region of Case (iii).

$$
\begin{aligned}
&= \int_{2^{R_c H(P_e)}-1}^{2^{R_c}-1} \int_{2^{R_c[1+H(P_e)]}-1-\gamma_1}^{+\infty} p(\gamma_1)\, p(\gamma_2)\, d\gamma_2 d\gamma_1 \\
&= \frac{\Gamma_2}{\Gamma_1-\Gamma_2} \exp\left[-\frac{1}{\Gamma_1}\left(2^{R_c}-1\right)\right] \\
&\quad \cdot \exp\left\{-\frac{1}{\Gamma_2}\left[2^{R_c H(P_e)+R_c}-2^{R_c}\right]\right\} \\
&\quad - \frac{\Gamma_2}{\Gamma_1-\Gamma_2} \exp\left\{-\frac{1}{\Gamma_1}\left[2^{R_c H(P_e)}-1\right]\right\} \\
&\quad \cdot \exp\left\{-\frac{1}{\Gamma_2}\left[2^{R_c H(P_e)+R_c}-2^{R_c H(P_e)}\right]\right\}. \qquad (14)
\end{aligned}
$$

It is expected that in this case the helpers most likely have the contribution to the source.

Case (ii):

The admissible rate region in Case (ii) is shown in Fig. 8, which corresponds to the situation where $H(u_1) < \frac{1}{R_c}C(\gamma_1)$ and $H(u_2 \mid u_1) \le \frac{1}{R_c}C(\gamma_2)$. The average occurrence probability of the case can be derived mathematically in an explicit form, as

$$
\begin{aligned}
&P_{in,case(ii)} \\
&= \Pr\left\{H(u_1) < \frac{1}{R_c}C(\gamma_1);\, H(u_2 \mid u_1) \le \frac{1}{R_c}C(\gamma_2)\right\} \\
&= \int_{\Phi[H(u_1)]}^{+\infty} \int_{\Phi[H(u_2|u_1)]}^{+\infty} p(\gamma_1)\, p(\gamma_2)\, d\gamma_2 d\gamma_1 \\
&= \int_{2^{R_c}-1}^{+\infty} \int_{2^{R_c H(P_e)}-1}^{+\infty} p(\gamma_1)\, p(\gamma_2)\, d\gamma_2 d\gamma_1 \\
&= \exp\left\{-\frac{1}{\Gamma_2}\left[2^{R_c H(P_e)}-1\right]\right\} \cdot \exp\left[-\frac{1}{\Gamma_1}\left(2^{R_c}-1\right)\right]. \qquad (15)
\end{aligned}
$$

It is expected that the contributions of both the source and the helper to P_{in} are large in this case.

Case (iii):

The admissible rate region in Case (iii) is shown in Fig. 9, which corresponds to the situation where $H(u_1) < \frac{1}{R_c}C(\gamma_1)$ and $\frac{1}{R_c}C(\gamma_2) < H(u_2 \mid u_1)$. When $\gamma_1 \gg \gamma_2$, Case (iii) happens most likely. The average occurrence probability of the case can be derived mathematically in an explicit form, as

$$
\begin{aligned}
&P_{in,case(iii)} \\
&= \Pr\left\{H(u_1) < \frac{1}{R_c}C(\gamma_1);\, \frac{1}{R_c}C(\gamma_2) < H(u_2 \mid u_1)\right\} \\
&= \int_{\Phi[H(u_1)]}^{+\infty} \int_{\Phi(0)}^{\Phi[H(u_2|u_1)]} p(\gamma_1)\, p(\gamma_2)\, d\gamma_2 d\gamma_1 \\
&= \int_{2^{R_c}-1}^{+\infty} \int_{0}^{2^{R_c H(P_e)}-1} p(\gamma_1)\, p(\gamma_2)\, d\gamma_2 d\gamma_1 \\
&= -\exp\left[-\frac{1}{\Gamma_1}\left(2^{R_c}-1\right)\right] \\
&\quad \cdot \left\{\exp\left[-\frac{1}{\Gamma_2}\left(2^{R_c H(P_e)}-1\right)\right]-1\right\}. \qquad (16)
\end{aligned}
$$

It is expected that even though the average occurrence probability is very small, however, still the contribution of the helper to P_{in} does not vanish in this case.

IV. Numerical Results

A. Outage Probabilities

Fig. 10 shows the outage probability obtained from explicit expressions shown in subsection III-C and through the Monte-Carlo technique [16]. It is found that the theoretical and Monte-Carlo results are exactly matched.

Fig. 11 compares outage probabilities of SW-MAC and h-SW-MAC with symmetric geometric gains. It is found that outage probabilities of h-SW-MAC is smaller than that of SW-MAC. For example, about 2 dB average SNR reduction can be achieved with the outage probability being 10^{-3}. This is because the admissible rate region is increased with a helper. Also, the joint decoders for both the h-SW-MAC and SW-MAC aim to exploit the correlation between u_1 and u_2, resulting in the 2^{nd} order diversity. The 2^{nd} order diversity can be more clearly seen in low average SNR value range and high correlation cases (small P_e). However, the diversity order converges into one when the average SNR value becomes large. This is because the correlation is not exactly one ($P_e \neq 0$).

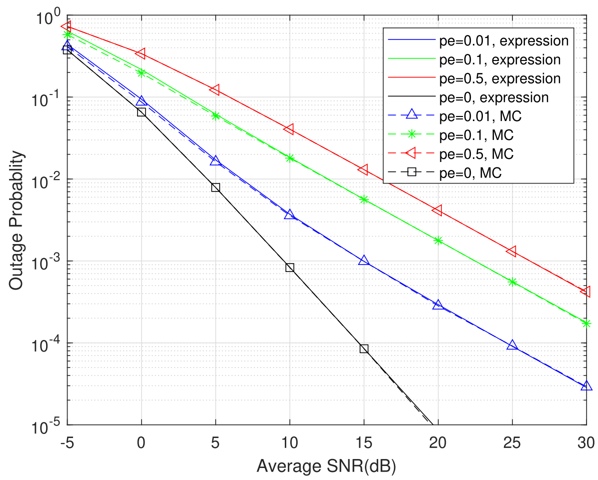

Fig. 10. Comparison of the outage probabilities calculated from explicit expressions and the Monte-Carlo technique.

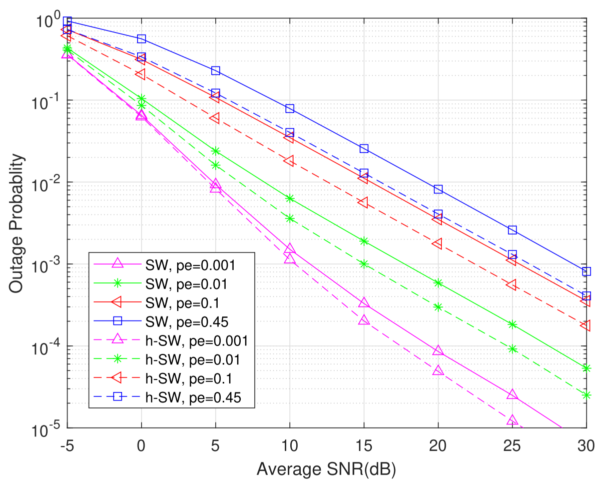

Fig. 11. Comparison of outage probabilities of SW-MAC and h-SW-MAC, where the symmetric geometric gain is assumed.

Fig. 12 and Fig. 13 indicate the outage probability with asymmetric geometric gains, given by

$$P_H = P_S + \alpha, \tag{17}$$

in dB, where P_S and P_H represent the geometric gains of the source and the helper links, respectively. The factor α is set at -5dB and 5dB in Fig. 12 and Fig. 13, respectively. The results indicate that lower outage probability can be achieved when the geometric gain of the source link is larger than that of the helper link.

B. Average Occurrence Probabilities of Three Cases

Fig. 14 shows the occurrence probability of Case (i) for $P_e = 0.1$. In Case (i), $\gamma_1 \ll \gamma_2$, but still intersection happens. Since the entropy of u_1 is fixed to be unity, the larger the average SNR, the less unlikely the Case (i) happens. Hence the occurrence probability due to Case (i) decreases in high average SNR value regions. In this case, since $\frac{1}{R_c} C(\gamma_1) \leq H(u_1)$, it is necessary to perform joint decoding at the destination for the successful recovery.

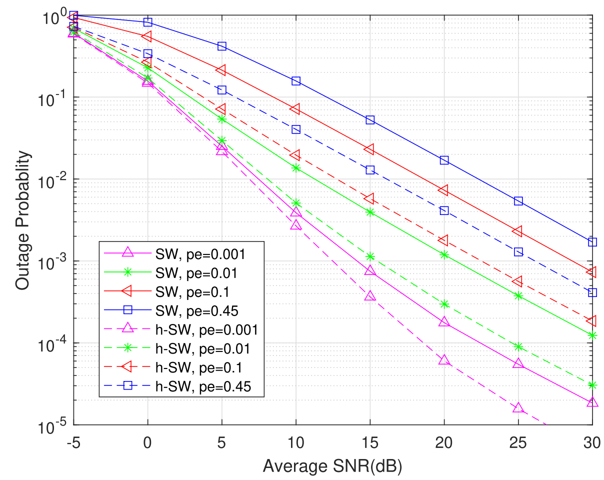

Fig. 12. Comparison of outage probabilities of SW-MAC and h-SW-MAC, where asymmetric geometric gain is assumed, i.e., the geometric gain of the helper-destination link is 5dB lower than that of the source-destination link.

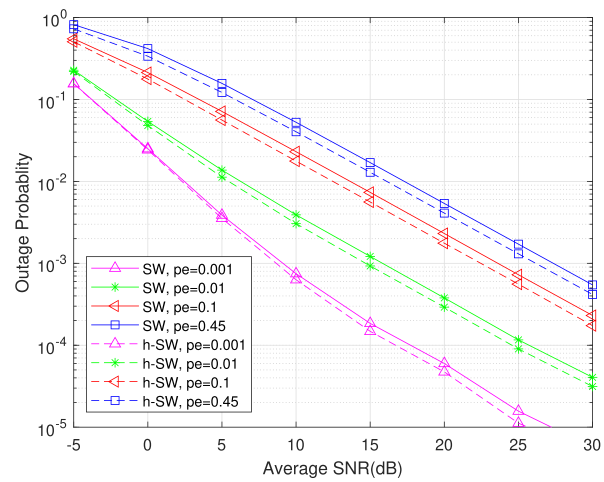

Fig. 13. Comparison of outage probabilities of SW-MAC and h-SW-MAC, where asymmetric geometric gain is assumed, i.e., the geometric gain of the helper-destination link is 5dB higher than that of the source-destination link.

In Case (ii), joint decoding is needed to recover the source u_1 when $\frac{1}{R_c} C\left(\frac{\gamma_1}{1+\gamma_2}\right) < H(u_1)$. Since the h-SW region is fixed with the information correlation P_e given while the capacity increases with the average SNR becomes large, the intersection of the h-SW and the MAC region also increases. As a result, the probability of Case (ii) approaches to unity with the increased average SNR, as shown in Fig. 15.

In Case (iii), since $\frac{1}{R_c} C\left(\frac{\gamma_1}{1+\gamma_2}\right) \geq H(u_1)$, the source u_1 can be recovered without joint decoding. However, performing the joint decoding further utilizes the helper information, resulting in even higher probability of successful recovery. The occurrence probability decreases as the average SNR increases, as shown in Fig. 16. Nevertheless, outage does not happen in Case (iii) because the intersection still happens.

The observations of Fig. 14-16 agree with the explanatory sentences shown in subsection III-D. Fig. 14-16 include the difference of the geometric gains between the source and

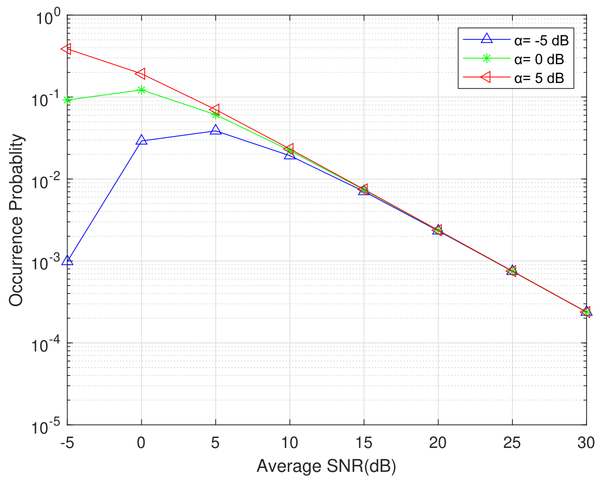

Fig. 14. Occurrence probability of Case (i) with $P_e = 0.1$.

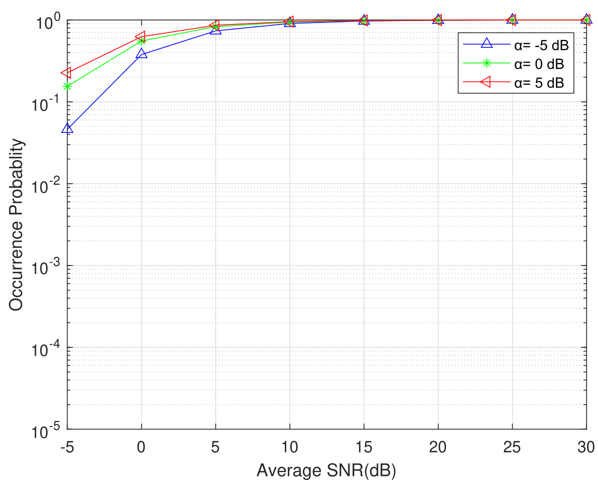

Fig. 15. Occurrence probability of Case (ii) for $P_e = 0.1$.

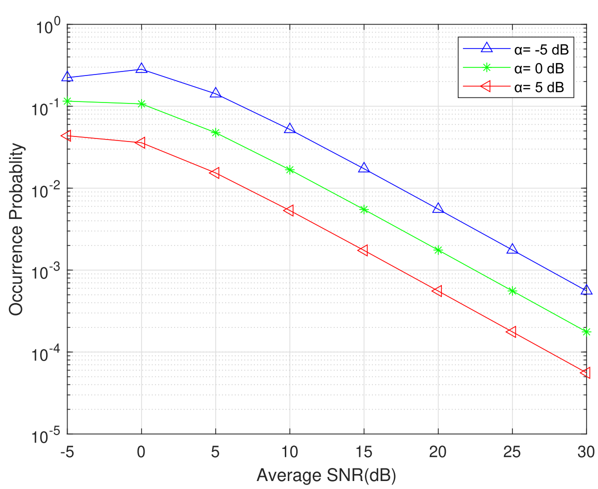

Fig. 16. Occurrence probability of Case (iii) for $P_e = 0.1$.

the helper links, 0dB, −5dB, and 5dB as a parameter. Such geometric gain difference makes the performance tendency more clearly.

V. Correlated Fading

The results shown in Section IV are under the scenario that information sequences are transmitted over independent block Raleigh fading channels. In this section, we investigate outage probabilities of correlated block fading channels. The fading variation correlation of the source-destination and the helper-destination link is defined as $\rho = \langle h_1 h_2^* \rangle$ with the normalization $\langle |h_1|^2 \rangle = \langle |h_2|^2 \rangle = 1$, and $|\rho| = [0, 1]$ [17]. If the transmission links are independent or fully correlated, $|\rho| = 0$ and $|\rho| = 1$, respectively. With $|\rho| \neq 0$, the $P_{out,1}$ and $P_{out,2}$ can be calculated by

$$\begin{aligned} P_{out,1} &= \Pr\left\{\frac{1}{R_c} C(\gamma_1) < H(u_1 \mid u_2)\right\} \\ &= \int_{\Phi(0)}^{\Phi[H(u_1|u_2)]} \int_{\Phi(0)}^{\Phi(+\infty)} p(\gamma_1, \gamma_2)\, d\gamma_2 d\gamma_1 \\ &= \int_{0}^{2^{R_c H(P_e)}-1} \int_{0}^{+\infty} p(\gamma_1, \gamma_2)\, d\gamma_2 d\gamma_1, \end{aligned} \tag{18}$$

and

$$\begin{aligned} P_{out,2} &= \Pr\left\{\begin{array}{c} H(u_1|u_2) \le \frac{1}{Rc} C(\gamma_1) < H(u_1); \\ \frac{1}{Rc} C(\gamma_1 + \gamma_2) < H(u_1, u_2) \end{array}\right\} \\ &= \int_{\Phi[H(u_1|u_2)]}^{\Phi[H(u_1)]} \int_{\Phi(0)}^{\Phi[H(u_1,u_2)]-\gamma_1} p(\gamma_1, \gamma_2)\, d\gamma_2 d\gamma_1 \\ &= \int_{2^{R_c H(P_e)}-1}^{2^{R_c}-1} \int_{0}^{2^{R_c[1+H(P_e)]}-1-\gamma_1} p(\gamma_1, \gamma_2)\, d\gamma_2 d\gamma_1, \end{aligned} \tag{19}$$

where $p(\gamma_1, \gamma_2)$ is the joint probability density function of γ_1 and γ_2, given by

$$\begin{aligned} p(\gamma_1, \gamma_2) &= \frac{1}{\Gamma_1 \Gamma_2 (1-\rho^2)} I_0\left(\frac{2|\rho|\gamma_1\gamma_2}{\sqrt{\Gamma_1\Gamma_2}(1-|\rho|^2)}\right) \\ &\quad \cdot \exp\left[-\frac{1}{1-|\rho|^2}\left(\frac{\gamma_1}{\Gamma_1} + \frac{\gamma_2}{\Gamma_2}\right)\right], \end{aligned} \tag{20}$$

with

$$I_o(x) = \sum_{m=0}^{\infty} \frac{(-1)^m}{(m!)^2} \left(\frac{x}{2}\right)^{2m}, \tag{21}$$

being Bessel function. Hence, $p(\gamma_1, \gamma_2)$ can be expanded into

$$p(\gamma_1, \gamma_2) = a \sum_{m=0}^{M} B_m \gamma_1^{2m} \exp\left(c \cdot \frac{\gamma_1}{\Gamma_1}\right) \gamma_2^{2m} \exp\left(c \cdot \frac{\gamma_2}{\Gamma_2}\right), \tag{22}$$

With

$$a = \frac{1}{\Gamma_1 \Gamma_2 (1-\rho^2)}, \tag{23}$$

$$b = \frac{2|\rho|}{\sqrt{\Gamma_1\Gamma_2}(1-|\rho|^2)}, \tag{24}$$

$$c = -\frac{1}{1-|\rho|^2}, \tag{25}$$

$$B_m = \frac{(-1)^m}{(m!)^2} \left(\frac{b}{2}\right)^{2m}, \tag{26}$$

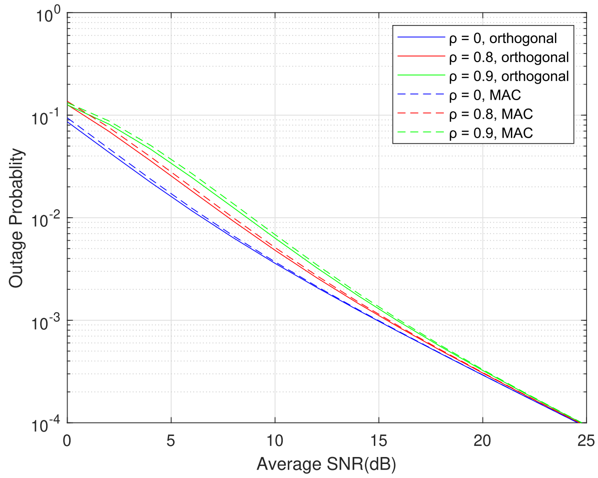

Fig. 17. Outage probability of correlated MAC and orthogonal transmissions with $P_e = 0.01$.

where M is determined empirically such that the summation in (19) converges. In this article, $M = 3$ is found to be enough. Then, $P_{out,1}$ and $P_{out,2}$ can be calculated by

$$P_{out,1} = a\sum_{m=0}^{M}\left\{\begin{array}{c} B_m\left[\int_0^{2^{R_c H(pe)}-1}\gamma_1^{2m}\exp\left(c\cdot\frac{\gamma_1}{\Gamma_1}\right)d\gamma_1\right] \\ \cdot\left[\int_0^{+\infty}\gamma_2^{2m}\exp\left(c\cdot\frac{\gamma_2}{\Gamma_2}\right)d\gamma_2\right]\end{array}\right\}, \tag{27}$$

and

$$P_{out,2} = a\sum_{m=0}^{M}\left\{B_m\int_{2^{R_c H(P_e)}-1}^{2^{R_c}-1}\int_0^{2^{R_c[1+H(P_e)]}-1-\gamma_1} P d\gamma_2 d\gamma_1\right\}, \tag{28}$$

with

$$P = \gamma_2^{2m}\exp\left(c\cdot\frac{\gamma_2}{\Gamma_2}\right)\gamma_1^{2m}\exp\left(c\cdot\frac{\gamma_1}{\Gamma_1}\right). \tag{29}$$

Fig. 17 shows for $P_e = 0.01$ the outage probability with the fading variation correlation $|\rho|$ as a parameter. The outage probabilities of the MAC and the orthogonal transmissions [18] are indicated by the dotted and solid curves, respectively. The 2^{nd} order diversity can be achieved in low average SNR value range and the outage curves finally approach the 1^{st} order diversity as the average SNR increases because $P_e \neq 0$. The larger the $|\rho|$ value, the smaller the value of the average SNR is, where the 2^{nd} order diversity can be achieved. Also, the orthogonal transmission can achieve lower outage probability, but the difference is very minor.

VI. Conclusions

In this article, the system of one source with one helper transmitted over a block Rayleigh fading MAC has been investigated. For successful transmission, the transmission rate should satisfy both the h-SW and MAC regions, which is a sufficient condition. Outage probabilities with different factors, including information correlations, geometric gains and channel variation correlations, have been provided. To have in-depth insights of the contributions of three cases to the successful transmission, we also derived the explicit expressions of the occurrence probabilities of the three cases.

It has been shown that compared with SW-MAC, the outage probability further reduces by h-SW-MAC. Numerical results indicated that the 2^{nd} order diversity can be achieved in the low average SNR value range in the case information sequences transmitted from the source and the helper are highly correlated. Lower outage can be achieved if the geometric gain of the source link is larger than that of the helper link. Most significantly, with MAC, the throughput efficiency can be twice as large as that with orthogonal transmissions. The numerical analysis shown in this article provide a theoretical basis for one source with one helper transmitted over a block Rayleigh fading MAC. The scenarios of one source with multiple helpers as well as multiple sources with one helper are left as a future study.

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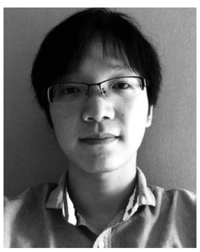

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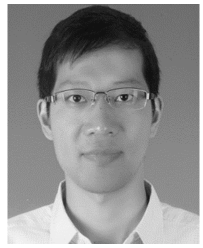

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